Chapter 1
Sustainability Complex Network

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Abstract We introduce the concept of *quantitative evaluation* of a complex network made up of researchers operating in different disciplines and different sectors belonging to life and hard sciences or social science and humanities or industrial and entrepreneurial activities, which, in addition to their disciplinary research, interact within each other in interdisciplinary scientific collaborations on sustainability projects. The complex network that we consider in this paper is of the *small-world* type, which has been already used in the study of several other biological, technological and social complex systems. This kind of network has a flexible structure which is in between those of the completely regular and the completely random networks. Similarly, the increase of interdisciplinary collaborations amongst scholars having a large and recognized experience in a given disciplinary sector may be favoured by random links arising in facing up specific issues of sustainability. Numerical results are given for a few unweighted networks having up to ten research groups with up to hundred researchers each.

Introduction

In this paper we address the problem on how scientific methodologies can help policy makers, industrial managers, entrepreneurs to handle in the best possible way and soon the seventeen UN 2030 Sustainable development Goals (SdGs), which for better clarity are reported and discussed in Appendix "Sustainable Development Goals and Disciplinary Sectors".

Given the fact that each SDG is strongly interdisciplinary if not trans-disciplinary, the question we try to answer is whether one should promote interdisciplinary research from being only instrumental of the traditional one to become the base of these scientific methodologies when dealing with sustainability issues.

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The answer we give to this question is somewhat intermediate between the full disciplinary science which provides blindly the instruments to be used by sustainability makers and a full interdisciplinary science whose individual researchers are characterized more by the soft skills necessary to create collaboration networks in macro-areas, rather then the capability of deepening specific scientific problems.

On one side it should not be forgotten the extraordinarily successful results obtained so far by the traditional disciplinary research, which has led to a deeper and deeper understanding of each single discipline. Let us quote a sentence declared by Abdus Salam during his Nobel Lecture on the eight December 1979

Scientific thought and its creation are the common heritage of mankind

At the same time, however, these successful results have also led to higher and higher specializations, giving rise to sub-disciplines, sub-sub-disciplines, and so on. Today, according to the Italian academical system, Life and Hard Sciences (LHS) and Social Sciences and Humanities (SSH) are divided all together into 15 disciplinary Areas, each of them is subdivided into several disciplinary sectors, for a grand total of about 370 disciplinary sectors [1]. Not to forget the applied research which is carried out by various industrial areas, which introduces other disciplinary sectors.

On the contrary interdisciplinary research requires the collaboration of scholars belonging to different disciplinary sectors. At present, such type of research is only an extremely small percentage of the whole research output, and, what is more important, it has never been planned in a methodological way, except for very few cases.

What we have in mind is a Laboratory made of *disciplinary researchers* who address problems which need to be solved by wide collaborations. In such a Laboratory, which may be viewed as a virtual Institute, the *disciplinary researchers* are grouped in N_C *macro-area Clusters*, which in turn interact amongst themselves. Therefore, in such *interdisciplinary laboratory* there is a first level of interactions, namely the *neighbouring interactions* between the members of the same *macro-area Cluster*, and a second one between the Clusters. These second kind of interaction is either of the *global* type between the coordinators of the clusters or of the *random* type between two individual researchers belonging to different Clusters.

We represent such a interdisciplinary laboratory, as a complex *small-world* network [2–7], that we denote as *Sustainability Complex Network (SCN)*.

The structure of a generic complex *small-world* network is presented on the next section, whereas section "The Sustainability Complex Network" is devoted to define the SCN structure.

Results on the characteristic path length and the clustering coefficient of large regular and random SCNs are presented and discussed on section "From a Small to Large Networks".

The goal of the present study is to develop an operational platform to evaluate the efficiency of the SCN, and correspondingly that of the interdisciplinary Laboratory, at any given time of its functioning in terms of both the creation of new links and the response given to a specified task. Section "From Unweighted to Weighted SCN" presents the theoretical scheme to build up such platform.

A discussion of the perspectives of the present study and conclusive remarks are left to section "Conclusions and Perspectives".

The Small–World Complex Network

Any complex system in nature can be modeled as a network, where vertices or nodes are the elements of the system and links represent the interactions between them. Coupled biological and chemical systems, neural networks or the Internet are just a few of such examples [8]. The characterization of the structural properties is of fundamental importance to understand the complex dynamics of these systems. In a recent paper [2] it has been shown that the connection topology of some biological and social networks is neither completely regular nor completely random. These networks have been named *small–world* from the concept of *small–world* phenomenon developed in social psychology [9] in the sixties. They are highly clustered as the regular lattices in spite of having characteristic path lengths like random graph. In several examples they have been shown to be both locally and globally efficient.

Let us reexamine the original formulation of *small–world* network given by Watts and Strogatz [2].

In that paper the authors consider a generic graph **G** with K links and N nodes, which has the following properties:

unweighted graph: the links are all equal;

sparse graph: $K \ll K_{MAX}$, with $K_{MAX} = \frac{N(N-1)}{2}$;

connected graph: there exists at least one path connecting any two nodes with a finite number of steps;

regular and random graphs: a regular graph is a graph where all the nodes R_α have the same *degree* k_α which is defined as the number of links $l_{\alpha\beta}$ with $\beta \neq \alpha$. A random graph is obtained by applying to a regular one a random rewiring procedure to a limited number r of links. The fraction $\rho = \frac{r}{K}$ measures the randomness. If $\rho = 1$ the graph is completely random;

links matrix: the graph representation is given by the matrix $[a_{\alpha\beta}]$, where $a_{\alpha\beta}$ is equal to 1 or 0 weather or not it exists the link $l_{\alpha\beta}$ or not.

The average of the node degrees is given by

$$\langle k_\alpha \rangle \equiv k = \frac{2K}{N}. \qquad (1.1)$$

Given the links matrix $[a_{\alpha\beta}]$ one can calculate the shortest path length $[d_{\alpha\beta}]$. Since **G** is connected, $d_{\alpha\beta}$ is always positive and finite for any $\alpha \neq \beta$.

Let us introduce, as in Ref. [2], the **characteristic path length** L and the **clustering coefficient** C

$$L = \frac{1}{N(N-1)} \sum_{\alpha \neq \beta} d_{\alpha\beta} = \frac{1}{N} \sum L_{\alpha}, \qquad (1.2)$$

where

$$L_{\alpha} = \frac{1}{N-1} \sum_{\beta \neq \alpha} d_{\alpha\beta}. \qquad (1.3)$$

The quantity L can be viewed as the average distance between any two nodes.

$$C = \frac{1}{N} \sum_{\alpha} C_{\alpha}, \qquad (1.4)$$

where C_{α} is defined as number of links existing in $\mathbf{G}_{\alpha}$, which is the sub-graph of the neighbors of α normalized with its maximum possible number given by $\frac{k_{\alpha}(k_{\alpha}-1)}{2}$.

The Sustainability Complex Network

The Sustainability Complex Network (SCN) is a complex network of the *small–world* type having N_C clusters, each addressing a given Cluster Sustainability Task T_i with $(i = 1, \ldots, N_C)$. Each task T_i refers to a subset $S_i = [t_1, \ldots, t_{S_i}]$ of the full set of the disciplinary sectors $S = [t_1, \ldots, t_S]$ as discussed in Appendix.

A given cluster C_i is made of M_i nodes with

$$M_T = \sum_{i}^{N_C} M_i \qquad (1.5)$$

being the total number of nodes. Each node R_{ij} is associated with an individual researcher, with $(i = 1, N_C)$ and $j = (1, M_i)$, and carries a weight function $W_{ij}[S]$, obtained by the evaluation of both his disciplinary research and other interdisciplinary works, addressing one or more t_m of the set $[S]$. The weight functions W_{ij} depend on the N_S disciplinary sectors t_m, and is given by a set of N_S values, $W_{ij}(t_m)$, one for each t_m, ranging from 0 to 1. The way this value is calculated is explained in section "From Unweighted to Weighted SCN".

Each Cluster C_i has a *central node* R_{i1} graphically represented by a white dot (a small circle) which is directly linked with the remaining nodes of the Cluster, each of them represented by black dots. The white nodes correspond to the coordinators of the Cluster.

Any two black dots R_{ij} and R_{kl}, with $(ij) \neq (kl)$ are connected with a link $L_{ij;kl}$ if and only if the two corresponding researchers have a documented common interest in one or more disciplinary sectors or, equivalently, if the link weight function $W_{ij;kl}(t_m)$ given by

$$W_{ij;kl}(t_m) = W_{ij}(t_m) \times W_{kl}(t_m),\tag{1.6}$$

for all the disciplinary sectors variables t_m of the set $[S]$, is not the null function.

The N_C white dots R_{i1}, besides having their own weight function, are also associated with the weight functions $W_i^C[S]$ of their own Clusters C_i, given by the normalized sum of the weight functions of all of their nodes, namely

$$W_i^C(t_m) = \sum_{j=1}^{M_i} W_{i,j}(t_m).\tag{1.7}$$

The sub-graph constituted by the white dots is intrinsically fully connected trough its own $\frac{N_C}{N_C-1}$ links, because it represents the coordination committee and two any members R_{i1} and R_{j1} need to directly interact within each other.

There are three types of link:

cluster link a link between two black dots of the same cluster.
coordinator link a link between white dots.
random link a link connecting a black or a white dot of a cluster with another black dot of a different cluster.

In the following, for the sake of simplicity, we may use Greek letter, $\alpha, \beta, \cdots$ to label the nodes R_{ij}. The various graph elements of a SCN obey the following rules

rule 1 The white dots, are all linked among each other through *coordinator links*.
rule 2 The white dots are connected with all the black dots of its cluster through *cluster links*.
rule 3 Any connection of a black or a white dot with a second black dot of a different cluster is of the random type
rule 4 A network with no random links is defined as a **regular SCN**. Note that such definition of regularity differs from that given in section "The Small–World Complex Network". In fact, the degree k_α of the node α, giving the number links incident with it cannot be equal within each other, because the white dots must obey rules 1 and 2.
rule 5 A *generic SCN* is denoted as $SCN_m^{(r)}$, where the subscript m labels the generic network structure and the upper-script (r) gives the number of its random links. Characteristic properties are the number of cluster N_C, the number M_i of nodes of each clusters, where i runs from 1 to N_C, the total number of links K, the total number of random links l.
rule 6 The random links are obtained by reconnecting a cluster link of a given cluster of the underling regular $SCN_m^{(0)}$ to a different node belonging to a different cluster. The reconnecting procedure leaves the total number of links unchanged. The rewiring is done satisfying periodicity,

Let us first consider in Fig. 1.1, just for the sake of clarifications of the above rules, an example of SCN with a very simple structure. The network considered is of
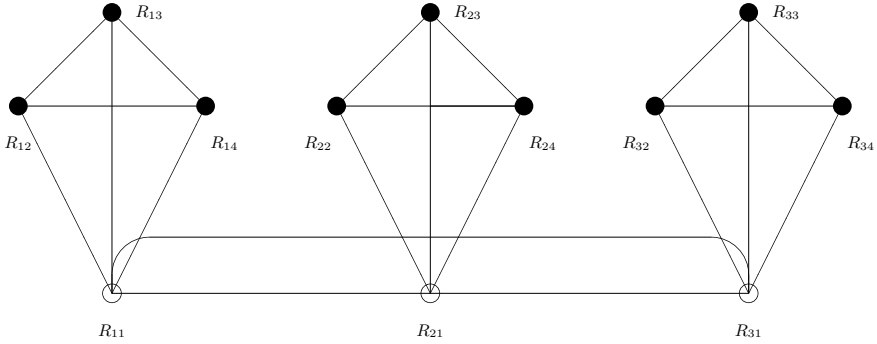
Fig. 1.1 Example of a network having three 4-node clusters. The three white dots represents the coordinators of the clusters. Each coordinator is linked to all the other members (black dots) of his own cluster and amongst themselves. The graph is denoted in the test as $SCN_{Fig}^{(0)}$. It has $N_C = 3$ and $M_T = 12$ and $K = 21$

the regular type, having three clusters, each with 4 nodes, which will be denoted as $SCN_{Fig}^{(0)}$. The three white dots R_{11}, R_{22} and R_{33} are connected within each other and characterizes the coordination of the network. The total number of nodes is $M_T = 12$. Each cluster is connected to the graph only through its own white dot (rule 2). As a consequence, the white dots are separability points of the graph which therefore is *separable*. The total number of links is $K = 21$.

In Fig. 1.2 we show one of the possible random networks, denoted as $SCN_{Fig}^{(3)}$, where we have introduced three random links, the first one connecting R_{14} with R_{22} in place of R_{13}, the second one connecting R_{24} with R_{32} in place of R_{23} and last one connecting R_{34} with R_{12} in place of R_{33}. The randomness is of the periodical type, as required by *rule 6*. Moreover, the white dots are no more separability points, which makes the graph *not separable*.

Moreover, we assume that the two graphs of Figs. 1.1 and 1.2 are unweighted, namely that the connection matrix elements $[a_{\alpha\beta}]$ are either 1 or 0, depending whether there a link $l_{\alpha\beta}$ exists or not. One can easily verify that, in the case of the regular network, each of the 3 white dots has the same degree, given by 5. The remaining nodes have all degree 3 for an average degree $\langle k \rangle$ given by 3.5, in accordance with the equality $\langle k \rangle = \frac{2K}{N}$.

It is worth noticing that, the example presented in this Section is not completely representative of the more general case of the SCN we are proposing in this paper for two following main reasons. The first one is that the number $M_\alpha = 4$ of nodes is too small for being representative of our general case. We will see that the minimum number of nodes is 6, one more of that given by the white dot, plus the two next neighbouring nodes and plus the other two next to next neighbouring nodes. This will be discussed in length in the next Section. The second reason is this network is unweighted. How to include weight to the links will be discussed further on in section "From Unweighted to Weighted SCN".
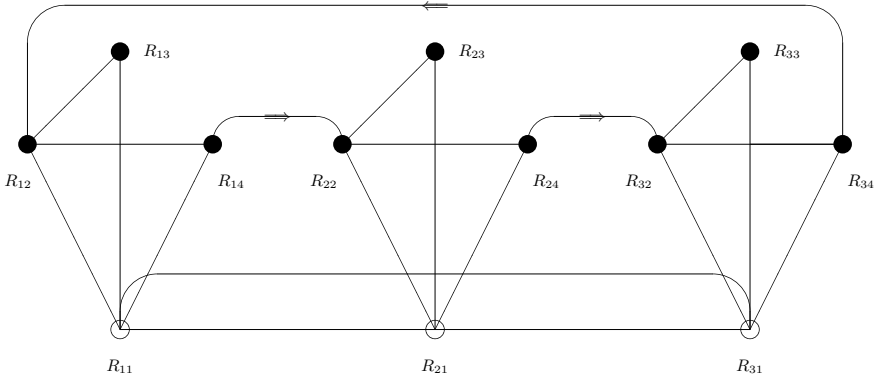
Fig. 1.2 Example of a $SCN_{Fig}^{(3)}$ having three 4-node clusters and three random links between the three clusters. The basic structure of the graph is given by $SCN_{Fig}^{(0)}$. The random links are $l_{14,22}$ and $l_{24,32}$ and $l_{34,12}$. The total number of links is $K = 21$, the same as for $SCN_{Fig}^{(0)}$

Table 1.1 Results for the characteristic path length L_α of the four nodes of the cluster 1. Clusters 2, and 3 provide similar results. In the left part of the Table, 1-p, 2-p and 3-p count the number of paths with 1, 2, and 3 links respectively for the regular graph, with Reg giving L_α. Analogously, the central part of the Table displays the results for the random graph *Ran1* ($\rho = 0.14$), whereas the left part is reserved to a random graph *Ran2* having the double of random links $\rho = 0.28$. The extra random links are the following: $l_{13;12} \rightarrow l_{13;23}$, $l_{23;22} \rightarrow l_{23;33}$ and $l_{33;22} \rightarrow l_{33;13}$

α	1-p	2-p	3-p	Reg	1-p	2-p	3-p	Ran1	1-p	2-p	3-p	Ran2
R_{11}	5	6	0	$\frac{17}{11}$	5	6	0	$\frac{17}{11}$	5	6	0	$\frac{17}{11}$
R_{12}	3	2	6	$\frac{25}{11}$	4	4	3	$\frac{21}{11}$	3	5	3	$\frac{22}{11}$
R_{13}	3	2	6	$\frac{25}{11}$	2	4	5	$\frac{25}{11}$	3	4	4	$\frac{23}{11}$
R_{14}	3	2	6	$\frac{25}{11}$	3	6	2	$\frac{21}{11}$	3	5	4	$\frac{22}{11}$

Let us give the results of the calculations of the quantities L_α, defined in Eq. 1.2, and $C\alpha$, defined in Eq. 1.4 for both the regular and random graphs displayed in Figs. 1.1 and 1.2 respectively. Let us first consider the characteristic path length L. The results for L_α are given in Table 1.1.

The results for the clustering coefficient C_α are given in Table 1.2.

In Table 1.3 we give the results for the characteristic path length L and the clustering coefficient C. As already mentioned in section "The Small–World Complex Network", L measures the typical separation between two nodes in the graph, *which is a global property*. On the contrary, the clustering coefficient measures the cliquishness of a typical neighborhood, *which is a local property*. For researcher networks we may give to the two quantities the following meanings: L is the average number of collaborations or common interests in the shortest chains connecting any two researchers; instead, C_α reflects the extent to which the collaborators of α collaborates with each other. The quantity C is an average of C_α and is defined in Eq. 1.4.

Table 1.2 Results for the clustering coefficient C_α of the four nodes of cluster 1. Clusters 2, and 3 provide similar results. In the left part of the Table, the three columns display the results for the regular graph Reg of the degree k_α, the number of links in the sub graph $\mathbf{G}_\alpha$ and the clustering coefficient C_α respectively. Analogous results are provided on the central part of the Table for the random graph $Ran1$, and on the left part for $Ran2$. See also the caption in Fig. 1.1

α	k_α^{Reg}	G_α^{Reg}	C_α^{Reg}	k_α^{Ran1}	G_α^{Ran1}	C_α^{Ran1}	k_α^{Ran2}	G_α^{Ran2}	C_α^{Ran2}
R_{11}	5	4	0.40	5	3	0.30	5	2	0.20
R_{12}	3	3	1.00	4	1	0.17	3	1	0.33
R_{13}	3	3	1.00	2	1	1.00	3	1	0.33
R_{14}	3	3	1.00	3	1	0.33	3	1	0.33

Table 1.3 Results for the characteristic path length L and the clustering coefficient C of the two graphs of Figs. 1.1 and 1.2. The fourth line gives the ratios of both L and C between the random and the regular graphs

Graph	L	C
Regular	2.09	0.85
Random (0.14)	1.91	0.45
Random (0.28)	1.91	0.30
Ratio (0.14)	0.91	0.52
Ratio (0.28)	0.91	0.35

Note that for a fully connected graph $L = C = 1$. In the case of the graph displayed in Fig. 1.2 $\rho = \frac{3}{21} = 0.14$.

The results of Table 1.3 show that the randomness diminishes both the characteristic path length and the clustering coefficient. The ratios indicates that the effect is significantly larger for C then for L.

From a Small to Large Networks

Let us first proceed in this Section to a first step towards a general, still unweighted SCN of the type proposed and calculated in the previous Section.

We consider an unweighted SCN having N_C clusters, each with $M_1, M_2, \ldots M_{N_C}$ nodes. The Total number of nodes, M_T is given by Eq. 1.5. The structure of each cluster is the same as that of Fig. 1.1 for the case of the regular network, namely that of a ring, with the white dot linked to all the black points. In addition the two neighbouring black dots of the white one are linked between themselves.

The randomness is done in the same way as in Fig. 1.2, namely the link $l_{1M_1;1(M_1-1)}$ is opened up towards the node R_{22}, the link $l_{2M_2;2(M_2-1)}$ is opened up towards the node $R_{32}, \ldots$, the link $l_{N_C M_{N_C};N_C(M_{N_C}-1)}$ is opened up towards the node R_{12}. See Fig. 1.3 for an example of such random graph.
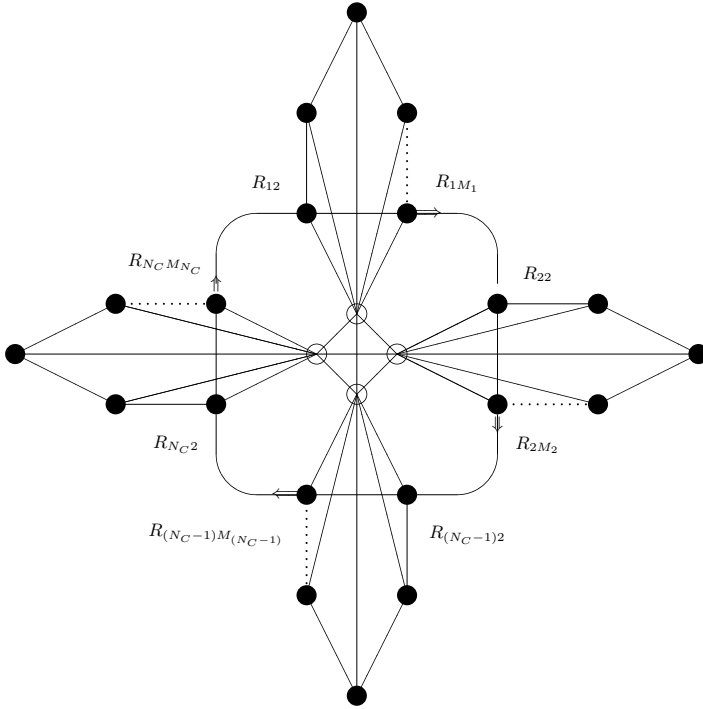
Fig. 1.3 Scheme of a generic $SCN^{(N_C)}$ having N_C clusters and N_C random links between them. The basic structure of the graph is the same of the graphs shown in Figs. 1.1 and 1.2. The clusters $3 \cdots N_C - 2$ are omitted to make the figure as simple as possible without loss of clarity. The dotted lines in the random graph are rewired into the corresponding oval links

The randomness is shown in the figure by a transversal ring of links, where the oval links are the rewiring of the dotted ones. One can imagine a third, a fourth, $\cdots$, rings of randomness to increase ρ.

The rationale of the network structure is the following. The set of SCN agents is made of experts in disciplinary sectors of LHS, SSH and Science Diplomacy, joined with components from industries, all working in a quantitative and transdisciplinary way on themes regarding sustainability. The agents are grouped in a number of clusters each having a *coordinator agent* (the white dots) and two deputy coordinators. The coordinators of the various clusters interact between each other. They are stable members of the scientific committee of SCN. They meet periodically and take decisions together, reporting the outcome of their cluster and defining new tasks for the network. Each coordinator and his two deputy coordinators interact each other as an internal coordination committee. Therefore they are linked together. The coordinators are linked to all the agents of their own clusters.

The topological structure of each cluster is that of ring of nodes, with the first and the last black dots of the ring also joined between themselves, and the white dots $R_{\alpha 1}$

represented by a fully connected sub-graph. The ring topology is suggested by the fact the (i) collaborations between researchers are mainly pairwise; (ii) the first and the last black dots of the ring constitute an executive committee of the cluster group and (iii) the second and the last but one black dots are influenced by the rewiring procedure. For this reason the minimum number of nodes in a cluster is 6.

Such a structure allows, already in its regular form, that no more than three direct links are necessary to a given agent to reach any other agent in three steps Because of this the proposed SCN structure has to be considered of the *small world network type*. The introduction of randomness leads to a diminishing of both the characteristic path length L and the clustering coefficient C.

The number K of links, which is the same for both the regular and the random networks, is given by

$$K = \frac{N_C(N_C - 1)}{2} + M_T - N_C + \sum_{i=1}^{N_C}(M_i - 1) = \frac{1}{2}(N_C^2 - 5N_C + 4M_T), \quad (1.8)$$

which has to be confronted with maximum number of links, given by $K_{MAX} = \frac{M_T(M_T-1)}{2}$. The ratio p between K and K_{MAX} measure how sparse is the network. In the case of $N_C = 3$ and $M_T = 12$, $K = 21$, $p = 0.32$.

Let us calculate the characteristic path length L and the clustering coefficient C for both the regular and random graphs of Fig. 1.3. The regular graph is obtained by considering the dotted links rather then the oval links. Because the four neighbouring black dots of a given white dots $R_{\alpha 1}$, $R_{\alpha 2}$, $R_{\alpha M_\alpha}$, $R_{\alpha 3}$ and $R_{\alpha(M_\alpha - 1)}$ have different counting properties after the randomization assumed in this paper. Obviously, if we want to proceed to further levels of randomization, the minimum number of $m + M_\alpha$ has to grow to 8, 10, *cdots*.

Regular Graph

Let us give in this section the results of the lengths L_α for the regular graph of Fig. 1.3, namely the one with the dotted links and without the oval ones.

$$L_{11} = L_{21} = \cdots = L_{N_C 1} = 2M_T - N_C - M_1,$$

$$\Sigma_C = \sum_{\alpha=1}^{N_C} L_{\alpha 1} = 2N_C M_T - N_C^2 - M_T, \quad (1.9)$$

$$L_{12} = L_{13} = \cdots = L_{1M_1} = 3M_T - N_C - M_1 - 4,$$

$$\Sigma_1 = \sum_{\beta \neq 1}^{M_1} L_{1\beta} = (M_1 - 1)L_{12},$$

$$L_{22} = L_{23} = \cdots = L_{2M_2} = 3M_T - N_C - M_2 - 4,$$

$$\Sigma_2 = \sum_{\beta \neq 1}^{M_2} L_{2\beta} = (M_2 - 1)L_{22}, \tag{1.10}$$

and similarly for $\Sigma_3, \cdots, \Sigma_{N_C}$.

Putting all together we get the following expression for the characteristic path length L

$$L = \frac{1}{M_T(M_T - 1)} \left(\Sigma_C + \sum_{\mu=1}^{N_C} \Sigma_\mu \right)$$

$$= \frac{1}{M_T(M_T - 1)} \left(3M_T^2 - 2M_T N_C - 4(M_T - N_C) - \sum_{i=1}^{N_C} M_i^2 \right). \tag{1.11}$$

In the case of $N_C = 3$ and $M_1 = M_2 = M_3 = 4$ one gets $L = 2.09$, in accordance with the results displayed on Table 1.3.

In order to calculate the clustering coefficient one needs to compute the degree k_α of each node R_α, which is given by

$$k_{\alpha 1} = (N_C - 1) + (M_\alpha - 1), \quad (\alpha = 1, N_C),$$
$$k_{\alpha \beta} = 3, \quad (\beta \neq 1). \tag{1.12}$$

The number $c_{\alpha \beta}$ of the links in the sub-graph $\mathbf{G}_{\alpha \beta}$ of the neighbours of $R_{\alpha \beta}$ is given by

$$c_{\alpha 1} = (M_\alpha - 1) + \frac{(N_C - 1)(N_C - 2)}{2}, \quad (\alpha = 1, N_C),$$
$$c_{\alpha \beta} = 2, \quad (\beta \neq 1), \tag{1.13}$$

with the exception of the case $M_i = 4$, for which $c_{i\beta} = 3$, with $\beta \neq 1$. Using Eqs. 1.12 and 1.13 on gets the following result for $C_{\alpha \beta}$

$$C_{\alpha 1} = \frac{2(M_\alpha - 1) + (N_C - 1)(N_C - 2)}{(N_C + M_\alpha - 2)(N_C + M_\alpha - 3)}, \quad (\alpha = 1, N_C),$$
$$C_{\alpha \beta} = \frac{2}{3}, \quad (\beta \neq 1), \tag{1.14}$$

with the exception of the case $M_i = 4$, for which $C_{i\beta} = 1$, with $\beta \neq 1$. Using the above equation Eqs. 1.12 and 1.13 on gets for $C_{\alpha\beta}$ the following result for the average clustering coefficient C

$$C = \frac{1}{M_T} \left\{ \frac{2}{3}(M_T - N_C) + \sum_{\alpha=1}^{N_C} \frac{2(M_\alpha - 1) + (N_C - 1)(N_C - 2)}{(N_C + M_\alpha - 2)(N_C + M_\alpha - 3)} \right\}. \quad (1.15)$$

Random Graph

Let us now consider the random graph of Fig. 1.3, in which we switch from the dotted links to the oval ones. The characteristic path length for the white dots, $L_{\alpha1}^{Ran}$, leads to the same expression as for the case of the regular graph (see L_{11} and $\Sigma_{\alpha1}$ given in Eq. 1.9)

$$L_{11}^{Ran} = L_{21}^{Ran} = \cdots = L_{N_C1}^{Ran} = 2M_T - N_C - M_1,$$

$$\Sigma_C^{Ran} = \sum_{\alpha=1}^{N_C} L_{\alpha1}^{Ran} = 2N_C M_T - N_C^2 - M_T. \quad (1.16)$$

For the two neighbouring black dots of a given white dots $R_{\alpha1}$, $R_{\alpha2}$ and $R_{\alpha M_\alpha}$, and the two next neighboring dots $R_{\alpha3}$ and $R_{\alpha(M_\alpha-1)}$ we get the following results

$$L_{12}^{Ran} = 3M_T - N_C - M_1 - 8,$$
$$L_{13}^{Ran} = 3M_T - N_C - M_1 - 5,$$
$$L_{14}^{Ran} = = L_{15}^{Ran} = \cdots = L_{1(M_1-2)}^{Ran} = 3M_T - N_C - M_1 - 4,$$
$$L_{1(M_1-1)}^{Ran} = 3M_T - N_C - M_1 - 3,$$
$$L_{1M_1}^{Ran} = 3M_T - N_C - M_1 - 8, \quad (1.17)$$

Summing over all the black dots of the various clusters we get the following expressions

$$\Sigma_\alpha^{Ran} = \sum_{\beta\neq1}^{M_1} L_{1\beta}^{Ran} = (3M_T - N_C - M_\alpha)(M_\alpha - 1) - 24 - 4(M_\alpha - 5) \quad (1.18)$$

with $\alpha = 1, N_C$. Using Eqs. 1.16, 1.17 and 1.18 we get the following result for the average characteristic path L^{Ran}

Table 1.4 Results for the average characteristic path length L of two graphs of the type of from Fig. 1.3 in its regular form (with dotted lines) and in its random form (oval lines). The number of nodes in each cluster for any choice of M_α is taken to be for the sake of simplicity. The randomness parameter of the three random network are, from top-down, $\rho = 0.91$, $\rho = 0.91$ and $\rho = 0.91$, respectively

Graph	N_C	$M_\alpha = 4$	$M_\alpha = 6$	$M_\alpha = 10$	$M_\alpha = 20$	$M_\alpha = 100$
Regular	3	2.09	2.27	2.42	2.55	2.64
Regular	6	2.30	2.47	2.62	2.72	2.81
Regular	10	2.38	2.55	2.69	2.79	2.88
Random	3	1.91	2.19	2.40	2.54	2.64
Random	6	2.22	2.44	2.60	2.72	2.81
Random	10	2.33	2.53	2.68	2.79	2.88

$$
\begin{aligned}
L^{Ran} &= \frac{1}{M_T(M_T - 1)} \left(\Sigma_C^{Ran} + \sum_{\mu=1}^{N_C} \Sigma_\mu^{Ran} \right) \\
&= \frac{1}{M_T(M_T - 1)} \left(\left(3M_T^2 - 2M_T N_C - 4(M_T + N_C) - \sum_{i=1}^{N_C} M_i^2 \right) \right).
\end{aligned}
\tag{1.19}
$$

In the case of $N_C = 3$ and $M_1 = M_2 = M_3 = 4$ one gets $L = 1.91$, in accordance with the results displayed on Table 1.3.

The results for the average path length of few regular and random networks are displayed on Table 1.4. The total number of nodes is given by the product of N_C with M_α.

One can see that the characteristic length L slightly increases with the number of clusters. The same effect is observed by increasing M_α. The randomness, as expected, reduces L in a visible away for values of ρ of the order of 1% (see also Table 1.6).

Let us calculate the clustering coefficient. The degrees of the white dots are given by

$$
\begin{aligned}
k_{\alpha 1}^{Ran} &= (N_C - M_\alpha - 2), \quad (\alpha = 1, N_C), \\
k_{\alpha \beta}^{Ran} &= 3, \quad (\beta \neq 1 \; and \; (M_\alpha - 1),
\end{aligned}
\tag{1.20}
$$

and $k_{\alpha(M\alpha-1)}^{Ran} = 2$ for any value of α.

The numbers $c_{\alpha\beta}^{Ran}$ of the links in the sub-graph $\mathbf{G}_{\alpha\beta}^{Ran}$ of the neighbours of $R_{\alpha\beta}$ are given by

Table 1.5 Results for the *clustering coefficient C* of two graphs of the type given in Fig. 1.3, one regular and the other one random. See also capture of Fig. 1.4

Graph	N_C	$M_\alpha = 4$	$M_\alpha = 6$	$M_\alpha = 10$	$M_\alpha = 20$	$M_\alpha = 100$
Regular	3	0.60	0.60	0.62	0.64	0.66
Regular	6	0.61	0.61	0.62	0.64	0.66
Regular	10	0.65	0.63	0.63	0.64	0.66
Random	3	0.49	0.54	0.59	0.62	0.66
Random	6	0.52	0.55	0.58	0.62	0.66
Random	10	0.56	0.57	0.60	0.62	0.66

$$
\begin{aligned}
c_{\alpha 1}^{Ran} &= (M_\alpha - 2) + \frac{(N_C - 1)(N_C - 2)}{2}, \\
c_{\alpha 2}^{Ran} &= c_{\alpha (M_\alpha - 1)}^{Ran} = c_{\alpha M_\alpha}^{Ran} = 1, \\
c_{\alpha 3}^{Ran} &= c_{\alpha 4}^{Ran} = \cdots = c_{\alpha (M_\alpha - 2)}^{Ran} = 2.
\end{aligned} \tag{1.21}
$$

By using Eqs. 1.20 and 1.21 on gets the following result for $C_{\alpha\beta}$

$$
\begin{aligned}
C_{\alpha 1}^{Ran} &= \frac{2(M_\alpha - 2) + (N_C - 1)(N_C - 2)}{(N_C + M_\alpha - 2)(N_C + M_\alpha - 3)}, \\
C_{\alpha 2}^{Ran} &= C_{\alpha M_\alpha}^{Ran} = \frac{1}{3}, \\
C_{\alpha 3}^{Ran} &= \frac{2}{3}, \\
C_{\alpha 4}^{Ran} &= C_{\alpha 5}^{Ran} = \cdots = C_{\alpha (M_\alpha - 1)}^{Ran} = \frac{2}{3}, \\
C_{\alpha 3}^{Ran} &= 1.
\end{aligned} \tag{1.22}
$$

Using the above equations one gets the following result for the average clustering coefficient C

$$
C^{Ran} = \frac{1}{M_T} \left\{ \frac{2}{3} M_T - N_C + \sum_{\alpha=1}^{N_C} \frac{2(M_\alpha - 2) + (N_C - 1)(N_C - 2)}{(N_C + M_\alpha - 2)(N_C + M_\alpha - 3)} \right\}. \tag{1.23}
$$

Results for the clustering coefficient of regular and random networks are displayed on Table 1.5.

The clustering coefficient C show very minor variations within the various combinations of N_C and M_α considered. The only visible effects come from the randomness, which amount to be of the same order of ρ in percentage.

Table 1.6 Results for the number K of links and, in parenthesis, for the randomness index ρ in connection with various combinations of N_C and M_α. All the graphs considered are random. Regular and random graphs for a given combination have the same value of K

Graph	N_C	$M_\alpha = 4$	$M_\alpha = 6$	$M_\alpha = 10$	$M_\alpha = 20$	$M_\alpha = 100$
Random	3	21 (.14)	33 (.091)	57 (.053)	117 (.027)	597 (.0050)
Random	6	51 (.11)	75 (.080)	123 (.049)	243 (.025)	1203 (.0050)
Random	10	105 (.095)	145 (.069)	225 (.044)	425 (.024)	2025 (.0050)

We display in Table 1.6 the results for the for the quantity K, giving the number of links, and the randomness index ρ.

From Unweighted to Weighted SCN

In this Section we discuss how to give weights to the links of the SCN and how the calculation of the unweighted network given in the previous sections can be generalized to take the into account.

Let us first consider the function $W_\alpha(t_m)$ associated with node $\alpha \equiv R_{ij}$, where t_m label the macro-sectors defined in Appendix "Sustainable Development Goals and Disciplinary Sectors". The description of the indicators that will be used to evaluate any agent of the network is given in Table 1.7

The function $W_\alpha(t_m)$ can be represented by an Histogram, with N_S bins each corresponding to a given t_m with $0 \leq m \leq N_S$. The height and the width correspond to the evaluation of the disciplinary (x-indicators) and the interdisciplinary research (y-indicators) respectively, both ranging fro 0 to 1. Therefore $W_\alpha(t_m)$ is a two value function given by the height and the width of the mth bin, namely by the pair (h_m, w_m). An example of histogram with only four bins out of the sixty one, is given in Fig. 1.4.

The weight function of link is formally given in Eq. 1.6 which we re-write in the following equation using the Greek labelling for the nodes, namely

$$W_{\alpha\beta}(t_m) = W_\alpha(t_m) \times W_\beta(t_m),$$
$$(h_m(\alpha\beta), w_m(\alpha\beta)) = (h_m(\alpha), w_m(\alpha) \times (h_m(\beta), w_m(\beta)), \tag{1.24}$$

where α stands for the pair ij and β for kl. The *convolution product of the two vectors* results from the geometrical averages of the heights and the widths, namely

$$h_m(\alpha\beta) = \sqrt{h_m(\alpha)\, h_m(\beta)},$$
$$w_m(\alpha\beta) = \sqrt{w_m(\alpha)\, w_m(\beta)}. \tag{1.25}$$

Table 1.7 Indicators used in the evaluation of the function $W_\alpha(t_m)$. The x-indicators are for the disciplinary research of type LHS or SSH, or for industrial activity (IA). The y-indicators are for interdisciplinary research

Indicator	Description	Activity area
x_1	Number of papers in journal indexed in Web of sciences or Scopus	LHS
x_2	Number of citations in Web of sciences or Scopus	LHS
x_3	Number of articles and chapters in books with ISDN	SSH
x_4	Number of articles published in in A-rated journals	SSH
x_5	Number of patents with their economical evaluation	IA
x_6	Number of employers	IA
y_1	Number of Google citations	All
y_2	Number of books and of articles on science and technology	All
y_3	Number of interdisciplinary conferences	All
y_4	Number of public events on science and technology	All

The corresponding histogram has the same structure of that shown in Fig. 1.4. The contribution to the weight of the link $l_{\alpha\beta}$ coming from disciplinary sector t_m is given by the modulus of the vector $W_{\alpha\beta}(t_m)$

$$|W_{\alpha\beta}(t_m)| = \sqrt{h_m^2(\alpha\beta) + w_m^2(\alpha\beta)}, \qquad (1.26)$$

whose maximum value is $\sqrt{2}$ when both the values of h and w are equal to 1. It is convenient to normalize $|W_{\alpha\beta}(t_m)|$ to unity. The weight to the link $\omega_{\alpha\beta}$ is then given by

$$\omega_{\alpha\beta} = \frac{1}{\sqrt{2}N_S} \sum_{m=1}^{N_S} |W_{\alpha\beta}(t_m)|, \qquad (1.27)$$

so that its maximum value is 1 when all the h_m and w_m factors have their maximum possible value. It is necessary to fix a cutoff value, ω_{min}, for $\omega_{\alpha\beta}$, below which the link $l_{\alpha\beta}$ does not exist. In fact, this condition substitutes that used for the unweighted
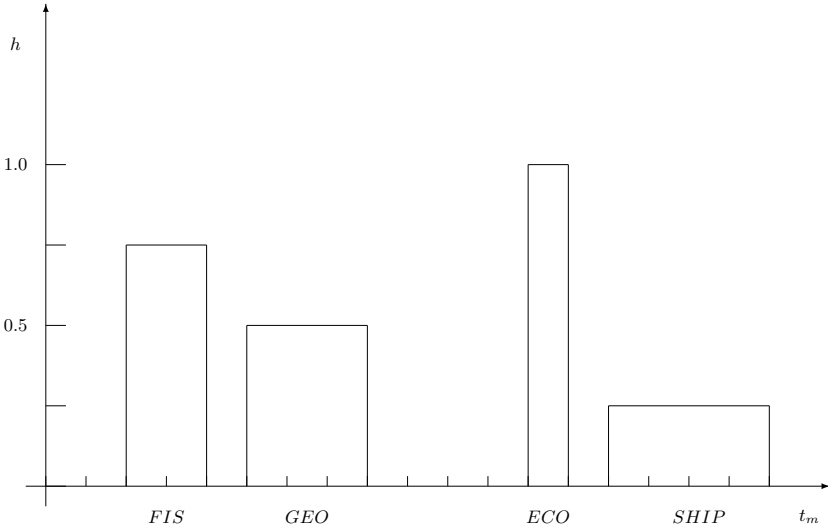
Fig. 1.4 Example of an histogram representing the two-value function $W_\alpha(t_m)$. The y-axis gives the h-value, whose maximum possible value is 1.0. The x-axis displays the width of the bins, providing the w-value, which also has a maximum value of 1.0. The spacing between the tics is 0.25. For instance, the disciplinary sector FIS has $h = 0.75$ and $w = 0.5$

networks for which $d_{\alpha\beta}$ is equal to 0 or 1 and a link can exist if and only if it is equal to 1. One can fix $\omega_{min} = 0.1$.

In the calculation of the minimum path one is faced with a path with one, two or at most three steps. The two- and the three-step paths are made of two links having one node in common, say μ and three links having μ and ν as intermediate nodes, respectively.

One can use the standard procedure to count their contributions, namely summing up the $\omega_{\alpha\mu}$ and the $\omega_{\mu\beta}$ values in the two-step process and the $\omega_{\alpha\mu}$, $\omega_{\mu\nu}$ and the $\omega_{\nu\beta}$ values in the three-step one.

Alternatively, one can consider the following average procedure

$$(h_m(\alpha\mu\beta),\, w_m(\alpha\mu\beta) = ((h_m(\alpha)h_m(\mu)h_m(\beta))^{1/3},\, (w_m(\alpha)w_m(\mu)w_m(\beta))^{1/3}) \qquad (1.28)$$
$$(h_m(\alpha\mu\nu\beta),\, w_m(\alpha\mu\nu\beta) = ((h_m(\alpha)h_m(\mu)h_m(\nu)h_m(\beta))^{1/4},\, (w_m(\alpha)w_m(\mu)w_m(\nu)w_m(\beta))^{1/4}).$$

The weights to consider in this case for the two- and the three-steps processes are given by the module of the vectors $W_{\alpha\mu\beta}(t_m)$ and $W_{\alpha\mu\nu\beta}(t_m)$ respectively. This second procedure could be preferable if one wishes to emphasize the feature that, because of dealing with a network made by researchers, the occurrence of multiple links of a path carries a potential better efficiency rather then a loss.

The evaluation of the histograms values for the various nodes can be done by using the methods described in Ref. [1].

The h_m and w_m values for the white nodes may be taken without making a specific evaluation, given the role of the coordinators in the structure of the network, which is mainly of interdisciplinary nature. We may take $w = 1$ for all the t_m, and some median value, say 0.5, for h.

The calculation of the average characteristic length L and of the clustering coefficient C, as well of that of other quantities, like for instance *the efficiency* [3], can be carried out by following the procedure described in this paper for the unweighted network.

The *efficiency* and the *normalized efficiency* of a node are defined as follows

$$e_\alpha = \frac{1}{N-1} \sum_{\alpha \neq \beta} \frac{1}{d_{\alpha\beta}}. \tag{1.29}$$

The global efficiency is given by

$$E_{glob} = \frac{1}{N} \sum_\alpha e_\alpha. \tag{1.30}$$

The global efficiency is usually normalized in such a way that the maximum global efficiency is 1, in the case of a perfect efficiency. Such an ideal case is obtained by a completely connected graph with the minimum possible distance, namely $min(d_{\alpha\beta}) = min(\omega_{\alpha\beta}) = \omega_{min}$ for all the pairs $(\alpha \neq \beta)$. The global efficiency of such an the ideal graph is given by $E_{glob}(ideal) = \frac{1}{\omega_{min}}$. Therefore the normalization of the efficiencies is obtained by dividing them with ω_{min}.

Conclusions and Perspectives

In this paper we analyze the possibility of representing a laboratory of interdisciplinary an trans-disciplinary research devoted to the science of sustainability with a complex network of the *small world* type. The structure of the network is designed in such a way that only three steps are at most necessary for an agent to reach any other one. The network may have any number of thematic clusters, and each of them may be composed by any number of agents, or black nodes of the graph, except for a special one, which is white and represents the coordinator of the cluster. The coordinator is linked to all the black dots of the cluster and has two neighbouring black dots, which represent its deputy coordinators. The coordinators of the various clusters constitute the scientific council of the laboratory and, therefore, are fully connected within each other. Any two agents of a cluster can reach each other in at most two step and any agent of a cluster can reach another of another cluster in three steps. The network is moderately sparse, with the number of links being only few per cent, or less, of the maximum possible number. We considered the possibility of

introducing randomness in the graph keeping the number of links fixed and study the behaviour of the characteristic path length L and of the clustering coefficient C as a function of the randomness index ρ. A quantitative analysis has been done for unweighted networks deriving general formulas of L and C as a function of the number of cluster and of the cluster nodes. The results obtained confirm the feature that randomness increases the characteristic path length and reduces the clustering coefficient, even in presence of special nodes, the coordinator nodes, which have a different degree from all the other nodes.

A second part of the paper has been devoted to give weights to the nodes and to the links in the particular case of net of researchers doing interdisciplinary studies devoted to the science of sustainability.

The seventeen SDGs of the UN 2030 project have been confronted with 26 disciplinary macro-sectors extracted from the 369 disciplinary sectors introduced by the Italian academical legislation for Life and Hard Science and for Social sciences and Humanities plus other 35 coming from industrial sectors. Six indicators have been suggested for evaluating the research quality of the agents for their disciplinary activity of and other four have been identified to evaluate the interdisciplinary work. These indicators are supposed to be used to produce for every agent of the laboratory an histogram of 61 bins, one for each disciplinary macro-sectors. Each bin is characterized by the height h, corresponding to the evaluation of the disciplinary research produced by the agent and a width w giving a measure of his interdisciplinary activity. They have to be considered as two component vectors whose modules give the weights for each bin. The procedure to calculate the weight of a generic link and that to evaluate the minimum path between any two nodes are also explained in detail. The necessary elements to compute the characteristic path length, the clustering coefficient and the global efficiency of the SCN at any running time are also described.

In the initial evaluation of the network the data to be used come from the last ten years research production of its agents, as resulting from their curricula. The successive monitoring, should include the interdisciplinary research activity performed to face up the tasks assigned to the various clusters.

Some of the methods developed in this paper for the SCN can be adopted in other research or educational activities. For instance, a school complex could be facing with given tasks, for which the students should show the ability of problem solving together with that of soft skills and team working capability. Similarly, an innovation center may find useful to adopt the type of evaluation suggested in this paper for the laboratory devoted to quantitative sustainability scientific research.

Possible extensions of the network are under current study. One of these regards the inclusion of various levels of randomness in a systemic way. Another dynamical behaviour of the SCN is the proliferation of new activities, in the form of the black dots becoming new coordinators of new tasks and therefore new white dots.

Sustainable Development Goals and Disciplinary Sectors

First of all let us give a schematic description of the historical path that the recognition of sustainability as a fundamental issue for the life of the Planet has followed. The first action against the risk generated by the human activity has been taken by a group of about 30 between scientists, educators, economists, humanists, industrial managers and policy makers during an informal encounter in April 1968 at the Accademia dei Lincei in Rome. This group became later on a virtual Institute, known as the *Rome's Club* which has been working on the limits of the growth [10]. About twenty years later, in 1987, the Brundtland Commission, in their report [11], gave the modern definition of *sustainable development*:

> *Sustainable development is development that meets the needs of the present without compromising the ability of the future generation to meet their own needs*

It is only in 2000 that the United Nations during the Millennium Summit proclaimed the Sustainable development Goals, the 17 *SDGs* of the UN 2030 project given in Fig. 1.5.

1. *Poverty eradication.*
2. *Zero hunger.*
3. *Good health and well being.*
4. *Quality education.*
5. *Gender equality.*
6. *Clean water and sanitation.*
7. *Affordable and clean energy.*
8. *Decent work and economic growth.*
9. *Industry, innovation, infrastructure.*
10. *Reduced inequalities.*
11. *Sustainable cities and communities.*
12. *Responsible consumption and production.*
13. *Climate actions.*
14. *Life below water.*
15. *Life and land.*
16. *Peace and justice strong institutions*
17. *Partnership to achieve the goals*

In the following we discuss them in the perspective of developing an interdisciplinary scientific approach to understand how far we are from the achievements of the SDGs and in which manner the risk of *compromising the ability of the future generation to meet their own needs* can be scientifically quantified.

Fig. 1.5 The seventeen Sustainable Development Goals of the UN 2030 project

First of all we need to confront the SDGs with the Disciplinary Sectors on which the evaluation of scientists, humanists, industrial researchers, financial experts, cultural operators, science journalists, politicians and the various actors of the social and environmental development are usually evaluated.

We propose to use the same evaluation categories for the agents of the SCN. Such evaluation is the key to give a weight to the links of the network [1]. According to the Italian academical system, Life and Hard Sciences (LHS) and Social Sciences and Humanities (SSH) are divided all together into 15 disciplinary Areas, each of them is subdivided into several disciplinary sectors, for a grand total of about 370 disciplinary sectors [1]. The these sectors we have to add the industrial sectors, namely those coming from the primary goods production (primary sector), from the material goods production (secondary sector) and from the service industry Tertiary&advanced tertiary sector). We define in the following the N_S macro-sectors t_m of the set S that will be used to give a weight to the nodes and to the links of the SCN. The acronyms that are used for LHS and SSH coincide with those used in the Italian legislation. Let us first consider the **LHS Disciplinary Sectors**

1. Mathematics: MAT/(01–09)
2. Informatics: INF/01
3. Physics: FIS/(01–08)
4. Chemistry: CHIM/(01–11)
5. Earth sciences: GEO/(01–12)
6. Biology: BIO/(01–19)
7. Medicine: MED/(01–50)
8. Agricultural sciences: AGR/(01–20)
9. Veterinary sciences: VET/(01–10)
10. Civil engineering and architecture: ICAR/(01–22)
11. Industrial engineering: ING-IND/(01–35)
12. Information engineering: ING-INF/(01–07)

Let us now proceed with the **SSH Disciplinary Sectors**

1. Antiquities studies: L-ANT/(01–09)
2. Linguistic studies: L-LIN/(01–21)
3. Philology studies: L-FIL-LET/(01–15)ART
4. Art history: L-ART/(01–08)
5. Oriental studies: L-OR/(01–23)
6. History: M-STO/(01–09)
7. Philosophy: M-FIL/(01–08)
8. Pedagogy: M-PED/(01–04)
9. Psychology: M-PSI/(01–08)
10. Physical Education: M-EDF/(01–02)
11. Law: IUS/(01–21)
12. Economics: SECS-P/(01–13)
13. Statistics:SECS-S/(01–06)
14. Political and social sciences: SPS/(01–14)

Let us now consider the three industrial areas. We list the disciplinary sectors belonging to each area. The acronyms associated with them are not referring to any previous labelling. They only follows the criteria that have been used for the LHS and SSH disciplinary sectors, just for the sake of uniformity.

Let us first consider the **primary goods production** area.

1. Agri-food industry: PR-AFI/(01–10)
2. Forestry economics: PR-FE/01
3. Fishing industry: PR-FI/01
4. Mining industry: PR-MI/(01–05)
5. Ceramics and Glass industries: PR-CG/(01–03)
6. Energy production industries: PR-EN/(01–05)

let us continue with the sector devoted to the **material goods production** area

1. Construction industry: SEC-EC/(01–03)
2. Chemical product industry: SEC-CH/(01–10):
3. Graphics industry: SEC-GRA/(01–03)
4. Paper converting industry: SEC-PC/01
5. Wood industry: SEC-WOO/01
6. Furniture industry: SEC-FUR/01–05)
7. Textile industry: SEC-TEXT/(01–03)
8. Engineering industry: SEC-ENG/(01–05)
9. Electronic industry: SEC-ELEC/(01–05)
10. Steel industry: SEC-STE/01
11. Shipbuilding industry: SEC-SHIP/(01–03)
12. Aeronautical industry: SEC-AER/(01–03)
13. Biochemical and Pharmaceutical industry: SEC-BIO-PH/(01–010)
14. Biomedical industry: SEC-BIOM/(01–05)

Let us finally consider the tertiary&advanced tertiary sectors, devoted to **services**.

1. Transport industry: TER-TRA/(01–10)
2. Logistic engineering: TER-LOG/(01–05)
3. Educational industry: TER-EDU/(01–10)
4. Culture industry: TER-CULT/(01–05)
5. Health industry: TER-HE/(01–10)
6. Commerce: TER-COM/(01–10)
7. Banks and Insurances services: TER-BIN/(01–05)
8. Environmental services: TER-ENV/(01–05)
9. Domotics: TER-DOM/01
10. Robotics: TER-ROB/01
11. Digital industry: TER-DIG/(01–05)
12. Telematic services: TER-TEL/01
13. Internet: TERT-INT/01
14. Journalism and Science journalism: TER-JOU/(01–05)
15. Science and innovation diplomacy: TER-S-DIP/(01–03)

We have defined a set of 61 Disciplinary sectors 26 of which coming from LHS and SSH and the remaining ones collecting up the industrial product chains. An example of product chains is provided by the North Adriatic industrial Union, which aggregates more than 1300 enterprises, grouped into 14 industrial product chains and include about sixty thousands employees in the industrial sector.

The SCN tasks necessarily refers to a number of SDGs and require the activity of researchers of few disciplinary sectors. The definition of the sustainable development goals and the disciplinary sectors for any given task is necessary in order to make evaluations of the behavior of the network and to measure its efficiency.

Let us make an example taken from the SCN Trieste Laboratory on quantitative sustainability (TLQS) [12]. This include seven clusters, one of which is *The Blue Planet and the sustainability of the sea economy*. Sees, oceans, coastal and internal waters are vital for our societies and the future of the Planet. They are sources of food, energy, biological resources, communication routs, work opportunities, leisure, cultural stimuli and they may also viewed as future new dimensions of human life.

In order to pursue a realistic strategy of the *blue prosperity* it is however fundamental to understand and make quantitative measurements and evaluations on the functioning of the oceans and the marine ecosystems as well as to learn their response to the antropic impact. Such a task requires the development and the deepening of several disciplinary aspects related to physical oceanography, marine biology, ecology, physical chemistry, environmental, economy, social sciences systems theory, engineering and others.

The SDGs related to this task are primarily number 6, 7, 9, 13 and 14. The macro-sectors to be associated with the cluster are the following: LHS/01–06, LAS/11–12, SSH/11–14, PRI/04–06, PRI/06, SEC/08, SEC/11, TER/01–02, TER/06–08, TER/10–11 and TER/15.

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