Chapter 4
Frequency and Wavelength

If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.

—Nikola Tesla

4.1 EMC and Frequencies

In general, *conducted EMC emission and immunity tests* take place at lower frequencies than *radiated EMC emission and immunity tests*.

- **Conducted RF Emissions.** It is assumed that connected cables to electric equipment could act as antennas (in case of conducted emissions from the equipment induced into these cables). Therefore, conducted RF emission limits exist to avoid radiation of connected cables (power, communication). Usually, conducted RF emissions are specified for frequencies $f = 150\,\text{kHz}$ to $30\,\text{MHz}$ [1].
- **Radiated RF Emissions.** Radiated RF emissions tend to occur at higher frequencies (in the megahertz range, depending on the equipment's maximum dimensions [m]). Therefore, regulators usually specify the maximum RF radiation limits for frequencies from $f = 30\,\text{MHz}$ to $f = 6\,\text{GHz}$ [1].
- **Conducted RF Immunity.** As stated above, connected cables are thought to act as antennas. Therefore, conducted RF immunity tests exist to evaluate the functional immunity of electrical and electronic equipment when subjected to conducted disturbances induced to connected cables by RF fields. Conducted RF immunity tests are usually performed with frequencies from $f = 150\,\text{kHz}$ to $80\,\text{MHz}$ [4].
- **Radiated RF Emissions.** Radiated RF immunity tests are intended to demonstrate the immunity of electrical and electronic equipment when subjected to wireless devices like mobile phones and other radiated interference. Radiated RF immunity tests are usually performed with frequencies from $f = 80\,\text{MHz}$ to $6\,\text{GHz}$ [3].
- **Radiated Magnetic Field Immunity.** Magnetic field immunity tests are usually performed at mains power frequency: 50 and $60\,\text{Hz}$ [5].

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Table 4.1 ITU frequency bands and their corresponding wavelengths in free-space [11]

Band number	Symbols	Frequency range	Corresponding wave length	Example Uses
1	Extremely low frequency (ELF)	3–30 Hz	100'000–10'000 km	Submarine communication
2	Super low frequenc (SLF)	30–300 Hz	10'000–1'000 km	Submarine communication
3	Ultra low frequency (ULF)	300–3'000 Hz	1'000–100 km	Communication within mines, submarine
4	Very low frequency (VLF)	3–30 kHz	100–10 km	Navigation, time signals, submarine
5	Low frequency (LF)	30–300 kHz	10–1 km	Navigation, time signals, AM, amateur radio
6	Medium frequency (MF)	300–3'000 kHz	1'000–100 m	AM, amateur radio, avalanche beacons
7	High frequency (HF)	3–30 MHz	100–10 m	Shortwave broadcast, amateur radio
8	Very high frequency (VHF)	30–300 MHz	10–1m	FM, TV, aircraft, amateur radio, weather radio
9	Ultra high frequency (UHF)	300–3'000 MHz	1–0.1 m	WiFi, mobile phones, Bluetooth, GPS, TV
10	Super high frequency (SHF)	3–30 GHz	100–10 mm	WiFi, mobile phones, astronomy, satellites
11	Extremely high frequency (EHF)	30–300 GHz	10–1 mm	Astronomy, remote sensing, weapons
12	Tremendously high frequency (THF)	300–3'000 GHz	1–0.1 mm	THz time-domain spectroscopy, tomography

- **Common-Mode Low-Frequency Disturbance**. In some areas, there are also conducted EMC immunity tests specified from $f = 0\,\text{Hz}$ to $f = 150\,\text{kHz}$ [2]. This is intended to demonstrate the immunity of electrical and electronic equipment when subjected to conducted common-mode disturbances such as those originating from power line currents, frequency converters, and return leakage currents in the earthing/grounding system.

The radio spectrum managed by the *International Telecommunication Union (ITU)* goes up to 3000 GHz[1] and is divided into 12 *ITU frequency bands*. Table 4.1) shows the ITU frequency bands with their corresponding wavelength and potential applications.

4.2 Wavelength vs. Frequency

4.2.1 Wavelength in Any Media

The *frequency* f [Hz] of a sinusoidal electromagnetic wave (Fig. 4.1) and its *wavelength* λ [m] have the following relationship [6]:

$$\lambda = \frac{v}{f} = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{\frac{(\epsilon'\mu' - \epsilon''\mu'')}{2} \cdot \left(\sqrt{1 + \left(\frac{\epsilon'\mu'' + \epsilon''\mu'}{\epsilon'\mu' - \epsilon''\mu''}\right)^2} + 1\right)}} \tag{4.1}$$

[1] Radio waves are electromagnetic waves of frequencies arbitrarily lower than 3000 GHz (3 THz), propagated in space without artificial guide. 3 THz is already in the infrared frequency band (300 GHz–430 THz); visible light starts at 430 THz.

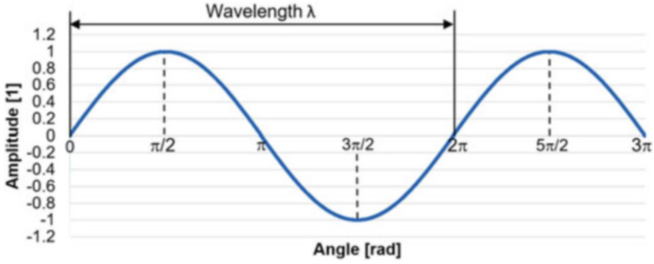

Fig. 4.1 Wavelength λ of a sine wave signal

where:

v = propagation velocity of the signal in [m/sec]
f = frequency of the sinusoidal signal in [Hz]
β = propagation constant in [1/m]; see Eq. 7.53
ϵ' = real part of the complex permittivity $\underline{\epsilon} = \epsilon' - j\epsilon''$ of the medium through which the wave is traveling in [F/m]
ϵ'' = imaginary part of the complex permittivity $\underline{\epsilon} = \epsilon' - j\epsilon''$ of the medium through which the wave is traveling in [F/m]
μ' = real part of the complex permeability $\underline{\mu} = \mu' - j\mu''$ of the medium through which the wave is traveling in [H/m]
μ'' = imaginary part of the complex permeability $\underline{\mu} = \mu' - j\mu''$ of the medium through which the wave is traveling in [H/m]

4.2.2 *Wavelength in Insulating Media*

In case of an insulator ($\mu'_r = 1$) and negligible dielectric and magnetic losses ($\epsilon'' = 0$, $\mu'' = 0$), the wavelength of a sinusoidal electromagnetic wave with frequency f [Hz] can be written as:

$$\lambda = \frac{v}{f} = \frac{1}{f\sqrt{\mu_0\epsilon_0\epsilon'_r}} = \frac{c}{f\sqrt{\epsilon'_r}} \tag{4.2}$$

where:

v = propagation velocity of the signal in [m/sec]
f = frequency of the sinusoidal signal in [Hz]
$c = 1/(\sqrt{\mu_0\epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light
$\mu_0 = 12.57 \cdot 10^{-7}$ H/m = permeability of vacuum, absolute permeability
$\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m = permittivity of vacuum, absolute permittivity
ϵ'_r = relative permittivity, dielectric constant of the insulator

4.2.3 Wavelength in Vacuum

For vacuum (and approximately air), the calculation of the wavelength of a sinusoidal electromagnetic wave reduces to [10]:

$$\lambda = \frac{c}{f} = \frac{1}{f\sqrt{\mu_0 \epsilon_0}} \tag{4.3}$$

where:

$c = 1/(\sqrt{\mu_0 \epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light
f = frequency of the sinusoidal signal in [Hz]

4.2.4 Wavelength in Good Conducting Media

In case the electromagnetic sinusoidal wave travels through a good conductor (through and not along(!), e.g., through a shield) with negligible magnetic losses ($\mu'' = 0$), the wavelength can be calculated as [10]:

$$\lambda = \sqrt{\frac{4\pi}{f\mu'\sigma}} \tag{4.4}$$

where:

f = frequency of the sinusoidal signal in [Hz]
$\mu' = \mu'_r \mu_0$ = real part of the complex permeability ($\underline{\mu} = \mu' - j\mu''$) in [H/m]
σ = specific conductance of the medium where the wave is propagating through in [S/m]

4.3 Wavelength of Signals Along Wires, Cables, and PCB Traces

It is important to understand that the signal propagation velocity v [m/sec] depends on the transport medium through which the electromagnetic field is traveling. Therefore, the same signal with the same frequency f [Hz] has a different wavelength λ [m] in a blank wire (surrounded by air) than in a cable or PCB trace (surrounded by one or multiple insulation materials). The wavelength λ of signals traveling along wires, cables, and PCB traces—where the dielectric and magnetic losses can be neglected ($\epsilon'' = 0$, $\mu'' = 0$) and the materials around the conductors are assumed to be nonmagnetic ($\mu'_r = 1$)—is given as [9]:

$$\lambda = \frac{v}{f} = \frac{c}{f\sqrt{\epsilon_{reff}}} = \frac{c}{f} \cdot \mathrm{VF} \tag{4.5}$$

where:

v = propagation velocity of the signal in [m/sec]
f = frequency of the sinusoidal signal in [Hz]
$c = 2.998 \cdot 10^8$ m/sec = speed of light
ϵ_{reff} = effective relative permittivity (dielectric constant) of the material(s) through which the electromagnetic field is propagating
VF = velocity factor

4.3.1 Wavelength of Signals Along Blank Wires

The *wavelength* λ [m] of a signal with frequency f [Hz] which travels along a blank wire (or antenna surrounded by air) depends only on the speed of light c [m/sec] and the signal frequency f [Hz] ($v = c$, because $\epsilon'_r = 1$ and $\mu'_r = 1$ and therefore VF = 1) [9]:

$$\lambda_{blankwire} = \frac{c}{f} \tag{4.6}$$

where:

$c = 1/(\sqrt{\mu_0\epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light
f = frequency of the sinusoidal signal in [Hz]

4.3.2 Wavelength of Signals Along Cables and PCB Traces

The wavelength λ of a signal with frequency f which travels along a wire, *cable*, or a *printed circuit board* (PCB) trace is [9]:

$$\lambda_{cable/PCBtrace} = \frac{c}{f \cdot \sqrt{\epsilon_{reff}}} \tag{4.7}$$

where:

$c = 1/(\sqrt{\mu_0\epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light
f = frequency of the sinusoidal signal in [Hz]
ϵ_{reff} = the effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

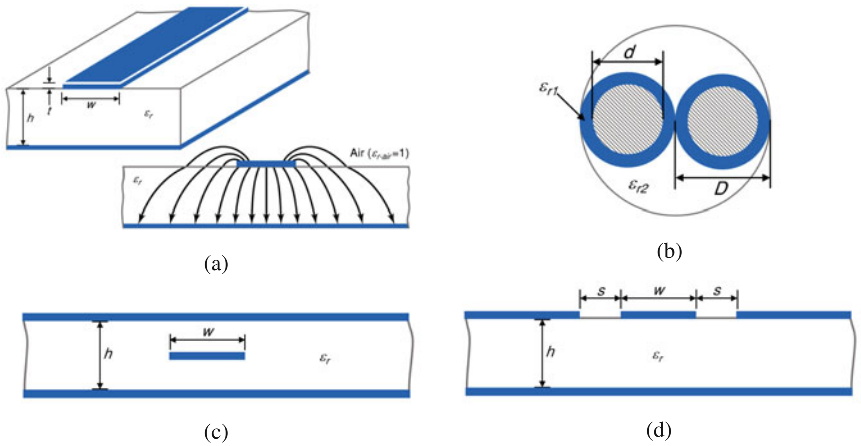

Fig. 4.2 Transmission line examples. (**a**) PCB trace: microstrip line. (**b**) Twisted pair cable. (**c**) PCB trace: stripline. (**d**) PCB trace: coplanar waveguide with reference plane

The effective dielectric constant ϵ_{reff} is defined as the uniform equivalent dielectric constant for a transmission line, even in the presence of different dielectrics (e.g., FR-4 and air for a microstrip line; see Fig. 4.2a). The relative permeability μ_r' is assumed to be equal 1.0 for cables and PCBs, because the insulation materials are nonmagnetic. Thus, the *velocity factor* (VF) depends primarily on the effective relative permittivity ϵ_{reff} through which the electromagnetic wave is propagating.

The calculation of the effective dielectric constant ϵ_{reff} depends on the insulation material and the geometry of the transmission line (e.g., ribbon cable, microstrip, coplanar waveguide, etc.), because the amount of electric field lines in the different media depends on the geometry of the transmission line. Figure 4.2 shows some common transmission lines and Table 4.2 the corresponding ϵ_{reff}

The velocity factor (VF) of a transmission medium is the ratio of the velocity v [m/sec] at which a wavefront of an electromagnetic signal passes through the medium, compared to the speed of light in vacuum c =2.998 $\cdot 10^8$ m/sec:

$$\mathrm{VF} = \frac{v}{c} \tag{4.8}$$

Thus, the smaller the velocity factor (VF), the smaller the wavelength λ [m].

During EMC emission measurement and troubleshooting, it is often necessary to determine the wavelength λ [m] of a certain unintended disturbance with frequency f [Hz] because once you know the wavelength of the disturbance, you can look for potential antennas of the disturbance (e.g., looking for cables with length $l = \lambda/4$ or $l = \lambda/2$ of the disturbance). Table 4.3 presents the velocity factors for different ϵ_{reff} and the resulting wavelength λ [m] for a given frequency f [Hz].

Table 4.2 Approximate velocity factor (VF) for different transmission lines and insulation materials. Calculation of ϵ_{reff} according to: [7, 8, 12]

Material	ε_r	Transmission line	ε_{reff}	VF
FR-4 (fiberglass epoxy of PCBs)	4.5	Microstrip line (w=0.3mm, h=0.5mm)	3.1	0.57
		Stripline (within FR-4 epoxy)	4.5	0.47
		Coplanar waveguide (w=0.3mm, h=0.5mm, s=0.5mm)	3.0	0.58
Polyethylene (PE)	2.26	Twisted pair cable	2.26	0.67
		Ribbon cable	1.3	0.88
Polyvinyl chloride (PVC)	3.0	Twisted pair cable	3.0	0.58
		Ribbon cable	1.5	0.82
Polytetrafluorethylene, Teflon (PTFE)	2.1	Twisted pair cable	2.1	0.69
		Ribbon cable	1.3	0.88
		Coaxial cable	2.1	0.69

Table 4.3 Wavelength λ [m] for given frequencies f [Hz] and dielectric constants ϵ_r'

Frequency [Hz]	λ [m] for free-space ε_r=1	λ [m] for ε_r=1.5, VF=0.82	λ [m] for ε_r=3.0, VF=0.58	λ [m] for ε_r=4.5, VF=0.47
1 Hz	300'000'000	246'000'000	174'000'000	141'000'000
10 Hz	30'000'000	24'600'000	17'400'000	14'100'000
100 Hz	3'000'000	2'460'000	1'740'000	1'410'000
1 kHz	300'000	246'000	174'000	141'000
10 kHz	30'000	24'600	17'400	14'100
100 kHz	3'000	2'460	1'740	1'410
1 MHz	300	246	174	141
10 MHz	30	25	17	14
100 MHz	3.0	2.5	1.7	1.4
1 GHZ	0.30	0.25	0.17	0.14
10 GHz	0.030	0.025	0.017	0.014
100 GHz	0.0030	0.0025	0.0017	0.0014
1 THz	0.00030	0.00025	0.00017	0.00014

4.3.3 Summary

- **Wavelength.** The wavelength λ [m] of a sinusoidal signal with frequency f [Hz] depends on the media through which the electromagnetic wave is propagating because the velocity v [m/sec] changes with the dielectric and magnetic properties $\underline{\epsilon}$ [F/m], $\underline{\mu}$ [H/m].
- **Wavelength of signals along conductors.**

$$\lambda = v/f = c/(f\sqrt{\epsilon_{reff}}) \tag{4.9}$$

where:

v = propagation velocity of the signal in [m/sec]
f = frequency of the sinusoidal signal in [Hz]
ϵ_{reff} = the effective dielectric constant (relative permittivity) through which the electromagnetic wave is propagating

- **Wavelength of electromagnetic waves in free-space.**

$$\lambda = c/f \tag{4.10}$$

where:

$c = 1/(\sqrt{\mu_0 \epsilon_0}) = 2.998 \cdot 10^8$ m/sec = speed of light
f = frequency of the sinusoidal electromagnetic wave in [Hz]

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