Chapter 10
Skin Effect

Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.

—Marie Curie

10.1 Skin Depth

Introduction and definitions of the term skin depth δ [m] (Fig. 10.1):

- **Conductors.** The skin depth δ [m] is defined as the distance from the conductor surface where the current density has fallen (caused by the skin effect) to 37% = 1/e = 1/2.72 of the current density at the surface of the conductor J_0 [A/m^2]. The current density J_d [A/m^2] at distance d [m] from the conductor surface is defined as [2]:

$$|J_d| = |J_0| \cdot e^{-\frac{d}{\delta}} \tag{10.1}$$

  where:

  $|J_d|$ = magnitude current density at distance d from the surface of the conductor in [A/m^2]
  $|J_0|$ = magnitude of the current density at the surface of the conductor in [A/m^2]
  d = distance from the conductor's surface in [m]
  δ = skin depth in [m]

- **Shielding.** Imagine an electromagnetic plane wave of field strength E_0 [V/m] and H_0 [A/m] entering an absorbing material (shield). The skin depth δ [m] is the distance an electromagnetic wave has to travel through that absorbing material until its field strength is reduced to 37% of E_0 or H_0 (Fig. 10.2). This means that the power of the plane electromagnetic wave is lowered by $10 \log_{10}\left(1/e^2\right) = 8.686\,\text{dB}$ after it has traveled the distance δ. The attenuation of an electromagnetic plane wave is defined like this [2]:

R. B. Keller, *Design for Electromagnetic Compatibility–In a Nutshell*,
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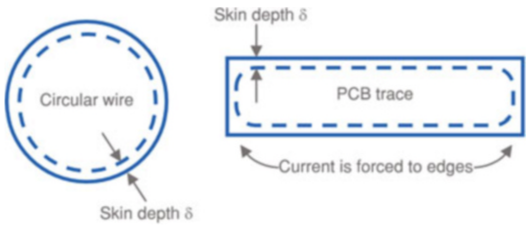
Fig. 10.1 Skin depth δ



$$|E_d| = \mathrm{Re}(|E_0| \cdot e^{-\underline{\gamma} d}) = \mathrm{Re}(|E_0| \cdot e^{-\alpha d} \cdot e^{-j\beta d}) = |E_0| \cdot e^{-\alpha d} = |E_0| \cdot e^{-\frac{d}{\delta}} \tag{10.2}$$

$$|H_d| = \mathrm{Re}(|H_0| \cdot e^{-\underline{\gamma} d}) = \mathrm{Re}(|H_0| \cdot e^{-\alpha d} \cdot e^{-j\beta d}) = |H_0| \cdot e^{-\alpha d} = |H_0| \cdot e^{-\frac{d}{\delta}} \tag{10.3}$$

where:

$|E_0|$ = electric field strength of an electromagnetic plane wave at the surface of a shield barrier, when entering that shield barrier in [V/m]
$|H_0|$ = magnetic field strength of an electromagnetic plane wave at the surface of a shield barrier, when entering that shield barrier in [A/m]
$|E_d|$ = electric field strength after the electromagnetic plane wave has traveled distance d through the shield barrier in [V/m]
$|H_d|$ = magnetic field strength after the electromagnetic plane wave has traveled distance d through the shield barrier in [A/m]
$\underline{\gamma}$ = the complex propagation constant of the shield barrier material [1/m]
α = attenuation constant of the shield barrier material in [1/m]
β = phase constant (or phase factor) of the shield barrier material in [rad/m]
d = distance from the shield barrier's surface in [m]
δ = skin depth in [m]

From Eqs. 10.2 and 10.3, we know that the skin depth δ [m] is defined as the inverse of the attenuation constant α [1/m] [1]:

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega \sqrt{\frac{(\epsilon'\mu' - \epsilon''\mu'')}{2} \cdot \left(\sqrt{1 + \left(\frac{\epsilon'\mu'' + \epsilon''\mu'}{\epsilon'\mu' - \epsilon''\mu''} \right)^2} - 1 \right)}} \tag{10.4}$$

where:

$\omega = 2\pi f$ = angular frequency of the signal in [rad/sec]
ϵ' = real part of the complex permittivity ($\underline{\epsilon} = \epsilon' - j\epsilon''$) in [F/m]
ϵ'' = imaginary part of the complex permittivity ($\underline{\epsilon} = \epsilon' - j\epsilon''$) in [F/m]
μ' = real part of the complex permeability ($\underline{\mu} = \mu' - j\mu''$) in [H/m]
μ'' = imaginary part of the complex permeability ($\underline{\mu} = \mu' - j\mu''$) in [H/m]

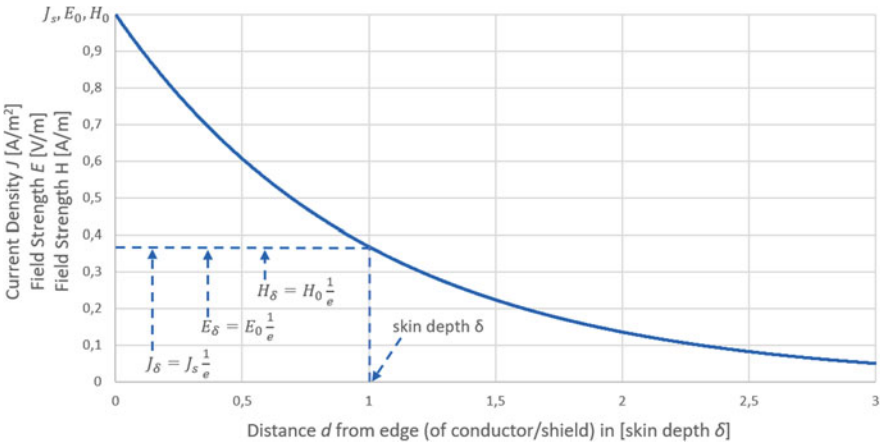


Fig. 10.2 Attenuation of current density J [A/m^2], electric field strength E [V/m], and magnetic field strength H [A/m] due to skin effect

For good conductors (with $\sigma \gg \omega\epsilon'$ and $\epsilon_r' = 1.0$) with negligible magnetic losses ($\mu'' = 0$), we can write [2]:

$$\delta = \sqrt{\frac{2}{\omega\mu_r'\mu_0\sigma}} = \sqrt{\frac{2}{\omega\mu_r'\mu_0\sigma}} = \frac{1}{\sqrt{\pi f \mu_r'\mu_0\sigma}} \tag{10.5}$$

where:

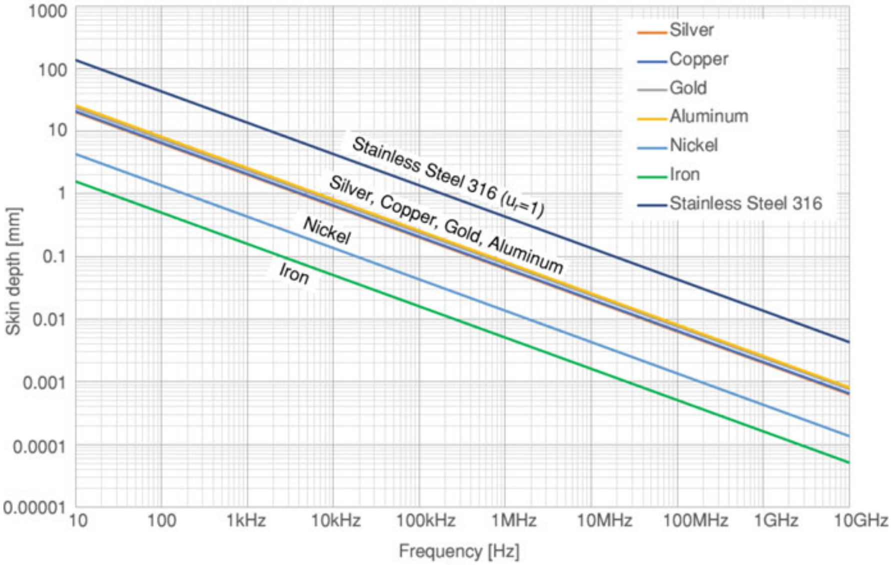
δ = skin depth in [m]
$\omega = 2\pi f$ = angular frequency of the signal in [rad/sec]
μ_r' = relative permeability of the material through which the signal current is flowing or through which the electromagnetic wave is traveling in [1]
$\mu_0 = 4\pi \cdot 10^{-1}$ H/m = 12.57 $\cdot 10^{-7}$ H/m = permeability of vacuum
σ = specific conductance of the material through which the signal current is flowing or through which the electromagnetic wave is traveling in [S/m]

Figure 10.3 presents some example values of skin depths for silver, copper, gold, aluminum, nickel, iron, and stainless steel 316. The following points have to be considered when calculating the skin depth for ferromagnetic metals (like nickel, iron):

- The relative permeability $\underline{\mu}_r$ depends on the specific material and alloy.
- Relative permeability $\underline{\mu}_r$ depends on the frequency f [Hz] and temperature T [K].

Parameters	Silver	Copper	Gold	Aluminum	Nickel	Iron	Stainless Steel 316
σ [S/m]	6,20E+07	5,82E+07	4,50E+07	3,80E+07	1,40E+07	1,00E+07	1,36E+06
μ_r [1]	1	1	1	1	100	1000	1
Frequency	**Skin depth [mm]**						
10 Hz	20,213	20,862	23,725	25,818	4,254	1,592	136,474
100 Hz	6,392	6,597	7,503	8,164	1,345	0,503	43,157
1 kHz	2,021	2,086	2,373	2,582	0,425	0,159	13,647
10 kHz	0,639	0,660	0,750	0,816	0,135	0,050	4,316
100 kHz	0,202	0,209	0,237	0,258	0,043	0,016	1,365
1 MHz	0,064	0,066	0,075	0,082	0,013	0,005	0,432
10 MHz	0,020	0,021	0,024	0,026	0,004	0,002	0,136
100 MHz	0,006	0,007	0,008	0,008	0,001	0,001	0,043
1 GHz	0,0020	0,0021	0,0024	0,0026	0,0004	0,0002	0,0136
10 GHz	0,0006	0,0007	0,0008	0,0008	0,0001	0,0001	0,0043

(a)



(b)

Fig. 10.3 Skin depth of some example metals. Permeability μ'_r is assumed to be constant. (**a**) Table with the assumed electrical conductivity σ and permeability μ'_r. (**b**) Skin depth δ as a function of frequency f

10.2 DC vs. AC Resistance

The resistance per-unit-length [Ω/m] for direct current (R'_{DC}) and for alternating current (R'_{AC}) of any conductor can be written as [2]:

$$R'_{DC} = \frac{\rho}{A} = \frac{1}{\sigma A} \tag{10.6}$$

$$R'_{AC} = \frac{\rho}{A_{eff}} = \frac{1}{\sigma A_{eff}} \tag{10.7}$$

where:

ρ = specific electrical resistivity of the conductor material in [Ωm]
σ = specific conductance of the conductor material in [S/m]
A = cross-sectional area of the conductor in [m^2]
A_{eff} = effective cross-sectional area of the conductor through which the current effectively flows in [m^2]

For direct current (DC, 0 Hz), the cross-sectional area A_{eff} [m^2] through which the DC current flows is equal to the conductor cross-sectional area A [m^2]. However, for high-frequency AC current with frequency f [Hz], the magnetic field—produced by current in the conductor—forces the current flow toward the outer surface of the conductor, and as a consequence of that, the current density increases exponentially from the core of the conductor toward the conductor's surface. The higher the signal frequency f [Hz], the smaller the cross section A_{eff} [m^2] through which the current effectively flows.

The accurate calculation of A_{eff} [m^2] is difficult but can be reasonably approximated by assuming that the current is uniformly distributed over the skin depth δ [m]. In case of $D \gg \delta$, the approximate $A_{effWire}$ [m^2] of a round conductor (circular wire) with diameter D [m] can be calculated as follows:

$$A_{effWire} \approx A - A_{noCurr} = \frac{D^2}{4}\pi - \frac{(D-2\delta)^2}{4}\pi = \left(D\delta - \delta^2\right)\pi \tag{10.8}$$

where:

A = cross-sectional area of the round conductor with diameter D in [m^2]
A_{noCurr} = the approximate cross-sectional area of the conductor where no current flows (as a consequence of the skin effect) in [m^2]
δ = skin depth in [m]
D = diameter of round conductor in [m]

For a PCB trace according to Fig. 10.4, the effective area $A_{effPCBtrace}$ [m^2] can be approximately calculated like this:

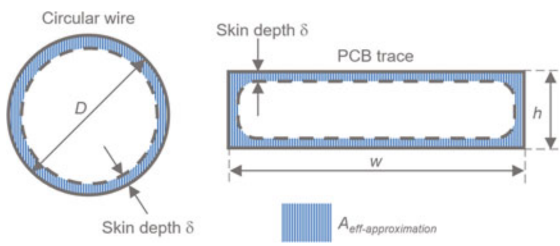


Fig. 10.4 Approximation of A_{eff} using the skin depth δ

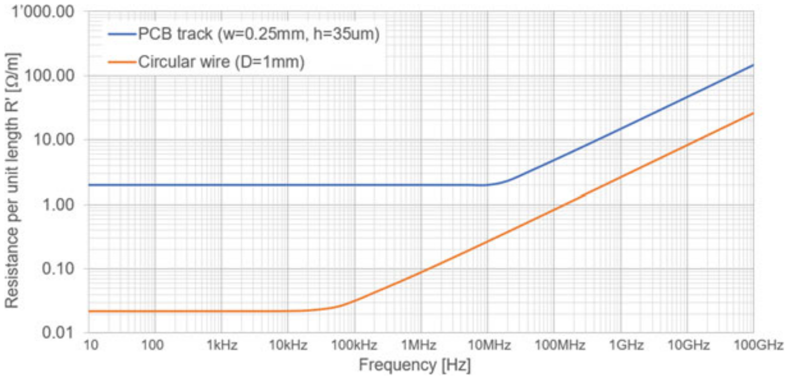


Fig. 10.5 Increased resistance due to skin effect: PCB copper trace (width $w = 0.25$ mm, height $h = 35$ µm = 1 oz) vs. round copper wire (diameter $D = 1$ mm)

$$A_{effPCBtrace} \approx A - A_{noCurr} = wh - (w - 2\delta)(h - 2\delta) = 2\delta(w + h) - 4\delta^2 \tag{10.9}$$

where:

A = cross-sectional area of the PCB trace with width w and height h in [m^2]
A_{noCurr} = the approximate cross-sectional area of the conductor where no current flows (as a consequence of the skin effect) in [m^2]
δ = skin depth in [m]
w = width of the PCB trace in [m]
h = height or thickness of the PCB trace in [m]

Figure 10.5 shows R'_{AC} [Ω/m] of a PCB trace versus a round copper wire. The calculations for R'_{AC} [Ω/m] are approximations and ignore the return current path (proximity effect) and assume a single conductor surrounded by air only. However, the diagram gives an idea of how the skin effect influences the resistance at higher frequencies.

10.3 Surface Resistance

The *surface resistance* (or *sheet resistance*) R_s [Ω/square] for a thin layer with height h [m]—like shown in Fig. 10.6—is defined as [4]:

$$R_s = \frac{1}{\sigma h} \tag{10.10}$$

where:

σ = specific conductance of the conductor material in [S/m]

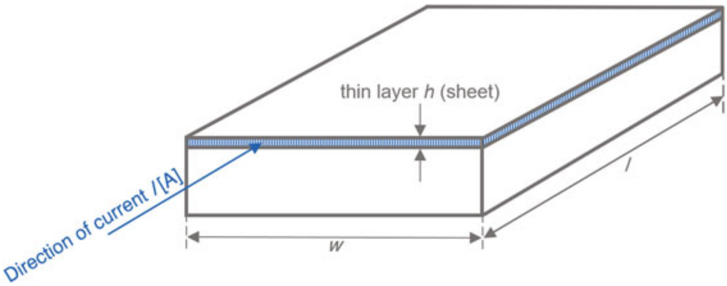


Fig. 10.6 Sheet (thin layer) of height of h, width w, and length l. Current flow is parallel to length l (not perpendicular to the sheet)

h = height or thickness of a thin conductor layer [m]

For a good conductor ($\sigma \gg \omega\epsilon'$), with negligible magnetic losses $\mu'' = 0$ and at high frequency (skin depth δ much smaller than conductor cross-sectional dimensions), the thickness of the conductor sheet h [m] can be set to the skin depth δ [m], and the surface resistance R_s [Ω/square] is defined as [3]:

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu'}{2\sigma}} \tag{10.11}$$

where:

σ = specific conductance of the conductor material in [S/m]
δ = skin depth in [m]
$\omega = 2\pi f$ = angular frequency of the signal in [rad/sec]
$\mu' = \mu'_r \mu_0$ = ability to store energy in a medium when an external magnetic field is applied = real part of the complex permeability ($\underline{\mu} = \mu' - j\mu''$) in [H/m]

At high frequencies, where the current flows at the surface of the conductor (skin effect), the resistance of a conductor with perimeter p [m] and length l [m] can be approximated as:

$$R \approx R_s \frac{l}{p} = \frac{l}{\sigma\delta p} \tag{10.12}$$

where:

R_s = surface resistance of the conductor material in [Ω/square]
l = length of the conductor in [m]
p = perimeter of the conductor in [m]
δ = skin depth in [m]

10.4 Summary

- **Skin Depth δ of Good Conductor.**

$$\delta = \frac{1}{\sqrt{\pi f \mu'_r \mu_0 \sigma}} \tag{10.13}$$

where:

μ'_r = relative permeability of the conductor material in [1]
$\mu_0 = 4\pi \cdot 10^{-1}$ H/m = 12.57 $\cdot 10^{-7}$ H/m = permeability of vacuum
σ = specific conductance of the conductor material in [S/m]

- **Resistance vs. Frequency.** Due to the skin effect, the resistance of a conductor increases with increasing frequency. As an approximation and in case the skin depth δ [m] is much smaller than the conductor's outer dimensions, it can be assumed that the current is uniformly distributed over the skin depth δ.
- **Surface Resistance**. Surface resistance R_s [Ω/square] of a good conductor at high frequency (skin depth δ much smaller than conductor cross-sectional dimensions):

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu'}{2\sigma}} \tag{10.14}$$

where:

σ = specific conductance of the conductor material in [S/m]
δ = skin depth in [m]
$\omega = 2\pi f$ = angular frequency of the signal in [rad/sec]
$\mu' = \mu'_r \mu_0$ = ability to store energy in a medium when an external magnetic field is applied = real part of the complex permeability ($\underline{\mu} = \mu' - j\mu''$) in [H/m]

References

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