Chapter 9
Introduction

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The focus of theme B is "analysing school mathematics curriculum reforms for coherence and relevance" (ICMI Discussion Document, 2018, p. 578). The introduction (Chap. 1) to this book highlights the diverse views and approaches to defining the term 'curriculum'. In that discussion, the notions of 'intended, implemented and attained' as 'curriculum levels' led to the concept of 'curriculum components' such as "goals, content, pedagogy, materials and assessment" (p. 15). These constructs underpin the working definition of the term 'curriculum' that is used in this section's analysis of the coherence and relevance of school curriculum reforms. These elements, along with wider resources and constraints (such as teacher capacity and societal values), are together referred to as the 'curriculum system'. This section highlights that, whatever the curriculum reform's intent, the achieved curriculum is highly contextualised by the entire system, at a variety of levels.

As an important constituent of the curriculum system the professional development of teachers and the conditions and constraints under which they work are critical to the translation of the *intended* curriculum into the *enacted*[1] curriculum. Whilst the preparation and quality of teachers is not directly part of this section, there is some commentary on its importance to faithful enactment of reformed curricula in mathematics, and so, particularly, to the coherence achieved between various curriculum components. This is intended both to illuminate the issues in practice and to emphasise the importance of the quality of teaching.

In relation to the *attained* curriculum, this ICMI study has elicited some quantitative and qualitative data about student attainment, both as a driver for mathematics

[1] The term 'enacted' is preferred to 'implemented'. The latter term suggests that it is faithful to some prior model, such as intended curriculum, whereas there are a number of valid enactments of an intended curriculum.

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curriculum reform and as a measure of success – or otherwise – of reforms. Where appropriate, these data are referenced in the analysis of particular reforms.

Definitions for Coherence and Relevance

At the most general level, we view *coherence* as 'internal' to the curriculum (including materials and technologies designed to support its implementation), with *relevance* seen as 'external' to it, as the interaction between the curriculum and needs and aspirations (of students/young people, the workplace, universities, society, etc.). Both terms require clarification in order to be meaningful – there needs to be some specificity in relation to 'coherence' of what with what, from whose perspective and for what purpose, and, similarly 'relevance' of what to what, or for whom and for what purpose.

A curriculum includes a complex system of components proposed by different agents at different moments of time, under different conditions, and for different purposes. This is why coherence is not always ensured. 'Coherence' might include the vertical and horizontal alignment[2] within the intended mathematics curriculum, as well as its relationship with the parent discipline, with teacher education, with assessment, with the rest of the curriculum. This might (or might not) be a purely mathematical coherence, including an alignment within and across content and processes. It could relate the coherence (or otherwise) of things like priorities, weightings, values, implicit messages across the curriculum system as a whole. We need at least to be clear which of these we are addressing.

There is another practical challenge that confronts identification and analysis of coherence in the context of mathematics curriculum reforms. Instances of lack of coherence between levels and components of mathematics curriculum can be quite apparent and therefore easy to recognise. Further, addressing such incoherence attracts attention as a potential focus for future reforms – we can potentially 'do something about it'. Coherence within the curriculum is, on the other hand, much more unremarkable to us because it is expected as the implicit intended state.

Relevance begs questions of 'relevant to whom, and at what stage? Relevant for what purpose(s)?' These are questions of values or beliefs, and dependent also on context: we need to take care to avoid imposing assumptions or values. We also need to identify and expose underlying assumptions and values adopted. The lexical term 'relevant' is legitimately used in two different ways in the context of mathematics curriculum. The first draws on the mathematical term 'relationship' and has a sense of direct connection that can seem to suggest that a change in one variable causes a change in other(s). The other usage of 'relevant' conveys the sense of 'useful to' and 'fits with'. This is particularly the case for materials designed to support

[2] 'Vertical alignment' is alignment over time as a student progresses in their experience of the mathematics curriculum. 'Horizontal alignment' is alignment of the different areas of the curriculum at a specific time.

student learning. There are many physical ways of modelling fractions. Some are more 'relevant' (i.e. more useful) than others when planning for the students to develop particular concepts about fractions, or for some particular use or application. This relevance is not 'causal', however, as there are likely to be other models and approaches that are also effective (i.e. relevant) in illuminating the concept(s).

Components of Mathematics Curricula

As indicated in the introduction to this book, "the term 'curriculum' is used with many different meanings and scholars have noted that it seems almost impossible to give a universally acceptable definition of 'curriculum'" (p. xx). However, discussing 'coherence' and 'relevance' in curricula in this section requires a certain consistency of language in describing components of mathematics curricula.

Building on the work of Kilpatrick (1994) and Niss (2016, 2018) proposed "to define a (mathematics) curriculum with respect to a given educational setting as *a vector with six components* [see below]" (p. 70; *italics in original*), which are very commonly evident in mathematics curricula. According to Niss, the components of a curriculum are:

- *goals* (the [declared] overarching purposes, desirable learning outcomes, and specific aims and objectives of the teaching and learning taking place under the auspices of this curriculum);
- *content* (the [names of the] topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);
- *materials* (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT systems employed in teaching and learning);
- *forms of teaching* (the tasks, activities and modes of operation of the teacher in this curriculum);
- *student activities* (the activities of, and the tasks and assignments for, the students taught according to this curriculum);
- *assessment* (the goals, modes, formats and instruments adopted for formative and summative assessment, respectively, in this curriculum).

After which he added, "Specifying a curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, *implementing* a given curriculum amounts to specifying it, as well as to *carrying it out*, i.e. putting all the six components into practice" (p. 70; *italics in original*).

Niss argues that the enactment of the curriculum requires all six components to be in place and evident. Curriculum authorities tend to retain control of the curriculum's goals and content, and often any summative and/or high-stakes assessment. These authorities may, or may not, devolve some or total control of the other components (materials, forms of teaching and student activities), as well as the formative components of assessment, to external agents (textbook writers, assessment developers) and educators at the local level (schools, consultants, teachers). In any

of these cases, there are matters of coherence with other curricular components that can and do emerge; even when there is top-down control of a component such as materials through a mandated textbook, there can be some level of choice. Teachers, researchers and developers can exploit this choice in ways that enhance coherence in the enacted curriculum (see, for example, Miyazaki et al., 2018). Alternatively, the choices made can be detrimental to this coherence.

This framework allows for identification and analysis of the coherence between any one of the components of a particular curriculum and, potentially, the five others, to the extent to which they are present or not in the formal documentation. It is the framework used throughout this theme B.

Theme B Analyses

There are two major themes that overlay the analyses in the main chapters (Chaps. 10, 11, 12 and 13) that follow.

Curriculum to Meet Needs (of Students, the Workplace, Higher Education, Society etc.)

The ICMI Study 24 discussion document identified a set of key questions in this territory:

> How are mathematics content and pedagogical approaches in reforms determined for different groups of students (for e.g. in different curriculum levels or tracks) and by whom? How do curriculum reforms establish new structures in content, stakeholders (e.g. students and teachers), and school organisations; and what are their effects? (p. 580)

Perhaps surprisingly, much of the content of mathematics curricula is very similar across the globe, a situation perhaps reflective of common perceived needs in terms of relevance to individual or societal good. The level of commonality evident currently has potentially been reinforced by the comparatively recently-emerging and influential international performance assessments such as TIMSS and PISA, addressed further in theme D of this volume. Because of the influence they have, developments in these assessments have potential to increase relevance – to society and to the individual – of curricula, globally.

For example, PISA 2021 enhances the assessed profile of digital analysis of elementary data sets, surely a key component of data literacy and so centrally relevant to societal needs in a twenty-first-century world. It will be surprising if many curriculum authorities do not adapt accordingly – though the coherence of what is achieved is another matter, and one key message of this section is the challenge in creating and sustaining curriculum coherence at scale. In this theme, we discuss some curriculum reforms that subvert established norms in their attempts to educate

young people for appreciation of wider societal challenges (Giménez & Zabala, 2018), or for cross-curricular thinking (Lupiáñez & Ruiz-Hidalgo, 2018), yet fundamental challenges to dominant curriculum norms are few, with Tarp (2018) representing a rare such attempt.

'Meeting needs' is very often a driver and informant of mathematics curriculum reforms. It is one lens on the coherence and relevance of those reforms. Some of the issues include:

- mathematics in compulsory education compared with mathematics when it is no longer compulsory;
- mathematics in different student pathways (science, literature, social sciences etc.), as well as in vocational and general education pathways;
- identification of the goals of specific mathematics reforms (for the citizen, for the future worker, for other disciplines, etc.);
- taking account of and responding to diversity in the classroom, and specific contexts in the classroom including:
  - cognitive diversity (need for diversity of teaching approaches);
  - cultural and social diversity (how to give meaning to mathematics for everybody and drawing on social and cultural aspects);
  - achievement (involving low achievers and extending the more able);
  - the kind of mathematics that gives meaning to the students' world given the geographical and social context of the classroom;
  - intended or possible student pathways for progression within and beyond schooling.

Each of the chapters in this theme address a range of these and other aspects of 'meeting needs' in their analyses of particular reforms.

'Top-Down' as Opposed to 'Bottom-Up' Curriculum Development and Reforms

These can be seen as oppositional, with OECD's Project 2030 and national curriculum reforms in many countries at one end of the spectrum ('top-down'), and reforms where teachers, students and the community play leading roles ('bottom-up') at the other. The reforms discussed in the chapters that follow are drawn from across this spectrum.

However, the level of specification and requirements in the curriculum levels and components of top-down approaches can allow – and even require – schools, teachers, students and communities to make choices and decisions that shape the enacted curriculum. This level of autonomy, where it exists, can lead to opportunities and risks in relation to both the coherence and relevance of mathematics curriculum reforms. These matters are analysed and reported on in what follows.

Theme B Chapters

This section is presented in six components, with the two cross-cutting themes integrated into the analyses and discussion:

Chapter 9 – Introduction
Chapter 10 – Analysis of a range of contemporary mathematics curriculum reforms
Chapter 11 – Reforms that focus on linking mathematics with other disciplines
Chapter 12 – Materials and technologies to support curriculum reforms
Chapter 13 – Theories and methodologies for studying mathematics curriculum reforms
Chapter 14 – Conclusion, achieving coherence and relevance in mathematics curriculum reforms: some guiding principles

The interpretation that coherence is largely 'internal' to the curriculum while relevance is 'external' as outlined above suggests that the balance of attention to coherence and relevance is different for each of the chapters. The main focus, both for Chap. 10 (general reforms) and for Chap. 11 (interdisciplinary, cross-curricular and STEM-inspired reforms) is largely the coherence and relevance of the *intended* curriculum – the documentation that specifies values and goals, and what is to be taught. Chapter 12 considers both the coherence and relevance of the materials and technologies used to translate the intended curriculum into use in schools and classrooms – the *enacted* curriculum. Chapter 13 analyses theories and methodologies used to study curriculum reforms in ways that identify issues of coherence and relevance that are evident in those reforms. Hence it provides insights into both coherence and relevance through the lenses of theory and methodology.

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