# Chapter 3
# Introduction


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Most of the studies presented in this book describe past experiences of curriculum reforms. What is then the specificity of theme A chapters? What kind of particular approach to past reforms do they propose? Two aspects can be distinguished. In what corresponds to the object of study considered, either they refer to reforms with a particular role or transcendence in the evolution of mathematics education across the world, or they approach more local reforms over a sustained period. In what concerns the types of questions addressed, these sometimes 'large' empirical objects are approached to identify some driving forces that explain the conditions and outcomes of the reforms' implementation, and also some barriers that seem to have constrained their development. Theme A uses the past as a *learning* strategy and as an experience from which one can draw some lessons.

Our approach is therefore not purely historical, even if a specific account of the different reforms is presented in each case: we are always relying on empirical material or previous studies – previous *narratives* – about them. The issue of the narrative, the perspective adopted about the reform is crucial for many reasons. Curriculum reforms are changes that are undertaken under certain conditions, to modify a given aspect of a country's or region's educational system. They are breaks or interruptions in the established curricula, motivated by a particular diagnostic of these curricula to improve or guide them in a new and revised direction. Once implemented, and before being modified by another reform, their effects are also assessed, to be used as rationales for subsequent change. Therefore, we do not only deal with curricula that have been changed, but also with the perspective of the

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different agents that took place in the change – politicians, mathematicians, educators, teachers, students, journalists, parents, etc. Their viewpoints, diagnosis and evaluations are also part of the unit of analysis.

The questions that motivated theme A were formulated in the following terms:

> What aspects of school mathematics curriculum reform carried out in the past decades have been taken to be the most important, for example, in content, pedagogy, and in the underpinning theoretical approaches?
>
> What potentially crucial aspects of mathematical curricula have not been considered, and even less, touched upon?
>
> Which goals and values in school mathematics curriculum reforms, carried out in the past decades, have been the most important (for example, in the selection and organisation of mathematics contents, or process aspects of mathematical activities)?
>
> How have the questions of content become linked to the notions of mathematical competencies, capabilities, and literacy; and how have these evolved to become a driving force in the curriculum development and reform initiatives?
>
> What has been the role and function of curriculum resources, materials, and technology, including digital curricula and textbooks in curriculum reforms and their implementation as driving forces or barriers?

These questions are strongly intertwined and did not suggest a set of problems to be approached independently. It was not surprising that the nine contributions to theme A addressed some of them and each one to a different extent. In preparing this section, we decided to organise them in four chapters. The first two chapters give an account of past curriculum reforms and provide an overview of the empirical field of study, with many shortcomings but enough diversity. In the other two chapters, specific perspectives are adopted: first, the cultural dimensions of curriculum reforms and the values they convey, related to the societies' specificities and the particular vision of mathematics proposed by each reform; and second, the question of the "content" and what is related to it is addressed. We present these four chapters in broad outline while the description of the main learnings from theme A are presented in the concluding chapter.

## International Co-operation and Influential Reforms

Chapter 4 presents four cases of national reforms. The first three were initiated in the 1960s in European countries, namely The Netherlands, Hungary and France, after the Royaumont conference in 1959 organised by the Organisation for European Economic Co-operation, which would soon become the OECD. It is interesting to see how the same initial starting point, motivated by what was seen at that time as, "a desire to bring school mathematics closer to the academic mathematics of the twentieth century" (Kilpatrick, 2012, p. 563), led to such diverse outcomes. Especially when they were all being piloted by prestigious mathematicians, like Hans Freudenthal and Tamás Varga for the case of the Netherlands and Hungary and sustained by newly created government commissions or institutions. The specific personality of the mathematicians who took part in the reform movements,

alongside the mathematical culture, epistemology or tradition they conveyed, seem to have been a critical driving force for the concrete implementation of the reform and its divergent continuations.

There are also two other significant elements that the brief accounts given in the chapter do not show in detail, but can be found in the original contributions of the authors (Doorman et al., 2018; Gosztonyi et al., 2018; Van Zanten et al., 2018). On the one hand, there were some previous local innovations in mathematics education during the beginning of the twentieth century, like Ehrenfest's and Van Hiele's developments for the teaching of geometry in the Netherlands, or the "teaching of mathematics by guided discovery" in Hungary. They can be seen as specific original conditions for the basis of the reforms, both for the educational resources they had already provided and for the tradition they represented.

On the other hand, even if the Royaumont seminar served as an impulse for European governments to launch curriculum reforms in mathematics, what made the difference is the concrete work organised by leading mathematicians in each country that resulted in the production of particular mathematical tasks and content organisations. The case of Spain, as portrayed by Ausejo (2010), is a good counter-example. Pedro Puig-Adam, the leading mathematician involved in the renewal of mathematics education who participated in the first meetings of the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) (and even organised the eleventh one in Madrid in 1957), died prematurely in 1960. The absence of leadership, together with the complex political situation of the country, ended up with a series of 'hybrid' textbooks where the most new-math oriented proposals in terms of sets and applications side-by-side coexisted with the more traditional knowledge organisations based on the classic arithmetic of quantities, ratios and proportions.

As in many other countries, the cases of France and Hungary show reforms that were somehow interrupted or 'counter-reformed', for reasons or principles that were not as explicit as those claimed in the New Math reforms. As a contrast, the work of Freudenthal in the Netherlands, the Realistic Mathematical Education (RME) movement has been described as a longstanding reform movement. To this respect, the 'period of educational engineering' in the 1970s that followed the emergence of the RME reform in the 1960s can be seen as a sustained effort made by the Institute for Development of Mathematics Education (IOWO, which later became the Freudenthal Institute) to create a wide variety of teaching materials to support curriculum development, and also teacher education and professional development. This effort continues to the present day, with some highs and lows in its original country but also with sustained dissemination internationally.

The chapter ends with a description of the Japanese case to counterbalance the European ones. What we can see through this case is continuous curriculum development, based on explicit principles, involved in and influenced by the international reform movements and organisations. By going back to the period before World War II, this account recalls that the domain of mathematics is also the fruit of a reform process that appears as a way to unify the traditional school distinction between arithmetic, algebra, geometry and analysis. It also shows how the

curriculum is developed in line with international movements, such as the Meran project, the New Math, the NCTM Common Standards or the subsequent OECD-PISA framework but is never a mere adaptation of outside proposals. Each reform period corresponds to the will of emphasising different learning goals linked to specific aspects of mathematical activities, along with the social evolutions of the country.

## Case Studies of Past 'Local' Curriculum Reforms in Mathematics

Chapter 5 extends the empirical landscape that is initially framed by the four cases of this chapter. It aims to address the research questions about the aspects of mathematics teaching and learning processes certain international reforms attend to and to identify the key stakeholders in curriculum reforms; factors that underpinned curriculum reforms, and barriers that inhibited reform efforts. It also aims to identify the universal lessons that can be taken from international reform efforts so that future curriculum reform movements can learn and build on past efforts.

To gain an insight into curriculum reform efforts in different countries internationally, a survey was designed by some theme A participants. The survey sought to gather information in relation to:

- how and why the reform movement came about;
- the ideologies underpinning the reform movement;
- the aim of the reform movement;
- the agents or stakeholders involved in the reform movement;
- the impact of the reform on mathematical content, mathematical teaching and mathematical assessment;
- the lessons learnt from the reform movement.

In total, six research colleagues from different countries (Brazil, Japan, Ireland, Italy, Serbia and South Africa) responded to the survey, ensuring a geographical spread across four continents. Three survey responses were analysed in detail for the purpose of this chapter. The countries selected were Ireland, Serbia and South Africa. These were selected for consideration due to the commonalities in some aspects of the curriculum reform efforts as well as quite unique differences. Many of the responses from these countries also reflected in part the reactions from the other respondents, and so the authors believed a comprehensive overview could be achieved with this limited sample.

The analysis showed that while the reasons behind the reform, the nature of the reform and the stakeholders involved in the reform differed slightly, many commonalities could also be found. Many of these commonalities were described as factors affecting curriculum reform in the framework proposed by Memon (1997). It led to the authors outlining a series of lessons that could be taken from these reform

movements to inform future reform efforts, hence highlighting how we can learn from the past and use past reform movements to overcome and avoid challenges or barriers in the future.

## The Role of Values and Culture in Past Mathematics Curriculum Reforms

Chapter 6 analyses the relationship between curriculum reforms and cultural values of countries or regions. The cases considered correspond to Italy and its most recent reform movement (2001–2018), with the implementation of 'mathematical laboratories'; Serbia and the changes brought in in relation to the nature of mathematics as a teaching subject (1970–1985); and Iran since the establishment of a formal educational system in 1920.

In the first two cases – Italy and Serbia – curricula from different periods illustrate how a reform conveys not only a body of content knowledge (notions, concepts, procedures, etc.) but also a specific way of considering mathematics, or of *valuing* it. Furthermore, they show that this conception cannot only be understood as it appears in the official texts and guidelines but in the specific proposals that are made. For instance, in the most recent global curriculum reform in Italy (2001), mathematics is conceived as having two fundamental functions, an instrumental one (for understanding reality and everyday life) and a cultural one (a coherent and systematic knowledge with a robust cultural unity). These two values assigned to the subject can just be stated, and they can be part of the discourse accompanying the reform. However, as is the case here, it can also give rise to a concrete instructional proposal – the mathematics laboratory – with its specific activities, where students will live mathematics as an empirical activity linked to many other disciplines (Bartolini Bussi et al., 2018).

The case of Serbia illustrates the changes of values about what mathematics is and how it is conceived through two examples of definitions (polygons and functions) in 1970 and 1985. It also shows how general principles about mathematics – under the New Math influence or in the 'counter-reform' – cannot be understood unless one approaches the concrete activities and tools that are proposed to the students and the way these activities and tools are structured (Milinkovic, 2018).

The case of Iran enlarges the perspective. It illustrates an interesting evolution of a curriculum that has been subjected to many political and cultural influences since 1920. It also shows the (positive and negative) effects of some of the decisions made. For instance, the centrality inherited from the French political influence in the 1920s resulted in the adaptation of a single national textbook that ensured students' access to educational resources. The study presents a rich illustration of how international movements can impact on a country with a strong cultural tradition, by adopting a specific shape and creating peculiar effects (Gooya & Gholamazad, 2018).

For instance, and this is a case that has been repeated in other countries, when the New Math curriculum was imported, the traditional geometry and trigonometry were maintained. More recently, Iran's participation in TIMSS opens the way for international co-operation and dissemination of recent movements in mathematics education. The way these global trends can permeate the educational system is always subject to political decisions and cultural circumstances. The chapter illustrates how driving forces and barriers that are not directly of a mathematical nature – but rather political, economic or social – can explain many of the changes that occur within educational systems and the way these changes are concretely operated.

## The Effects of Past Reforms on the Construction of the Knowledge to Be Taught

Chapter 7 focuses on the question of the mathematical content and how it is treated and affected by past curriculum reforms. It addresses this question from a concrete perspective, the anthropological theory of the didactic, and by modelling curriculum reforms in terms of *didactic transposition processes* (Chevallard, 1985; Chevallard & Bosch, 2020). A crucial element in this perspective is the notion of the *knowledge to be taught*, which can be approximated to the notions of 'intended' or 'official' curriculum. The notion of didactic transposition points to the existence of a complex process undertaken to elaborate the knowledge to be taught, usually from a raw material that is called *scholarly knowledge*. Selecting, structuring, labelling and elaborating on the concrete mathematical activities and conceptual organisations that are proposed to be carried out by the students corresponds to the *transposition work* that is undertaken – even if not always visible – any time a curriculum reform is proposed. Its analysis helps highlight the driving forces but also the difficulties met during this endeavour.

The first case study (Wijayanti & Bosch, 2018) illustrates the complexity of the transposition work by considering a "piece of content", the notion of proportionality, and by looking at the different mathematical organisations that have composed the knowledge to be taught. An interesting phenomenon appears, showing how reforms are very much indebted to the past. It is, of course, a titanic enterprise to elaborate from scratch the knowledge to be taught for a given discipline – like mathematics – and for a whole curriculum. This was partly what the New Math reform undertook in some countries, as a result of pressure from politicians and the impulse of some mathematicians: they built up new content organisations, with new topics, new definitions, new types of tasks and exercises, new procedures, etc.

Before the New Math reforms, the knowledge to be taught was the result of centuries of construction and remained rather stable, especially in the lower educational levels. The question is about what happened after the New Math reform when educational systems applied a 'back to the basics' strategy. How was the new knowledge to be taught elaborated and by whom? Where did the knowledge resources

come from? What we find when digging into it is a patchwork of pieces of knowledge coming from different layers in time: a particular structure resulting from the New Math remains, but many elements have been taken over from the previous organisation, which suddenly appears as a newly discovered world. However, the final result is not always a coherent construction, as the underdeveloped treatment of quantities in school mathematics shows.

The second case study of the chapter (Vu-Nhu, 2018), focuses on an even smaller piece of knowledge, the notion of integral, and shows its destiny across different curriculum reforms in Vietnam before and after the reunification of the country. The phenomenon described can be seen as a simple anecdote. Still, it illustrates a rather general situation, where big decisions are made by the high authorities as if the details of the transposition work were to follow naturally. When looking at the concrete activities that students are required to carry out, and the mathematical means we propose them to do so, we find important barriers that can explain the difficulties met in the implementation of curriculum reforms and in its outcomes. As in many other situations, involving curriculum reforms, the devil is in the details.

Chapter 7 illustrates another transposition phenomenon related to the elaboration and reception of the knowledge to be taught by the teachers. In a study about the most recent (2008–2010) curriculum reform in Ireland using Memon's (1997) framework, O'Meara et al. (2018) identify "the instruction time" – hours devoted to the content to be taught – as one of the barriers hindering the implementation of the reform from the perspective of the teachers. According to the didactic transposition approach, in teaching and learning processes *didactic time* is created when new elements of the knowledge to be taught are introduced in the sequence of elements that define it. When, in reform, the sequence and its elements are totally transformed or newly built, teachers cannot easily identify the new milestones that mark the path of the learning process and show the advance of time. Curriculum evolutions necessarily materialise in changes in the knowledge to be taught that are sometimes taken too much for granted by the curriculum developers themselves and might end up creating difficulties in the very concrete activities teachers and students carry out in their classrooms.

## Driving Forces and Barriers

Theme A chapters present key elements of reforms that took place in the distant and recent past, in countries of different cultural and educational traditions, with differing degrees of influence in the time and the space, with some still being in force and others reaching their conclusion many years ago. We approach these reforms from different perspectives, trying to understand the reasons that prompted them, the strategies used to implement them, their local and sometimes external effects, the values they conveyed, and the constraints that hindered their development. The chapters point at the commonalities and specificities of these reforms, from different perspectives and using various methodological tools. General descriptions about

the historical and social situations are complemented with a detailed analysis of the mathematical content organisations and the specific conditions of implementation. The gathering of all these experiences constitutes a valuable endeavour that can be used to draw some important lessons that we expect will be useful for the planning of future mathematical reforms.

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