Sound Pressure Field Reconstruction for Ultrasound Phased Array by Linear Synthesis Scheme Optimization

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Abstract. Ultrasound phased array is a device that is usually used to provide mid air tactile sensations like three-dimensional shape haptics images by generating various specific sound fields. Forming foci for the ultrasound phased array using the linear synthesis scheme (LSS) is a straightforward technique to induce tactile feeling. The matching phase set for each focal point is calculated separately in LSS, and then they are linearly superimposed to generate multiple focal points. Due to the fact that adding an arbitrary offset to the entire phase pattern has no effect on the generated focus patterns, adjusting the offset in linear summation may result in a superior sound field. In our study, we propose that optimize the offset before linear superposition. These offsets are determined based on the number of focal points, which means it will not cause an explosive increase in computing cost with the increase of transducers. To optimize the offset of each focus pressure generated by LSS, we used a greedy algorithm with a brute-force search optimization method. The computing cost of our proposed method is dictated by the number of foci after calculating the phase sets of LSS once. We demonstrate the proposed method's optimum performance in varied numbers of foci and transducers in this study.

Keywords: Foci field · Linear synthesis scheme · Optimization

1 Introduction

Ultrasound phased array, such as airborne ultrasound tactile display (AUTD) [1], can produce various tactile sensations remotely on a human skin surface. Monnai et al. used an ultrasound focus generated by AUTD to develop a mid-air interaction system that allows users to touch a floating virtual screen with non-contact tactile feedback [2]. Moreover, the foci field has been applied to the

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formation of three-dimensional (3D) haptic images, allowing people to perceive static 3D shapes by touching them with their fingers and hands [3]. As a result, the question of how to generate a specialized sound field with stronger sound pressure has been widely studied to provide a better tactile experience.

Many methods for optimizing the foci sound field have been proposed. Long et al. optimized the transmission matrix to achieve a 3D tactile sensation like virtual objects in the air [4]. GS-PAT, which can optimize the foci field using both CPU and GPU [5]. The preceding optimization approaches can all obtain a better sound field, but the computing cost rises dramatically as the transducers increases, and the calculation becomes more difficult. As a result, we anticipate discovering a strategy that is simple to calculate, while also optimizing the sound field effectively.

One of the simplest ways to generate a foci field is a linear synthesis scheme (LSS). When designing a foci sound field with LSS, the phases set for each focus are calculated separately, and then those sets are linearly synthesized. The phase pattern that forms each focal point has a degree of freedom in terms of phase offset. Because even if the phase pattern is given an arbitrary offset, the spatial pattern of the generated sound pressure will be the same, it is vital to give an appropriate offset for each focus when linear addition is conducted. We present a greedy algorithm with brute-force search in this study for generating a stronger sound field by optimizing the offset when synthesizing the phase patterns of multiple focal points.

Suzuki el al. have proven the superiority of the greedy algorithm in optimizing individual transducer phase in the ultrasonic phased array [6]. In this paper, we applied a greedy algorithm to the offsets optimization to improve the performance of LSS.

After calculating the phases sets of each focus field based on the LSS for once, we only have to explore the optimal offsets, and add them to the sets, which means that the computing cost for optimization is linearly proportional to the number of foci. As a result, the computing cost of creating a foci sound field will be lowered even when a huge number of transducers are used. Furthermore, the pressure of the sound field generated by LSS can be strengthened. When using ultrasound phased array to provide tactile sensation, this can deliver a better experience. In our study, all the experiments are based on simulations and the final phases are normalized while the amplitude of the transducers are set to be in the range of $(0, 1]$.

2 Methods

2.1 Linear Synthesis Scheme Optimization

Firstly, we introduce how LSS works. The linear synthesis scheme, as its name implies, generates the foci field by linearly synthesizing each focus signal. Thus, we must calculate the phase set which generates each focus at first.

Let the phase set be $\boldsymbol{q}(\phi_1, ..., \phi_M)$, defined as,

$$\boldsymbol{q}(\phi_1, ..., \phi_M) = [\mathrm{e}^{-\mathrm{j}\phi_1}, ..., \mathrm{e}^{-\mathrm{j}\phi_M}]. \tag{1}$$

where, ϕ_i is the phase of i-th transducer. Assuming the sound emitted from the transducer is a spherical wave, the acoustic field $p(\boldsymbol{r})$ generated by M transducers is expressed as:

$$p(\boldsymbol{r};\boldsymbol{q}) = \sum_{i=1}^{M} \frac{1}{4\pi\left|\boldsymbol{r}-\boldsymbol{r}_i\right|} \mathrm{e}^{\mathrm{j}(k|\boldsymbol{r}-\boldsymbol{r}_i|)} q_i, \tag{2}$$

where k is a wavenumber, and $\boldsymbol{r}_i$ is the position of i-th transducer. Note that, the amplitude of the transducers are set to be 1 in the study. From Eq. (2), when creating a focus at a position $\boldsymbol{r}_f$, we should set ϕ_i as follows:

$$\phi_i = k\left|\boldsymbol{r}_f - \boldsymbol{r}_i\right|. \tag{3}$$

Here, let the phase sets of the transducers that generates the focus at $\boldsymbol{r}_f$ be $\boldsymbol{q}(\boldsymbol{r}_f)$, i.e.,

$$\boldsymbol{q}(\boldsymbol{r}_f) = [\mathrm{e}^{\mathrm{j}k|\boldsymbol{r}_f-\boldsymbol{r}_1|}, ..., \mathrm{e}^{\mathrm{j}k|\boldsymbol{r}_f-\boldsymbol{r}_M|}]. \tag{4}$$

By linearly synthesizing the phase set that generates each focus, we can generate N foci located at $\boldsymbol{r}_{f_1}, ..., \boldsymbol{r}_{f_N}$. The human sense of touch cannot distinguish a phase difference of 40 kHz vibration at the focus, and thus, there is room to optimize the phase of each focus. Therefore, the phase set $\boldsymbol{q}$ which generate N foci is represented as,

$$\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{r}_{f_1})\mathrm{e}^{\mathrm{j}o_1} + \cdots + \boldsymbol{q}(\boldsymbol{r}_{f_N})\mathrm{e}^{\mathrm{j}o_N}, \tag{5}$$

where o_n is the phase offset of n-th focus (Fig. 1).

With Eq. (5), the calculation of LSS has been done, but we have to set the amplitude in the range of $(0, 1]$, we have to do the normalization in the final:

$$\boldsymbol{q}_{\text{normal}} = \frac{\boldsymbol{q}}{\max\{|q_i|\}} \tag{6}$$

In theory, setting all the offsets to 0 is the fastest way to form a foci field, but the sound pressure may become weaker. GS-PAT uses an iterative method to optimize the offsets, and we suggest a faster method: greedy algorithm with brute-force search, to deal with the optimization.

For each focus, we use the greedy algorithm to explore the optimal offset that generates the strongest sound pressure. As a result, the calculation cost of such LSS optimization is the order of the number of foci.

2.2 Greedy Algorithm with Brute-Force Search

Greedy algorithm is an intuitive algorithm that is used in optimization problems. The algorithm breaks the problem into sub problems, and searches for the best solution for each one to arrive at the overall best solution for the entire problem. Suzuki et al. treated each transducer in the ultrasonic phased array as a separate problem, sampling from the original continuous $[0, 2\pi)$ phase space in an equal and discrete manner. In the study, we focus on the offsets of the phase set from each focus and discretize the offset from $[0, 2\pi)$ as L division. Then we calculate

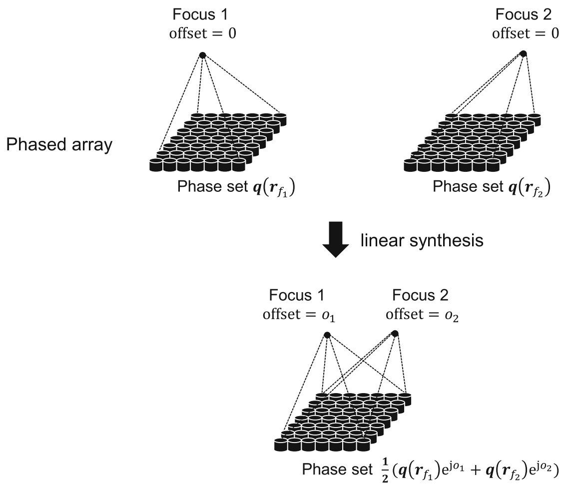


Fig. 1. Linear synthesis scheme

phase set $\boldsymbol{q}(\boldsymbol{r}_{f_n})$ of each focus and obtain $\boldsymbol{p}_n$, where $\boldsymbol{p}_n$ means the sound field vector of foci which is generated by the transducers with phase set $\boldsymbol{q}(\boldsymbol{r}_{f_n})$. Unlike Suzuki, we optimize the phase set offset of the foci in each simulation at random, which means we may receive different outcomes each time. In the loops of N foci and equal division L offsets, we explore the optimal phase offset o_n one by one. That is, we search optimal phase offset o_1 which obtained the maximum $\|\boldsymbol{p}^l\|$ on the focus position, and then keep its optimum and search next optimal offset o_2, and we continue this process for all phase offsets. The optimal phase set and sound field can be retrieved when the loop is completed. The algorithm we suggest in this study is depicted in the diagram below:

Algorithm 1. Greedy Pressure Field Reconstruction with Brute-force Search

Input: $\boldsymbol{q}_{\boldsymbol{r}_1}, ..., \boldsymbol{q}_{\boldsymbol{r}_N}$
Output: $\boldsymbol{q}$

discretize offset o as $\left\{ o^l \middle| o^l \in [0, 2\pi), l = 1, \cdots, L \right\}$
calculate $\boldsymbol{p}_n = [p(\boldsymbol{r}_{f_1}; \boldsymbol{q}(\boldsymbol{r}_{f_n})), ..., p(\boldsymbol{r}_{f_N}; \boldsymbol{q}(\boldsymbol{r}_{f_n}))]$ for $n = 1, ..., N$
Set $\boldsymbol{p}_\mathrm{t} = \boldsymbol{0}$
for $n = 1, ..., N$ **do**
    for $l = 1, ..., L$ **do**
        obtain $\boldsymbol{p}^l = \boldsymbol{p}_\mathrm{t} + \boldsymbol{p}_n \mathrm{e}^{\mathrm{j} o^l}$
    end for
    $o_n \leftarrow o^{l^*}$ s.t $l^* = \mathrm{argmax}_l \left\{ \|\boldsymbol{p}^l\| \right\}$
    $\boldsymbol{p}_\mathrm{t} \leftarrow \boldsymbol{p}_\mathrm{t} + \boldsymbol{p}_n \mathrm{e}^{\mathrm{j} o_n}$
end for
Substitute optimal offset $o_1, ..., o_N$ into Eq. (5)

3 Experiments

3.1 Outline

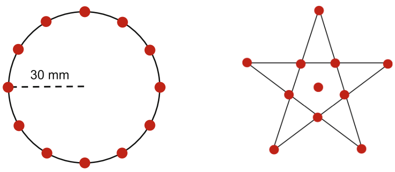
The experiments are run by a desktop computer that has an Intel(R) Core(TM) i9-9900X CPU @3.50 GHz CPU. And we run the code in C++ base on Eigen library. It should be noted that the computing cost of GS-PAT depends on the matrix calculation. Therefore, the calculation of GS-PAT would be faster by using a library or computational resources that can perform faster matrix calculations. The experiments mainly simulate the performance of each approach for generating sound fields with various foci number using a phased array of 18×14 transducers. The x-axis is the side with 18 transducers, and the y-axis is the side with 14 transducers. The number of x-axis and y-axis transducers is multiplied by K to generate a larger phased arrays, as the number of phased arrays grows, implying that the number of phased arrays will expand by K^2. We calculated the field in the plane of $x = (-50, 50)$ mm, $y = (-50, 50)$ mm at 150 mm above the center of the phased array vertically. The speed of sound is set to 340 m/s. Furthermore, the frequency of the ultrasound was set to 40 kHz, and thus the wavelength was 8.5 mm. In this study, we will evaluate our proposed method compared to the LSS without offset optimization and GS-PAT (CPU).

First, we will sequentially generate (2, 4, 6, 8, 10, 12)-foci fields on the periphery of a circle with a radius of 30 mm (Fig. 2a) based on one phased array unit, and evaluate the average sound pressure of each focus generated by the methods. Then, in three experiments, we will compare the computing cost, with the field of these experiments being the 12 foci field on the circle as shown in Fig. 2a. The first shows the computing cost of varying numbers of foci with one phased array unit with 252 transducers, whereas the second one increases the number of foci like the first one but with 16 phased array units which have 4032 transducers. The third experiment increases the number of phased array units in the situation of generating 12 foci field. The phase space of the offset in the greedy algorithm was divided into $L = 16$ equal parts and discretely sampled, the each offset of focus will be optimized in random order. To calculate the mean and standard deviation of the foci pressure, we ran ten simulations for each sound field.

Then, to evaluate the versatility of the proposed method, we also designed the other foci sound field that can be applied to the sense of tactile. A sound field of a five-pointed star similar to Inoue et al. [3] (Fig. 2b) with 11 foci based on one phased array. The outer foci of the five-pointed star are based on a concentric circle with a radius of 43 mm, the inner foci are based on a circle with a radius of 16 mm, and the center of the circle is the last point. Our proposed method, like the prior experiment, calculates each offset in random order.

3.2 Evaluations

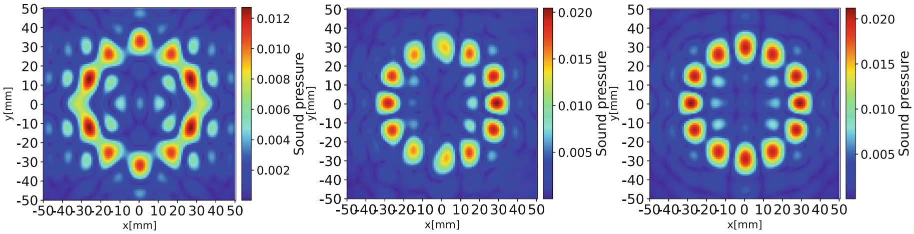
Due to space limitations, we only list the result field of the 12 foci situation in Fig. 3 (the result of LSS+Greedy is picked one of the ten simulations), and the evaluation of the average foci pressure in the other situations is listed in Fig. 4a.



(a) Foci on a circle (b) Five-point star

Fig. 2. Objective fields

The computing cost by the three methods to form a different number of foci fields with the 252 transducers in Fig. 4b, and 16 phased array units with 4032 transducers in Fig. 4c. The computing cost of increasing number of transducers while generating 12 foci is shown in Fig. 4d.



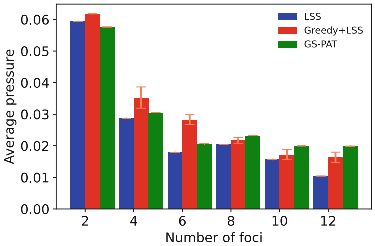
(a) LSS (b) LSS+Greedy (c) GS-PAT

Fig. 3. Average pressure of the 12 foci field on a circle

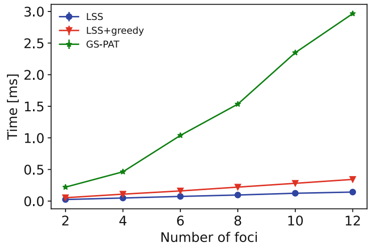
From the results, we can know Greedy algorithm has strengthened the sound field pressure generated by LSS without offset optimization, although the sound pressure is lower than GS-PAT while the foci is lager than 8, in some cases our proposed method obtained the strongest sound pressure.

In terms of computing cost, we can see that LSS without offset optimization took the least computing cost in any case. The computing cost of our proposed method is faster than GS-PAT under all the situations.

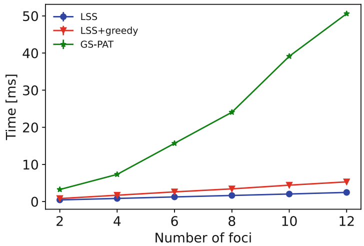
As Fig. 5 shows, LSS without offset optimization forms the field that the side lobe has other peaks as well, which may influence the tactile experience. After optimizing the field by our proposed method, it formed an 11 foci sound field clearly.



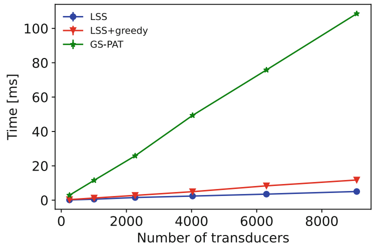
(a) Average pressure for the number of foci



(b) Computing cost for the number of foci with 252 transducers

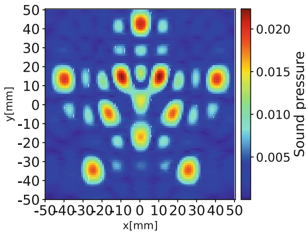


(c) Computing cost for the number of foci with 4032 transducers

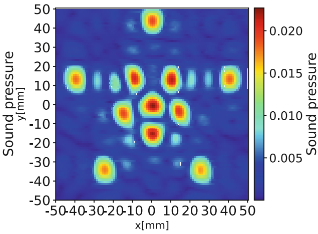


(d) Computing cost for the number of transducers

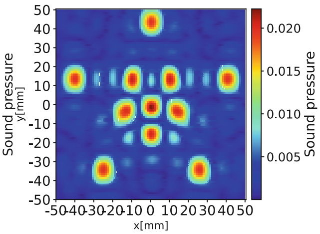
Fig. 4. Evaluation of computing cost



(a) LSS



(b) LSS+Greedy



(c) GS-PAT

Fig. 5. Fields of five-point star

4 Conclusion

To use the ultrasonic phased array to generate a foci sound field with stronger sound pressure to provide a tactile experience in a shorter time, we applied greedy algorithms with brute-force search to optimize the foci obtained by LSS. The solution we proposed has increased the sound pressure while also optimizing the side lobe. Even the computing cost is larger than LSS without offset optimization, our proposed method will not cause an explosive increase in computational cost with the increase of transducers. Furthermore, we will use the proposed method in the actual world to detect and evaluate pressure using the sensor. Moreover, we anticipate that greedy algorithms with brute-force search will be able to improve the performance of not just LSS, but also approaches such as GS-PAT. As a result, we will investigate the improvement of applying the greedy algorithm to other existing methods in the future.

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