Why Tontines? Why Now?

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In this chapter I provide some background on the reasons (I think) a traditional fund company might want to introduce a modern tontine as it relates to the unique challenges that people face managing their financial affairs towards the end of the human lifecycle. I also touch upon the difficulty retirees have in figuring out how to decumulate wealth and explain the difference between guaranteed income for life and the value of mortality and longevity credits.

1.1   Retirement vs. Decumulation

Australians retire with millions of dollars. It's not that Australia is necessarily wealthier than any other country, although they certainly rank high on a per capita basis. Rather, they happen to reach the traditional retirement age with millions of dollars in their retirement savings accounts. The source of (and credit for) their wealth is the Australian government who forces workers to save close to 10% of their salary in an investment account, and its employers mostly who are contributing to that pot. To put it in very simple terms, if your quoted salary is 100,000 AUD per year, your employer will guarantee and pay another 10,000 AUD which flows into your *super* (short for superannuation) *pot* as it's called. So, after a few decades of being forced to save that much money every year, and if the money is invested at a reasonable rate of return, it's not surprising it accumulates to millions of dollars at retirement. No other country has such a widespread system of forced retirement savings, also known as mandatory Defined Contribution (DC) Individual Account (IA) plans.

The problem with all these millions of AUD is that retiring Australians face a huge dilemma of what to do with all this money. Now sure that sounds like a super problem to have, but it's a scary one when you are looking at large sums that must finance your golden years of unknown length and duration. Australians can continue to invest the funds in the many different investment products they used during the

accumulation phase, or withdraw their money and spend it slowly, or they can yank it out and buy a sailboat, which some do. Australians have lot of choices to make, with complex income-tax and old-age pension implications, which can be rather paralyzing and often leads to some very peculiar outcomes.

The Australian scheme has been in place for almost three decades now and has offered plenty of time to gauge how typical retirees behave—and what they actually do with their accounts—as they age and progress thru the lifecycle. One very large employer who managed a very large *super* in Brisbane, a lovely city in Queensland on the east coast of Australia, has carefully tracked the financial behaviour of tens of thousands of retirees during the last thirty years. Needless to say, many of the people who retired in their late 60s and early 70s are no longer alive three decades later, in which case the money in those accounts are transferred and bequeathed to surviving spouses, children and beneficiaries. But, after digging into all that spending and investing data over three decades of retirement, researchers in Brisbane noticed something very peculiar—and is an insight at the core of this book.

The Brisbane "discovery"—which is what I'll call this, with a shout out to Brnic Van Wyk was that on *average* the amount of money left in people's retirement account when they died was *equal* to the original sum they had started with when they retired decades before. Members ended their retirement journey with an average balance equivalent to when they started the journey. If they began with a million dollars at age 65, they ended with a million dollars. If they only had half a million in their pot when they exited the labour force, they left this world with half a million dollars, etc.

Now, to be very careful, the Brisbane "discovery" was a loosely defined average and there were many exceptions to this result, but the behavioural implications were even more interesting. Remember that the pension super pot wasn't sitting under their mattress or deposited in a bank account earning little interest income. The money was allocated to stocks (shares), bonds and many other investment asset classes over the 10, 20 or even 30 years of retirement. They had proper investment portfolios much like their brethren who were still accumulating. So, these account values fluctuated over time, bobbing up and down with markets and interest rates. They might have been tilted a bit more conservatively, but these pots earned dividends, interest and realized capital gains over time, which means that they increased and decreased from day to day and year to year. But again on average retirees adjusted and fine-tuned withdrawals and spending, so that the balance of the pension pot followed a rather flat trajectory over the long-run. How exactly? Well, if the pot grew in one year, the owner spent a bit more. If the pot shrunk, the owner would cut back and perhaps have one less "shrimp on the barbie", to overuse the Australian phrase. In economic terms, this might help smooth wealth but not consumption.

*Australians had **retired**,*
*but they didn't **decumulate**.*

Decumulation is a relatively recent word, according to the Merriam Webster dictionary and is a noun defined as the *"disposal of something accumulated."* The

Objective: Decumulation vs. Retirement Income

1. Wealth over Time
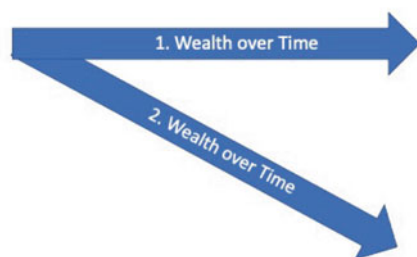
2. Wealth over Time

Fig. 1.1 What is your plan for the end of the life cycle?

word retirement is often used interchangeably with the word decumulation, but I hope you can see now that those are two very different terms (see Fig. 1.1). There are many experts in the field of economics, sociology, psychology, medicine and gerontology that study—and also give advice—on retirement. The former is about lifestyle choices, withdrawing from the labour force, spending quality time, etc. It's a massive field, well beyond the expertise of one person, one book or one department.

In contrast to that very large area, I am interested in decumulation, how people are doing it, how it should be done and whether there are things that governments and industry can do to make decumulation more efficient. And, to get to my main point, this book is about creating a new category of products—inspired by a very old product—that will help people do a much better and more efficient job of decumulation. That product is called a *Modern Tontine*. I claim that if modern tontines were widely available, it might be easier for Australians (and many others around the world) to both retire and decumulate. This book will explain how they work and how to build one. And, by the end of the book the hope and expectation is that you will see how this might help solve the Brisbane problem.

I should note that Australians aren't the only group who need help with decumulation, although they do provide an extreme and current example. In the USA, researchers have documented a similar albeit more nuanced phenomena, and one of the leading economists who has worked to uncover the drivers of decumulation is Professor James Poterba at MIT, with various colleagues. If I can quote from their article in the *Journal of Public Economics*, directly: "...The relatively modest age-related changes in wealth...suggest that the distribution of wealth near the end of life may be largely determined by wealth at age 65." Source: Poterba et al. [24]

Retirement and decumulation should be treated as distinct domains of activity and expertise. Technically speaking decumulation is an extraordinarily complex mathematical optimization problem for which insurance, risk management and stochastic control is the proper apparatus and modelling lens. Consumers who want (good, reliable) advice on this matter will have to pay more than they did for

simple, easy, passive accumulation advice. To be clear, retirement is a slow ongoing progression in which people gradually withdraw from the paid labour force, often involuntarily and abruptly, with many non-financial externalities.

Intermediaries in the financial services industry must be careful not to veer from the technical domain of decumulation assistance—also encompassing the timing of retirement benefits such as Social Security—into the nebulous region of retirement planning. If I may digress just a little bit from the main objective of this chapter, I would suggest that eager 30-year-old advisors, brokers and insurance agents with little in the way of life experiences or financial assets (to be very blunt) should refrain from counselling financially successful middle-aged couples in their 50s, 60s and 70s, on exactly when they can afford to stop working, withdraw from the labour force and/or disengage from their commercial network. I suggest they leave that sort of advice to experienced gerontologists, social workers, psychologists and perhaps even family members who are in the best position to assess non-financial externalities. In sum, I would encourage my readership here who are interested in decumulation planning to learn more about the *modern tontine.*

1.2   Annuity Benefits: Credits vs. Insurance

I'm stumbling into a growing cloud of confusion creeping into the dialogue around guaranteed lifetime income. This fog is pervasive and thick within the neighbourhood of rationales and reasons offered to retirees as to why they might want to include annuities in their aging portfolios. Generally speaking these well-intentioned conversations centre around the longevity risk that is associated with living an unexpectedly long time, or the financial cost of becoming a centenarian and the benefit of pooling resources with a large group of similar retirees. Indeed, if only one or two people from a starting group of 20 reach the age of 100, then if everyone collectively combines a portion of their financial portfolio to support the few who are lucky enough to beat the odds and reach advanced ages, it will be cheaper for the entire group, etc. I have no quibbles with this line of reasoning and in fact have used most of these tropes myself. But to be clear and quickly get to the essence of this essay, there really are two different things going on within the life annuity story: credits and insurance. As an industry and as a community we must agree not to conflate them.

One aspect of the annuity story is the financial benefit of risk pooling, and the other is the insurance benefit and comfort from having a guaranteed income that you can't outlive. Again, those are two quite distinct features. And, right now I'm growing in favour of the former (credits) over the latter (insurance), which I tried to illustrate graphically in Fig. 1.2. This all might sound rather jumbled and theoretical, so allow me to elaborate with a statement that some readers might find bizarre. If you are 75 years old with $100,000 in your retirement account and would like to **guarantee** a protected annual income for the rest of your life, there is absolutely no need to purchase a life annuity. There are other options.

This might sound like something odd for an *annuity advocate* to say. But the fact is that I can assure you that if you politely ask a non-insurance company investment-bank or your favourite broker-dealer, for example, they can grab some inventory and design a lovely portfolio of zero-coupon strip bonds that will do the job. That collection of bonds will generate $4,000 per year for the rest of your life, even if you reach the grand old age of 115. Ok, BDs need to eat too, so they may not do it for $100,000, but I'm sure that a lump-sum of $1,000,000 will pique their interest and in exchange you will get $40,000 per year. Scale it up and they will come.

Moreover and with these strips, if you don't make it all the way to the astonishing age of 115, they will continue to send those $4,000 (or $40K) to your spouse, children or favourite charity until the date you would have reached 115, if you had been alive. This collection of strips would be completely liquid, tradeable and fully reversable, although subject to the vagaries of bond market rates. For those readers who dream of numbers, I have assumed a conservative, safe and constant 2.5% discount rate across the entire yield curve, which isn't entirely unreasonable in this environment. Think of the 30-year US treasury rate as a proxy, perhaps with a smidgen of corporate credit risk.

Stated technically, the present value of the $4,000 annual payments, for the 40 years between age your current 75 and your maximum age 115, is exactly equal to $100,000 when discounted at 2.5%. Yes, those numbers and ages were deliberately selected so my numerical example rhymes with the infamous 4% rule of retirement planning but has absolutely nothing to do with it. Now, if you are still with me—and perhaps have been trained by a good annuity wholesaler—I'm sure you must be thinking (or even yelling) "Moshe, but what if you live beyond age 115, eh? You will run out of money!"

Touché. Let's unpack that common knee-jerk reaction to a non-insurance solutions for a moment. To start with, the probability of becoming a supercentenarian—that is reaching age 110—is ridiculously and unquantifiably low. There are only about 30 of them (verified) in the USA, out of a population of 330,000,000. The chances of reaching age 115, remember that is when your strips run out, are even lower.

Up until the summer of 2021, the oldest living man in the world was Emilio Flores, who lived in Puerto Rico and died at the age of 112 years old. That's three years short of the terminal strip. And, if you do happen to be the one in a 100 million (or perhaps billion) that reaches age 115, I suspect you will have other things on your murky mind. Personally and post-covid, there is a very long list of hazards that worries me more than beating Emilio's record. Alas, some might argue that I'm neglecting medical breakthroughs and the risk we become a nation of Emilios. However, I'm more of a mortality compression-ist than extension-ist, which I'll explain in another essay. In English, you won't live to 115.

More importantly, nobody really "runs out of money" in retirement in the twenty-first century. That is plain utter fear mongering nonsense. With national social security programs in all developed countries, all Emilios will continue to receive some income for as long as they live even if they have completely emptied every piggy bank on their personal balance sheet. In fact, with tax-based means-testing you might get more benefits if you actually do empty your bank accounts.

Ok, back to my prior claim, if you want a guaranteed (liquid, reversable, bequeathable) income for the rest of your life, you can exchange your $100,000 for a bunch of strip bonds and voila, you have created a protected pension plan. My point here is that the primary objective isn't a guaranteed lifetime of income—which anyone can create with a simple discount brokerage account and a DIY instruction manual.

The goal is to get the HIGHEST possible income
and at the LOWEST possible cost.

Here we go. Transferring the above-noted $100,000 into an insurance company sold income annuity would result in a guaranteed income of $9,000 per year, which is $5,000 more per year, even if they cut-you-off (and forget to send you further payments) at age 115. That more-than-double number assumes the insurance company uses the exact same 2.5% interest rate to price their products, which they don't. In fact, if I do the same simple math with a valuation rate of 3.5% instead of 2.5%, the $100,000 would generate $10,000 per year at age 75, with the income annuity. Back to my favourite investment-bank or broker-dealer, they would need to price strips at north of 8% to get me that sort of income, which they obviously can't do unless the bonds are floated by some DDD country that will default well before I need dentures.

The reason for this rather magical jump to $9,000 from a mere $4,000 is that via the income annuity I have pooled my resources with many other similar 75-year-olds but have given up the assets in the event of early-death in exchange for a subsidy to those living longer, etc. If you are willing to forfeit the money when the longevity coin falls on tails, then you can benefit from heads, etc. If you're reading to this point, you know the drill.

To repeat, the motivation and rationale for the life annuity is *not* to necessarily generate a guaranteed lifetime income to some ridiculous age. Again, I can do that with simple discount bonds. Nor should the rationale be driven by the fear of running out of money in retirement. We don't do that to people. Rather, the legitimate concern retirees have is that their accustomed standard of living might be forcefully and involuntarily reduced if the markets don't cooperate, and/or they live longer than anticipated. That can be mitigated by pooling resources and benefiting from mortality & longevity credits that accrue to those who are willing to share with their neighbours. Moreover, those credits will be more valuable in states-of-nature in which markets are performing poorly and the rest of my investments have taken a tumble. The implicit 8% return from a fixed-income product is the magic of longevity credits, a term that sounds better than mortality credits.

This distinction between longevity credits versus longevity insurance opens the door for a universe of pooling products that don't necessarily guarantee (or

even offer) to pay income for the rest of your life or guarantee anything for that matter. One can harvest mortality & longevity credits without requiring a rest-of-life horizon. For example, imagine a pooling arrangement that lasts for 25 or 30 years in which survivors inherit the investment assets of the deceased, thus acquiring mortality & longevity credits, but the entire fund is designed to be wound up when everyone reaches some predetermined age. Actuaries will recognize this as a temporary life annuity. That fund might not promise longevity insurance per se, or income for the rest of your life, or money that you can't outlive. But it would certainly include generous longevity credits. That is the *modern tontine.*

1.3 How Does This Book Differ from All the Other Books?

For starters, this isn't a book about seventeenth century financial or insurance history nor is it a collection of theorems and proofs about the mathematical properties of tontines. I have written both of those already, cited as Milevsky [20] and Milevsky and Salisbury [19], so that certainly isn't my intention. Indeed, I plan to stay very focused on the present day twenty-first century and will be using an absolutely minimal amount of mathematics. In that sense, I would describe what follows in the next 8–10 chapters as an instruction manual on how to build a modern tontine for decumulation. Those instructions or directions will be written in a language called **R**, which enables the users to copy-and-paste algorithms and generate their own results. Readers will be able to quickly and easily reproduce every result, number or figure using their own parameters and assumptions.

The growing media and commercial interest in *modern tontines* over the last decade has convinced me that this might be a good time to write a cook-book or instruction manual, accessible with a minimal level of technical background. If you can download R (which is free) and write some basic code (which everyone should), then you too can build your own *modern tontine.*

With the above in mind, the ideal audience for this book is practitioners or advanced students looking for a thesis project. The first group are financial engineers working for asset managers, financial services companies and even start-ups who are building *modern tontine* funds and would like to quickly and easily stress test various design and behavioural features. Perhaps the reader has been tasked with building or coding-up the expected payout from a *modern tontine.* What if more or less people die? What if returns are higher or lower? What if interest rates move up or down? What if investors surrender earlier or later? These and other questions will likely be on the mind of tontine sponsors, and the algorithms and R-scripts provided in this book will help shed light and perhaps even answer those questions. For those readers who are familiar with my prior book on retirement income recipes, cited as Milevsky [21], this work can be viewed as a sequel.

A secondary audience for this book is students—both graduate and undergraduates in finance, economics or even business—who are interested in quickly getting up to speed on how a *modern tontine* works in practice. Notice that I didn't mention insurance actuaries, who tend to use their own language, notation and framework for managing risk. In other words, if you know nothing about mortality

and longevity tables but realize that some minimal knowledge of those topics is absolutely necessary for properly thinking about *modern tontines*, then this book is for you. This book is therefore not meant as a comprehensive review of the scholarly literature on tontines. My objective isn't for this book to be *cited* by other scholars, but rather to be used by practitioners. And, the algorithms included with this book— albeit not very sophisticated or unique—can be considered my modest contribution to a free and open-source movement for *modern tontines*.

1.4   Outline and Plan

1. **Introduction and Motivation:** This Chapter You are Reading.
2. **Actuarial Background:** A Crash Course in Mathematics.
3. **Core Simulation in R:** Introducing the Secret Sauce.
4. **Statistical Risk Management:** What Drives the Outcomes?
5. **Introducing Death Benefits:** Promises without Guarantees.
6. **Richer Returns & Models:** More Realistic Investments.
7. **Mortality of the Future:** Benjamin Gompertz is Old.
8. **Running a Business:** Theory vs. Practice.
9. **Solutions and Tips:** End-of-chapter Questions Solved (with Joe Bisk).
10. **Conclusion:** Final Thoughts.

1.5   Tontine Literature: What (and Who) Else to Read

The traditional approach to writing a review of the literature is to create a (very) long list of articles that are pertinent to the topic being discussed and to provide a brief overview and summary of those articles. The objective of that classical activity is to embed the author's own work into the *literature*, offer credit where it is due and then progress to carving-out the newer contribution by the author. But within the context of this particular book, I believe such a list of articles might be unnecessary and redundant. This book really isn't intended to be a contribution to the scholarly literature and it certainly isn't a tontine history book, both of which have been attempted elsewhere and by others.

Rather, I am assuming that the reader(s) of this book—hopefully more than singular—are interested in building a *modern tontine* and as part of that process would like to learn who *else* they should be learning from. So, what follows is a list of writers, researchers and scholars who (I know) have spent much time and energy *thinking about* the design of *modern tontine* products. They have worked-on and contributed to the **technical** actuarial development of instruments with similar aims and objectives, but perhaps with different names. I list them here in alphabetical order and highlight one or two of their research papers or monographs. My intent here is not to be exhaustive or even to highlight the research paper they would most likely recommend. Rather my point with this list is to encourage you to google, carefully read and dig deeper for their latest *tontine thinking*.

1.5.1 A. Chen

In a series of articles with a number of her students and co-authors, the prolific
Ann Chen [7] introduces many innovations into the basic tontine design, including
options on tontines, tontines that allow for bequest and legacy, tontines that are
linked to health status and optimal allocations to a mixtures of tontines and
annuities. Under her tutelage, *Ulm University* in Germany has become something of
a modern factory for tontine research, and more generally strategies for managing
personal longevity risk.

1.5.2 J.M. Bravo

In an article cited as Bravo [6], this Portuguese professor of economics examines
something (he calls) *participating longevity-linked life annuities* (PLLA), in which
benefits are updated periodically based on the observed survival experience of
a given underlying population and the performance of an underlying investment
portfolio. As you might sense, this is a type of *modern tontine*. Likewise, in a series
of articles with well-known global pension expert R. Holzmann and various co-
authors they address how to incorporate longevity heterogeneity into the design of
modern retirement plans.

1.5.3 C. Donnelly

In a series of papers, including a recent one cited as Bernhardt and Donnelly [4],
the Scottish actuary and professor of mathematics has conducted and published a
number of research projects funded by the *UK Institute and Faculty of Actuaries* on
the optimal way for individuals to manage personal longevity risk. In particular, she
has helped further *tontine thinking* by emphasizing the costs embedded in traditional
(capital intensive) annuities and the point at which consumers are willing to "risk"
their retirement on *modern tontines* to avoid those fees.

1.5.4 R.K. Fullmer

An American practitioner who is actively engaged in designing and engineering the
next generation of *modern tontines*, the monograph cited as Fullmer [12] is a *CFA
Institute* publication that provides a detailed and practitioner-accessible explanation
of how tontines actually work. See his work with M.J. Sabin on tontine bond ladders,
and his work with law professor J.B. Forman on how *tontine thinking* can be legally
embedded inside modern pension funds.

1.5.5   M. Guillen

A well-known and prolific researcher in statistics and actuarial science based in Barcelona, Spain, M. Guillen has recently focused her attention on tontines and pooled annuity funds, which recall is yet another one of the many names for modern tontines. In a paper with J.P. Nielsen, cited as Bräutigam et al. [5] she compares pooled annuity overlay funds based on actuarial fairness, to equitable retirement income tontines and notes that "the market would appear to be ready for such innovations" and I wholeheartedly agree.

1.5.6   S. Haberman

One of the original deans of actuarial science research based in London, and focused on longevity risk management, S. Haberman together with co-authors, cited as Denuit et al. [8] suggested a number of innovative ways in which *modern* annuities can be designed, by sharing and pooling longevity risk. He has supervised many graduate students at Bayes Business School (formerly Cass Business School) at City University of London, which is yet another power-house of research in this area.

1.5.7   J. Piggott

The director of the Australian Research Council Centre of Excellence in Population Ageing at the University of New South Wales, Professor Piggott was one of the first classically trained economists to direct his attention to the design of (better) longevity risk pooling arrangements. In particular, the article cited as Piggott et al. [25] is the earliest to suggest how to build a pooled annuity fund which is another type of *modern tontine.* They called this arrangement *group self-annuitization*, explained how to mix cohorts of different ages into one large pool and were able to prove the existence and uniqueness of their design. That article has formed the basis of many follow-up studies and has been widely cited in this literature.

1.5.8   E. Pittacco

In a series of articles with his co-author A. Olivieri, cited as Olivieri and Pittacco [23], this Italian demographer, actuary and expert of longevity dynamics examines the many different ways in which longevity risk can be shared. His extensive research work also discusses capital requirements in the presence of guarantees, as well as the (complicated) problem of forecasting future mortality.

1.5.9 R. Rogalla

A researcher who is interested in the design of annuities, as well as the management of longevity risk over the lifecycle. In a series of articles, including the one cited as Gemmo et al. [14], their results indicate that early on in retirement, a tontine is an attractive investment option if the tontine funds are invested in a risky asset, which is precisely the approach taken in the next few chapters. He is based at the Maurice R. Greenberg School of Risk Management, Insurance and Actuarial Science (SRM) at St. John's University in New York.

1.5.10 T.S. Salisbury

A noted Canadian probabilist and mathematician based at York University, T.S. Salisbury developed an interest in actuarial science, annuities and then tontines while supervising graduate students (including the author of this book). In a series of research articles beginning with Milevsky and Salisbury [19], he helped rekindle a discussion about retirement income product that has been long neglected, and then leveraged economic theory and tools from mathematical finance to design the next generation of tontine annuities.

1.5.11 M.J. Sabin

An American entrepreneur M.J. Sabin (2010) proposed what he called a *fair tontine annuity* which is an arrangement that provides a lifetime payment stream whose expected present value matches that of a fair annuity. The article itself was never peer-reviewed and published in an academic journal (unfortunately) but is available as an SSRN research paper and has been quite influential within the literature on modern tontines. Although his original work is a based on a one-period model and static (known) mortality rates, his idea can easily be extended to multiple time periods.

1.5.12 M. Sherris

A prolific Australian mathematician who has supervised many students and projects on longevity risk. In particular, the article, cited as Qiao and Sherris [26], addresses challenges for designing group pooled schemes that include decreasing average payments when mortality improves significantly, decreasing numbers in the pool at older ages, and the impact of dependence from systematic mortality improvements across different ages of members in the pool. This article uses a multiple-factor stochastic mortality model in a simulation study to show how pooling can be made

more effective and to quantify the limitations of these pooling schemes arising from the impact of systematic longevity risk.

1.5.13 M.J. Stamos

In one of the earliest papers in this genre, cited as Stamos [28], and then in a series of papers under the supervision of R. Maurer at Goethe University in Germany, Stamos merged ideas from asset allocation and personal longevity risk management. He noted that pooled annuity funds insure very effectively against longevity risk even if their pool size is small. Furthermore, he showed (again, rather early on) that only very risk averse investors would prefer to pay a risk premium (i.e. higher costs) to access conventional life annuities that completely eliminate longevity risk.

1.5.14 J.H. Weinert

In research work that grew out of his Ph.D. dissertation in Germany under the supervision of his professor H. Grundel, and then published as Weinert and Grundel [29], he uses insights from behavioural economics and Prospect Theory to design better tontines. As part of his thesis work he conducted a comprehensive survey of *modern tontine* products that are available around the world in various guises and forms.

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