9

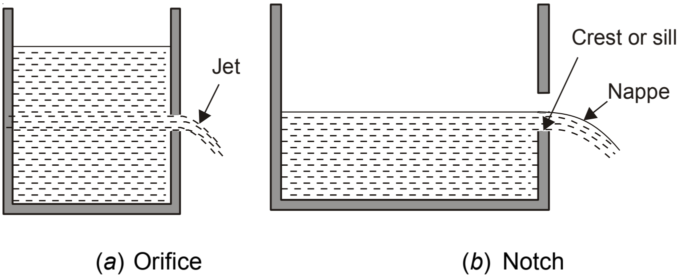
Notches and Weirs

9.1 INTRODUCTION

Notches and Weirs are used for measuring the rate of flow of liquid through open channel. Both are having same function *i.e.*, to measure the rate of flow. Difference between the two is their size and the quantity of discharge. **Notches** are small in size, hence used to measure small discharge through tank in laboratory. **Weirs** are big in size, thus measure large discharge in dams, rivers, *etc*.

A notch may be defined as an opening provided in the side of tank (or channel) in such a way that the liquid surface in the tank or channel is below the top edge of the opening. Normally, there is no need of upper edge of a notch. Liquid flows over a notch or weir while it passes through an orifice. The stream of liquid issuing from the orifice is called a jet while the stream of liquid issuing from a notch or weir is called a **nappe** or **vein**. The upper surface of the notch or weir over which the liquid flows is called the **crest** or **sill** of the weir.

9.2 DIFFERENCE BETWEEN NOTCH AND ORIFICE



(*a*) Orifice (*b*) Notch

Fig. 9.1 Orifice and notch.

S. Kumar, *Fluid Mechanics (Vol. 2)*,
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S.No.	Orifice	Notch
1.	An orifice is an opening provided in the side or bottom of the tank or vessel in a such a way that the level of liquid in the tank is above the top edge of the opening as shown in Fig. 9.1 (*a*).	A notch is an opening provided in the side of tank in a such a way that the level of liquid in the tank is below the top edge of the opening as shown in Fig. 9.1 (*b*).
2.	It can be provided on the side or at the bottom of a tank.	It can only be provided on the side of the tank.
3.	The liquid flowing out of orifice is called a jet.	The liquid flowing out over a notch is called nappe or vein.
4.	An orifice is used to measure discharge through the tank or vessel which contains liquid.	A notch is used to measure discharge through small open channels.
5.	In an orifice, the free surface of liquid is always above the top edge of orifice.	In a notch, the free surface of flowing liquid is always below the top edge of the notch.

9.3 DIFFERENCE BETWEEN A NOTCH AND A WEIR

The following are main difference between a notch and a weir.

S.No.	Notch	Weir
1.	A notch is an opening made of metallic plate.	A weir is made of masonary or concrete.
2.	A notch is small in size, hence used to large discharge.	A weir is big in size, hence used to measure large discharge.
3.	The edges of a notch are thin and sharp	The edges of a weir are much wider as compared to notch.
4.	It is used in tanks or small open channels in laboratory for the purpose of experimental flow.	It is used in an ancient or spillway, canal or river for the purpose of storing and regulating the flow.

9.4 CLASSIFICATION OF NOTCHES AND WEIRS

Notches may be classified:

1. According to the shape of the opening:
   - (*i*) Rectangular notch
   - (*ii*) Triangular notch or V-notch
   - (*iii*) Trapezoidal notch
   - (*iv*) Stepped notch.
2. According to the effect of the sides on the nappe:
   - (*i*) Notch with end contractions
   - (*ii*) Notch without end contractions or suppressed notch.

Weirs may be classified:

1. According to the shape of the opening:
   - (*i*) Rectangular weir
   - (*ii*) Triangular weir
   - (*iii*) Trapezoidal weir.
2. According to the effect of the sides of the nappe.
   - (*i*) Weir with end contractions
   - (*ii*) Weir without end contractions or suppressed weir.
3. According to the shape of the crest:
   - (*i*) Sharp-crested weir
   - (*ii*) Broad-crested weir
   - (*iii*) Narrow-crested weir
   - (*iv*) Ogee-shaped weir

9.5 DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

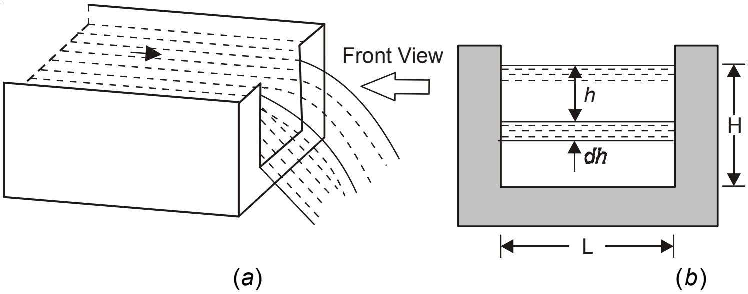


Fig. 9.2: Rectangular notch or weir.

Consider a rectangular weir from which the liquid is flowing as shown in Fig. 9.2.

Let L = length of the weir,

H = head of liquid over the crest of the weir.

Let us consider a horizontal strip of liquid of thickness dh at the depth h from the free surface of liquid as shown in Fig. 16.2 (*b*).

∴ Area of strip: $dA = Ldh$

and the theoretical velocity of liquid flowing through the elemental strip = $\sqrt{2gh}$

∴ Theoretical discharge through elemental strip:

$$dQ_{\text{th}} = \text{area of strip} \times \text{theoretical velocity}$$

$$= Ldh \times \sqrt{2gh} \qquad \ldots(9.5.1)$$

The total theoretical discharge, over the weir, may be determined by integrating the above Eq. (16.5.1) within the limits 0 and H, we get

$$Q_{\text{th}} = \int_0^H Ldh \times \sqrt{2gh}$$

$$= L\sqrt{2g}\int_0^H \sqrt{h}\,.dh = L\sqrt{2g}\left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3}L\sqrt{2g}\left[H^{3/2} - 0\right]$$

$$Q_{\text{th}} = \frac{2}{3}\sqrt{2g}LH^{3/2} = \frac{2}{3}\sqrt{2g}LH^{1.5}$$

Actual discharge: $Q = C_d\, Q_{\text{th}}$

where C_d = coefficient of discharge.

$$\boldsymbol{Q = \frac{2}{3}C_d\sqrt{2g}LH^{1.5}}$$

Note: The expression for discharge over a rectangular notch or weir is the same.

$$\boldsymbol{Q = \frac{2}{3}C_d\sqrt{2g}LH^{1.5}}$$

9.6 TRIANGULAR NOTCH OR V-NOTCH

Let H = head of liquid over the apex of the notch

θ = angle of notch

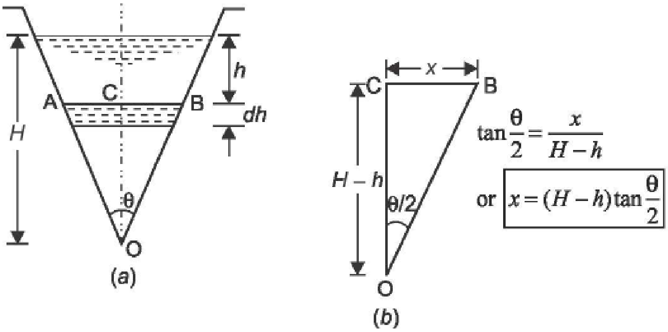


Fig. 9.3: Triangular notch.

Let us consider a horizontal elemental strip of liquid of thickness *dh* at the depth *h* from the free surface of liquid as shown in Fig. 9.3 (a).

Width of the elemental strip:

$$AB = AC + CB = 2\ CB \qquad \because AC = CB$$

$$= 2x = 2(H - h)\tan\frac{\theta}{2}$$

∴ Area of elemental strip:

$$dA = AB \times dh = 2(H - h)\tan\frac{\theta}{2} \times dh$$

$$= 2(H - h)\,dh\,\tan\frac{\theta}{2}$$

and the theoretical velocity of liquid flowing through the elemental strip = $\sqrt{2gh}$

$\therefore$ Theoretical discharge through elemental strip:

$$dQ_{th} = \text{area of strip} \times \text{theoretical velocity}$$

$$= 2(H - h)\, dh \tan\frac{\theta}{2} \times \sqrt{2gh}$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}(H - h)\sqrt{h}\, dh$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}(Hh^{1/2} - h^{3/2})\,dh \qquad \text{...(9.6.1)}$$

The total theoretical discharge, over the notch, may be determined by integrating the above Eq. (9.6.1) with the limits 0 and H, we get

$$Q_{\text{th}} = \int_0^H 2\sqrt{2g}\tan\frac{\theta}{2}(Hh^{1/2} - h^{3/2})\,dh$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}H^{5/2} = \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}H^{2.5}$$

Actual discharge: $Q = C_d\, Q_{\text{th}}$

where C_d = coefficient of discharge.

$$\mathbf{Q = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}H^{2.5}} \qquad \text{...(9.6.2)}$$

The expression for discharge in above Eq. (9.6.2) over a triangular notch or weir is the same.

$$\boldsymbol{Q_{\triangledown} = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}H^{2.5}}$$

Problem 9.1: Find the discharge over a rectangular notch of 4 m long when the head of liquid over the crest in 0.5 m. Take $C_d = 0.60$.

Solution: Given data for a rectangular notch:

Length of notch: $L = 4$ m

Head of liquid over the crest:

$H = 0.5$ m

$C_d = 0.60$

We know that the discharge over a rectangular notch:

$$Q_{\square} = \frac{2}{3}C_d\sqrt{2g}LH^{1.5} = \frac{2}{3}\times 0.60\times\sqrt{2\times 9.81}\times 4\times(0.5)^{1.5}$$

$$= \mathbf{2.50\ m^3/s}$$

Problem 9.2: The discharge over a rectangular notch is 120 litre/s when the water level is 300 mm above the sill. Find the length of a notch if the coefficient of discharge is 0.62.

Solution: Given data for a rectangular notch:

Discharge: $Q = 120$ litre/s $= 0.12$ m^3/s

$H = 300$ mm $= 0.3$ m

$L = ?$

$C_d = 0.62$

We know that $$Q_{\square} = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$

$$0.12 = \frac{2}{3} \times 0.60 \times \sqrt{2 \times 9.81} \times L \times (0.3)^{1.5}$$

or $L = 0.39888$ m $=$ **398.88 mm.**

Problem 9.3: A rectangular weir of 5 m long is used to measure the rate of flow of water. The head of water over the weir is 800 mm. If the available height of waterfall is 22 m, find the power of the waterfall. Take $C_d = 0.60$.

Solution: Given data for rectangular weir:

$L = 5$ m

$H = 800$ mm $= 0.8$ m

Available height of water fall:

$H_1 = 22$ m

$C_d = 0.60$

Discharge: $$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5} = \frac{2}{3} \times 0.60 \times \sqrt{2 \times 9.81} \times 5 \times (0.8)^{1.5}$$

$= 6.339$ m^3/s

Power of the waterfall:

$P = \rho\, Q\, g\, H_1$

$= 1000 \times 6.339 \times 9.81 \times 22$ W $= 1368082.9$ W

$=$ **1.368 MW.**

Problem 9.4: The maximum flow through a rectangular channel 1.5 m deep and 2 m wide is 1.5 m^3/s. It is proposed to install a full width, sharp-edged rectangular weir across the channel to measure the flow. Find the maximum height at which the crest of the weir must be placed in order that water may not overflow the sides of the channel. Take $C_d = 0.62$.

Solution: Given data for rectangular channel and rectangular weir:

Depth of flow in the channel:

$Z = 1.5$ m

Width of the channel = length of weir

i.e., $L = 2$ m

Discharge through channel:

$$Q = 1.5 \text{ m}^3/\text{s}$$

$$C_d = 0.62$$

Discharge: $$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$

$$1.5 = \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \times 2 \times H^{1.5}$$

or $$H^{1.5} = 0.40964$$

or $$H = 0.5516 \text{ m}$$

Let h = height of the crest of the weir above the bottom of the channel.

$\therefore$ $$Z = H + h$$

$$1.5 = 0.5516 + h$$

or $$h = \mathbf{0.9484\ m.}$$

Problem 9.5: The daily rainfall over a catchment area was found to 2.5×10^8 litre. It was observed that 25% of the rain water is lost due to evaporization and the remaining reaches the reservoir which passes over a rectangular weir: Find the length of the weir, if water over the weir will never rise more than 500 mm. Take coefficient of discharge as 0.62.

Solution: Given data:

2.5×10^8 litre rainfall in 24 hrs.

$\therefore$ Discharge: $$Q_1 = \frac{2.5 \times 10^8}{24} \text{ litre/hr}$$

$$= \frac{2.5}{24} \times \frac{10^8 \times 1}{1000} \text{ m}^3/\text{hr} = \frac{2.5}{24} \times 10^5 \text{ m}^3/\text{hr}$$

$$= \frac{2.5}{24 \times 3600} \times 10^5 \text{ m}^3/\text{s} = 2.89 \text{ m}^3/\text{s}$$

25% water is lost due to evaporization and the remaining discharge reaches the reservoir is 75%.

i.e., $Q = 0.75\ Q_1 = 0.75 \times 2.89\ \text{m}^3/\text{s} = 2.167\ \text{m}^3/\text{s}$.

Head of water over the sill of weir:

$$H = 500 \text{ mm} = 0.5 \text{ m}$$

$$C_d = 0.62$$

Discharge: $$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5}$$

$$2.89 = \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \times L (0.5)^{1.5}$$

or $$L = \mathbf{4.46\ m.}$$

Problem 9.6: A right-angled V-notch is used to measure the discharge of a pump. If the head of water over the sill is 300 mm, find the discharge over the notch in litre/s. Take $C_d = 0.61$.

Solution: Given data for V-notch:

Angle of notch: $\theta = 90°$ $\because$ Right-angled V-notch

$H = 300 \text{ mm} = 0.3 \text{ m}$

$C_d = 0.61$

We know that the discharge over the V-notch: Q

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

$$= \frac{8}{15} \times 0.61 \times \sqrt{2 \times 9.81} \times \tan\left(\frac{90°}{2}\right) \times (0.3)^{2.5}$$

$= 0.01863 \text{ m}^3\text{/s} =$ **18.63 litre/s.**

Problem 9.7: During an experiment in a laboratory, 140 litres of water flowing over a right-angled V-notch was collected in 30 seconds. If the head of water over the sill is 100 mm, find the coefficient of discharge of the notch.

Solution: Given data for V-notch:

140 litres of water flowing over a V-notch in 30 seconds.

i.e., Discharge: $Q = \frac{140}{30}$ litre/s = 4.66 litre/s = 0.00466 $\text{m}^3\text{/s}$

$\theta = 90°$

$H = 100 \text{ mm} = 0.1 \text{ m}$

We know that the discharge over the V-notch: Q

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

$$0.00466 = \frac{8}{15} \times 0.61 \times \sqrt{2 \times 9.81} \times \tan\left(\frac{90}{2}\right) \times (0.1)^{2.5}$$

$$0.00466 = 0.00747\, C_d$$

or $C_d =$ **0.623**

Problem 9.8: Water flows over a rectangular notch of 1m length over a head of water 200 mm. Then, the same discharge over a right-angled triangular notch. Find the height of water above the sill of the notch. Take C_d for the rectangular and triangular notches as 0.60 and 0.61 respectively.

Solution: Given data:

For rectangular notch	For right-angled triangular notch
$L = 1$ m	$\theta = 90°$
$H = 200$ mm $= 0.2$ m	$C_d = 0.61$
$C_d = 0.60$	

Discharge:

$$Q_{\square} = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$

$$= \frac{2}{3} \times 0.60 \times \sqrt{2 \times 9.84} \times 1 \times (0.2)^{1.5} = 0.1584 \text{ m}^3/\text{s}$$

Given condition:

$$Q_{\square} = Q_{\nabla} = 0.1584 \text{ m}^3/\text{s}$$

We know that

$$Q_{\nabla} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

$$0.1584 = \frac{8}{15} \times 0.61 \times \sqrt{2 \times 9.81} \times \tan\left(\frac{90}{2}\right) \times H^{2.5}$$

or $$H^{2.5} = 0.1099$$

or $$H = 0.41342 \text{ m} = \mathbf{413.42 \text{ mm.}}$$

Problem 9.9 Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge coefficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360 mm, find the depth of water over the rectangular weir.

Solution: Given data :

Right-angled triangular weir	**Rectangular weir width**
$\theta = 90°$	Width: $L = 1$ m
$C_d = 0.6$	$C_d = 0.7$
$H = 360$ mm	
$= 0.36$ m	

Discharge:

$$Q_{\nabla} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

$$= \frac{8}{15} \times 0.6 \sqrt{2 \times 9.81} \times \left(\tan\frac{90°}{2}\right) \times (0.36)^{2.5}$$

$$= 0.1102 \text{ m}^3/\text{s}$$

Given condition:

$$Q_{\square} = Q_{\nabla} = 0.1102$$

We know that

$$Q_{\square} = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5}$$

$\therefore$

$$0.1102 = \frac{2}{3} \times 0.7 \sqrt{2 \times 9.81} \times 1 \times H^{1.5}$$

or $H^{1.5} = 0.053312$

or $H = 0.14167 \text{ m} = \mathbf{141.67\ mm}$

9.7 DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

A trapezoidal notch is a combination of triangular notch and rectangular notch. Therefore, discharge over a trapezoidal notch is sum of discharge over triangular notch and discharge over rectangular notch.

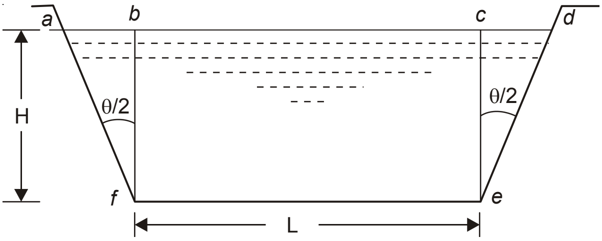
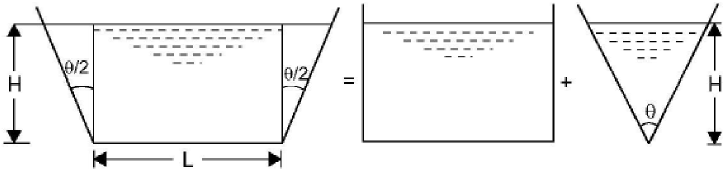


Fig. 9.4: Trapezoidal notch or weir.

Consider a trapezoidal notch *a b c d e f* as shown in Fig. 9.4. This trapezoidal section is dividing into rectangular and triangular sections. The rectangular section *b c e f* is a rectangular notch of length *L* and the height of liquid over notch *H*. Two triangular sections *a b f* and *c d e* are having angle of notch equal to θ/2 and height of liquid is *H*. The discharge through two triangular sections is equal to the discharge through single triangular notch with angle of notch θ and same height of liquid *H*.



(*a*) Trapezoidal notch (*b*) Rectangular notch (*c*) Triangular notch

Fig. 9.5: Trapezoidal section is dividing into rectangular and triangular sections

Discharge over trapezoidal notch = discharge over rectangular notch + discharge over triangular notch

$$Q = Q_{\square} + Q_{\nabla}$$

$$\boldsymbol{Q = C_{d1}\sqrt{2g}LH^{1.5} + \frac{8}{15}C_{d2}\tan\frac{\theta}{2}H^{2.5}} \qquad \ldots(9.7.1)$$

where C_{d1} = coefficient of discharge for rectangular notch

C_{d2} = coefficient of discharge for triangular notch

θ/2 = slope of the side of trapezodial notch.

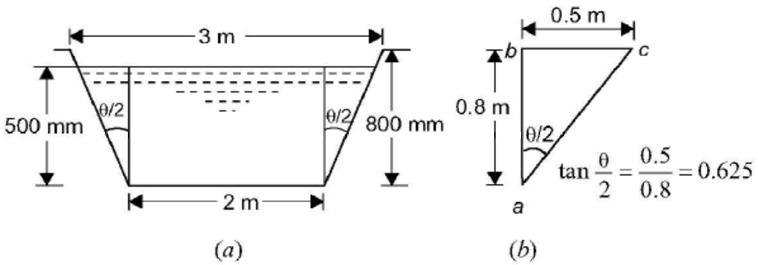
The expression for discharge in above Eq. (9.7.1) over a trapezoidal notch or weir is the same.

Problem 9.10: A trapezoidal weir of 3 m wide at the top and 2 m at the bottom is 800 mm high. Find the discharge over the weir, if the head of water is 500 mm. Take C_d = 0.61.

Solution: Given data for trapezoidal weir:

Width of weir at the top = 3 m

Width of weir at the bottom: L = 2 m



(*a*) (*b*)

Fig. 9.6: Schematic for Problem 9.10

Height of weir = 800 mm = 0.8 m

Head of water: H = 500 mm = 0.5 m

C_d = 0.61

From triangular section on one side of the weir as shown in Fig. 9.6 (*b*).

$$\tan\frac{\theta}{2} = \frac{0.5}{0.8} = 0.625$$

Discharge

$$Q_{\bigtriangledown} = Q_{\square} + Q_{\triangledown}$$

$$= \frac{2}{3}C_d\sqrt{2g}LH^{1.5} + \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}H^{2.5}$$

$$= \frac{2}{3}\times 0.61\times\sqrt{2\times 9.81}\times 2\times(0.5)^{1.5}$$

$$+\frac{8}{15}\times 0.61\times\sqrt{2\times 9.81}\times 0.625\times(0.5)^{2.5}$$

$$= 1.273 + 0.159 = \mathbf{1.432\ m^3/s}$$

9.8 DISCHARGE OVER A STEPPED NOTCH

A stepped notch is a combination of rectangular notches. The discharge over a stepped notch is equal to the sum of the discharges over separate notches.

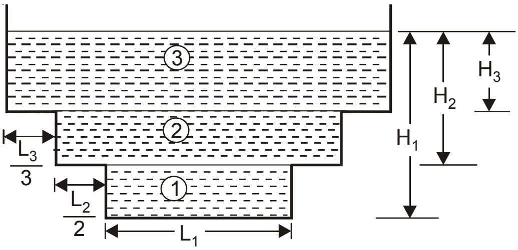


Fig. 9.7: Stepped notch.

Consider three steps notch as shown in Fig. 9.7.

Let H_1 = height of liquid in notch 1

L_2 = length of notch 1

Similarly H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

∴ Total discharge $Q = Q_1 + Q_2 + Q_3$

$$= \frac{2}{3}C_d\sqrt{2g}L_1H_1^{1.5} + \frac{2}{3}C_d\sqrt{2g}\,L_2\,H_2^{1.5} + \frac{2}{3}C_d\sqrt{2g}L_3\,H^{1.5}$$

$$Q = \frac{2}{3}C_d\sqrt{2g}\left[L_1H_1^{1.5} + L_2\,H_2^{1.5} + L_3H_3^{1.5}\right]$$

where C_d = coefficient of discharge is same for three notches.

9.9 ADVANTAGES OF TRIANGULAR NOTCH OVER RECTANGULAR NOTCH

Advantages of triangular notch over rectangular notch are listed below:

1. For low discharge, a triangular notch gives more accurate discharge than a rectangular notch. This is because, a triangular notch provides a greater head than the rectangular notch for same low discharge. Hence, head measurement can be done more accurately over the triangular notch than over the rectangular notch.
2. The coefficient of discharge for a triangular notch is independent of the head (*i.e.,* C_d = constant, for wide range of liquid head over triangular notch). Whereas in a rectangular notch, the coefficient of discharge is not constant (*i.e.,* $C_d = f(H)$).
3. No need of ventilation for the nappe of a triangular notch. But in a rectangular notch is necessary.
4. For a right angle V-notch, the expression for discharge becomes very simple:

   i.e., $\theta = 90°$, $C_d = 0.6$, $g = 9.81$ m/s^2

   then, $Q = 1.417\ H^{2.5}$.

Problem 9.11: Find the discharge over a stepped weir of the following dimensions:

Top section: 2 m × 0.5 m

Middle section: 1.5 m × 0.25 m

Bottom section: 1 m × 0.15 m

Take coefficient of discharge for three sections as 0.62.

Solution: Given data for stepped weir:

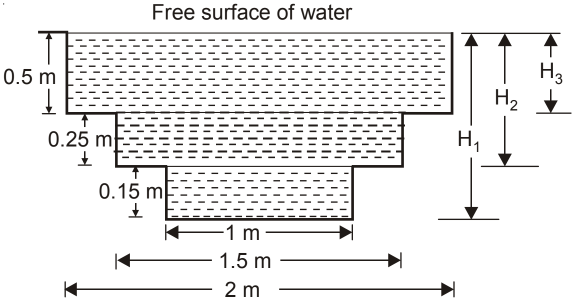


Fig. 9.8: Schematic for Problem 9.11

For bottom section: $L_1 = 1$ m

$H_1 = 0.5 + 0.25 + 0.15 = 0.9$ m

For middle section: $L_2 = 1.5 - L_1 = 1.5 - 1 = 0.5$ m

$H_2 = 0.5 + 0.25 = 0.75$ m

For top section: $L_3 = 2 - 1.5 = 0.5$ m

$H_3 = 0.5$ m

We know that the discharge through stepped weir:

$$\begin{aligned}
Q &= \frac{2}{3}C_d\sqrt{2g}\left[L_1H_1^{1.5} + L_2H_2^{1.5} + L_3H_3^{1.5}\right] \\
&= \frac{2}{3}\times 0.62\times\sqrt{2\times 9.81} \\
&\quad \left[1\times(0.9)^{1.5} + 0.5\times(0.75)^{1.5} + 0.5\times(0.5)^{1.5}\right] \\
&= 1.83 \times [0.8538 + 0.3247 + 0.1767] \\
&= 1.83 \times 1.355 = \mathbf{2.479\ m^3/s.}
\end{aligned}$$

9.10 EFFECT ON THE DISCHARGE OVER A NOTCH DUE TO AN ERROR IN THE MEASUREMENT OF HEAD

We know that the discharges over a rectangular and triangular notches are:

$$Q = \frac{2}{3}C_d\sqrt{2g}LH^{1.5} \qquad \text{for rectangular notch}$$

$$= \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}H^{2.5} \qquad \text{for triangular notch}$$

$Q \propto H^{1.5}$ for rectangular notch

$Q \propto H^{2.5}$ for triangular notch.

Thus, the accurate measurement of the head (*H*) of the water above the sill of the notch is very essential to know the accurate discharge over the notch. A small error in the measurement of head (*H*), will affect in the calculation of the discharge over the notch, and is generally expressed as the percentage of the discharge.

The following two cases of error in the measurement of head will be considered:

1. Over a rectangular notch,
2. Over a triangular notch.

9.10.1 For a Rectangular Notch

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$

$$Q = K_1 \; H^{1.5} \qquad \ldots(9.10.1)$$

where $K_1 = \frac{2}{3} C_d \sqrt{2g} L$

Differentiating above Eq. (12.10.1) with respect to H, we get

$$dQ = K_1 \times 1.5 \; H^{0.5} \; dH \qquad \ldots(9.10.2)$$

Dividing Eq. (12.10.2) by Eq. (12.10.1), we get

$$\frac{dQ}{Q} = \frac{K_1 \times 1.5 H^{0.5} dH}{K_1 \, H^{1.5}}$$

$$\boldsymbol{\frac{dQ}{Q} = 1.5 \frac{dH}{H}} \qquad \ldots(9.10.3)$$

Equation (9.10.3) shows that an error of 1% in measuring head (H) will produce 1.5% error in discharge over a rectangular notch (same as in case of weir).

9.10.2 For a Triangular Notch

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{2.5}$$

$$Q = K_2 \; H^{2.5} \qquad \ldots(9.10.4)$$

where $K_2 = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2}$

Differentiating above Eq. (9.10.4) with respect to H, we get

$$dQ = K_2 \times 2.5 \; H^{1.5} \; dH \qquad \ldots(9.10.5)$$

Dividing Eq. (9.10.5) by Eq. (9.10.4), we get

$$\frac{dQ}{Q} = \frac{K_2 \times 2.5 H^{1.5} dH}{K_2 \, H^{2.5}}$$

$$\boxed{\boldsymbol{\frac{dQ}{Q} = 2.5 \frac{dH}{H}}} \qquad \ldots(9.10.6)$$

Equation (9.10.6) shows that an error of 1% in measuring head (H) will produce 2.5% error in discharge over a triangular notch (same as in case of weir).

Problem 9.12: A rectangular weir of 500 mm long is used for measuring a distance of 140 litre/s. An error of 2.5 was made, while measuring the head over the weir. Find the percentage error in the discharge. Take $C_d = 0.61$.

Solution: Given data for rectangular weir:

Length of weir: $L = 500 \text{ mm} = 0.5 \text{ m}$

Discharge: $Q = 140 \text{ litre/s} = \frac{140}{1000} \text{ m}^3\text{/s} = 0.14 \text{ m}^3\text{/s}$

Error in head: $dH = 2.5 \text{ mm} = 0.0025 \text{ mm}$

$C_d = 0.61$

Discharge over a rectangular weir:

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$

$$0.14 = \frac{2}{3} \times 0.61 \times \sqrt{2 \times 9.81} \times 0.5 H^{1.5}$$

or $H^{1.5} = 0.15544$

or $H = 0.2891 \text{ m}$

Percentage error in discharge:

$$\frac{dQ}{Q} = 1.5 \frac{dH}{H} \times 100 = 1.5 \times \frac{0.0025}{0.2891} \times 100 = \mathbf{1.29\%}.$$

Problem 9.13: A discharge of 60 litre/s was measured over a right-angled triangular notch. While measuring the head over the notch, an error of 1.5 mm was made. Find the percentage of error in the discharge, if coefficient of discharge is 0.62.

Solution: Given data for right-angled triangular notch:

$Q = 60 \text{ litre/s} = 0.06 \text{ m}^3\text{/s}$

$dH = 1.5 \text{ mm} = 0.0015 \text{ m}$

$\theta = 90°$

$C_d = 0.62$

Discharge: $$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{2.5}$$

$$0.06 = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \tan \frac{90°}{2} \times H^{2.5}$$

or $H^{2.5} = 0.0496$

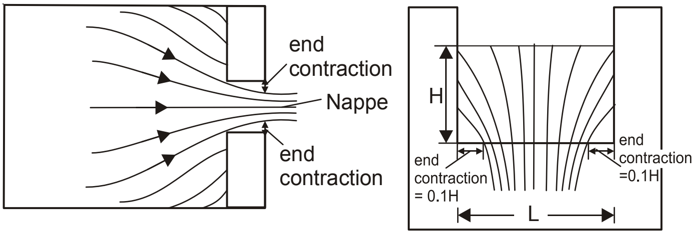
or $H = 0.3 \text{ m}$

Error in discharge: $$\frac{dQ}{Q} = 2.5 \frac{dH}{H} = 2.5 \times \frac{0.0015}{0.3} = 0.0125 = \mathbf{1.25\%}$$

9.11 CIPOLLETTI WEIR

We know that a discharge over rectangular weir is given by

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}$$



(*a*) No. of end contractions: *n* = 2 (*b*) Effective length = L – 0.2H

Fig. 9.9: Discharge over rectangular weir with ends contraction.

When the length (*L*) of the weir is less than the width of the channel in which liquid is flowing. Then, there will be end contraction at each end as shown in Fig. 9.9. According to Francis, each end contraction is equal to 0.1 H.

If the actual length of weir is *L*, then the effective length of the weir will be (L – 0.2H). Hence, the discharge is decreases due to formation of end contraction.

∴ Discharge through rectangular weir with end contraction:

$$Q = \frac{2}{3} C_d \sqrt{2g}(L - 0.2H)H^{1.5}$$

$$= \frac{2}{3} C_d \sqrt{2g} LH^{1.5} - \frac{0.4}{3} C_d \sqrt{2g}\, H^{2.5}$$

$$Q = Q - Q_1$$

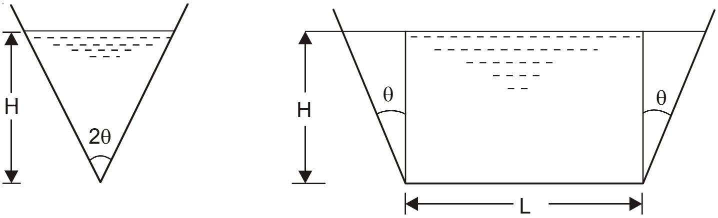
where $Q = \frac{2}{3} C_d \sqrt{2g} LH^{1.5}$,

discharge through rectangular weir without end contractions.

$$Q_1 = \frac{0.4}{3} C_d \sqrt{2g}\, H^{2.5},$$

reduction in discharge due to formation of end contraction.

This reduction in discharge Q_1 over rectangular weir is to be compensated by the addition discharge due to additional area at same base length *L* as shown in Fig. 9.10(*b*), is called cipolletti weir.



(*a*) Additional area acts as triangular weir if 2θ = 28° (*b*) Cipoletti weir if θ = 14°

Fig. 9.10: Cipolletti weir

Deduction in discharge Q_1 over rectangular weir = gain in discharge by adding area which acts as triangular notch

$$Q_1 = Q_\nabla$$

$$\frac{0.4}{3} C_d \sqrt{2g} H^{2.5} = \frac{8}{15} C_d \sqrt{2g} \tan\theta \, H^{2.5}$$

$$\frac{0.4}{3} = \frac{8}{15} \tan\theta$$

or $$\tan\theta = 0.25$$

or $$\theta = \tan^{-1}(0.25) \approx \mathbf{14°}$$

Cipolletti weir is a specific type of trapezoidal weir in which sloping side makes an angle of 14° (*i.e.,* $\theta = 14°$) with vertical on each side. In other words sloping sides have an inclination of 1 horizontal to 4 vertical. The cipolletti weir was invented by an Italian engineer Cipoletti.

The discharge over cipolletti weir is equal to discharge over rectangular weir without end contraction at same base length L.

$$\boldsymbol{Q = Q = \frac{2}{3} C_d \sqrt{2g} L H^{1.5}}$$

For experimental analysis, Cipolletti proposed following formula for discharge over cipolletti weir.

$$\mathbf{Q = 1.86 \ LH^{1.5}.}$$

9.12 FRANCIS'S FORMULA FOR RECTANGULAR WEIR WITH END CONTRACTIONS

Francis has given empirical formula for calculation of discharge

$$Q = 1.84\,(L - 0.2\ H)H^{1.5}$$

for single weir or number of contractions: $n = 2$

$$Q = 1.84\,(L - 0.1\ nH)H^{1.5}$$

where n = number of contractions

L = total length of weir

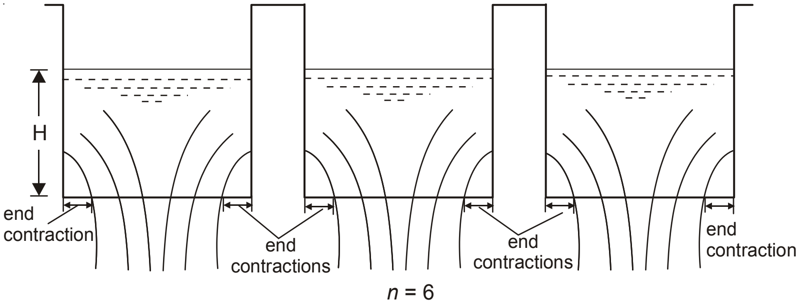


Fig. 9.11: Three rectangular weir with six end contraction.

Francis's formula for rectangular weir without end contractions or for cipoletti weir:

$$\mathbf{Q = 1.84 \ LH^{1.5}}$$

9.13 VELOCITY OF APPROACH

In the previous articles, the discharge equations are derived on the assumption that the water on the upstream side of the weir is not in motion. Therefore, the head of water considered in deriving the equations of discharge was taken as the height of the free surface of water above the sill of the weir. But in actual practice, the weir is provided across a river or a stream and the water approaching the weir which gets a certain velocity is called velocity of approach.

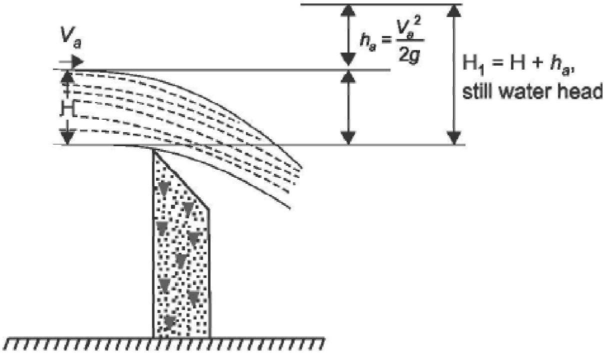


Fig. 9.12: Velocity of approach

Let V_a = velocity of approach

$\frac{V_a^2}{2g} = h_a$, head due to the velocity of approach.

Because of velocity of approach, the available total head, upstream the weir, is not the height of the free surface above the sill, but is equal to height of the free surface above the sill plus head due to the velocity of approach.

i.e., Total head: $H_1 = H + h_a$.

Therefore, the limits of integration for the discharge over a rectangular weir will be h_a to H_1 instead of 0 to H.

Discharge over rectangular weir if the velocity of approach considered:

$$Q_{va} = \frac{2}{3} C_d \sqrt{2g}\ L\ (H_1^{1.5} - h_a^{1.5})$$

where $H_1 = H + h_a$, called still water head

$$h_a = \frac{V_a^2}{2g}$$

The velocity of approach (V_a) can be determined by using continuity equation.

Discharge: $Q = A\,V_a$

or Velocity of approach:

$$V_a = \frac{Q}{A}$$

where Q = discharge over the weir is determined by neglecting the velocity of approach.

A = cross-sectional area of flow of water in the stream on the upstream side of the weir.

Discharge over rectangular weir with end contraction.

$$Q = \frac{2}{3} C_d \sqrt{2g}\ (L - 0.1nH_1)\ (H_1^{1.5} - h_a^{1.5})$$

where n = number of end contractions

$H_1 = H + h_a$, total head over the sill of weir.

Francis's formula for rectangular weir with end contraction.

$$Q = 1.84\ (L - 0.1\ n\ H_1)\ (H_1^{1.5} - h_a^{1.5})$$

Problem 9.14: Water is flowing over a Cipolletti weir 5 m long under a head of 1.5 m. Find the discharge, if the coefficient of discharge for the weir is 0.60.

Solution: Given data for a Cipoletti weir:

Length: $L = 5$ m

Head: $H = 1.5$ m

$C_d = 0.60$

Discharge: $$Q = \frac{2}{3} C_d \sqrt{2g}\ LH^{1.5} = \frac{2}{3} \times 6.0 \times \sqrt{2 \times 9.81} \times 5 \times (1.5)^{1.5}$$

$= \mathbf{16.27\ m^3/s.}$

Problem 9.15: Find the length of a Cipolletti weir required for a flow of 500 litre/s, if the head of water is not to exceed one-tenth of its length. Use Francis's formula for the weir.

Solution: Given data for Cipoletti weir:

Discharge: $Q = 500$ litre/s $= 0.5$ m^3/s

$$H = \frac{1}{10} L$$

or $L = 10H$

Francis's formula for a Cipoletti weir:

$$Q = 1.84\ L\ H^{1.5} = 1.84 \times 10\ H\ .\ H^{1.5}$$

$$0.5 = 1.84 \times 10 \times H^{2.5}$$

or $H^{2.5} = 0.02717$

$H = 0.2364$ m

and $L = 10\ H = 10 \times 0.2364 = \mathbf{2.364\ m.}$

Problem 9.16: Find the discharge over a rectangular weir 20 m in length with a head of 2 m. Take the velocity of approach as 1.2 m/s and $C_d = 0.59$.

Solution: Given data for a rectangular weir:

$L = 20$ m

$H = 2$ m

$V_a = 1.2$ m/s

$\therefore$ $$h_a = \frac{V_a^2}{2g} = \frac{(1.2)^2}{2 \times 9.81} = 0.073\,\text{m}$$

$\therefore$ Total head: $H_1 = H + h_a = 2 + 0.073 = 2.073$ m.

$C_d = 0.59$

Discharge over the rectangular weir,

$$Q = \frac{2}{3} C_d \sqrt{2g}\; L\,[H_1^{1.5} - h_a^{1.5}]$$

$$= \frac{2}{3} \times 0.59 \times \sqrt{2 \times 9.81} \times 20 \times [(2.073)^{1.5} - (0.073)^{1.5}]$$

$$= 34.84 \times [2.984 - 0.019] = \mathbf{103.30\ m^3/s}$$

Problem 9.17: A weir of 25 m long is divided into 10 equal bays by vertical posts each of 500 mm width. Find the discharge over the weir, if the head over the crest is 1.5 m and the velocity of approach is 2 m/s.

Solution: Given data for a rectangular weir:

Total length of the weir = 25 m

Number of bays = 10

Width of each post = 500 mm = 0.5 m

$H = 1.5$ m

$V_a = 2$ m/s

We know that the number of end contractions,

$n = 10 \times 2 = 20$ ($\because$ each bay contains two end contractions)

For 10 bays, number of vertical posts = 9

Length of the weir: $L = 25 - 9 \times 0.5 = 20.5$ m

Head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{(2)^2}{2 \times 9.81} = 0.203\,\text{m}$$

$\therefore$ Total head: $H_1 = H + h_a = 1.5 + 0.203 = 1.703$ m

$\therefore$ Discharge over the weir according to Francis's formula,

$$Q = 1.84\,(L - 0.1\,nH_1)\,[H_1^{1.5} - h_a^{1.5}]$$

$$= 1.84\,(20.5 - 0.1 \times 20 \times 1.703)$$

$$[(1.703)^{1.5} - (0.203)^{1.5}]$$

$$= 31.45 \times [2.222 - 0.091] = \mathbf{72.74\ m^3/s.}$$

Problem 9.18: A sharp crested rectangular weir of 1.4 m height extends across a rectangular channel of 4 m width. If the head of water over the weir is 0.5 m, find the discharge over the weir. Consider velocity of approach and take coefficient of discharge as 0.61.

Solution: Given data for rectangular weir:

Height of weir from base of channel:

$$h = 1.4 \text{ m}$$

Length of weir: $L = 4$ m

Head of water over the weir:

$$H = 0.5 \text{ m}$$

Height of water in the channel:

$$= h + H = 1.4 + 0.5 = 1.9 \text{ m}$$

$$C_d = 0.61$$

For calculation of the velocity of approach.

Discharge of rectangular weir without considering velocity of approach,

$$Q = \frac{2}{3} C_d \sqrt{2g}\ L H^{1.5} = \frac{2}{3} \times 0.61 \times \sqrt{2 \times 9.81} \times 4 \times (0.5)^{1.5}$$

$$= 2.547 \text{ m}^3\text{/s.}$$

Wetted cross-sectional area of the channel:

$$A = \text{width} \times \text{heights of water in the channel}$$

$$= 4 \times 1.9 = 7.6 \text{ m}^2$$

$\therefore$ Velocity of approach: $V_a = \dfrac{Q}{A} = \dfrac{2.547}{7.6} = 0.335 \text{ m/s}$

and head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{(0.335)^2}{2 \times 9.81} = 0.0057 \text{ m}$$

$\therefore$ Total head: $H_1 = H + h_a = 0.5 + 0.0057 = 0.5057$ m

Now the discharge over rectangular weir with velocity of approach,

$$Q_{va} = \frac{2}{3} C_d \sqrt{2g}\ L\ (H_1^{1.5} - h_a^{1.5})$$

$$= \frac{2}{3} \times 0.61 \times \sqrt{2 \times 9.81} \times 4 \times [(0.5057)^{1.5} - (0.0057)^{1.5}]$$

$$= \mathbf{2.588\ m^3/s.}$$

9.14 VENTILATION OF WEIRS

Some cases, a suppressed weir (*i.e.,* the length of weir is equal to the width of the channel) is provided in a channel to measure the discharge of water. Then, the nappe touches the side walls of the channel.

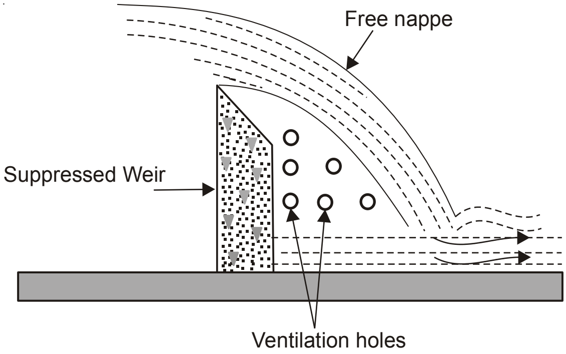


Fig. 9.13: Ventilation of weir

Because of this, a hollow space is maintained among the nappe, the weir, bottom and side walls of the channel in downstream of the weir as shown in Fig. 9.13. This space is occupied by air. The air, thus entrapped is slowly carried away by the flowing water, thus creating a vacuum pressure below nappe. This vacuum pressure below nappe draws the lower nappe towards the downstream surface of the weir. Because of the vacuum pressure (*i.e.,* pressure below the atmospheric pressure) below the nappe, the discharge over weir slightly increases.

In order to keep the atmospheric pressure in the space below the nappe, holes are provided in the side walls of the channel in the space below the lower nappe, so that the space is connected to free atmosphere and air is supplied continuously for the amount of air carried away by flowing water. The number of holes act like ventilators and are called ventilation holes, and the weir where ventilation holes are provided on the side walls of the channel in downstream of the weir, is called ventilated weir.

If the weir is well ventilated and the atmospheric pressure is maintained below the nappe, then it is called free nappe as shown in Fig. 9.13.

If a weir is not properly ventilated, the amount of air which is carried away by water is more than the quantity of air supplied through ventilation holes. Then, partial vacuum is created below the nappe. As a result, the nappe is gradually drawn towards the downstream surface of a weir. Such a nappe is called depressed nappe as shown in Fig. 9.14. It has been observed that the discharge over weir with depressed nappe is increased by 6 to 7% as compared with free nappe.

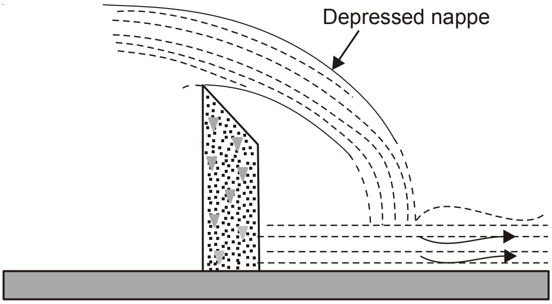


Fig. 9.14: Depressed nappe.

If there is no air left between the lower nappe and the down stream surface of the weir, then the lower nappe adheres the down stream surface of the weir. Such nappe which adheres to the down stream surface of the weir is called adhering or clinging nappe as shown in Fig. 9.15. The discharge over clinging nappe is 20 to 30% more than the free nappe.

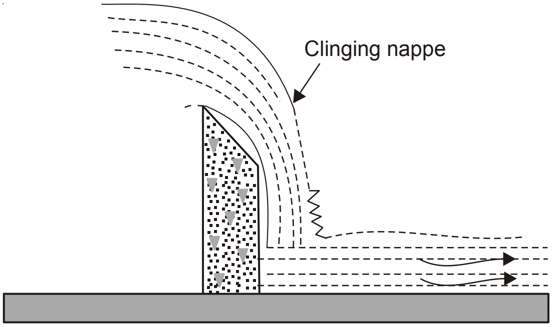


Fig. 9.15: Clinging nappe

9.15 DISCHARGE OVER A BROAD CRESTED WEIR

A weir with a wide crest is known as broad crested weir. The width of the crest B is greater than 0.5 H. This type of weir is shown in Fig. 9.16.

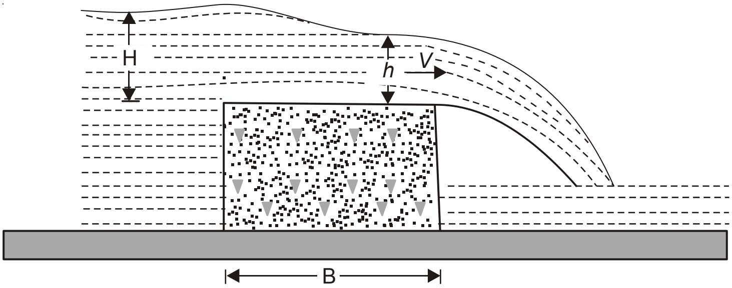


Fig. 9.16: Broad crested weir.

Let H = head of water on the upstream side of the weir.

h = head of water on the downstream side of the weir.

V = velocity of water on the downstream side of the weir.

Applying the Bernoulli's equation to the crest of the weir on the upstream side and downstream side:

$$H + 0 + 0 = h + \frac{V^2}{2g} + 0$$

or
$$H = h + \frac{V^2}{2g}$$

or
$$\frac{V^2}{2g} = H - h$$

or $$V = \sqrt{2g(H-h)} \qquad \ldots(9.15.1)$$

∴ Discharge over the weir:

$$Q = C_d \times \text{area of flow} \times \text{velocity}$$

$$= C_d \times Lh \times V = C_d\,L\,hV \qquad \ldots(9.15.2)$$

where C_d = coefficient of discharge

L = length of the weir

Substituting the value of V from Eq. (9.15.1) in above Eq. (9.15.2) we get

$$Q = C_d\,Lh\sqrt{2g(H-h)}$$

$$= C_d\,L\sqrt{2g}\sqrt{(Hh^2-h^3)} \qquad \ldots(9.15.3)$$

The above Eq. (9.15.3) shows that the discharge will be maximum, when $(Hh^2 - h^3)$ is maximum. Therefore, the condition of discharge for constant head H may be obtained by equating $\frac{d}{dh}(Hh^2-h^3)$ to zero.

$$\therefore \quad \frac{d}{dh}(Hh^2-h^3) = 0$$

$$2Hh - 3h^2 = 0$$

$$2H - 3h = 0$$

or $$\boldsymbol{h = \frac{2}{3}H}$$

Substituting the value of h in Eq. (9.15.3), we get

$$Q_{\text{max}} = C_d\,L\sqrt{2g}\sqrt{H\left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^2}$$

$$= C_d\,L\sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d\,L\sqrt{2g}\sqrt{\frac{4}{27}H^3}$$

$$= C_d\,L\sqrt{2g}\times\frac{2}{3}\sqrt{\frac{H^3}{3}} = \frac{2}{3}C_d\,L\sqrt{2g}\,\frac{H^{3/2}}{\sqrt{3}}$$

$$= \frac{2}{3\sqrt{3}}C_d\,L\sqrt{2g}\,H^{1.5} = \frac{2}{3\sqrt{3}}\times\sqrt{2\times 9.81}\;C_d\,L\,H^{1.5}$$

$$\boldsymbol{Q_{\text{max}} = 1.71\;C_d\;L\;H^{1.5}}$$

9.16 DISCHARGE OVER A SUBMERGED WEIR

When the water level on the down stream side is above the crest of the weir, then the weir is said to be submerged or drowned weir. It is used for large discharge capacity. The discharge over submerged weir may be divided into two portions as discussed below:

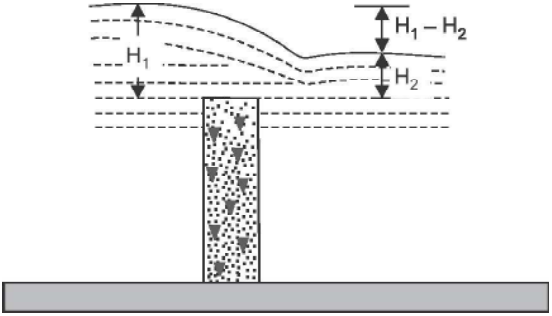


Fig. 9.17: Submerged weir.

Let H_1 = height of water on the upstream side of the weir, and

H_2 = height of water on the downstream side of the weir.

The portion between upstream and downstream water surface may be treated as a freely discharge weir for the available head equal to $(H_1 - H_2)$.

∴ The portion of discharge over the freely discharge weir:

$$Q_1 = \frac{2}{3} C_d \sqrt{2g}\, L(H_1 - H_2)^{1.5}$$

The portion between downstream water surface and the crest of the weir may be considered as a submerged orifice.

∴ The portion of discharge through a submerged orifice.

$$Q_2 = C_d\, L H_2 \sqrt{2g(H_1 - H_2)}$$

where C_d = coefficient of discharge

L = length of the weir.

∴ Total discharge over submerged weir:

$$Q = Q_1 + Q_2$$

$$= \frac{2}{3} C_d \sqrt{2g}\, L(H_1 - H_2)^{1.5} + C_d\, L H_2 \sqrt{2g(H_1 - H_2)}$$

9.17 OGEE WEIR

In case of a sharp crested weir, the nappe as it leaves the crest, rises slightly at the lower surface. The space below the bottom surface of the nappe is filled with masonry or concrete. In this manner a new weir formed beside a sharp crested weir is called an ogee weir. Thus in an ogee weir, the solid boundary of the weir remains in contact with the bottom surface of the nappe of the sharp crested weir under designed head.

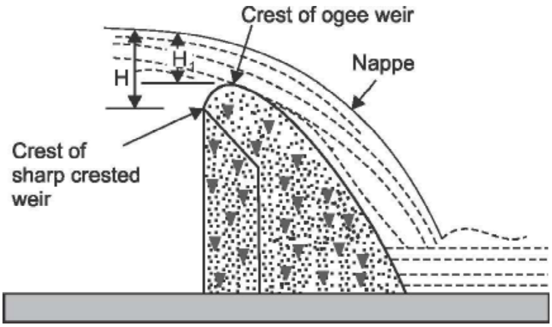


Fig. 9.18: An ogee weir.

Discharge over an ogee weir:

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5} \qquad \ldots(9.17.1)$$

According to Francis's formula;

$$Q = 1.84\ L\ H^{1.5} \qquad \ldots(9.17.2)$$

where H = head over the sharp crest weir.

Eqs. (9.17.1) and (9.17.2) same as discharge over rectangular weir.

If H_1 = head above the crest of ogee weir be considered, then the discharge is given by,

$$\boldsymbol{Q = 2.20\ L H_1^{1.5}.}$$

Problem 9.19: A submerged sharp crested weir 0.5 m high stands clear across a channel having vertical sides and a width of 2 m. The depth of water in the channel is 1.2 m. The depth of water is 0.8 m in downstream and 5 m from the weir. Find the discharge over the weir. Assume C_d = 0.6.

Solution: Given data:

Length of weir: L = 2 m

$C_d = 0.6$

$H_1 = 1.2 - 0.5 = 0.5$ m

$H_2 = 0.8 - 0.5 = 0.3$ m

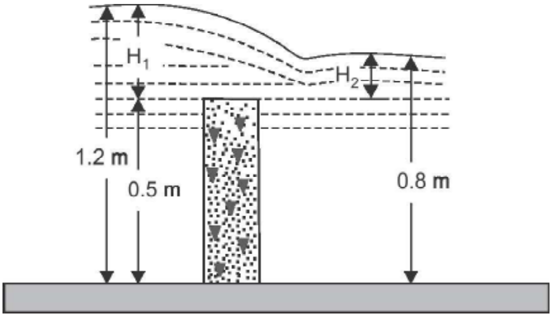


Fig. 9.19: Schematic for Problem 9.19

Discharge over the freely discharge weir:

$$Q_1 = \frac{2}{3} C_d \sqrt{2g}\, L (H_1 - H_2)^{1.5}$$

$$= \frac{2}{3} \times 0.6 \times \sqrt{2 \times 9.81} \times 2(0.5 - 03)^{1.5} = 0.3169 \text{ m}^3\text{/s}$$

Discharge through a submerged orifice:

$$Q_2 = C_d\, L H_2 \sqrt{2g(H_1 - H_2)}$$

$$= 0.6 \times 2 \times 0.3 \times \sqrt{2 \times 9.81(0.5 - 03)} = 0.7131 \text{ m}^3\text{/s}$$

∴ Total discharge over submerged weir:

$$Q = Q_1 + Q_2 = 0.3169 + 0.7131 = \mathbf{1.03\ m^3/s.}$$

Problem 9.20: An ogee weir 5 m long has 400 mm head of water. Find the discharge over the weir. Take $C_d = 0.61$.

Solution: Given data:

$L = 5$ m
$H = 0.4$ m
$C_d = 0.61$

We know that the discharge over an ogee weir:

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5} = \frac{2}{3} \times 0.61 \times \sqrt{2 \times 9.81} \times 5 \times (0.4)^{1.5}$$

$$= \mathbf{2.278\ m^3/s}$$

SUMMARY

1. Notches and weirs are used for measuring the rate of flow of liquid through open channel. Both are having same function *i.e.*, to measure the rate of flow. Difference between the two is their size and the quantity of discharge. Notches are small in size, hence used to measure small discharge through tank in laboratory. Weirs are big in size, thus measure large discharge in dams, rivers, *etc*.
2. Classification of notches:
   (*a*) According to the shape of the opening:
      (*i*) Rectangular notch
      (*ii*) Triangular notch or V-notch
      (*iii*) Trapezoidal notch
      (*iv*) Steeped notch.

Contd...

(*b*) According to the effect of the sides on the nappe:
   (*i*) Notch with end contractions
   (*ii*) Notch without end contractions or suppressed weir.

3. Classification of weirs:

(*a*) According to the shape of the opening:
   (*i*) Rectangular weir
   (*ii*) Triangular weir
   (*iii*) Trapezoidal weir.

(*b*) According to the effect of the sides of the nappe.
   (*i*) Weir with end contractions
   (*ii*) Weir without end contractions or suppressed weir.

(*c*) According to the shape of the crest:
   (*i*) Sharp-crested weir
   (*ii*) Broad-crested weir
   (*iii*) Narrow-crested weir
   (*iv*) Ogee-shaped weir

4. The discharge over a rectangular notch or weir:

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5}$$

where C_d = coefficient of discharge.
L = length of the notch or weir.
H = head of liquid over the crest of the notch or weir.

5. The discharge over a triangular notch or weir.

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

where θ = angle of notch or weir
H = head of liquid over the apex of the notch or weir.

6. The discharge over a trapezoidal notch or weir.

$$Q = \frac{2}{3} C_{d1} \sqrt{2g}\, L H^{1.5} + \frac{8}{15} C_{d2} \tan\frac{\theta}{2} H^{2.5}$$

where C_{d1} = coefficient of discharge for rectangular notch.
C_{d2} = coefficient of discharge for triangular notch.
$\frac{\theta}{2}$ = slope of the side of trapezoidal notch.

Contd...

7. The effect on the discharge over a notch due to an error in the measurement of head:

$$\frac{dQ}{Q}=1.5\frac{dH}{H} \quad \text{for a rectangular notch or weir}$$

$$=2.5\frac{dH}{H} \quad \text{for a triangular notch or weir.}$$

For a rectangular notch or weir, an error of 1% in measuring head (*H*) will produce 1.5% error in discharge.

For a triangular notch or weir, an error of 1% in measuring head (*H*) will produce 2.5% error in discharge.

8. Cipoletti weir is a specific type of trapezoidal weir in which sloping side makes an angle of 14° (*i.e.,* $\theta = 14°$) with vertical on each side. In the other words sloping sides have an inclination of 1 horizontal to 4 vertical. The cipoletti weir was invented by an Italian engineer Cipoletti.

Discharge over a Cipoletti weir: $Q=\frac{2}{3}C_d\sqrt{2g}\,L\,H^{1.5}$

9. Francis's formula for rectangular weir with end contractions

$$Q = 1.84\,(L - 0.2H)\,H^{1.5} \quad \text{for } n = 2$$

$$= 1.84\,(L - 0.1\,nH)\,H^{1.5}$$

where n = number of contractions,

L = total length of weir,

10. Francis's formula for rectangular weir without end contractions or for Cipolletti weir:

$$Q = 1.84\,LH^{1.5}$$

11. The discharge over rectangular weir if the velocity of approach considered:

$$Q_{av}=\frac{2}{3}C_d\sqrt{2g}\,L\,(H_1^{1.5}-h_a^{1.5})$$

where $H_1 = H + h_a$, called still water head.

$$h_a=\frac{V_a^2}{2g}$$

V_a = velocity of approach

ASSIGNMENT - 1

1. Define the following terms:
  (*i*) Notch
  (*ii*) Crest
  (*iii*) Nappe.

2. What is a weir? Differentiate between a notch and weir.

3. Differentiate between a notch and a orifice.

4. Derive an expression for the discharge over a rectangular weir.

5. Prove that the discharge over a triangular notch is given by:

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{2.5}$$

where H = head of water over the sill of a notch.
θ = angle of notch.

6. Prove that the discharge over a rectangular weir is given by:

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{1.5}$$

where C_d = coefficient of discharge
L = length of weir
H = head of water over the sill of a weir.

7. What are the advantages of triangular weir over rectangular weir?

8. Prove that the error in discharge due to the error in the measurement of head over a rectangular weir is given by

$$\frac{dQ}{Q} = 1.5\frac{dH}{H}$$

where Q = discharge over a rectangular weir, and
H = head over the crest of a weir.

9. Prove that the error in discharge due to the error in the measurement of head over a triangular weir is given by:

$$\frac{dQ}{Q} = 2.5\frac{dH}{H}$$

where Q = discharge over a rectangular weir, and
H = head over the crest of a weir.

10. What is a Cipolletti weir? How does it differ from the rectangular weirs?

11. What do you mean by end contractions of a rectangular weir? How can the loss of discharge due to end contractions be compensated?

12. Define the velocity of approach.

13. How does the velocity of approach affect the expression for discharge over a weir?

14. What are the ventilation holes?
15. What is a nappe of weir? Describe the free, depressed and clinging nappes with the help of sketches. State how do they effect the discharge measurement in case of weir.
16. Explain clearly the following weirs:
    (*i*) Broad crested (*ii*) Submerged (*iii*) Ogee.

ASSIGNMENT - 2

1. Find the discharge over a rectangular notch 5 m long when the head of liquid over the sill is 0.4 m. Take $C_d = 0.62$. **Ans.** 2.315 m^3/s
2. The discharge over a rectangular notch is 90 litre/s when the water level is 250 mm above the crest. Find the length of a notch if the coefficient of discharge is 0.61. **Ans.** 399.71 mm
3. A rectangular weir of 4 m long is used to measure the rate of flow of water. The head of water over the weir is 500 mm. If the available height of waterfall is 12 m, find the power of the waterfall. Take $C_d = 0.62$. **Ans.** 304.78 kW
4. The maximum flow through a rectangular channel 1 m deep and 1.5 m wide is 0.9 m^3/s. It is proposed to install a full width, sharp-edged rectangular weir across the channel to measure the flow. Find the maximum height at which the crest of the weir must be placed in order that the water may not overflow the sides of the channel. Take $C_d = 0.6$. **Ans.** 514.12 mm
5. A right-angled V-notch is used to measure the discharge in a small channel. If the depth of water at V-notch is 200 mm, find the discharge over the notch in litre per second. Take $C_d = 0.62$. **Ans.** 26.20 litre/s
6. Water flows over a rectangular notch of 1.2 m length over a head of water 300 mm. Then, the same discharge over a right-angled triangular notch. Find the height of water above the sill of the notch. Take C_d for the rectangular and triangular notches as 0.60 and 0.62 respectively. **Ans.** 563.65 mm
7. A trapezoidal weir 4 m wide at the top and 3 m at the bottom is 1 m high. Find the discharge over the weir, if the head of water is 600 mm.

   Take $C_d = 0.60$. **Ans.** 2.667 m^3/s
8. Find the discharge over a stepped notch of the following dimensions:

   Top section: 1000 mm × 150 mm

   Middle section: 800 mm × 100 mm

   Bottom section: 600 mm × 800 mm

   Take coefficient of discharge for three sections as 0.62.**Ans.** 275.28 litre/s
9. A rectangular notch 400 mm long is used for measuring a discharge of 30 litre/s. An error of 1.5 mm was made, while measuring the head over the sill of the notch. Find the percentage error in the discharge. Take coefficient of discharge as 0.60. **Ans.** 1.85 %

10. A right-angled V-notch is used for measuring a discharge of 30 litre/s. An error of 2 mm was made while measuring the head over the notch. Find the percentage error in the discharge.

Take coefficient of discharge as 0.62. **Ans.** 2.36%

11. Water is flowing over a Cipoletti weir 4 m long under a head of 1 m. Find the discharge, if the coefficient of discharge for the weir is 0.62. **Ans.** 7.32 m^3/s

12. Find the length of a Cipoletti weir required for a flow of 500 litre/s, if the head of water does not exceed one-tenth of its length. Use Francis's formula for the weir. **Ans.** 2.36 m

13. Water is flowing in a rectangular channel 1m wide and 0.75 m deep. Find the discharge over a rectangular weir of sill length 600 mm if the head of water over the sill of weir is 200 mm and water from channel flows over the weir. Take C_d = 0.62, neglecting end contraction, but considering the velocity of approach. **Ans.** 98.81 litre/s

14. A rectangular weir is constructed across a channel of 770 mm width with a head of 390 mm and the sill 600 mm above the bed of the channel. Find the discharge over a rectangular weir neglecting the end construction and considering the velocity of approach. Assume C_d = 0.62. **Ans.** 0.3555 m^3/s

15. A submerged sharp crested weir 800 mm high stands clear across a channel having vertical sides and a width of 3 m. The depth of water in the channel is 1.25 m. The width of water is 1 m in the downstream and 10 m from the weir. Find the discharge over the weir. Take C_d = 0.6. **Ans.** 0.6644 m^3/s

16. An ogee weir of 3 m long has 800 mm head of water. Find the discharge over the weir. Take C_d = 0.62. **Ans.** 3.93 m^3/s

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