Autonomous Wireless Sensors via Graded Elastic Metamaterials

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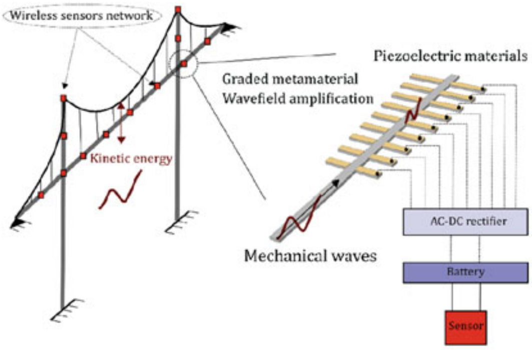
Abstract Amongst the 17th Sustainable and Development Goals (SDGs), it's crucial to ensure access to sustainable and modern energy, as emphasized by the Goal 7. This is not only relevant for large utilities, but also for tiny devices such as wireless sensors that can ubiquitously found in our information driven society. Recent advances in low-power consumption circuitry have enabled ultrasmall power integrated circuits, which can run with extremely low amount of power. For these reasons, energy harvesting can be used to self-power small electronic devices, using ambient waste energy from vibrations. Recent metamaterial technologies allow to dramatically increase the energy available for harvesting, and the operational bandwidth. A large-scale application of metamaterial-based energy harvesting could increase the sustainability in the global energy mix as well as provide improvement in energy efficiency.

Graphical Abstract



Keywords Affordable and clean energy · Metamaterials · Energy harvesting · Piezoelectricity · Elastic waves · Graded resonators · Sensors

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1 Preliminary Comments and Outlines

Goal 7 of the United Nations SDGs aims at ensuring access to affordable, reliable, sustainable and modern energy. The targets also emphasize the importance of using renewable energy in the global energy mix as well as the improvement in energy efficiency. The possibility to use new forms of clean energy is not only relevant for large utilities, but also tiny devices such as wireless sensors, which generate the data and allow further functionality from self-monitoring and self-configuration to condition monitoring of complex processes. In recent years the world is facing an extraordinary diffusion of the Internet of Things (IOT) concept which is the idea of building smart and autonomous sensors networks which can help us in sensing, understanding, and controlling our environment. For this idea to be effective, new sensors should be small, barely costless, and autonomous. The reduced power requirements of recent small electronic components, makes on-chip energy harvesting solutions a promising alternative to batteries or complex wiring. Amongst others, vibration-based energy harvesting solutions are particularly attractive due to the numerous and continuous sources of vibration present in the environment. However, due to the low amount of energy involved in common ambient vibrations, it is interesting to focus, or trap, waves from a larger region outside the device into a confined region in the near vicinity of the sensor. For these reasons, in order to fully take advantage of this form of energy, it is required a device that: (i) *focus or confine waves*: it is possible to increase the absorbed energy since it comes from a larger spatial region, or due to confinement in specific positions; (ii) *work in a broadband regime*: the energy of common ambient spectra can be completely used, and the performance is less affected by input changing; (iii) *can be easily manufactured*: mass scale production is possible with affordable costs. Several works have been reported in the literature to partially or totally address the aforementioned key requirements. Most of them rely on the design of structuring materials and metamaterials, i.e. engineered systems able to show efficient wave manipulation properties; once the wave is localised, by using electromagnetic, electrostatic, or piezoelectric effects, efficient conversion from elastic to electric energy can be achieved.

To introduce the reader into this field, the basic theory of elastic wave propagation, metamaterials and piezoelectric materials is introduced.

1.1 Mechanical Waves

A *wave* can be defined as the propagation of a disturbance with oscillations about a stable equilibrium configuration. As the disturbance propagates, it carries along amounts of energy that can be transmitted over considerable distances. A *mechanical wave* is a local strain that propagates in a deformable body from particle to particle, by creating local stresses. To create a mechanical wave, two opposed forces that simultaneously counteract and restore equilibrium are required. This is done by the *inertia* and *elastic* forces which correspond, energetically speaking, to the *kinetic* and *potential* energies. If we lose the elastic force, it is not a wave, but motion of mass. If the wave is defined by nodes and anti-nodes, i.e. with fixed peaks amplitude positions in space, it is called *stationary*; contrary, it is called *travelling* or *propagating*. A particular wave is the *plane wave*, that is a wave with constant amplitude for any plane perpendicular to its direction of propagation. It is worth to notice that the direction of propagation does not necessarily coincide with the direction of oscillation of the particles: if equal, the wave is called *longitudinal*, otherwise *transverse*.

An elastic, homogeneous and isotropic continuum, is defined by three parameters: density ρ, Young (or Elastic) modulus E, and Poisson ratio ν. It is reasonable to assume that, in this medium, the wave velocity is $c = c(\rho, E, \nu)$. By adopting the form $c = \xi(\nu)\rho^{\alpha}E^{\beta}$, due to dimensional considerations we get: $c = \xi(\nu)\sqrt{E/\rho}$, which means that increasing the elastic parameter or decreasing the inertial one, the wave velocity increases.

The waves supported by an elastic medium, can be identified through the *dispersion relation*, which is a relation between the angular frequency and the wavenumber. If this relation is linear, the medium is called *non-dispersive*, contrary it is called *dispersive*. An important quantity involved in the dispersion relation is the wave velocity, which can be distinguished into *phase velocity* and *group velocity*. The phase velocity is the rate at which the phase of the wave propagates in space, and it is defined as $c_{ph} = \omega/\kappa$. The group velocity is the velocity with which the overall envelope shape of the wave's amplitudes propagates through space (wave packet velocity); it is defined as $c_g = \partial\omega/\partial\kappa$, and it is just the same as the velocity of energy transport of a monochromatic wave [1].

1.2 Metamaterials

The word *metamaterial* etymologically means, from the greek prefix $\mu\epsilon\tau\alpha$, a material with properties beyond what we expect to find in naturally occurring, or conventional materials. Firstly, it is important to notice that there is a strong dependance on the level at which the phenomenon is observed. In wave propagation phenomena (the ones considered here), it is reasonable to take as a reference scale of observation the *wavelength* λ, i.e. the wave spatial period. In other terms, we can adopt a material Representative Volume Element (RVE) of the size of λ for the *homogenised* material, i.e. a homogenous material with equivalent global properties. If λ is much larger than the scale of variation of the internal microstructure (called *unit cell*, in analogy with atoms at smaller scale) we can consider the RVE as homogeneous, as typically done in continuum mechanics. If this RVE shows unusual physical properties (with respect to conventional materials), we denote it as a metamaterial.

Using this convention, a material is considered as a metamaterial if shows unusual properties at strong subwavelength scale. On the contrary, it is simply an inhomogeneous medium. In the setting of elasticity, inhomogeneous media with a periodic structure are usually called Phononic Crystals (PnC), coming from the term phonon, i.e. the quantum vibration, and crystal which suggest the idea of something regularly repeated in space. Specifically, the term phononic is used to say that the phenomenon involves phonons, i.e. vibrations, and that it occurs at the wavelength scale of the order of the unit cell size. The simplest example of a phononic crystal is a spring mass chain [2, 3].

Since the mechanical concept of continuum is meaningful if the behaviour is strongly subwavelength, metamaterials can be based in essence only on resonance effects, in accord with [4]. This is coherent with the seminal work of the group of Ping Sheng at Hkust [5], that provided the first numerical and experimental evidence of a localised resonant structure for elastic waves propagating in three-dimensional arrays of thin coated spheres. Adopting this interpretation, metamaterials, contrary to phononic crystals, can be even aperiodic. However, they are usually periodically defined, due to the peculiar properties given by periodicity, as well as reduced computational complexity and the existence of analytical closed form solutions. The work of Liu in acoustics opens the door to the design of elastic metamaterials, but this concept was preceded by important discoveries in electromagnetism and optics.

In 1967, the Russian physicist Victor Veselago published a visionary paper [6] in which electromagnetic media with simultaneously negative permittivity ϵ and magnetic permeability μ were shown to be characterized by a negative refractive index of refraction. He showed that a slab of a negative refractive index material can act as a flat convergent lens that images a source on one side to a point on the other. This discovery remained an academic curiosity for almost three decades, until the British physicist. John Pendry [7, 8] proposed effective designs of structuring materials with negative ϵ and μ, and experimental demonstrations were done at the GHz frequencies by a handful of photonic groups in the United States (2000) [9]. As previously emphasised, these materials are structuring at subwavelength scale (typically $\lambda = 10$), hence it is possible to regard them as nearly homogeneous. The term metamaterial, coined by Walser [10], describes such periodic structures when one can average their properties, which are strongly dispersive and anisotropic [4].

Metamaterials, and in general structuring materials, due to their unique properties to guide the propagation of elastic waves and focus their energy, can be adopted to enhance vibration-based energy harvesting [11]. These features have also been exploited for the design of innovative actuators and sensors, and elements of logic circuitry based on the propagation of elastic waves [12, 13].

1.3 Piezoelectricity

Piezoelectricity is the ability of a material to develop an electric charge in response to an applied mechanical stress (direct piezoelectric effect) and vice-versa (inverse piezoelectric effect). The term comes from the Greek words $\pi\iota\epsilon\zeta'\epsilon\iota\nu$, which means to squeeze or press and $\eta'\lambda\epsilon\kappa\tau\rho o\nu$, meaning amber, an ancient source of electric charge. It was firstly discovered by the French physicists Jacques and Pierre Curie in 1880, which demonstrated the direct piezoelectric effect in quartz and in other crystalline materials in the natural state. The piezoelectric effect is due to the peculiar crystalline structure of such materials, with no inversion symmetry. Electrical dipoles within the piezoelectric material are responsible for the creation of a potential difference across the material, when the top and bottom layers are connected to electrodes. When the material is in the unstressed state, it is neutrally charged, since the positive and negative charges balance each other. Contrary, the application of a stress, changes the position of the charges, thus modifying the dipole moment.

The most common piezoelectric materials are ceramics, and specifically Aluminium. Nitride (AIN) and Lead Zirconate Titanate (PZT) due to their piezoelectric and manufacturability qualities. In order to describe the behaviour of piezoelectric materials in the setting of continuum mechanics, the electromechanical coupling enters in the constitutive laws [14]:

$$\begin{cases} T_{ij} = c^E_{ijkl} S_{kl} - e_{kij} E_k \\ D_i = e_{ikl} S_{kl} + \varepsilon^S_{ik} E_k \end{cases} \tag{1}$$

where c^E_{ijkl} is a 4th order elastic stiffness symmetric tensor evaluated at constant electric field; e_{kij} is a 3rd order tensor of the piezoelectric stress constants, and ε^S_{ik} is a 2nd order tensor of the dielectric constants at constant strain. Equation (1) is the *e-form* of the piezoelectric constitutive equations, in which the strain and the electric field are used as coupling variables. Alternatively, it is possible to use the stress instead of the strain, obtaining the *d-form* of the piezoelectric constitutive equations:

$$\begin{cases} S_{ij} = s^E_{ijkl} T_{kl} + d_{kij} E_k \\ D_i = d_{ikl} T_{kl} + \varepsilon^T_{ik} E_k \end{cases} \tag{2}$$

where s^E_{ijkl}, d_{kij} and ε^T_{ik} are the 4th order elastic symmetric compliance tensor at constant electric field, the 3rd order tensor of the piezoelectric strain constants and the 2nd order tensor of the dielectric constants at constant stress respectively. The peculiarity of piezoelectricity is given by 3rd order tensors e_{kij} and d_{kij}, that couple mechanical and electrical quantities.

2 Graded Elastic Metamaterials

A crucial aspect to manipulate waves is the possibility to create *band gaps*, i.e. frequency ranges for which the wave propagation is forbidden. From a different perspective, in those frequency ranges, only *evanescent waves* exist. This means that the wave does not propagate in the medium, and the energy is spatially concentrated in the vicinity of the source. A hallmark of an evanescent wave is that there is no energy flow in the considered region, with a Poynting vector (i.e. the directional energy flux) equal to zero. By operating within or close to a band gap, waves can be guided and focused, and this can be useful to locally enhance the harvestable energy.

Several approaches have been reported in the literature to focus energy using structuring materials, dynamic induced anisotropy or metamaterials, usually combined with smart materials for energy harvesting and sensing. Most of them rely on the creation of mirrors and funnels [11], defects in phononic crystals [15, 16], local resonators [17–19] or lenses [20, 21].

The intensive research on this topic reveals its relevance inside the scientific community, motivating novel approaches to overcome current missing or limits of the existing solutions. While parabolic mirrors, funnels and lenses are able to focus elastic energy from large regions of space, one of the main drawbacks is the capability to use them for finite structures, where waves are intrinsically confined, and boundary reflections can partially or totally alter the wavefield. On the other hand, broadband behaviour is difficult to be achieved; a change in frequency reflects significantly on the wavelength, resulting in ineffective interactions between the wave and the device. On the contrary, devices based on local resonance are less affected by boundary effects and allow for an interaction with large wavelengths. While this is a strong advantage for real applications, the periodic repetition of identical resonators reduces the broadband capability of these systems. Moreover, close to a local resonance bandgap, only the resonators near the input region are expected to store most of the energy, reducing the overall efficiency of the device. For these reasons, the usage of different types of resonators to control and confine waves looks promising, due to their capabilities to take energy from broadband signals at strong subwavelength scale. Moreover, by gradually varying the medium effective properties, it is possible to modify the waves, that are spatially compressed and amassed, with a strong amplitude enhancement.

The advantages of such designs, based on the so-called *rainbow effect* [22–27] have been demonstrated in different contexts, from electromagnetism to acoustics and elasticity, but not extensively for vibration-based energy harvesting. Theoretical models, together with numerical finite elements analyses and experiments, demonstrate that graded metamaterials are excellent candidates for vibration-based energy harvesting. These findings, numerically demonstrated and experimentally validated at meso scale, can be even extended to micro scale, for the implementation of next generation vibration energy harvesting devices.

2.1 Numerical Analyses on Slow Flexural Waves

A graded array is formed by smoothly varying a particular parameter or set of parameters of neighboring elements in space through a specific design of consecutive unit cells. Figure 1a shows an elastic beam with attached an array of cantilevers of linearly increasing lengths.

The beam and the resonators are made of aluminium with Young modulus $E_a =$ 70GPa, Poisson ratio $\nu_a = 0.33$ and density $\rho_a = 2710\text{kg/m}^3$. The beam is 500mm

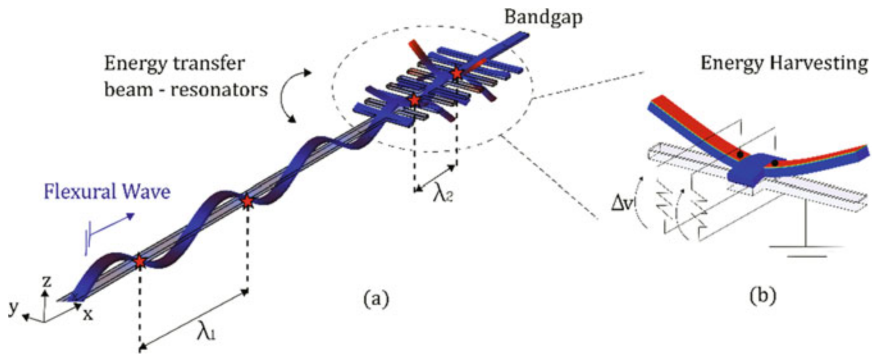

Fig. 1 Schematic of the graded linear array of resonators for energy harvesting. **a** By exciting the waveguide with a flexural wave, energy is efficiently transferred to the array of resonators. Such interaction reduces both the amplitude and the wavelength of the waves along the waveguide within the array. Leveraging on this energy transfer mechanism, the elastic energy in the resonators can be used for piezoelectric energy harvesting (**b**)

long, 7mm wide and 2mm thick. The array is made of 9-unit cells of size $a = 15$mm, with a linear grading law for the lengths of the resonators, from 16.75 to 27.75mm, resulting in a grading angle of approximately 5.2°.

By spatially varying the resonance frequency of the resonators attached to the beam [28–32] waves slow down with a reduction of both amplitude and wavelength. Differently with respect to the acoustic wave compression [24], the array of resonators progressively absorbs energy from the beam, allowing for a wave amplitude reduction in the beam inside the array [29].

The concurrent amplitude and wavelength reduction is a hallmark of energy transfer between the main structure and the resonators and can be used for energy harvesting purposes. Figure 1b shows the arrangement of piezoelectric patches and the electric circuit employed to transduce electric due to resonators motion in a tailored position along the beam. This is obtained exploiting the 31-mode of the piezoelectric patches connected to a resistive load, to effectively harvest the elastic energy stored inside the target resonators.

The wave propagation properties of the system can be rigorously inferred by looking at the dispersion curves of a given cell inside the array. Provided the grading is gentle enough and provided the number of unit cells is sufficient, the global behaviour of the whole array can be deduced from the local dispersion curves of the constituent elements [26]; in this way, the desired spatial selection by frequency, i.e. the rainbow behaviour of the system, is determined from the locally periodic structure at a given position. Figure 2a shows the numerical dispersion curves for the cell number 7 (where the cell numbering in the array goes from 1 for the shortest to 9 for the longest). These dispersion curves are computed along the 1D irreducible Brillouin Zone using the finite elements software Abaqus®, that incorporates the Bloch phase shift via Bloch-Floquet periodic boundary conditions. The spatial properties of the wavefield can be deduced from the local dispersion curves at a given frequency,

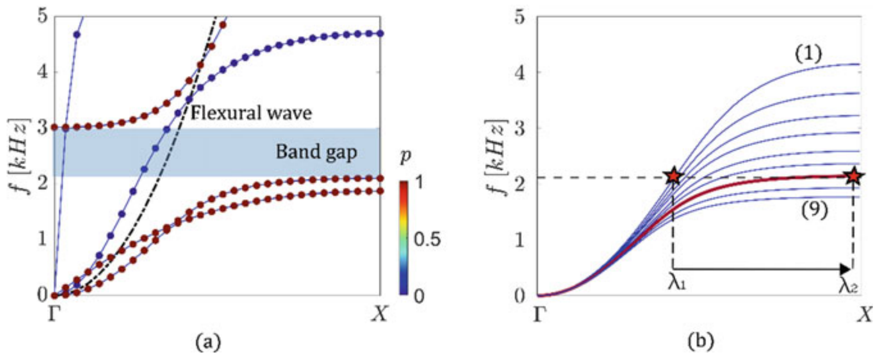

Fig. 2 **a** Numerical dispersion curves for a periodic array of identical resonators of fixed length (in this case the resonator length is 25 **mm**); scatter points colors represent the wave polarization (red corresponding to the vertical motion, i.e. bending of the resonator). **b** In-phase bending mode for different resonators inside the array. Moving from short (1) to long (9) resonators at a given frequency the group velocity and the wavelength decrease, until the bandgap opening

as shown in Fig. 2b. By increasing the length of the resonators along the spatial dimension, i.e. moving from (1) to (9), the dispersion curves shift towards lower frequencies. As a result, by fixing the frequency, the group velocity, $c_g = \partial\omega/\partial\kappa$, smoothly reduces until zero. Such effect allows to slow down elastic waves inside the array and to confine waves in different positions depending on frequency. In addition, since the zero-group velocity mode occurs at the band edge, it can couple with a backward propagating mode, which is typical of rainbow reflection [30].

2.2 Experiments on Graded Waveguides for Energy Harvesting

A peculiar property of this system is the capability to slow down array guided waves as they transverse the array. Such phenomenon allows for a longer interaction between the wave and the resonators, locally increasing the amplitude of the wavefield inside the resonators [32]. To validate this effect, and the implications in terms of energy harvesting, experimental tests are performed in narrow and broad-band frequency regime. Figure 3 shows the experimental setup used for testing.

At the right boundary, a LDS v406 electrodynamic shaker is rigidly connected to the beam through a thick aluminium plate with high strength adhesive, to provide excitation. At the opposite boundary, the structure is suspended through elastic cables that do not affect the dynamics of the system. The wavefield on the elastic beam is measured through a Polytec 3D Scanner Laser Doppler Vibrometer (SLDV), which is able to separate the out-of-plane velocity field in both space and time. The input is synchronously start with the acquisition which, in turn, is averaged in time to decrease the noise. We experimentally show the rainbow effect in the linear array by

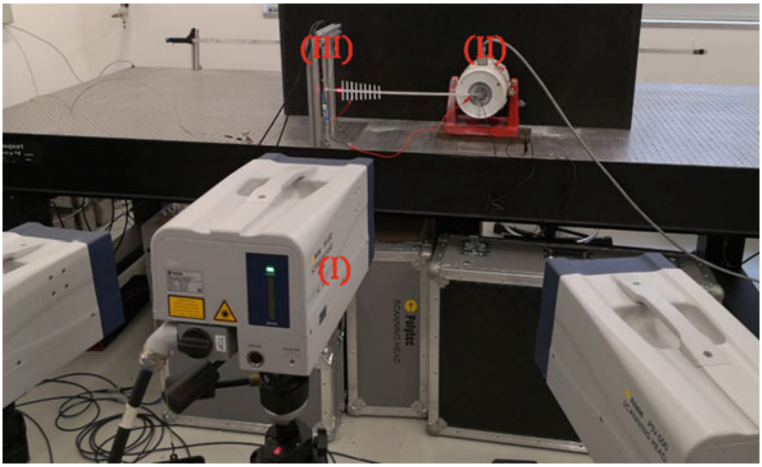

Fig. 3 Experimental setup. The wavefield is measured using a 3D laser vibrometer (I). The waveguide is excited using an electrodynamic shaker (II), while the opposite side is suspended through elastic cables (III)

applying a broadband frequency sweep in the range 1.6−4.2 kHz. Figure 4a shows the space-frequency analysis of the experimental data. Depending on the frequency, waves stop at different spatial positions, corresponding to the band gap opening.

Moreover, we notice that the amplitude and the wavelength of the mode shapes decrease inside the array, until the amplitude vanishes in correspondence of the position of the resonating element, which is well predicted by numerical results (dashed white line). We then quantify the advantages of such mechanism for energy

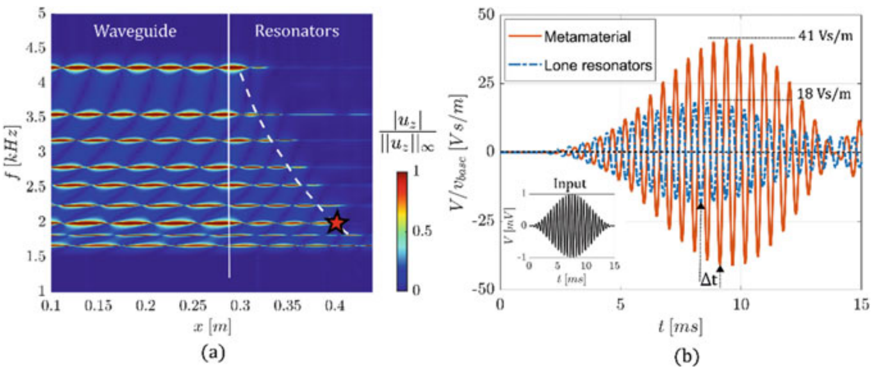

Fig. 4 Space-frequency analysis of the experimental data superimposed to numerical predictions (dashed white line) from dispersion curves. **b** Open circuit output voltage for the graded array and single cell configuration (i.e. the same harvester but without the graded metamaterial), corresponding to the input excitation frequency marked with the red star in (**a**). The graded metamaterial amplifies the output voltage, i.e. the energy harvesting capabilities of the device

harvesting by placing piezoelectic PZT-5H patches ($E_p = 61$ GPa, $\nu_p = 0.31$, $\rho_p =$ 7800 kg/m^3, dielectric constant $\frac{\varepsilon_{33}^T}{\varepsilon_0} = 3500$, and piezoelectric coefficient $e_{31} = -9.2$ C/m^2) at the position of the 7th cell, denoted with the red star in Fig. 4a.

Figure 4b shows the mean output open circuit voltage for the single cell (i.e. the same resonators but without the graded metamaterial), and graded array, normalized by the measured input velocity, to make sure that the results are displayed under the same conditions. We observe that the graded array gives a mean normalized peak voltage of 41 Vs/m which is 56% higher than the single cell. We notice that such peak is reached with a delay Δt of approximately 1.3 ms, which is justified by the smooth reduction of the group velocity inside the array.

3 Conclusions

In conclusion, potential advantages in using graded metamaterials for efficient elastic energy confinement have been demonstrated both numerically and experimentally. The graded metamaterial capability of slowing down waves enables a strong energy transfer to the resonators, which then reflects in enhanced energy harvesting performances. This mechanism can be adopted to create more efficient energy harvesting systems able to self-power, or at least compensate, the power consumption of small electronic devices, using ambient waste energy from vibrations. The possibility to substitute batteries and their chemical waste, with fully sustainable vibration-based energy harvesting systems can play an important role toward Goal 7 of the SDGs for the next generation of affordable and clean energy sensors and small electronic devices.

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