Knowing by Drawing: Geometric Material Models in Nineteenth Century France

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Introduction

The university collections of mathematical models have aroused a growing interest since the turn of the 21th century. In Paris, Institut Henri Poincaré has recently enhanced the collection it had inherited when it was created in 1928 from the older cabinet of mathematics of the Sorbonne.[1] Several models have been restored through crowdfunding processes, both permanent and temporary exhibitions have been set up, the models that had fascinated the surrealists Man Ray and Max Ernst in 1934 have been loaned to several art museums,[2] the publication of a collective volume has been supported by the institute,[3] as well as the production of a documentary film (see Figs. 1, 2 and 3).[4]

[1] About this collection, see: Jean Brette, "La collection de modèles mathématiques de la bibliothèque de l'IHP," *Gazette des mathématiciens* 85 (July 2000): 4–8.

[2] See especially: the temporary exhibition "Le surréalisme et l'objet" set up at Centre Pompidou in Paris from 30 October 2013 to 3 March 2014. On Man Ray's photographs and paintings of several models displayed at IHP, see: Isabelle Fortuné, "Man Ray et les objets mathématiques," *Études photographiques* 6 (May 1999): 1–12; Edouard Sebline and Andrew Strauss, "Man Ray à l'Institut Henri Poincaré: des objets mathématiques aux équations shakespeariennes," in *Objets mathématiques*, ed. Institut Henri Poincaré (Paris: CNRS Éditions, 2017), 152–62.

[3] Institut Henri Poincaré, ed., *Objets mathématiques* (Paris: CNRS Éditions, 2017).

[4] *Man Ray et les équations shakespeariennes*, directed by Quentin Lazzarotto (Paris: Institut Henri Poincaré, 2019).

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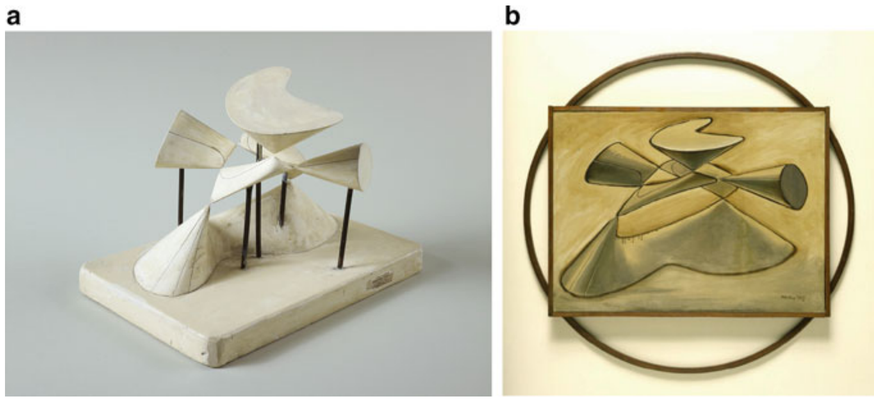

Fig. 1 **a** Model of a Kummer surface with eight double points, edited by Brill-Schilling in Halle. This object is one of the models of the collection of Institut Henri Poincaré which was photographed by Man Ray in 1934 and named after a play by Shakespeare: "King Lear" © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: © Anne Chauvet. All rights reserved **b** Man Ray, "Shakespearean Equations: King Lear," 1948, oil on canvas, Hirshhorn Museum and Sculpture Garden, Washington, DC. © Man Ray2015 Trust/ VG Bild-Kunst, Bonn 2021, all rights reserved

Fig. 2 Model of a quartic surface with nine real double points, designed by Joseph Caron in Paris. This model was photographed by Man Ray in 1934 and named after Shakespeare's play "All's Well That Ends Well." © Collections de l'Institut Henri Poincaré, all rights reserved

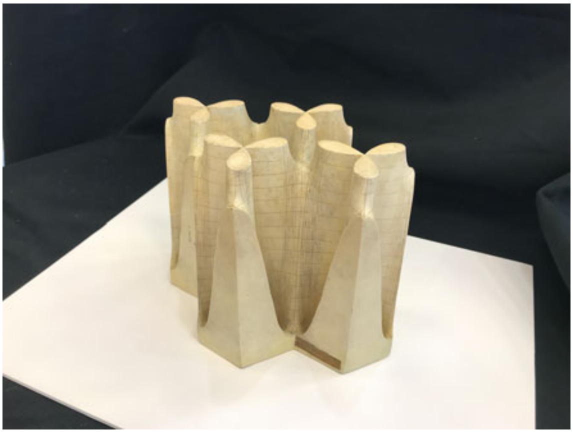

Fig. 3 Model of an elliptic function, designed by Ludwig Brill in Darmstadt. This model was photographed by Man Ray in 1934 and named “The Merry Wives of Windsor.” © Collections de l’Institut Henri Poincaré. Photo: Frédéric Brechenmacher, all rights reserved

In addition to the efforts of the mathematical community for preserving, enhancing, and publicizing collections of models, several publications and conferences have tackled various issues raised by the history of these collections. Most of these works have focused on what we shall designate in this paper as the ‘models of higher mathematics’ that were designed at the turn of the twentieth century and gave a material form to mathematical objects that were taught at the highest levels of mathematical education, such as the university lectures on the ‘higher geometry’ of cubic surfaces and their applications to mechanics.[5] These publications have usually identified two distinct periods during this golden age of mathematical models. The first, in the 1860 and 1870s, saw the emergence of models of higher geometry thanks to the growing individual commitment of various practitioners of mathematics, including several prominent mathematicians, especially in the United Kingdom, with James Joseph Sylvester, Arthur Cayley, Olaus Henrici, or Alicia Boole Stott, and in Germany, with Julius Plücker, Ernst

[5] See: Gerd Fischer, ed., *Mathematische Modelle* (Braunschweig: Vieweg + Teubner, 1986); Peggy Kidwell, “American Mathematics Viewed Objectively: The Case of Geometric Models,” in *Vita Mathematica*, ed. Ronald Calinger (Washington: Mathematical Association of America, 1996), 197–208; Herbert Mehrtens, “Mathematical Models,” in *Models: The Third Dimension of Science*, ed. Soraya de Chadarevian and Nick Hopwood (Stanford: Stanford University Press, 2004), 276–306; Irene Polo-Blanco, “Theory and History of Geometric Models” (Phd diss., University of Groningen, 2007); Jeremy Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe, ed. “History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences,” *Oberwolfach Reports* 12, no. 4 (2015): 2767–858; Livia Giarcardi, “Models in Mathematical Teaching in Italy (1850–1950),” in *Mathematics and Art III. Visual Art and Diffusion of Mathematics*, ed. Claude P. Bruter (Paris: Cassini, 2015), 11–38; François Apéry, “Caron’s Wooden Mathematical Models,” in *Mathematics and Art III. Visual Art and Diffusion of Mathematics*, ed. Claude P. Bruter (Paris: Cassini, 2015), 39–48; Anja Sattelmacher, “Präsentieren: Zur Anschauungs- und Warenökonomie mathematischer Modelle,” in *Sammlungsökonomien. Vom Wert wissenschaftlicher Dinge*, ed. Nils Güttler and Ina Heumann (Berlin: Kadmos, 2016), 131–55; Michael Friedman, *A History of Folding in Mathematics: Mathematizing the Margins* (Basel: Birkäuser, 2018), 127–205.

Eduard Kummer, Christian Wiener, Alfred Clebsch, Hermann Amadeus Schwarz, Felix Klein, and Alexander Brill. During the second period, the manufacturing of models of higher mathematics developed in Germany, starting with the publishing house of Ludwig Brill in Darmstadt in the 1880s, later merged with the editor Martin Schilling in Halle in 1899.[6] The production of models eventually culminated at the beginning of the twentieth century with semi-industrial manufacturers such as Brill/Schilling, Teubner, Mehrmittel anstalt, J. Ehrhard & Ci, and Polytechnisches Arbeits-Institut. These editors have disseminated mathematical models in universities and technological institutes all over Europe and the U.S.A. They boasted very large and diversified catalogues, which covered all branches of mathematics, such as geometry, mechanics, topology, and analysis, as well as of their applications to electricity, thermodynamics, shipbuilding, gearing, etc.

Several historical investigations have tackled the issue of the motivations that led to the development of such large and diversified collections. As a matter of fact, the heuristic value of models of higher mathematics for academic research seems to have been very limited, aside from a few, even though iconic, examples, such as the error of reasoning Felix Klein discovered by observing a model,[7] the counter-example Georges Brunel exhibited in a public demonstration in 1896 to a theorem of topology recently stated by Henri Poincaré,[8] or the key role played by the concrete folding manipulations of cardboard models in Henri Lebesgue's work on integration and developable surfaces.[9] Yet, most models of higher mathematics were actually designed only after algebraic research had been performed with pen and ink on the plane surfaces of papers or blackboards.[10]

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[6] Martin Schilling, ed., *Catalog mathematischer Modelle für den höheren mathematischen Unterricht* (Halle: Martin Schilling, 1903).

[7] David Rowe, "On Franco-German Relations in Mathematics, 1870–1920," in *Proceedings of the International Congress of Mathematicians Rio de Janeiro 2018,* ed. Boyan Sirakov, Paulo Ney de Souza, and Marcelo Viana (Singapore: World Scientific, 2018), 21–36.

[8] In a public demonstration at the Bordeaux Society of physical sciences on 23 January 1896, Brunel gave a counter example to Poincaré's statement that any closed surface is a two-sided surface by displaying a model of a closed surface with just one side. See: Pierre Duhem, "Georges Brunel," *Association amicale des anciens élèves de l'Ecole normale supérieure* (1901): 103–16.

[9] Lebesgue was especially influenced by Darboux's lectures on higher geometry at the Sorbonne which, as shall be seen later, went along with sessions of practical works on mathematical models designed by Darboux's assistant, Joseph Caron. On Lebesgue's cardboard folding approach to integration, see: Sébastien Gandon and Yvette Perrin, "Le problème de la définition de l'aire d'une surface gauche: Peano et Lebesgue," *Archive for History of Exact Sciences* 63 (2009): 665–704.

[10] For a discussion of this issue in connection with the models of the 27 lines of a cubic surface, see: François Lê, "Around the History of the 27 Lines upon Cuvic Surfaces: Uses and Non-uses of Models," in *History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences. Oberwolfach Reports* 14, no. 4, ed. Jeremy J. Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe (2015): 2794–98, and Anja Sattelmacher, "Zwischen Ästhetisierung und Historisierung: Die Sammlung geometrischer Modelle des Göttinger mathematischen Instituts," *Mathematische Semesterberichte* 61, no. 2 (2014): 131–43.

In contrast to the limited heuristic value of models for mathematical research, most historical works have highlighted specific pedagogical values attributed to models, especially the ones of vizualization and manipulation. To be sure, the pedagogical values of models have been highly praised by several mathematicians,[11] especially in the international institutions that have been established at the turn of the twentieth century for promoting debates on mathematical education, such as the journal *L'Enseignement mathématique*, founded in 1899, and the International Commission on Mathematical Instruction established in 1908. Yet, historical sources about the actual pedagogical use of models of higher mathematics are scarce. Moreover, these sources are not as apologetic as public discourses. The use of models of higher geometry in classrooms or amphitheatres did not only raise practical difficulties—since models were often bulky, fragile, and costly—but the value of vizualization associated with them also conflicted with the important preliminary knowledge most models required from the students before they would be able to vizualize anything. Further, several teachers opposed the value of vizualization with the one of rigour associated with mathematical proofs performed on the blackboard.[12] This situation makes it difficult to assess the collective dimension of the pedagogical use of models of higher mathematics, beyond the individual commitment of iconic individuals, such as Klein, who had very strong and specific ideals about the roles of vizualization and experimentation, not only in mathematical research and education, but also in the very epistemological nature of mathematics.[13]

Mathematical models therefore call for further historical investigation on the social and cultural practices associated with models beyond the roles played by a few individuals at the turn of the twentieth century. This paper aims at shedding new light on such collective dynamics by investigating a period of time longer than the one of the golden age of models of higher mathematics. But this broader time scale forces us to restrict our corpus to a specific national setting. We shall, therefore, focus on the use of mathematical models in France from the late eighteenth century to the turn of the twentieth century.

[11] See in particular: Walther Dyck's opening speech in the first ever individual mathematics exhibition of models, apparatus, and instruments he organized in Munich on the occasion of the Annual Conference of the German Mathematician Society in 1893. In this speech, Dyck distinctly conceded the boundaries and restrictions of mathematical artifacts by explaining that, although many of the models did not have a practical use, they had an instructional purpose. Ulf Hashagen, *Walther von Dyck (1856–1934): Mathematik, Technik und Wissenschaftsorganisation an der TH München* (Stuttgart: Steiner, 2003), 431–36.

[12] See: Stanislas Meunier, "Reliefs à pièces mobiles destinés à l'enseignement de la géométrie descriptive," *La nature* 37 (Febrary 1874): 166–67; Paul Staeckel, "La préparation mathématique des ingénieurs dans les différents pays. Rapport général," *L'enseignement mathématique* 16 (1914): 307–28, here: 320.

[13] The importance attributed by Klein to geometric models as an *Anschauungsmittel* (illustration aid) in research and teaching of mathematics has especially been well studied. See David E. Rowe, "Klein, Hilbert, and the Göttingen Mathematical Tradition," *Osiris* 2nd Series 5 (1989): 186–213; David E. Rowe, "Mathematical Models as Artefacts for Research: Felix Klein and the Case of Kummer Surfaces," *Mathematische Semesterberichte* 60, no. 1 (2013): 1–24; Sattelmacher, "Zwischen Ästhetisierung und Historisierung;" Sattelmacher, "Präsentieren."

The renewed interest for mathematical models since the beginning of the 21th century has raised new issues about the specific situation of France. Several papers have highlighted the leading role of German mathematicians and manufacturers in the development of collections of models after the 1860s, while previous historical works had emphasised the legacy of Gaspard Monge's descriptive geometry, in the tradition of which Klein himself claimed he had been raised by his professor Plücker,[14] as well as the innovative models designed by Théodore Olivier in the 1840.[15] Both Monge and Olivier have therefore tended to be considered as French precursors of a movement that would eventually blossom in Germany, in a transition that may be understood in view of the larger historiographical perspective of a shift in the balance of power after the 1870 Franco-Prussian war. With regard to models, this idea of a transition between France and Germany seems to be underpinning the popularity acquired by the episode of Klein's trip to Paris in 1870,[16] during which the latter expressed his enthusiasm when discovering the collection of Olivier's string models displayed at the Conservatoire national des arts et métiers.[17] Even so, other collective dimensions have to be considered in between the very small scale of the individual experience and the very large one of the balance of European powers.

In this paper, we shall discuss the use of models in France in the framework of a broad mathematical practice, which was far from limited to the innovations of individuals such as Monge and Olivier. Over the course of the eighteenth and nineteenth centuries, the design and the use of mathematical models resulted from the strong belief that teaching geometry to engineers and technicians required to practice drawing, and more precisely model drawing, in contrast to other pedagogical methods such as plenary lectures: "it is in the drawing room that the master will judge the fruits of his teaching; it is there that he will recognize if the seeds he sowed from the pulpit chair has fallen on a good ground or on a stony soil."[18]

Teaching geometry, it was believed, required to "educate the hand and the eye," which was precisely the main pedagogical value attributed to the practice of drawing. On this issue, let us quote Jean-Jacques Rousseau's 1762 *Émile, or on*

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[14] René Taton, *L'œuvre scientifique de Monge* (Paris: Presse Universitaires de France, 1951), 240.

[15] Joël Sakarovitch, "Théodore Olivier, Professeur de Géométrie descriptive," in *Les professeurs du Conservatoire national des arts et métiers, dictionnaire bibliographique 1794–1955*, ed. Claudine Fontanon and André Grelon (Paris: INRP/CNAM, 1994), 326–35.

[16] Felix Klein, *Gesammelte mathematische Abhandlungen,* vol. 2: *Anschauliche Geometrie. Substitutionsgruppen und Gleichungstheorie. Zur mathematischen Physik*, ed. Robert Fricke and Hermann Vermeil (Berlin: Springer, 1922), 3.

[17] Klein, *Gesammelte mathematische Abhandlungen*, vol. 2. The reviewer of this volume for *L'enseignement mathématique* especially focused on the episode of the discovery of the Olivier collection. See Grace Chisholm Young, "F. Klein–Gesammelte mathematische Abhandlungen," *L'enseignement mathématique* 24 (*1924–1925*): 167–69. See also: Felix Klein, *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, vol. 1, ed. Richard Courant and Otto Neugebauer (Berlin: Springer, 1979 [1926]), 63–93.

[18] Alphonse Bernoud, "Chr. Beyel. Ueber den Unterricht in der darstellenden Geometrie," *L'enseignement mathématique* 2 (1900): 63. All translations from French to English by the author, Frédéric Brechenmacher, if not stated otherwise.

Education, one of the main inspiration of the new national system of education set up during the French Revolution:

> One cannot learn to estimate the extent and size of bodies without at the same time learning to know and even to copy their shape; for at bottom this copying depends entirely on the laws of perspective, and one cannot estimate distance without some feeling for these laws. All children in the course of their endless imitation try to draw; and I would have Émile cultivate this art; not so much for art's sake, as to give him exactness of eye and flexibility of hand. Generally speaking, it matters little whether he is acquainted with this or that occupation, provided he gains clearness of sense-perception and the good bodily habits which belong to the exercise in question. So I would take good care not to provide him with a drawing master, who would only set him to copy copies and draw from drawings. Nature should be his only teacher, and things his only models […]. I would even train him to draw only from objects actually before him and not from memory, so that, by repeated observation, their exact form may be impressed on his imagination, for fear that he should substitute absurd and fantastic forms for the real truth of things and lose his sense of proportion and his taste for the beauties of nature.[19]

The above quotation exemplifies a very strong pedagogical ideal associated with model drawing: the idea that the knowledge of forms and proportions required a direct contact with Nature with no mediation by any teacher, in contrast with the forms of knowledge transmitted by reading textbooks and listening to lectures. This ideal would have a lasting influence on the teaching of mathematics. From the eighteenth century to the turn of the twentieth century, mathematical models were usually considered as substitutes to natural forms and supported pedagogical methods that promoted action learning, relegated the role of the teachers to the one of supervisors, or even praised the mutual instruction of students by students: "Nature should be the only teacher." Geometric models, therefore, challenged the role of teachers in the teaching of mathematics.

Exactness of eye was a key issue in the philosophy of the Enlightenment: it was required for both the observation of sensible objects and the sense of proportion, which were the main instruments of knowledge for John Locke or Étienne

[19] Jean-Jacques Rousseau, "Émile, ou De l'éducation," in Jean-Jacques Rousseau, *Œuvres complètes de J.-J. Rousseau*, vol. 5, ed. Louis Barré (Paris: J. Bry, 1856), 97–98: "On ne saurait apprendre à bien juger de l'étendue et de la grandeur des corps, qu'on n'apprenne à connaître aussi leurs figures et même à les imiter; car au fond cette imitation ne tient absolument qu'aux lois de la perspective ; et l'on ne peut estimer l'étendue sur ses apparences, qu'on n'ait quelque sen timent de ses lois. Les enfants, grands imitateurs, essaient tous de dessiner : je voudrais que le mien cultivât cet art, non précisément pour l'art même, mais pour se rendre l'oeil juste et la main flexible; et, en général, il importe fort peu qu'il sache tel ou tel exercice, pourvu qu'il acquière la perspicacité du sens et la bonne habitude du corps qu'on gagne par cet exercice. Je me garderai donc bien de lui donner un maître à dessiner, qui ne lui donnerait à imiter que des imitations, et ne le ferait dessiner que sur des dessins : je veux qu'il n'ait d'autre maître que la nature, ni d'autre modèle que les objets. […] Je le détournerai même de rien tracer de mémoire en l'absence des objects, jusqu'à ce que, par des observations fréquentes, leurs figure exactes s'impriment bien dans son imagination; de peur que, substituant à la vérité des choses des figures bizarres et fantastiques, il ne perde la connaissance des proportions et le goût des beautés de la nature.". English translation: Jean-Jacques Rousseau, "Rousseau's *Emile, or On Education*," trans. Barbara Foxley (Dover Publications: New York, 2013 [1911]), 128–29.

Condillac. Seeing was directly associated with intelligence: exactness of eye was a preliminary to the faculty of judgment because judging required comparison. The teaching of drawing was therefore intimately associated to that of geometry throughout the eighteenth and nineteenth century. As stated by the French report on the exhibition of geometric drawing at the 1862 world fair in London: "it is not sufficiently understood that drawing trains the eye, develops powers of observation, makes the finger more delicate […]. Intimately linked with a few notions of geometry, drawing is useful in both the field and the workshop."[20] Further, as claimed by the mathematician and pedagogue Sylvestre François Lacroix, learning geometry through model drawing aimed at "training both judgment and the eye."[21]

Learning by drawing was associated with pedagogical issues very different from the values of visualization and manipulation that would be attributed to the models of higher geometry after the 1860s. Actually, the emergence of these models broke up with the long tradition of associating geometry with drawing, even though this rupture was less sudden in France where one of the main proponents of the models of higher geometry, Gaston Darboux, introduced the practice of drawing in the University of Paris in the 1870s. As we shall see in the first section of this paper, the specificity of the use of models in France was largely due to the distinction between universities and grandes écoles, and more precisely to the centrality of École polytechnique. This school indeed played a key role in the organization of the mathematical instruction in France because, on the one hand, of its centralized and national character as an institution, and, on the other hand, because its alumni dominated the mathematical sciences in France throughout much of the nineteenth century.

Several detailed investigations have already been devoted to the connections between the teaching of mathematics and drawing in France in the eighteenth and nineteenth centuries.[22] These works have especially shown the key role geometry played in the decline of the pedagogical approach promoted by the Académie des Beaux-Arts, which consisted in placing the model of the human figure at the core of the teaching of drawing. Yet, little attention has been paid to the specific models that were designed for the teaching of geometric drawing: it is on this specific issue that we shall focus on this paper. In order to investigate the specific French tradition of learning mathematics by model drawing, we shall pay a specific attention to the materiality of mathematical models. As we shall see in the second section of this paper, steel and string models were intertwined with industrialization, while plaster ones inherited from the arts of fortification, cardboard

[20] Jean Rapet, "Situation de l'enseignement chez les diverses nations représentées à l'exposition. Matériel scolaire," in *Exposition universelle de Londres de 1862. Rapports des membres de la section française du jury international sur l'ensemble de l'exposition,* vol. 6, ed. Michel Chevalier (Paris: N. Chaix, 1862), 16–79, here 69.

[21] Silvestre François Lacroix, *Essais sur l'enseignement en général et sur celui des mathématiques en particulier*, 4th ed. (Paris: Bachelier, 1838), 321: "On exerce ainsi le jugement en même temps que l'œil […]".

[22] For a synthetic monograph on the teaching of drawing and of geometry in the eighteenth and nineteenth century, see Renaud d'Enfert, *L'enseignement du dessin en France. Figure humaine et dessin géométrique (1750–1850)* (Paris: Belin, 2003).

models supported the ideal of raising the mathematical instruction of the greatest number of children, while wooden models were, for a time, associated with the idea that the élite mathematicians of École normale supérieure had to be trained in handling saws and planes.

Specific attention to the materiality of models is also required by the very nature of the practices associated with models, such as drawing, fabricating, manipulating, observing, surveying, leveling, etc. Because of this practical nature, the use of models was usually not formalized by any textual knowledge. It is mainly for this reason that, as noted above, historical sources about the actual uses of models are scarce, especially when looking at the usual sources for the history of mathematics in the nineteenth century, such as books, periodical publications, and epistolary correspondence. To be sure, material models and épures, i.e. geometric drawings, form a rich pool of historical sources. But we shall nevertheless also look for textual historical sources by investigating the many reports that were devoted to technical innovations, crafts, and skills by the institutions involved in the industrialization of France, such as the Société d'encouragement pour l'industrie nationale, as well as local industrial exhibitions, and world fairs.

In the case of model drawing, in particular, the very epistemological essence of this activity was the transmission of a non-textual form of geometric knowledge, one that required practical work, apprenticeship and companionship. Reading texts or attending lectures could not subsume the knowledge associated with drawing: drawing was knowing. The investigation of skills, know-how, tacit knowledge, procedures, and scientific practices is a vivid field of research in the history of science. It has raised specific issues about historical sources as well as specific methodologies, such as that of reproducing experiments in order to access the material conditions, interpretations, and outcomes that emerge through investigations into matter. Such methodologies may be well adapted to the epistemological investigation of the type of mathematical knowledge associated with model drawing, in the interplay between the act of reading and attending lectures on descriptive geometry, and that of imitating, drawing and experimenting on models. This issue nevertheless goes beyond the scope of the present paper. But we shall emphasize a classical result in the history of non-textual knowledge: because of the absence of texts, and especially textbooks, non-textual knowledge cannot be dissociated from cultural practices and communities, it especially requires a direct transmission and may thus decline rapidly. In the third section of this paper, we shall therefore pay specific attention to identifying the communities associated with the practice of learning mathematics by model drawing, and whether this practice declined with the emergence of other forms of interplays between mathematics, models, and vizualization, such as those promoted by models of higher mathematics after the 1860s.

These discussions will eventually lead us in the fourth, and final, section of this paper: raising the issue of how the history of mathematical models may contribute to mathematical modelization. The etymology of the French 'épures,' which comes from 'épurer,' i.e. to refine, points to a typical activity in craftsmanship which consists in removing impurities or unwanted elements and which, when applied to drawing, involves a form of mathematisation that was theorized by Monge with the creation of descriptive geometry. We shall especially discuss how models were associated with a specific evolution of mathematisation in the view of the emergence of 'the graphical method,' which, at the turn of the twentieth century, would cover a very large range of graphical techniques, instruments, forms of vizualization, and knowledge.

Geometry and Model Drawing

The etymological origin of the word 'model' in the latin 'modello,' which derived from 'modulus,' i.e. measure or rhythm, highlights the ancient connections between mathematics and the arts in the uses of models for drawing, engraving, painting, sculpting, or constructing.[23] Renaissance humanism especially promoted such connections in the training of engineers. Construction drawing was developed in intimate connection with geometry and applied in various major concerns such as architecture, fortifications, cartography, wood and stonecutting, shading, shipbuilding, bridge building, and others in both civil and military engineering.[24] Model drawing thus came to be associated with a specific type of mathematical education through companionship and apprenticeship, which promoted practice and activity as opposed to, or as a complement to, reading books and attending lectures.

Drawing, Models, and Analysis

In the eighteenth century, drawing was considered as a critical factor for the progress of industry.[25] It participated in the promotion of manual work by the Encyclopedists who especially aimed at raising the value of the mechanical arts to the status of liberal arts. Drawing aimed not only at representing but also at explaining an operating process or a manufacturing process, it came to be considered as a kind of universal language and, until the nineteenth century, engineers had therefore to be "artist-engineers."[26]

The tradition of 'compagnonnage,' or fraternity, associated with 'corporations,' or guilds, was called into question during the age of Enlightenment. The creation of drawing schools and engineering schools played a key role in this interrogation: these schools indeed aimed at providing a vocational training for craftsmen and

[23] Peter Jeffrey Booker, *A History of Engineering Drawing* (London: Northgate, 1979); Antoine Picon, "Architecture, sciences et techniques. Problématiques et méthodes," *Les cahiers de la recherche architecturale et urbaine* 9–10 (January 2002): 151–60; Anne Coste and Joël Sakarovitch, "Construction History in France," in *Construction History. Research Perspectives in Europe*, ed. Antonio Becchi, Massimo Corradi, Federico Foce, and Orietta Pedemonte (Turin: Kim Williams Books, 2004).

[24] Joël Sakarovitch, *Epures d'architecture: de la coupe des pierres à la géométrie descriptive, XVIe-XIXe siècle* (Basel: Birkhäuser, 1998).

[25] See d'Enfert, *L'enseignement du dessin*, 32–35.

[26] Antoine Picon and Michel Yvon, *L'ingénieur artiste, dessins anciens de l'École des ponts et chaussées* (Paris: Presses des Ponts et Chaussées, 1989).

engineers as a complement to apprenticeship in workshops.[27] Yet the pedagogical method of fraternity would remain vivid in these schools even after the abolition of guilds and would especially play a key role in the mathematical training of engineers.

French military engineering schools, in particular, attributed a central role to mathematics in both the selection and the training of their students. The view that among all the sciences necessary to military engineers, mathematics have the most considerable rank became common in eighteenth century France.[28] The interest in mathematics arose not only because of its direct usefulness: mathematics, and especially instruction in mathematics, was seen to have valuable moral uses. It sharpened powers of reasoning and inculcated an orderly manner of thinking. Furthermore, the learning process of mathematics was considered to foster habits of work, self-control, and discipline. Mathematical education was also instrumental to the hierarchy between engineers and craftsmen while both were trained in model drawing.[29] The teaching of drawing actually aimed at both raising the qualification of the workforce and at disciplining it[30]: the practice of model drawing, in particular, was associated with the values of accuracy, heed, assiduity, and obedience.[31]

In the eighteenth century, the teaching of drawing was normalized as a progression from the simple to the complex. Models played a key role in a three-step progression: the students had first to copy drawings or prints, i.e. models of two dimensions, in order to acquire exactness of eye ("coup d'œil juste"), before passing to the "ronde-bosse," i.e. three dimensional models, and eventually to living and natural models. When their training was complete, students were supposed to be able to decompose a complex figure into a series of simple elements, corresponding to the models they had been trained with, and to recompose a complex drawing from its elementary parts (see Fig. 4).[32] This pedagogical method therefore followed the process of decomposition/recomposition that was formalized as the method of 'analysis' by Enlightenment philosophers such as Locke and Condillac, and which would be especially influential for the development of mathematical education.

In the tradition of the Académie des Beaux-Arts, the human figure played a key role in the teaching of drawing and most of the models that were used in drawing

[27] d'Enfert, *L'enseignement du dessin*, 43–44.

[28] Paris de Meyzieu, "Ecole royale militaire," in *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers, par une Société de Gens de lettres*, vol. 5, ed. Denis Diderot and Jean Le Rond d'Alembert (Paris: Briasson, David, Le Bretton, Durand, 1755), 307–13.

[29] On the training of craftsmen in the eighteenth century, see: Antoine Léon, "Une forme typique de l'enseignement technique à la fin du XVIII[e] siècle: Les écoles des dessin," *Bulletin du C.E.R.P.* 12, no. 1 (1963): 67–69; Arthur Birembaut, "Les écoles gratuites de dessin," in *Enseignement et diffusion des sciences en France au XVIII[e] siècle*, 2nd ed., ed. René Taton (Paris: Hermann, 1986), 441–76; Yves Deforge, "Des écoles de dessin en faveur des arts et métiers," *Les Cahiers d'histoire du CNAM* 4 (July 1994): 14–30.

[30] Steven L. Kaplan, "L'apprentissage au XVIII[e] siècle: le cas de Paris," *Revue d'histoire moderne et contemporaine* 40, no. 3 (1993): 436–79.

[31] d'Enfert, *L'enseignement du dessin*, 47.

[32] Ibid., 56.

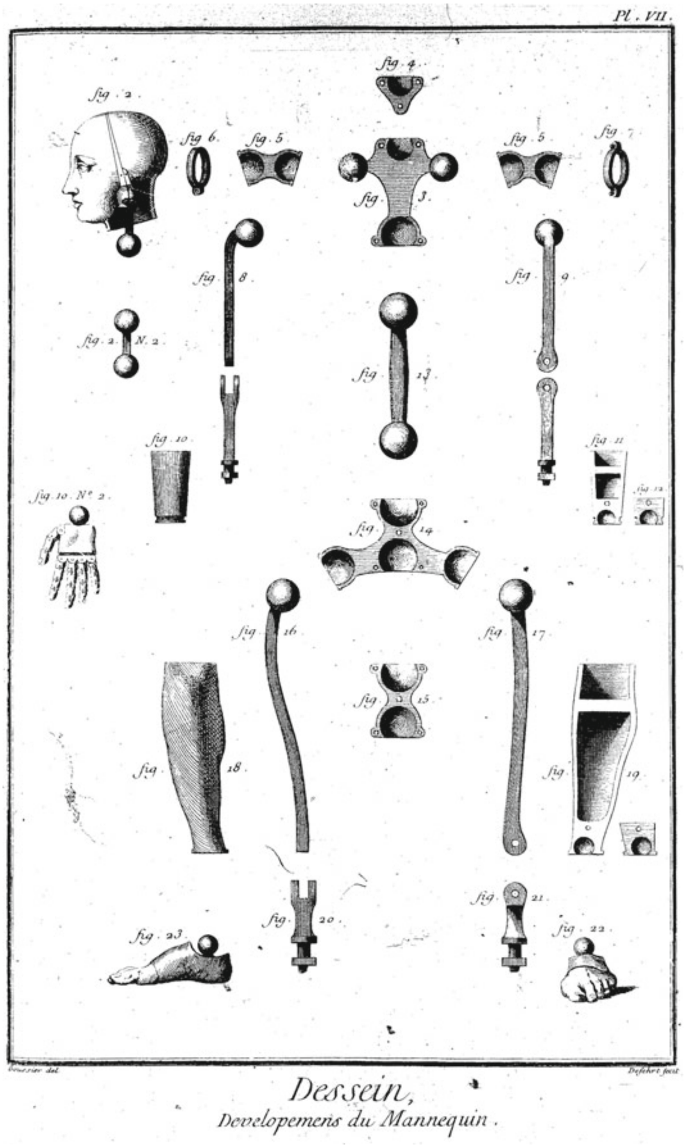

Fig. 4 Robert Bénard, "Dessein, Dévelopemens du Mannequin," following the design by Louis Jacques Goussier. From "Recueil de planches sur les sciences, les art libéraux, et les arts méchaniques, avec leur explication [1763]," Plate VII; accompanying Denis Diderot and Jean-Baptiste le Rond d'Alembert, *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers* (Paris: Briasson, 1762–1772). © Bibliothèque nationale de France, all rights reserved

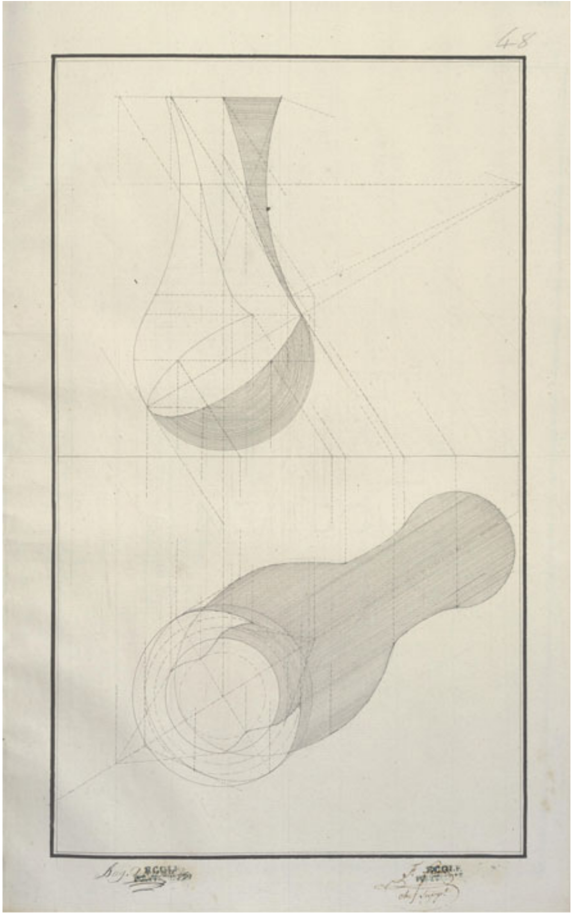

Fig. 5 Épure from the portfolio of Auguste Dupau, a student at École polytechnique in 1802. © Collections École polytechnique, Palaiseau, all rights reserved

schools were devoted to its representation and decomposition. Yet, other types of models were designed for special professions along the same analytical process of decomposition/recomposition, such as with the molds, capitals, and balustrades in architecture. At the turn of the nineteenth century, the model role of the human figure was challenged by geometric figures as well as by the mathematical models designed for the teaching of descriptive geometry (see Fig. 5).[33] For its creator Gaspard Monge, descriptive geometry embodied the "esprit d'analyse,"[34] not only

[33] Ibid., 98.

[34] Sakarovitch, *Epures d'architecture*, 214.

in the sense that its teaching could be organized from the simple to the complex, but also because it provided a heuristic "method for finding the truth."[35] As claimed by Lacroix in his 1805 *Essais sur l'enseignement*, in contrast with the slavish imitation of the human figure, models of geometric figure should be promoted because they formed the elementary parts of all the objects used, or manufactured, by craftsmen. Geometry, thus, provided models of a "more general usefulness" than the human figure.[36] For the same reason, the "dessin linéaire," created by the mathematician Louis-Benjamin Francœur in 1819, made only use of geometric models "for people's benefit."[37]

Geometric Drawing in the Royal Engineering Schools

The practice of model drawing played a key role in the first engineering schools established in France, such as the École royale des ponts et chaussées (Royal School of Bridges and Roadways; see Fig. 6), established in 1747,[38] and the École royale du génie de Mézières (Royal School of Military Engineering in Mézières), founded in 1748. The first was designed as a school without any professor. The students were mainly trained by drawing the models of various constructions that were deposited by visiting engineers, and second year students were supposed to advise first year students. More advanced students also trained their peers in mathematics by the use of textbooks such as the ones of Alexis Claude Clairaut, Charles Étienne Louis Camus, Charles Bossut, and Étienne Bézout. But most of the time was devoted to project-based learning in view of yearly competitions in mathematics, mechanics, architecture, stonecutting, planing and leveling, etc., which all required to perform drawings.[39] As we shall see later in greater detail, this important role given to companionship and to apprenticeship prefigured the method of mutual instruction that would develop in Europe in the beginning of the nineteenth century.

In contrast with the École des ponts, the Mézières school did include a professor of mathematics with the nomination of Charles Bossut in 1752. Yet the main

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[35] Gaspard Monge, "Programme liminaire à la *Géométrie descriptive,* 20 janvier 1795," in *Leçons de mathématiques: Laplace, Lagrange, Monge*, ed. Jean Dhombres, vol. 1: *L'École normale de l'an III* (Paris: Dunod, 1992), 306: "un moyen de rechercher la vérité […]."

[36] Lacroix, *Essais sur l'enseignement,* 359.

[37] Louis-Benjamin Francœur, *Le dessin linéaire d'après la méthode de l'enseignement mutuel* (Paris: Colas, 1819), 2: "[…] mais limité à la seule partie qui soit à l'usage du peuple, c'est-à-dire, l'enseignement du dessin linéaire."

[38] See: Antoine Picon*, L'invention de l'ingénieur moderne. L'École des Ponts et Chaussées 1747–1851* (Paris: Presses de l'École nationale des Ponts et Chaussées, 1992); André Grelon, Anousheh Karvar, and Irina Gouzevitch, *La formation des ingénieurs en perspective. Modèles de référence et réseaux de médiation, XVIIIe-XXe siècles* (Rennes: Presses Universitaires de Rennes, 2004); Joël Sakarovitch, "The Teaching of Stereotomy in Engineering Schools in France in the XVIIIth and

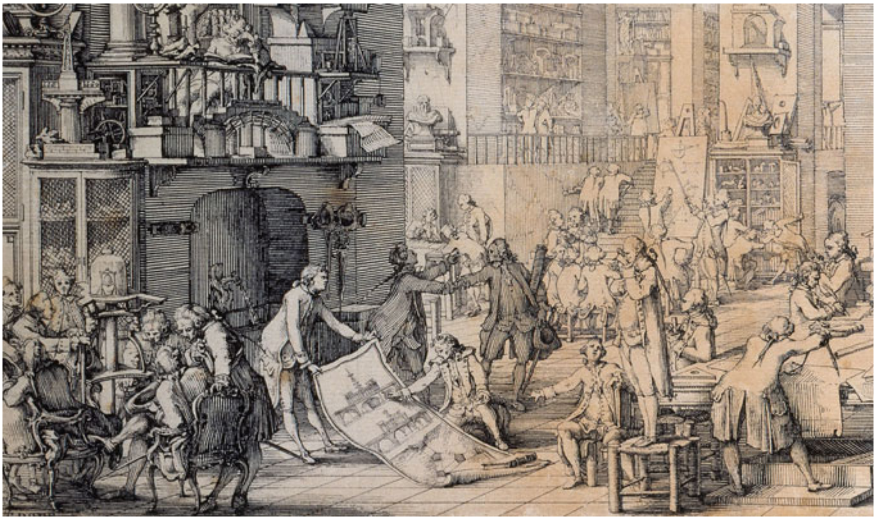

Fig. 6 Louis-Jean Desprez, "Vue imaginaire de l'École des Ponts et chaussées" (detail), circa 1750 (Musée Carnavalet, Paris). CC0 1.0 Universal (CC0 1.0) Public Domain Dedication (https://creativecommons.org/publicdomain/zero/1.0/)

role of this professor was not to lecture.[40] With regard to mathematical training, both schools relied mostly on the practice of model drawing and on mutual instruction.[41] At Mézières, the instruction was however less associated with project-based learning, and more and more organized in successive steps. It started with the construction of two épures of geometry. Next, this basic training in the elements of geometry was applied to the construction of épures in more special fields such as stonecutting, woodcutting, perspective, shadow drawing, and, in the second year of instruction, to fortification, survey work, buildings and machines. Because the

XIXth Centuries: an Application of Geometry, an 'Applied Geometry,' or a Construction Technique?" in *Entre Mécanique et Architecture. Between Mechanics and Architecture*, eds. Patricia Radelet-de Grave and Eduardo Benvenuto (Basel: Birkhäuser, 1995), 204–18.

[39] Bruno Belhoste, Antoine Picon, and Joël Sakarovitch, "Les exercices dans les écoles d'ingénieurs sous l'Ancien Régime et la Révolution," *Histoire de l'Éducation* 46 (1990): 56–73.

[40] Bossut only gave a series of short lectures on elementary mathematics, mechanics, and hydrostatics three days a week over a period of six months between 1752 and 1777, when the course of elementary mathematics was eventually cancelled. Further, Bossut's lectures on mathematics were hardly more than a repetition since they were based on the four volumes of Camus' textbook, which the students had already studied for passing the competitive entrance exam to the school. At the École des ponts, the students were, similarly, supposed to attend a series of lectures on chemistry and physics given by the professors of the Museum d'histoire naturelle. See René Taton, "L'École royale du Génie de Mézières," in *Enseignement et diffusion des sciences en France au XVIII^{e} siècle*, ed. René Taton 559–615.

[41] Belhoste et al., "Les exercices," 53.

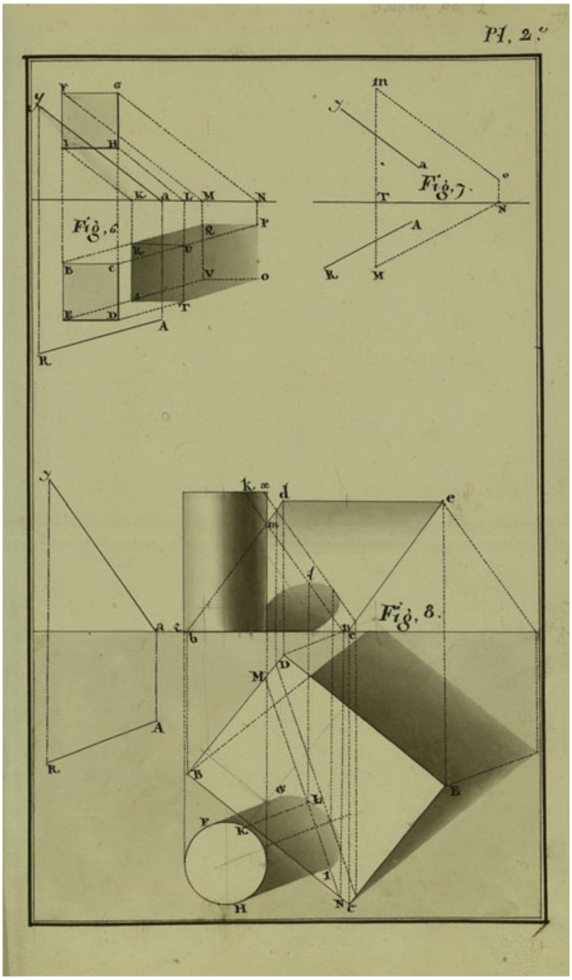

Fig. 7 Épure by Marchal, a student of Gaspard Monge at Mézières. From Gaspard Monge, "Petit traité des ombres à l'usage de l'école du genie." © Collections École polytechnique, Palaiseau, all rights reserved

drawing of actual buildings or fortifications required time consuming outside activities, models played an important role at all stages of the education in the royal engineering schools.

Monge, who succeeded Bossut in Mézières, formalized the mathematical nature of model drawing with the creation of the 'dessin géométral,' which would later be renamed as descriptive geometry and would become one of the major branches of the mathematical sciences in the nineteenth century (see Fig. 7).[42] Descriptive geometry allows one too make "the intimate and systematic link between three-dimensional and planar figures:"[43] a three dimensional object is represented by two

[42] Taton, *L'œuvre scientifique de Monge.*

[43] Michel Chasles, *Aperçu historique sur l'origine et le développement des méthodes en géométrie* (Bruxelles: Hayez, 1837), 191: "l'alliance intime et systématique entre les figures à trois dimensions et les figures planes." See also: Victor Poncelet, *Traité des propriétés projectives des figures*

planar projections into mutually perpendicular directions; each of the two adjacent planar figures shares a full-scale view of one of the three dimensions of space. These figures may serve as the beginning point for a third projected view, such as of ‘shadows’ which facilitates the visualization of volumes. As Monge himself phrased it in his very first series of lectures on descriptive geometry at the École normale de l’an III in 1795:

> The purpose of this art [descriptive geometry] is two-fold. First it allows one to represent three-dimensional objects susceptible of being rigorously defined on a two-dimensional drawing […]. Second […], by taking the description of such objects to its logical conclusion, we can deduce something about their shape and relative positioning […]. [It is] a language necessary for the engineer to conceive a project, for those who are to manage its execution, and finally for the artists who must create the different components.[44]

Monge had initially been hired at the Mézières school as a draughtsman in 1765 and assigned to the “atelier de la Gâche,” a workshop devoted to the construction of models made of stucco. The construction of épures of fortifications provided Monge the opportunity to prove his mathematical abilities and he was elevated in 1766 to the position of *répétiteur* of mathematics, i.e. adjoint to Bossut, and eventually to the position of professor in 1769. Yet, as said before, Monge’s role was not so much to lecture but to assist the students in their drawings. It was for the purpose of this companionship training that Monge established descriptive geometry as providing a mathematical formulation to the diversity of the graphical techniques of engineers.[45] On this issue, let us quote the historian Joël Sakarovitch:

> Descriptive geometry has been two-faceted from the time it was created. It is on the one hand an entirely new discipline […] [which] offers an unprecedented manner of tackling three-dimensional geometry or, to be more exact, linking planar geometry with spatial geometry […]. But it simultaneously appears as the last stage of a tradition that is losing momentum, as the ultimate perfecting of previous graphical techniques and, in that capacity, marks the endpoint of an evolutionary process as much as the birth of a new branch of geometry. As such, it can also be viewed as a transition discipline that allowed a gentle evolution to take place: from the ‘artist engineer’ of the Old Regime, whose training was based

(Paris: Bachelier, 1822); Jules de la Gournerie, *Discours sur l’art du trait et la géométrie descriptive* (Paris: Mallet-Bachelier, 1855).

[44] Gaspard Monge, quoted in Joël Sakarovitch, “Gaspard Monge: Géométrie Descriptive, First Edition (1795),” in *Landmark Writings in Western Mathematics, 1640–1940*, ed. Ivor Grattan-Guinness (Amsterdam: Elsevier Science, 2005), 225–241, here: 226. See: Gaspard Monge, “Programme liminaire à la Géométrie descriptive”, 305–6: “Cet art a deux objets principaux. Le premier est de représenter avec exactitude, sur des dessins qui n’ont que deux dimensions, les objets qui en ont trois, et qui sont susceptibles de définition rigoureuse. Sous ce point de vue, c’est une langue nécessaire à l’homme de génie qui conçoit un projet, à ceux qui doivent en diriger l’exécution, et enfin aux artistes qui doivent eux-mêmes en exécuter les différentes parties. Le second objet de la géométrie descriptive est de déduire de la description exacte des corps tout ce qui suit nécessairement de leurs formes et de leurs positions respectives.” See also: Gaspard Monge, “Stéréotomie,” *Journal de l’École polytechnique* 1 (1794): 1–14.

[45] Sakarovitch, *Epures d’architecture*; Bruno Belhoste, “Du dessin d’ingénieur à la géométrie descriptive: l’enseignement de Chastillon à l’École royale du génie de Mézières,” *In Extenso* 13, (June 1990): 103–35.

> on the art of drawing rather than scientific learning, to the 'learned engineer' of the 19th century, for whom mathematics—and algebra in particular—is going to become the main pillar of his training.[46]

As we shall see in this paper, the growing importance of geometric models calls for reassessing the evolutions of descriptive geometry in the nineteenth century and its role in the interplay between textual knowledge and knowing by drawing.

In the royal schools of military engineering such as Mézières, the key role played by the practice of model drawing highlights a clear-cut distinction between the practical mathematical training provided within these schools and the more textual initial mathematical instruction of the students. One major feature of the royal military schools was indeed the distinction made between teaching and examining. Mathematics served as the dominant criterion in the entrance examinations, which took the form of an oral examination by a member of the Paris Academy of Science, such as Bossut, Bézout, or Pierre-Simon Laplace. These examinations were notoriously difficult and selective.[47] Lazare Carnot, for instance, succeeded to enter Mézières at his second attempt while Claude Rouget de Lisles did not succeed before his fifth attempt. Most candidates had received an elementary instruction in a Jesuit college, which included elements of arithmetic and of geometry in the tradition of Euclid. But the preparation for the entrance examinations of the royal military school was an individual affair based on the study of classical textbooks, such as Bezout's,[48] and required more advanced knowledge in arithmetic, algebra, geometry, and differential calculus.

The Foundation of École Polytechnique

In 1793, the schools of instruction and teaching were disorganized by the war that opposed revolutionary France to a coalition of European nations. In 1794, Jacques-Élie Lamblardie, director of the École des ponts, who lost a great number of his pupils, thought of creating a preparatory school for bridges and roads, and then for all engineers. Monge was enthusiastic about this idea and convinced several members of the Comité de Salut Public (French Public Welfare Committee) and the Convention. Under support of figures such as the chemist François Fourcroy, a decree of March 11, 1794 created the Central school of public works, which would be renamed École polytechnique one year later, on September 1, 1795.[49]

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[46] Sakarovitch, "Gaspard Monge," 240.

[47] Roger Chartier, "Un recrutement scolaire au XVIIIe siècle: l'école royale de génie de Mézières," *Revue d'Histoire moderne et contemporaine* 20, no. 3 (1973): 369–75.

[48] Lilianne Alfonsi, *Étienne Bézout (1730–1783): mathématicien des Lumières* (Paris: L'Harmattan, 2011).

[49] On the history of the creation of Polytechnique, see: Ambroise Fourcy, *Histoire de l'École polytechnique* (Paris: Impr. de l'École polytechnique, 1828); Janis Langins, *La République avait besoin de savants: les débuts de l'Ecole polytechnique, l'Ecole centrale des travaux publics et les cours révolutionnaires de l'an III* (Paris: Belin, 1987); Jean Dhombres and Nicole Dhombres, *Naissance*

Its mission was to provide its students with a well-rounded scientific education with a strong emphasis on mathematics, physics, and chemistry. The Comité de Salut Public entrusted Monge, Lazare Carnot, and several other scholars with enlisting, by means of a competitive recruitment process, the best minds of their time, and teaching them science for the benefit of the French Republic. In 1795, all the other engineering schools were reorganized as special application schools for the students who had graduated from École polytechnique. The latter therefore acquired both a central and national role in the French educational system. It would spread its standards and pedagogical practices to other schools and would play a key role in imposing national standards of mathematical instruction in France and abroad through Napoléon's efforts to create a centralized, uniform system of education.

When the school was founded in 1794, its main features were the competitive entrance examination, the importance of mathematics, and the association of technical and mathematical education with military issues.[50] Monge elaborated the content of the first plan of instruction on two axes: the mathematics and the physics acquired by the experiment in the laboratories. The teaching of mathematics was divided between descriptive geometry, on the one hand, analysis and mechanics, on the other. Descriptive geometry had the most prominent role and was intimately associated to applications to the 'description of forms.' It started with stereotomy, i.e. the mathematical principles of descriptive geometry, and was then applied to architecture and fortifications. By contrast, the teaching of analysis was initially very limited, and focused on applications to the 'description of motions' in mechanics, hydrostatics and machines.

As was already the case in Mézières, the practice of model drawing in the teaching of geometry was given a prominent place. In the initial plan of instruction, about four fifths of the time (74 h) was devoted to practical activities (61 h) which consisted mainly in graphical activities in geometric drawing (30 h) and figure or landscape drawing (12 h). On a typical day, the short morning lecture mainly aimed at providing the students with the "knowledge, the instructions, and the methods" required for the graphical constructions of the day. As Monge phrased it in 1795: "the drawings constitute the ostensible work of the student […] they require meditations, but there will not be any specific time devoted to these meditations, which will develop during the constructions, and the student who will

d'un nouveau pouvoir: sciences et savants en France, 1793–1824 (Paris: Payot, 1989); Ivor Grattan-Guinness, *Convolutions in French Mathematics, 1800–1840: From the Calculus and Mechanics to Mathematical Analysis and Mathematical Physics* (Basel: Birkhäuser, 1990); Charles C., Gillispie, *Science and Polity in France: The Revolutionary and Napoleonic Years* (Princeton: Princeton University Press, 2004); Bruno Belhoste, *La formation d'une technocratie. L'École polytechnique et ses élèves de la Révolution au Second Empire* (Paris: Belin, 2003).

[50] École polytechnique only became a military school in 1804 but military issues were already very strong in 1794. For a short synthesis on the role played by mathematics in the educational purpose of the school from 1794 to 1850, see: Ivor Grattan-Guinness, "The 'Ecole Polytechnique,' 1794–1850: Differences over Educational Purpose and Teaching Practice," *The American Mathematical Monthly* 112, no. 3 (2005): 233–50.

have trained simultaneously his intelligence and his skills using hands, will get, as the price of his double work, the exact description of the knowledge he will have acquired."[51]

The idea that mathematics established a hierarchy between engineers and craftsmen, while drawing was a 'common language' between them, shows continuity in the training of engineers before and after the French Revolution. As Antoine Lavoisier phrased it in his "reflections on public instruction:"

> Drawing is a sensitive language which speaks to the eye, gives shape to our thought and therefore expresses more than language; it is a mean of communication between the one who conceives or who commands and the one who executes. Considered as a language, drawing is an instrument for perfecting one's thoughts; drawing is therefore the primary education of those aiming at [a career in] the arts [i.e. the techniques].[52]

Yet, in contrast with the military schools in the Ancien Régime, the students of École polytechnique had to attend lectures of mathematics even though, according to Monge, the new school initially attached "much more importance to the works done by students with their own hands than to what they can learn by listening to professors or reading books. It is indeed the best method for fixing in the mind the knowledge that is acquired, for making it accurate, and for one to be certain that he fully possesses this knowledge."[53] The founding professors ('instituteurs') of analysis and mechanics were Joseph-Louis Lagrange and Gaspard Riche de Prony. Descriptive and differential geometry was in the hands of Monge, who also served as Director for two short periods. Each *instituteur* had an

[51] *Procès-verbaux du conseil d'administration de l'École centrale des Travaux publics*, séance du 20 pluviôse an III, Archives of École polytechnique, https://journals.openedition.org/sabix/703 (accessed April 15, 2022): "[…] ces constructions graphiques, c'est dans des dessins que consistera tout le travail ostensible des choses, ces Dessins, ces Constructions exigent de leur part des méditations ; mais il n'y aura aucun tems purement consacré à ces méditations ; elles auront eu lieu pendant toute la Durée des Constructions, et l'élève qui aura en même tems exercé son intelligence et l'adresse de ses mains, aura pour prix de ce double travail, la Descriptionexacte de la connoissance qu'il aura acquise."

[52] Antoine Lavoisier, *Réflexions sur l'instruction publique*, *présentées à la Convention nationale par le bureau de consultation des arts et métiers* (Paris: Du Pont, 1793), 15: "[…] le dessin est un langage sensible qui parle aux yeux, qui donne de l'existence aux pensées, et sous ce point de vue, il exprime plus que la parole ; c'est un moyen de communication entre celui qui conçoit ou qui ordonne un ouvrage et celui qui l'exécute, enfin considéré comme langue, c'est un instrument propre à perfectionner les idées : le dessin est donc la première étudie de ceux qui se destinent aux arts."

[53] Gaspard Monge, "Avant propos," *Journal de l'École polytechnique* 1 (1794): iii–viii, here iv: "Il faut dire encore que l'école est tellement montée, que l'on s'y attache bien plus au travail que l'élève exécute de ses propres mains, qu'à ce qu'il peut apprendre en écoutant les professeurs, ou en étudiant dans des livres. C'est en effet la meilleure méthode pour fixer dans l'esprit les connaissances que l'on acquiert s'assurer de leur justesse, et être certain qu'on les possède complètement."

assisting adjoint, who were named "répétiteurs" after 1798.[54] Among early notable adjoints or *répétiteurs* in mathematics, one may cite Jean Nicolas Pierre Hachette in geometry (another former member of the Mézières school who had been hired as a draughtsman and elevated to *répétiteur*) and Joseph Fourier in analysis. One major feature was the distinction made between teaching and examining so examiners were also appointed. For mechanics and analysis the initial examiners were Bossut and Laplace.

While the school had originally been conceived as the one and only institution to train engineers, the impracticability of the vision was soon recognized and the role of the school was thus changed in 1795 to that of a preparatory institution for the other schools, which were organized into a collection of 'écoles d'application,' such as École d'application de l'artillerie et du génie in Metz (School of Artillery and Engineering Applications), École des mines (School of Mining), and École nationale des ponts et chaussées (National School of Bridges and Roadways). This change would have important consequences on the roles attributed to mathematics at Polytechnique, especially through the influence of Laplace.[55] For six weeks in 1799 Laplace acted as Minister of the Interior. He proposed that the school have a governing council, the "Conseil de perfectionnement," to supplement the "Conseil d'Instruction" on teaching details, and a "Conseil d'Administration" for management.[56] Laplace was one of its founding members; and he exercised much influence there, in particular reducing the time given to descriptive geometry and transferring much of it to mechanics and analysis.[57] This opposition was led mainly by Laplace's desire to confine the programs at École polytechnique to teaching general theories, which would then be applied in the more specialized other schools. This kind of difference over curriculum policy in the school would continue for a long time: the archives of the reports of the school's councils highlight that the issue of the roles attributed to mathematics fuelled a never ending tension in the school curriculum, between the general and the special, and between the theoretical and the applied.[58] This tension would play an important role in the

[54] On the *répétiteurs* at Polytechnique in the nineteenth century, see: Yannick Vincent, "Les répétiteurs de mathématiques à l'École polytechnique de 1798 à 1900" (Phd diss., École polytechnique, 2019).

[55] Roger Hahn, "Le rôle de Laplace à l'École polytechnique," in *La formation polytechnicienne: 1794–1994*, ed. Bruno Belhoste, Amy Dahan Dalmedico, and Antoine Picon (Paris: Dunod, 1994), 54.

[56] Belhoste, *La formation d'une technocratie*, 50–51.

[57] Joël Sakarovitch, "La géométrie descriptive, une reine déchue," in *La formation polytechnicienne: 1794–1994*, ed. Bruno Belhoste, Amy Dahan Dalmedico, and Antoine Picon (Paris: Dunod, 1994), 77–93.

[58] See: Bruno Belhoste, "The École polytechnique and Mathematics in Nineteenth-Century France," in *Changing Images in Mathematics*, ed. Umberto Bottazzini and Amy Dahan (London: Routledge, 2001), 15–30. For the long shadow cast by this tension on the teaching of mathematics, see: Jean-Luc Chabert and Christian Gilain, "Debating the Place of Mathematics at the École polytechnique around World War I," in *The War of Guns and Mathematics: Mathematical Practices and Communities in France and Its Western Allies around World War I*, ed. David Aubin and Catherine Goldstein, vol. 42: *History of Mathematics* (Providence: American Mathematical Society, 2014), 275–306.

evolution of mathematical models in the nineteenth century, as shall be seen later in greater details.

From 1794 to 1800, École polytechnique thus passed from the "École de Monge" to the "École de Laplace."[59] Mathematics was given an increasing importance in the school's curriculum, from 50% in 1794 to 65% in 1800, while the teaching of applications was much reduced. Moreover, analysis came to play a more and more important role in the curriculum at the expense of descriptive geometry: while the respective proportions of descriptive geometry and analysis amounted to 50% and 8% of the curriculum in 1794, they amounted to 26% and 29% in 1800.[60]

But even though the first plan of instruction conceived by Monge was called into question one year after the creation of the school,[61] the intimate connection between the teaching of descriptive geometry and the practice of drawing would have a lasting influence at École polytechnique. Actually, geometric drawing was not much affected by the reduction of the teaching of descriptive geometry, and therefore of the lectures devoted to mathematical drawing: in the legacy of the pedagogical practices developed in the royal engineering school, drawing was indeed much more a practical activity at Polytechnique than a matter of plenary lecture. The industrialization of France in the century would even strengthen the importance of geometrical drawing with the creation of lessons on machinery distinct from the one of descriptive geometry.[62]

Mutual Instruction Versus Academic Pedantry

In 1794 École polytechnique was established under the label of the strong ideals that had been developed during the Enlightenment and which had called for the development of scientific education. Mathematics was especially valued as a way of emancipation because it was considered to provide results closer to the truth

[59] Théodore Olivier, "Monge et l'École polytechnique," *Revue scientifique et industrielle* 38 (1850): 64–68.

[60] To this proportion of 29%, one should actually add the 17% amounting to the teaching of mechanics in 1800 (in 1794, analysis and mechanics were not yet distinguished one from the other).

[61] On the evolutions of École polytechnique in the first half of the nineteenth century, see Bruno Belhoste, "Un modèle à l'épreuve. L'École polytechnique de 1794 au Second Empire," in *La formation polytechnicienne*, *1794–1994*, ed. Bruno Belhoste, Amy Dahan Dalmedico, and Antoine Picon (Paris: Dunod, 1994), 9–30.

[62] Konstantinos Chatzis, "Mécanique rationnelle et mécanique des machines," in *La formation polytechnicienne: 1794–1994*, ed. Bruno Belhoste, Amy Dahan Dalmedico and Antoine Picon (Paris: Dunod, 1994), 95–108; Jean-Yves Dupont, "Le cours de machines de l'École polytechnique, de sa création jusqu'en 1850," *Bulletin de la Société des amis de l'École polytechnique* 25 (2000): 3–79.

than any other science.[63] For Nicolas de Condorcet in particular, mathematical education had a moral value: it aimed at ensuring the continuation of progress, not only in science and technology, but also in the morality of the younger generations. Further, the ideal of universality associated with mathematics was at the core of the evolution of the system of competitive recruitment process for the Ancient Regime royal military engineering school into a new system, which aimed at replacing hereditary privileges by individual merit, which was to be proved by solving mathematical problems.[64] Not only did the new system of competitive exams abolish any prerequisite of nobility but it also reduced the expectation of a prerequisite knowledge, acquired by studying textbooks, in order to favor intelligence over cramming.[65]

These ideals went along with the goal to create a new pedagogy that would promote both theoretical and practical knowledge. Inspired by the teaching of geometry by model drawing at Mézières as well by the pedagogical innovations made in mining schools such as the one of Schenitz in Hungary, the founders of École polytechnique aimed at promoting science activities and experiments.[66] This plan required both a library and a collection of scientific instruments. Both were initially constituted from the property seizures that had been taken under the exigencies of revolution in three waves from 1789 to 1793, especially in private library collections from the aristocracy and clergy. This collection quickly expanded with the publications and the new apparatus that were invented by the professors, often alumni of Polytechnique, for their teaching.

The important role attributed by Monge to action learning through the practice of drawing thus participated to a more general plan for articulating practice and theory. For the same purpose that laboratories were created for promoting experiments in chemistry, Monge introduced a distinction between the "grandes salles," where the *instituteurs* lectured, and the "petites salles," (see Fig. 8) in which the students were divided into several "brigades."[67] The students actually spent most of their time in the "petites salles," or the "salles d'études," at least five hours a

[63] Jean Antoine Nicolas de Caritat, marquis de Condorcet, "Discours de réception à l'Académie française," *Recueil des harangues prononcées par Messieurs de l'Académie française dans leurs réceptions et en d'autres occasions* 8 (1782): 413–49.

[64] On the continuities in the role devoted to mathematics in the entrance examinations of the royal military school and of École polytechnique, see: Janis Langins, "The Ecole polytechnique and the French Revolution: Merit, Militarization, and Mathematics," *Llull: Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas* 13, no. 24 (1990): 91–105; Janis Langins, "La préhistoire de l'Ecole polytechnique," *Revue d'histoire des science*s 44, no. 1 (1991): 61–89.

[65] Circular to examiners in 1794 quoted in Fourcy, *Histoire de l'École polytechnique*, 34.

[66] Antoine-François Fourcroy, *Rapport sur les mesures prises pour l'établissement de l'école centrale des travaux publics, fait par Fourcroy au nom des comités réunis de salut public, d'instruction publique et des travaux publics. Du 3 vendémiaire an III* (Paris: Imprimerie du Comité de salut public, 1794).

[67] Monge, "Stéréotomie," 3–6; Belhoste, *La formation d'une technocratie*, 200.

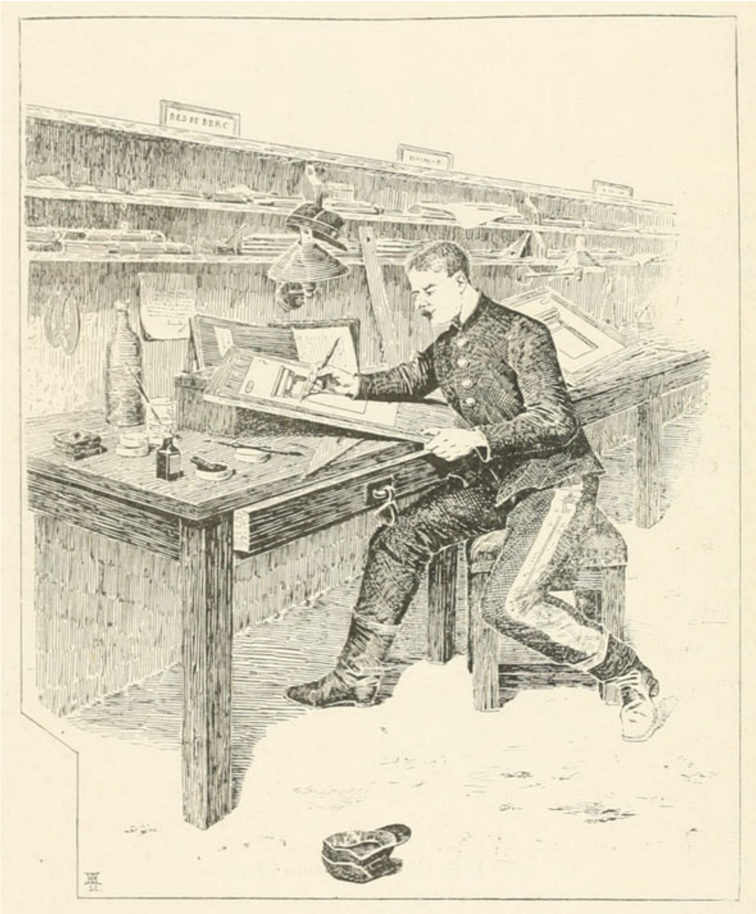

Fig. 8 A student working in a 'petite salle.' From Gaston Claris, *Notre École polytechnique* (Paris: Librairies-imprimeries réunies, 1895), 140

day, and this time was mostly devoted to model drawing, studying daily lectures, and preparing for the regular oral examinations ("répétitions").[68]

[68] In the initial schedule established by Monge, the daily lectures of descriptive geometry took place from eight to nine in the morning and were followed by practical drawing exercices from 9 am to 2 pm. Four of the six weekly afternoon sessions, which took place from 5 pm to 8 pm, were devoted to drawing. See: Gaspard Monge, "Développement sur l'enseignement adopté pour l'école centrale des travaux publics", in Janis Langins, *La République avait besoin de savants: les débuts de l'Ecole polytechnique, l'Ecole centrale des travaux publics et les cours révolutionnaires de l'an III* (Paris: Belin, 1987), 227–69. On the evolution of this schedule in the beginning of the nineteenth century, see Fourcy, *Histoire de l'École polytechnique*, 376–79. Several testimonies of students on their schedule at the school are also available, such as the ones Jacques Louis-Rieu in

To fully understand the importance of the practice of drawing in the activities of the students, one has to recall the predominance of oral teaching and examinations at École polytechnique: until the 1830s the students were not expected to produce any mathematical writings other than the use of the blackboard during the examinations.

The promotion of the activity of the students was also motivated by the rejection of the model of university education—universities were abolished during the revolution—accompanied by a mistrust of professors and their "inevitable appetency for pedantry." As claimed by the chemist Antoine-François Fourcroy, one of the founders of Polytechnique, the new system of republican instruction should aim at "populating classrooms with students for avoiding the risk of populating them with professors."[69] The important role devoted to the practice of model drawing at Polytechnique therefore highlights, once again, the long-term legacy of companionship and mutual instruction in the transmission of the mathematical crafts of the engineers.

A form of mutual instruction was institutionalized at Polytechnique with the selection of a few 'chefs de brigades' among the best students who had passed the first entrance examination in 1794. Each of these 'chefs de brigades' was responsible for helping a group of students in their work on geometric drawing in one of the "petites salles."[70] A few years later, the 'chefs de brigades' were selected between the young graduates from the school rather than from the students themselves:

> […] this disposition provides the opportunity to stay in Paris to the young men who may benefit the most from continuing their studies […]. By making these positions temporary […] we are protecting them against the pedantry, from which tenured professors so often fail to spare themselves.[71]

This experimentation of mutual instruction at Polytechnique would decline after 1798 when Laplace created the function of *répétiteurs*, i.e. adjunct professors who were in charge of the oral examinations of the students and of supervising the 'chefs de brigades' who, as a consequence, lost their autonomy and saw their role limited to the one of maintaining discipline in the "petites salles." Yet, several former 'chefs de brigades' and alumni of Polytechnique, such as Francœur and

1806 and of Auguste Comte in 1815. Auguste Comte, *Lettres d'Auguste Comte à M. Valat, professeur de mathématiques et ancien recteur de l'académie de Rhodez 1815–1844* (Paris: Dunod, 1870), 1–2; Jean-Louis Rieu, *Mémoires de Jean-Louis Rieu, ancien premier syndic de Genève* (Geneva: H. Georg, 1870), 16.

[69] Antoine-François Fourcroy, "Discours prononcé au corps législatif par Fourcroy, sur l'instruction publique, du 20 floréal an X," *Recueil des lois et règlements concernant l'instruction publique* 2 (May 10, 1802): 244.

[70] See Monge, "Stéréotomie," 4.

[71] Monge, "Développement," 241: "[…] Cette disposition fournit aux jeunes gens les plus en état d'en profiter, les occasions de continuer leurs études à Paris, et de perfectionner leur instruction. […] En rendant ces places passagères, […] on les met en garde contre le pédantisme, dont il est bien difficile que les instituteurs à poste fixe puissent se garantir."

Edme François Jomard, would get much involved in the movement for mutual instruction in France.[72] As will be seen later in greater detail, this movement would play a major role for the development of the mathematical instruction of the emerging working class through model drawing.

Monge's "Cabinet Des Modèles"

As mentioned above, models were instrumental in the distinction between the three main forms of pedagogical methods for teaching mathematics at École polytechnique: the plenary lectures of the *instituteurs*, the individual oral examinations of the *répétiteurs*, and the autonomous activities of groups of students in the petites salles. At the foundation of the school in 1794, Monge created a cabinet of models composed of the collection of all the drawing models that were to be used for the practical activities in the *petites salles*.[73] This collection consisted initially of models similar to the ones that had been previously used in the royal schools of engineering, such as épures, maps, models of architecture or fortifications, mechanical devices, etc.[74] As with the library and the scientific instruments of the laboratories, the cabinet of models was originally furnished with property seizures, especially in abolished royal institutions (such as the Mézières school),[75] but quickly expanded with the new publications and apparatus produced by students and professors. A drawing office with twenty-five draughtsmen was created for designing new models for the teaching of descriptive geometry and stereotomy.[76]

Monge's collection of models has unfortunately been lost, except a series of cardboard models of polyhedrons that were used for both the teaching of geometry and crystallography in physics and chemistry (see Fig. 9). The detailed inventory of the cabinet is known only for the year 1794, but historical sources document that Monge had commissioned a series of string models made of silk for the practice of geometric drawing. In 1814, the cabinet included two large-scale string models, one of the line generation of a revolution hyperboloid of one sheet, and the other of the line generation of a hyperbolic paraboloid (see Fig. 10).[77] But it is likely

[72] Renault d'Enfert, "Jomard, Francœur et les autres… Des polytechniciens engagés dans le développement de l'instruction élémentaire (1815–1850)," *Bulletin de la Sabix* 54 (2014): 81–94.
[73] Fourcy, *Histoire de l'École polytechnique*, 17.
[74] "Cabinet des modèles de l'École centrale. Citoyen Lesage, conservateur. Compte décadaire. 30 nivôse an 3, Lesage, conservateur" January 19, 1795; "Cabinet des modèles de l'École centrale. […] État de situation. Compte décadaire," Archives of École polytechnique. Fonds Prieur de la Côte-d'Or.
[75] Belhoste et al., "Les exercices," 98.
[76] Jean Nicolas Pierre Hachette, *Traité de géométrie descriptive*, 2nd ed. (Paris: Corby, 1828), xvii; Belhoste et al., "Les exercices," 101–107.
[77] Arthur Morin, *Conservatoire national des arts et métiers. Catalogue des collections publié par ordre de M. le Ministre de l'Agriculture et du Commerce*, 2nd ed. (Paris: de Guiraudet et Jouaust, 1855), 24.

Fig. 9 Cardboard model of a crystal. This set of cardboard models dates back to the creation of Monge's cabinet of models at École polytechnique. © Collections École polytechnique, Palaiseau, all rights reserved

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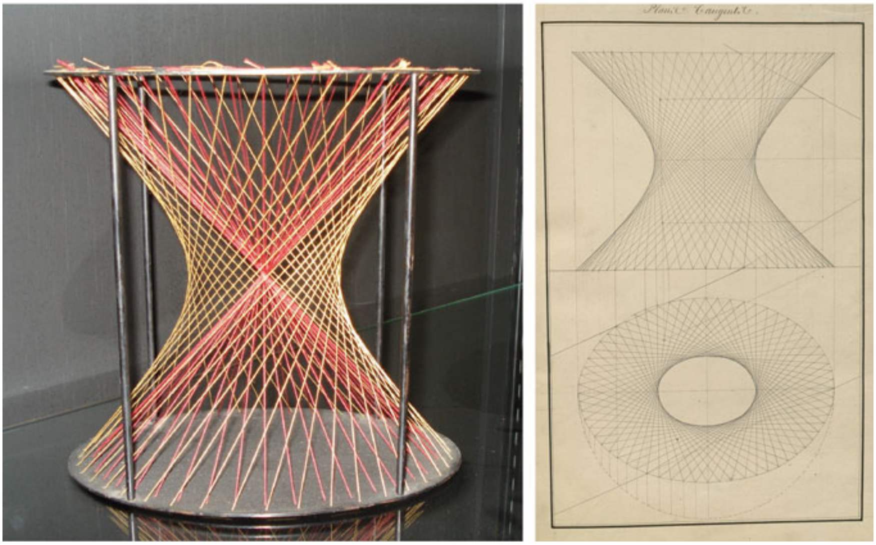

Fig. 10 **a** The revolution hyperboloid of one sheet in Gaspard Monge's cabinet may have been similar to the one above © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: Frédéric Brechenmacher. **b** Drawing of the string model of the line generation of a revolution hyperboloid of one sheet in Monge's cabinet. Léon Duflos de Saint Armand, Épures 1823–1824. © Collections École polytechnique, Palaiseau, all rights reserved

that other types of models were designed since several craftsmen were attached to the *instituteur* of descriptive geometry: a fitter ('appareilleur'), a carpenter, a joiner, a locksmith, and a plaster modeler.

The development of the cabinet des modèles is also documented by the nomination in 1813 of Louis Brocchi as curator of the models ("conservateur des modèles"),[78] a position renamed in 1816, as "artist keeper of the cabinet of models," and again in 1820 as "artist curator of the cabinet of models."[79] Born in Veroli, Italy, Brocchi had arrived in Paris in 1799.[80] He had been at first hired temporarily by École polytechnique for restoring several models of Monge's cabinet that had been damaged by the students.[81] He was eventually offered a permanent position on June 4, 1813 with a larger scope of responsibilities and kept this position until his death in 1837.[82] His act of nomination provides rare information about the role of the models in the organization of the teaching:

> He is in charge of keeping the models of machines, architecture, woodwork, stonecutting, topography, &c, the brass models and the collections of épures that have to be distributed to the students.
> He receives instructions from various professors for maintaining his cabinet, for printing plates, for distributing épures and paper to students; for installing the models and drawings in the amphitheaters and the study rooms.
> He remains in his office during the hours devoted to graphical work in order to furnish the students with the paper they may need.
> He maintains in condition the plaster models of stonecutting; he restores the objects entrusted to him.[83]

In 1813, the students' graphical activities required Brocchi to remain in his office every morning, from Monday to Friday (between 8:30 to either 12:30 or 2:30) and

[78] "Le gouverneur de l'Ecole impériale Polytechnique nomme le sieur Brocchi…," 4 June 1813. Archives of École polytechnique, VI1d2.

[79] Letter from the director of École polytechnique to the minister of interior, 9 December 1820, Archives of École polytechnique, VI1d2.

[80] Act of naturalisation of Brocchi by the mayor of the 12th arrondissement of Paris, 9 January 1815, Archives of École polytechnique, VI1d2.

[81] "État supplémentaire de proposition de gratification en faveur d'agents attachés à la Direction des études," Archives of École polytechnique, 24 January 1814, VI1d2.

[82] "État pour servir à la liquidation de la pension revenant à Madame Antoinette Jeanne Darley, veuve de M. Louis Marie François Brocchi," 1837, Archives of École polytechnique, VI1d2.

[83] "Brocchi, artiste gardien du cabinet des modèles," 1816, Archives of École polytechnique, VI1d2: "Il a sous sa responsabilité la garde des modèles de machines, d'architecture, charpente, coupe des pierres, topographie &c, les cuivres et les collections d'épures qui doivent être distribuées aux élèves.

il prend les instructions des différens professeurs pour la tenue de son cabinet; pour le tirage des planches; les distributions d'épures et de papier à faire aux élèves; pour le placement des reliefs et dessins aux amphithéâtres et dans les salles d'étude.

il se tient à son cabinet pendant ses heures de travail graphique pour donner aux élèves le papier dont ils auraient besoin.

il maintien en bon état les modèles de coupe des pierres en plâtre; soigne les objets qui lui sont confiés; met de l'ordre dans son cabinet; provoque des mesures conservatrices et rend compte de l'emploi des objets de consommation qu'il distribue."

from 12: to 2:30 on Saturday.[84] Brocchi also enriched the cabinet by designing new models and instruments, such as a compass of stereotomy for measuring the dimension of a body in regard with three orthogonal planes. This instrument was based on the founding principles of descriptive geometry, i.e. orthogonal projections and the generation of a surface by the motion of a variable line along parallel planes corresponding to the sections of the surface by parallel planes.[85] It was especially used for designing geometric models of several types of mouldboards for modern plows. Brocchi also completed Monge's collection of string models, such as with a model of the line generation of a non-revolution hyperboloid of one sheet. He also designed plaster stonecutting models and molds of topographic landforms.[86] Several reports written by the administration of the school praise Brocchi's talent for designing new models of descriptive geometry "for the instruction of the students."[87] Brocchi's talent earned him an important reputation[88]: foreign engineering school such as the one of Saint-Petersbourg purchased several of his models.[89]

A Polytechnic Culture of Drawing

As seen before, the important role devoted to the practice of drawing in the teaching of mathematics at École polytechnique did not suffer from the relative decline of descriptive geometry in the curriculum.[90] The drawing skills of the students graduating from the school were actually a constant matter of preoccupation of the 'conseil de perfectionnement,' in which the schools of applications were represented. The school of artillery and engineering applications in Metz, in particular, often complained about the limited drawing skills of the students who entered Metz after having graduated from École polytechnique and pleaded for strengthening further the practice of model drawing. The épures of descriptive geometry were indeed crucial for both artillery and military engineering: they laid the ground

[84] "M. Brocchi, conservateur du cabinet des modèles," June 2, 1813, Archives of École polytechnique, VI1d2.

[85] Jean Nicolas Pierre Hachette, "Rapport fait par M. Hachette, au nom du Comité des arts mécaniques, sur plusieurs instrumens de mathématiques présentés à la société," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 228 (November 1823): 145–60.

[86] The existence of these models is documented by a reclamation in which Brocchi's sons complained that École polytechnique made use of plaster models of topographies without having purchased the molds designed by their father for these models. "Auguste Brocchi à l'administrateur de l'École polytechnique," January 27, 1840, Archives of École polytechnique, VI1d2.

[87] Letter from the director of École polytechnique to the minister of interior for awarding a wage increase to Brocchi, december 20, 1818, Archives of École polytechnique, VI1d2.

[88] Brocchi was also charged by Louis Visconti, the architect of the Royal library, for restoring ancient vases and etruscan antiques.

[89] Letter from the count Johann von Sievers to Brocchi, October 4, 1818, Archives of École polytechnique, VI1d2.

[90] On the decline of descriptive geometry in the nineteenth century, see: Joël Sakarovitch, "La géométrie descriptive, une reine déchue," 77–93.

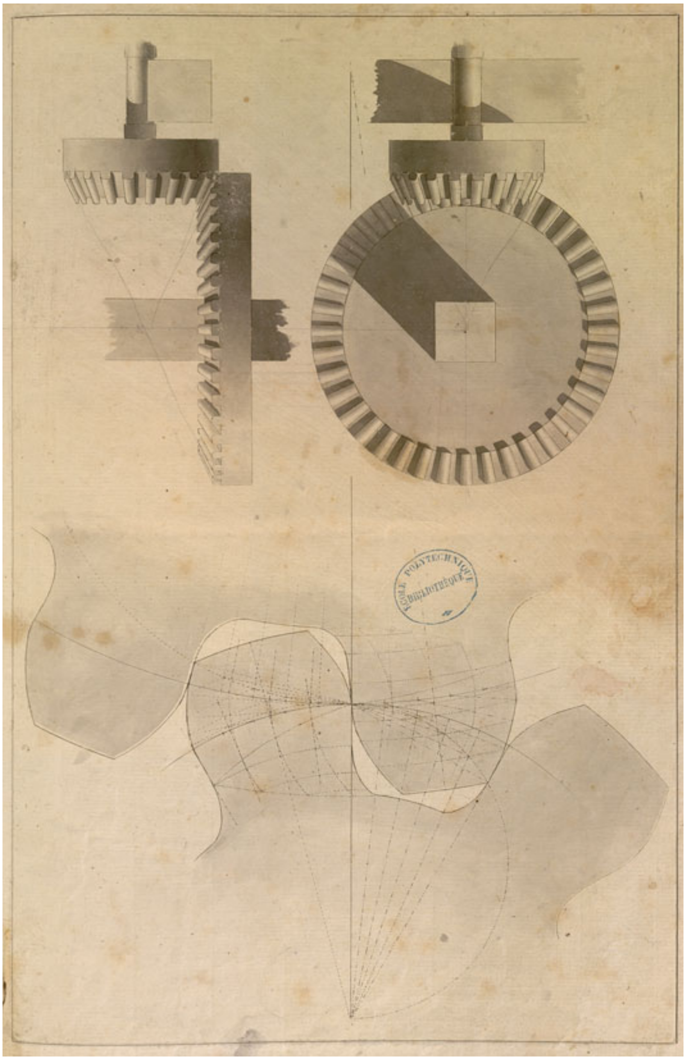

Fig. 11 Épure of a conical gearing by a student of École polytechnique in 1807. Epures 1794–1850. Cours de Jean-Nicolas-Pierre Hachette Lavis noir et blanc sur planche imprimée. © Archives de l'École polytechnique, all rights reserved

for the sciences of fortifications and topography and were necessary for designing various kinds of machinery and weaponry (see Fig. 11).

As a consequence, a drawing examination was added to the entrance examination of Polytechnique in 1804: it was the unique non-oral examination, and one of the two non-mathematical examinations along with a test in the French language.[91]

[91] Bruno Belhoste, "Anatomie d'un concours. L'organisation de l'examen d'admission à l'École polytechnique de la Révolution à nos jours," *Histoire de l'éducation* 94 (2002): 141–75.

Further, several épures were required for graduating from the school, which established a ranking of the students and therefore decided of the applications schools in which they would complete their training: four lavished épures of architectures, four lavished épures of machines, six épures of fortifications, represented by both descriptive geometry and perspectives, and six épures of maps. These requirements were even consolidated in 1812 with the addition of the elements of descriptive geometry in the program of entrance examinations, as well as of a special examination consisting in performing a geometric construction with only straightedge and compass.

Further, the teaching of drawing at Polytechnique was not limited to descriptive geometry and its various applications. As a matter of fact, engineers did not always have the adequate conditions to perform rigorous and precise geometrical drawings. They therefore also had to be trained in more classical forms of drawing, such as figure and landscape drawing, which involved professors trained at the École des Beaux-Arts (the Academy of Fine Arts) and carried humanistic values associated with the beaux arts, such as educating the "bon goût," i.e. the artistic sense of the students. These various forms of drawing and painting were commonly designated as "imitation drawing" (or "monkey drawing" in the student's slang) which, once again, highlights the key role models played. Imitation drawing was indeed learned by drawing various types of models,[92] including master drawings taken from revolutionary deposits, such as Jacques-Louis David's Bélisaire,[93] still life compositions, buildings, landscapes, and, after 1818, living models (see Fig. 12).[94]

The very large scope of the practice of model drawing at École polytechnique laid the ground for a common culture that exceeded the curriculum of the school. From 1818 to 1929, a yearly event was organized by the students, the "séances des Ombres," which consisted in a theater of Chinese shadows made of cardboard caricatures accompanied with songs written and performed by the students. Drawing caricatures was thus at the core of the social activities associated with the "Ombres," whose name also referred to the issue of shadow drawing in the teaching of both geometric and imitation drawings (see Fig. 13). This culture of drawing can also be seen in the richly illustrated student journal *Le petit crapal*, which was published from 1896 to 1932. Moreover, and more important for the topic of this paper, drawing played a central role in the professional careers of most students, whether they became engineers, military officers, scholars or, even, for a few of them, painters.

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[92] François-Marie Neveu, "Dessin: Compte rendu par l'Instituteur de Dessin, relativement à cette partie de l'enseignement," *Journal de l'École polytechnique* 1 (1794): 78–80; François-Marie Neveu, "Cours préliminaire relatif aux Arts de Dessin," *Journal de l'École polytechnique* 1 (1794): 81–91.

[93] David's Bélisaire is a preparatory drawing for the 1794 painting, Bélisaire demandant l'aumône (Belisarius Begging for Alms).

[94] Jean-Baptiste Regnault, "Correspondance relative à l'introduction de l'étude du modèle vivant dans le cours de dessin, 22 avril 1818," Archives of École polytechnique, VIIb2.

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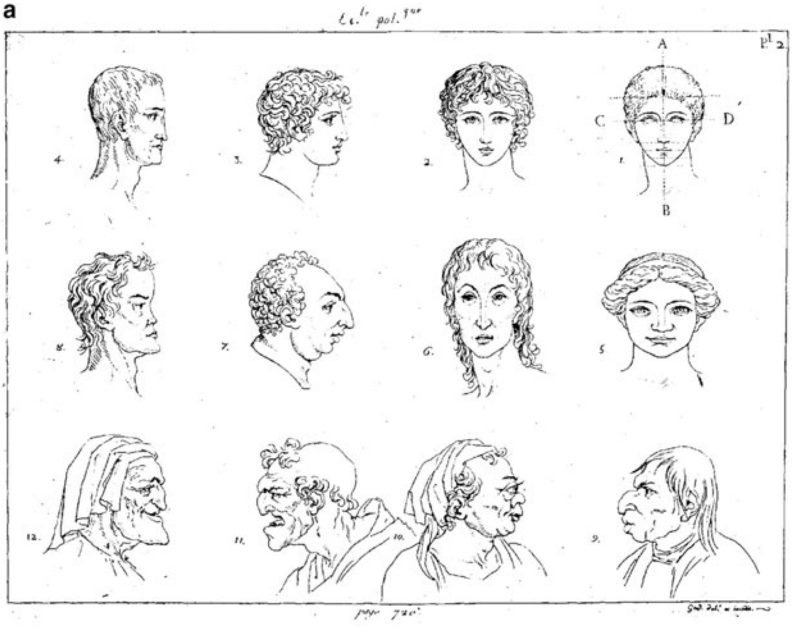

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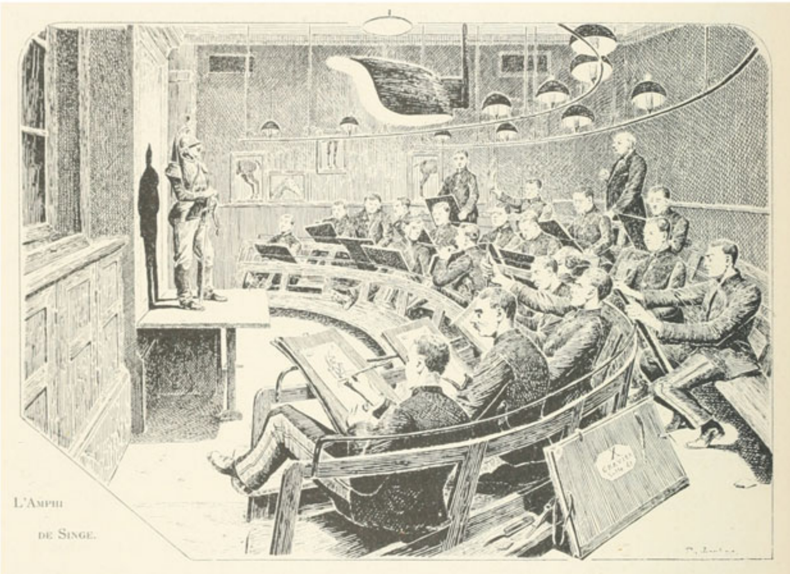

Fig. 12 **a** François Marie Neveu, “Suite du cours préliminaire relatif aux arts de dessin,” *Journal de l'École polytechnique*, 4 (1796), n.p. © Bibliothèque nationale de France, all rights reserved. **b** An amphitheatre of life drawing, “L'amphi de singe.” From Gaston Claris, *Notre École polytechnique* (Paris: Librairies-imprimeries réunies, 1795), 134

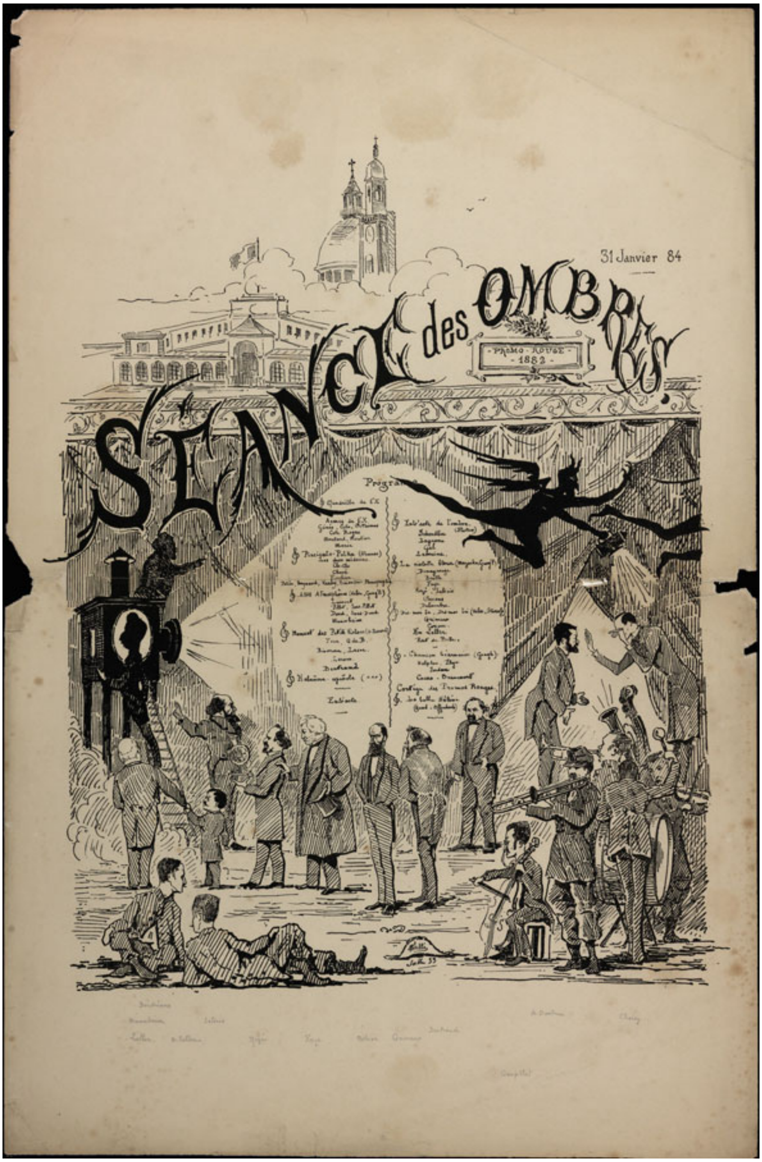

Fig. 13 Séance des ombres, 1882.

The Canons of Geometric Drawing: Models and the Artillery School

Monge's emphasis on descriptive geometry in the first plan of instruction of École polytechnique had laid the basis for a very coherent articulation between theory and practice as well as between the general and the special. The effectiveness of this plan was demonstrated during the French Campaign in Egypt and Syria (1798–1801), which included an important contingent of engineers and scholars assigned to the invading French force, 167 in total.[95] These scholars included several founding members of École polytechnique, such as Monge, as well as professors of the school, such as Fourier, and many alumni and students of the first promotions. They founded the Institut d'Égypte with the aim of propagating Enlightenment values in Egypt through interdisciplinary work. In this context, the young polytechnicians applied Monge's descriptive geometry for establishing fortifications in Cairo, surveying battle fields, mapping the cartography of Egypt, and for describing in minute details the monuments of Ancient Egypt, which gave rise to fascination with Ancient Egyptian culture in Europe and the birth of Egyptology:

> The most outstanding scholars were accompanied with engineers and architects of the highest merit in charge of surveying battlefields, cities, and the magnificent monuments of the pharaohs. We did not forget to arrange for them to have a staff of skillful draughtsmen working with them, and it even often happened that the skills of scholar, engineer, and draughtsman came together in the same operator […]. Two of the youngest members of this Institut d'Égypte […], Caristie and Jomard, who had just graduated from the new École polytechnique […] told everyone that their colleagues and themselves had never separated the two fundamental elements of their task: the precision of measurements of all kinds of surveys and the artistic effect of the monuments, represented in perspective with the surrounding landscape as a frame.[96]

As we have seen before, Monge's plan was challenged as early as 1795 when the school was assigned the new role to provide a general instruction that would be specialized in the various application schools ('écoles d'applications'). The mathematical curriculum at the Polytechnique came to be conceived as fundamental

[95] Yves Laissus, *L'Égypte, une aventure savante* (Paris: Fayard, 1998); Patrice Bret, ed., *L'expédition d'Egypte, une entreprise des Lumières, 1798–1801* (Paris: Technique & Documentation, 1999).

[96] Aimé Laussédat, *Recherches sur les instruments, les méthodes et le dessin topographiques*, vol. 2. part 1 (Paris, Gauthier-Villars, 1901): 4-5: "Ainsi, en nous en tenant à l'expédition d'Égypte, où les plus illustres savants étaient accompagnés d'ingénieurs et d'architectes du plus grand mérite chargés de lever les plans des champs de bataille, des villes, des magnifiques monuments des pharaons, on n'avait pas oublié de leur adjoindre d'habiles dessinateurs, et il arrivait même souvent que les talents de savant, d'ingénieur et d'artiste se trouvaient réunis chez le même opérateur. […] Deux des plus jeunes membres de cet Institut d'Égypte […] Caristie et Jomard […] déclaraient chacun à qui voulait l'entendre que leurs collègues comme eux-mêmes n'avaient jamais séparé les deux conditions essentielles de l'œuvre entreprise la précision des mesures et des relevés de toute sorte et l'effet artistique des monuments représentés en perspective ainsi que des paysages qui les encadraient le plus souvent."

instruction, which had to be theoretical and general in order to be applied, later on, in a great variety of special professions. This reformulation resulted in the growing importance of analysis at the expense of descriptive geometry and its applications. It therefore promoted an articulation between theory and application, as well as between the general and the special, very different from the one that had been designed by Monge. Yet, we shall see that Monge's legacy would remain vivid in the Metz school of artillery and engineering applications.

The Alliance Between Practice and Theory

Founded in 1794, the Metz artillery school would become the principal application school of École polytechnique after Napoléon merged it with the Mézières school of engineering in 1802.[97] In the first decades of the century, several alumni of the Metz school played a prominent role in the development of descriptive geometry and of its teaching in France, among whom the mathematician Jean-Victor Poncelet (who would become professor at Metz in 1825),[98] and two important promoters of geometric models, Théodore Olivier and Libre Bardin. As for the *Mémorial de* l'artillerie, a journal founded in 1824 and attached to both the school and the corps of artillery, it soon became a major periodical publication on descriptive geometry. Quite often, this journal published applications to engineering issues of more theoretical memoirs published in the *Journal de l'École polytechnique,* such as with the interplay of the publications of the colonel Lefevre on models of racks, pinions, and gearings, and of Olivier's mathematical research on space curves.

Bardin and Olivier graduated from Metz in 1816 and 1815 respectively, as lieutenants of artillery.[99] The first would quit the army in 1820 for experimenting with some business activities for a few years while the second would remain in the Metz school for a couple of years as adjunct to the *instituteur* of mathematical sciences and physics, before moving to Sweden between 1821 and 1826 for organizing the polytechnic instruction at the Royal School of Marienberg. They would meet again in the late 1820*s* at École polytechnique, where Bardin would be named professor of drawing and fortification (and would later be charged with managing all graphical works, in 1852), Olivier as *répétiteur* of descriptive geometry.

As most of the followers of Monge and supporters of geometry, Olivier and Bardin were active proponents of the industrialization of France, especially in

[97] Bruno Belhoste and Antoine Picon, eds., *L'École d'application de l'artillerie et du génie de Metz 1802–1870: Enseignement et recherches* (Paris: Musée des Plans-Reliefs, 1996).
[98] On Poncelet's teaching in Metz, see: Konstantinos Chatzis, "Les cours de mécanique appliquée de Jean-Victor Poncelet à l'École de l'Artillerie et du Génie et à la Sorbonne, 1825–1848," *Histoire de l'Éducation* 120 (2008): 113–38.
[99] On Olivier, see: Sakarovitch, "Théodore Olivier," 326–35; Eugène-Melchior, Peligot, Albert Perdonnet, Alexandre Dumas, "Funérailles de M. Théodore Olivier," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 591 (September 1853): 502–10; Eugène Rouché, "La vie et les travaux d'Olivier," *Annales du Conservatoire des arts et métier*s 2, no. 8 (1896): 21–22.

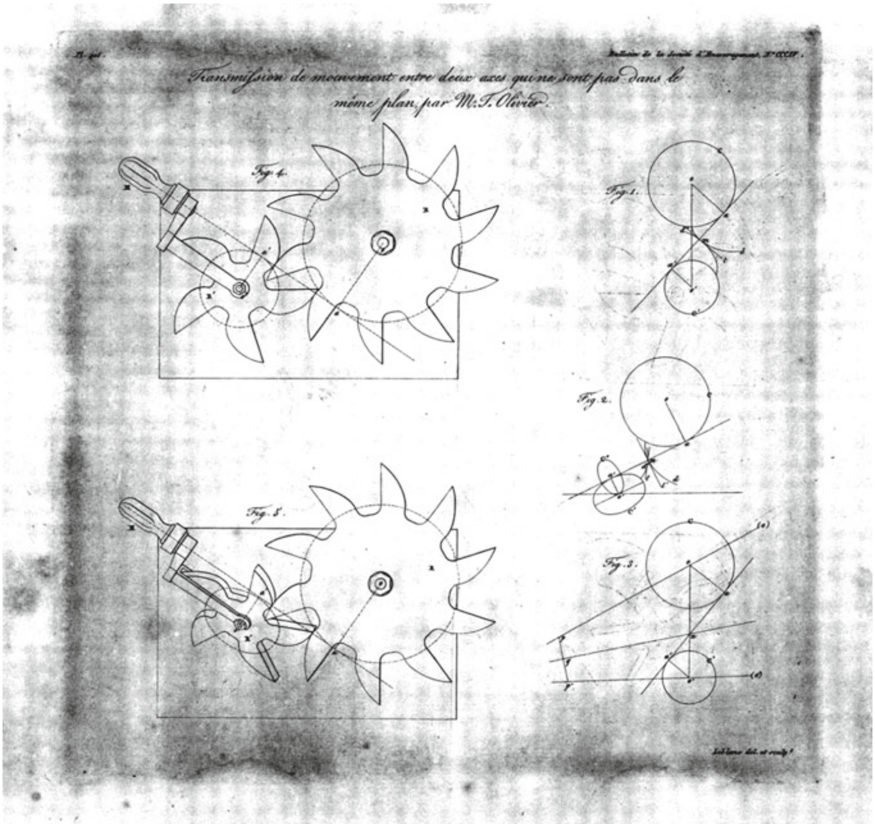

Fig. 14 Théodore Olivier, "Note sur un mode de transmission de mouvement entre deux axes qui ne sont pas dans un même plan," *Bulletin de la société d'encouragement nationale*, t. 304 (1829): 431

the Societé d'encouragement pour l'industrie nationale (Society for Encouraging National Industry), an organization established in 1801 to promote French industry.[100] The Société d'encouragement especially promoted innovations, by awarding prizes to inventors, and supported the development of technical education (see Fig. 14). Both Bardin and Olivier were especially active in the creation

[100] On the connection between the promoters of synthetic geometry and the industrialists, see: Lorraine Daston, "The Physicalist Tradition in Nineteenth Century French Geometry," *Studies in History and Philosophy of Science Part A* 17, no. 3 (1986): 269–95. On the technocratical role played by the engineers trained at Ecole polytechnique, see: Bruno Belhoste and Konstantinos Chatzis, "From Technical Corps to Technocratic Power: French State Engineers and their Professional and Cultural Universe in the First Half of the nineteenth century," *History and Technology* 23, no. 3 (September 2007): 209–25.

of new courses of descriptive geometry, which they conceived as the "writing of the engineer"("l'écriture de l'ingénieur"):

> […] the one who knows how to read space can visit a factory or a manufactory without taking any note; after returning home, he can draw the tool and the machine that he has rightly seen and understood.[101]

Olivier, in particular, was a fierce opponent to the theoretical turn taken by École polytechnique under the influence of Laplace.[102] He blamed the "theoreticians," such as Laplace and Augustin Louis Cauchy, "who fashion themselves as pure scholars and consider that they form an aristocratic corp with the legitimacy to command and dominate practitioners."[103] Faithful to Monge, Olivier often used the Societé d'encouragement as a tribune for vindicating the articulation between practice and theory promoted in the first plan of instruction of École polytechnique. He never stopped insisting that "it is only through materialization that one can use the truths discovered by intelligence:"

> Without theory, practice is blind; theory is the torch that guides us. Without practice, the truths obtained by theoretical research are no more than idealities, which are useless to man's terrestrial condition, and which may only charm humans because they are intelligent beings […], practice must precede theory. It is only through materialization that one can use the truths discovered by intelligence. Such is, always and everywhere, the law of useful labor […]. Do not forget ever the principle, so powerful and fruitful, of the *alliance between practice and theory*.[104]

For Olivier, this alliance even involved political issues. He especially attributed the political turmoil of the 1848 revolution to utopian idealities and contrasted the love of vainglory and excessive freedom with the morality resulting from the love of work and of useful science. In 1849, he concluded a vibrant plea for developing a more important and diversified use of instruments in the teaching of geometry by claiming that:

> An education that would be limited to theoretical ideas, and in which science would only be studied from an abstract point of view, will produce a people of ideologues and dreamers; such an education will never train useful citizens. The most beautiful ideas are useless to

[101] Olivier, quoted in Sakarovitch, "Théodore Olivier," 328.

[102] Théodore Olivier, *Mémoires de géométrie descriptive, théorique et appliquée* (Paris: Carillan-Gœury & Dalmont, 1851), i-xxiii.

[103] Ibid. See also: Sakarovitch, "Théodore Olivier," 327.

[104] Théodore Olivier, "Notices industrielles," *Bulletin de la société d'encouragement pour l'industrie nationale* 580 (October 1852): 716-17, here: 717: "Sans la théorie la pratique est aveugle la théorie est le flambeau qui nous guide Sans la pratique les vérités dues aux recherches de la théorie ne sont que des idéalités inutiles à l'homme en sa condition terrestre et qui seulement peuvent le charmer parce qu'il est un être intelligent. […] La pratique doit précéder la théorie Ce n'est qu'en les matérialisant que l'on peut utiliser les vérités découvertes par l'intelligence Telle est en tout et partout la loi du travail utile […] N'oubliez jamais en toute chose le principe si puissant et si fécond de l'*alliance de la pratique et de la théorie*."

> man until they are *materialized*. What is the use of moral truths to humanity, until they are put into practice, that is into traditions and into laws? It is only by making *use* of it that one can recognize whether a *thing* is good or bad; in order to know whether an *idea* is good or bad, one has therefore to *materialize* it, so that men can use it and appreciate its value. A materialized idea is the mind assuming a body, it *is the Word being made man.*[105]

As is exemplified by Olivier's discourses, the proponents of industrialization who supported the development of a technical education for the working class often associated popular education with moral issues.[106] The teaching of geometric drawing, in particular, was associated with the values of order, discipline and with the taste for work well done. The diffusion of geometric drawing in primary and technical education in the 1830 therefore participated to both the conservative political agenda of the constitutional monarchy and to the ideal of emancipation through education in mathematics.[107]

Because of his frustration with the evolution of his alma mater, which he blamed as having turned into an "École monotechnique" by focusing on analysis, Olivier participated in the foundation of the École centrale des arts et manufactures in 1826 (Central school for arts and manufactures),[108] in which he would be named professor of descriptive geometry and 'directeur des études' (dean of studies) in 1828, a position that would later be attributed to Bardin as well, from 1839 to 1841. Later on, Bardin and Olivier would both become professors at the Conservatoire national des arts et métiers (National Conservatory of Arts and Crafts), and Olivier would even be named director of the Conservatoire from 1852 to his sudden death in 1853. It was there that both Olivier and Bardin promoted the use of models for teaching descriptive geometry by designing innovative mathematical models. But before investigating these models further, we shall first discuss the specific educational model of the Conservatoire.

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[105] Théodore Olivier, "Rapport fait par M. Théod. Olivier, au nom d'une commission spéciale, sur une nouvelle méthode de géométrie pratique, sans instruments, de M. Martin Chatelain, professeur à l'Athénée national," *Bulletin de la société d'encouragement pour l'industrie nationale* 544 (October 1849): 481–85, here: 485: "Un enseignement par lequel on ne donnera que des idées théoriques, dans lequel on n'étudiera les sciences que sous le point de vue abstrait, donnera un peuple d'idéologues et de rêveurs ; un pareil enseignement ne formera jamais de citoyens utiles. Les plus belles idées sont inutiles à l'homme tant qu'elles ne sont pas matérialisées. En quoi sont utiles à l'humanité les vérités morales, tant qu'elles ne sont pas passées dans la pratique, c'est-à-dire dans les meurs et dans les lois ? Ce n'est qu'à l'user qu'on peut reconnaitre si une chose est bonne ou mauvaise ; pour savoir si une idée est bonne ou mauvaise, il faut donc de, toute nécessité la matérialiser, pour que les hommes puissent d'abord s'en servir et ensuite en apprécier la valeur. Une idée matérialisée, c'est la pensée qui revêt un corps c'est le Verbe fait homme."

[106] d'Enfert, *L'enseignement du dessin*, 105.

[107] Pierre Rosanvallon, *Le moment Guizot* (Paris: Gallimard, 1985).

[108] Francis Potier, *Histoire de L'Ecole centrale des arts et manufactures* (Paris: Delamotte, 1887), 25–27.

Learning by Drawing at the Conservatoire and Beyond

The Conservatoire national des arts et métiers is one of the grandes écoles established by the National Convention during the French Revolution, along with École polytechnique and École normale.[109] In contrast with Polytechnique, the Conservatoire was not designed as a school but as a "depository for machines, models, tools, drawings, descriptions and books in all the areas of the arts and trades."[110] The Conservatoire was therefore charged with the collections of inventions, in which models and drawings played an important role (see Fig. 15). It did not originally provide lectures but was rather a place that could be visited, especially for the purpose of drawing the models of its collections. This activity was strictly regulated: the Conservatoire was opened to the public on Thursday and Sunday but the permission to practice drawing as well as to access the drawings in its archives required addressing a request to its director.

In 1798 though, Claude-Pierre Molard, the administrator of the Conservatoire, designed the project to create a "free school of drawing applied to the arts" (i.e. the techniques).[111] This school would eventually be created in 1806 with four professors, one of arithmetic and elementary geometry, one of descriptive geometry and its application to carpentry, stonecutting, etc., one of elementary architecture and drawing applied to mechanics, and one of figure drawing. In contrast with École polytechnique, the Conservatoire drawing school was designed for workers and not for engineers. It therefore did not include any lecture on higher mathematics, in accordance with the usual role played by mathematical knowledge in the hierarchy and the management of the French industrial and scientific institutions.[112] But the Conservatoire nevertheless appropriated the pedagogical method developed at Polytechnique for teaching descriptive geometry through model drawing.[113] Molard ordered several models of hyperbolic paraboloid designed by Hachette and fabricated by Brocchi for formalizing the moldboard plow attributed to Thomas Jefferson, a model of obtuse angle applied to a ship rudder designed after a drawing made by Poncelet and based on Hachette's *Traité des machines*, as well as Brocchi's stereotomy compass and it's applications to modern moldboards.

[109] Claudine Fontanon, "Les origines du Conservatoire national des arts et métiers et son fonctionnement à l'époque révolutionnaire (1750–1815)," *Les cahiers d'histoire du CNAM* 1 (1992): 17–44.

[110] Aimé Laussédat, "Le Centenaire du Conservatoire des Arts et Métiers," *Annales du Conservatoire des arts et métiers* 3, no. 1 (1899): 1.

[111] Alain Mercier, "Les débuts de la 'petite école.' Un apprentissage graphique au Conservatoire sous l'Empire," *Les cahiers d'histoire du CNAM,* 4 (July 1994): 27–55.

[112] On the similar role played by mathematics in the hierarchy and the management of observatory sciences in the nineteenth century, see: David Aubin, "Observatory Mathematics in the Nineteenth Century," *Oxford Handbook for the History of Mathematics*, ed. Eleanor Robson and Jacquelin Stedall (Oxford: Oxford University Press, 2009), 273–98.

[113] The first professor of mathematics at the Conservatoire, Louis Gautier, was an alumnus of polytechnique and a former student of Hachette. On the influence of Monge's followers on the CNAM, see: Émmanuel Grison, "L'École de Monge et les Arts et Métiers," *Bulletin de la Sabix* 21 (1999): 1–19.

Fig. 15 Émile Bourdelin (drawer) & Eugène Mouard (printer), "Salles rénovées du Conservatoire national des arts et métiers." From *Le Monde illustré*, (9 May 1863): 301

In turn, the Conservatoire was especially influential for the development of the teaching of geometric drawing in the other "écoles d'arts et métiers," which were created in the first half of the nineteenth century in Châlons, Angers, and Aix for providing medium level qualifications, especially to workshop foremen and machine-shop crew chiefs,[114] as well as in the practical mining schools of Alès and Saint-Étienne. Already in 1793, Monge had designed a project of establishing schools for workers and craftsmen,[115] but Monge's plan was not followed by the Convention and the teaching of descriptive geometry to workers would mainly be driven by the Conservatoire. After the 1820s, the education of the working class in geometry was especially supported by the idea that modern industry required exact drawing, and therefore the systematic use of orthogonal projection at the expense of perspective. Republican and Saint-Simonian theories about the intellectual improvement of the French working class renewed the interest in universal education that had achieved its first peak during the Revolution.

In 1819, the Conservatoire created three new public courses applied to the arts: in mechanics, chemistry, and industrial economy respectively. The course of applied mechanics was attributed to Charles Dupin, another former student of Monge at École polytechnique, who had graduated from the École d'application du Génie Maritime (naval engineering application school).[116] The academic work on geometry that Dupin had developed on the side of his activity as a naval engineer had earned him to be nominated to the body of the Paris Academy of Sciences one year before his nomination at the CNAM.

In 1825, Dupin promoted the creation of free public courses of applied geometry and mechanics in 57 cities.[117] These courses were inspired by the schools recently established in the United Kingdom for developing the instruction of workers in applied sciences. They targeted the various professions of the 'industrial class,' such as architects, carpenters, joiners, bricklayers, sculptors, painters, engravers, or even surgeons or anatomists. The charge of 20 of these 57 courses was attributed to former students of Ecole polytechnique, "true followers of the

[114] In Châlon, the "chef de travaux" was a former student of Polytechnique: François Emmanuel Molard. See: Charles R. Day, *Les écoles d'Arts et Métiers. L'enseignement technique en France, XIXe-XXe sicèle* (Paris: Belin, 1991).

[115] René Taton, "Un projet d'écoles secondaires pour artisans et ouvriers, préparé par Monge en septembre 1793," in *L'École normale de l'an III*, vol. 1: Leçons de mathématiques: Laplace, Lagrange, Monge, ed. Jean Dhombres (Paris: Dunod, 1992), 574–82.

[116] Konstantinos Chatzis, "Charles Dupin, Jean-Victor Poncelet et leurs mécaniques pour 'artistes' et ouvriers," in *Charles Dupin (1784–1873). Ingénieur, savant, économiste, pédagogue et parlementaire du Premier au Second Empire*, ed. Carole Christen, François Vatin (Rennes: Presses universitaires de Rennes, 2009), 99–113; Carole Christen, "Les cours pour les ouvriers adultes au Conservatoire des arts et métiers dans le premier XIX^{e} siècle," *Cahiers de RECITS* 10 (2014): 33–56; Robert Fox, "Un enseignement pour une nouvelle ère: le Conservatoire des arts et métiers, 1815–1830,"*Cahiers d'histoire du CNAM* 1 (1992): 75–92.

[117] Charles Dupin, "Prospectus d'un cours de géométrie et de mécanique appliquées aux arts et métiers, à l'usage des chefs et sous-chefs d'ateliers et de manufactures," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 255 (September 1825): 299–300.

illustrious Monge who will spread in the industrial class the enlightenment they have received from the genius of their master."[118] Even though the proportion of polytechnicians in these schools fell to about 25% in 1830,[119] many alumni of Polytechnique supported the development of free courses of geometry for workers. The Association Polytechnique, founded shortly after the 1830 Revolution, espoused the goal of raising the instruction of the working class. Its prototype was classes given by polytechnicians.[120]

The few actors, such as Dupin, Olivier, Bardin, or the colonel Arthur Morin,[121] who held teaching positions at the Conservatoire were therefore part of a much larger movement of engineers and artillery officers actively involved in the industrialization of France. Along the line of Saint-Simonian philosophy, they considered that industrial prosperity required putting the 'useful innovations' made by scholars in the service of the nation by increasing the instruction of the industrial class. In doing so they participated in spreading the pedagogical practices developed at Polytechnique, especially model drawing, as well as the ideals the school had inherited from the Enlightenment.

Geometry was indeed promoted by the polytechnicians as a mean of emancipation, by which workers would avoid the fate of being reduced to machines and thus the risk of proletarianization. Already in his 1793 project, Monge had promoted the project of teaching descriptive geometry to workers and craftsmen for developing not only rigor and exactness, but also the faculty of judgment, intelligence, and the "esprit d'analyse" ("analytical spirit"). As the mathematician Francœur phrased it when reporting on the textbook Dupin had designed for workers: "should the working people remain sunken in ignorance, they would badly serve the intelligence of the men who hire them, they could only be employed as a kind of machine, and would regress even further under the burden of a life much similar to the one of animals, that is limited to the exercise of physical strength […]. The manufacturing

[118] Charles Dupin, "Exposé fait à la Société d'Encouragement sur les progrès du nouvel enseignement de la géométrie et de la mécanique, appliquées aux arts et métiers, en faveur de la classe industrielle," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 257 (December 1825): 374–80.

[119] Renaud d'Enfert, "L'offre d'enseignement mathématique pour les ouvriers dans la première moitié du XIX[e] siècle: concurrences et complémentarités," *Les Études Sociales* 159 (2014): 85–101.

[120] Gérard Bodé, "Les Associations polytechnique et philotechnique entre 1830 et 1869," in *Le Paris des Polytechniciens. Des ingénieurs dans la ville, 1794–1964*, ed. Bruno Belhoste, Francine Masson, and Antoine Picon (Paris: DAAVP, 1994), 63–68.

[121] An alumnus of the Polytechnique and of the Metz application school, Arthur Morin was nominated as professor of descriptive geometry at the Conservatoire in 1839 after having worked in Metz from 1829 to 1834 as adjunct to Poncelet. See: Claudine Fontanon, "Arthur Morin (1795–1880). Un ingénieur militaire au service de l'industrialisation," *Cahiers d'histoire et de philosophie des sciences* 29 (1990): 90–117.

and industrial prosperity of the realm will result from popular education […].”[122] Dupin himself claimed that his textbook, which required no other prerequisites than the capacity to read and count, aimed not only at “accessing by simple steps to the intelligence of the methods of geometry and mechanics that are the most useful for the various branches of the industry,” but also at “developing the most precious faculties of intelligence, comparison, reflection, judgment, and imagination as well as to allow the workers to execute their work more effectively and less painfully.” In sum, it aimed at “preparing a new welfare for workers” and at “raising their morality by impressing in their mind the ideas and the habits of order and reason, which lay the most reliable ground for public peace and general happiness.”[123]

Olivier's String Models

Théodore Olivier's scientific interests were mainly focused on the mechanical theory of gearing,[124] and more precisely on the mathematical determination of the shape of gear teeth, which involved investigations on space curves and therefore fundamental research in geometry, which he also applied to the tracing of railroads.

After 1825, he started publishing both on mathematics, with memoirs sent to the Paris Academy of sciences or to the *Journal de l'École polytechnique*, and on

[122] Louis Benjamin Francœur, “Rapport fait par M. Francœur sur la publication d'un ouvrage de M. Charles Dupin, de l'Academie des Sciences, destiné à répandre dans la classe des ouvriers l'enseignement des élémens de géométrie et de mécanique,” *Bulletin de la Société d'encouragement pour l'industrie nationale* 257 (November 1825): 372–74, here 373: “Le peuple même des ouvriers, s'il demeure plongé dans l'ignorance, servira mal l'intelligence des hommes qui l'emploient, et ne pouvant même être employé que comme une espèce de machine, s'abrutira davantage sous le fardeau d'une existence trop semblable à celle des animaux dont sa force tient lieu. […] La prospérité manufacturière et industrielle d'un royaume est donc la conséquence de l'instruction populaire.”

[123] Dupin, “Exposé,” 374-5: “[…] conduire […] à l'intelligence des méthodes de géométrie et de mécanique les plus utiles aux différentes branches de l'industrie […]. Un second but que le nouvel enseignement doit atteindre, c'est de développer dans les industriels de toute classe, et même dans les simples ouvriers, les facultés les plus précieuses de l'intelligence, la comparaison, la réflexion, le jugement et l'imagination ; c'est de leur offrir des moyens d'exécuter leurs travaux d'une manière moins pénible et plus fructueuse ; c'est de leur préparer un nouveau bien être ; c'est de rendre leur conduite plus morale en imprimant dans leurs esprits des idées et des habitudes d'ordre et de raison, qui sont les plus sûrs fondemens de la paix publique et du bonheur général.”

[124] Hachette suggested Olivier start his research by providing a mathematization in descriptive geometry to a special kind of spur or level gear which had been presented in Paris in 1810 by White, a British engineer, as a frictionless gear. For a description of Olivier's works on gearings, see: Sakarovitch, “Théodore Olivier,” 329 and Jacques M. Hervé, “Théodore Olivier (1793–1853),” in *Distinguished Figures in Mechanism and Machine Science*, ed. Marco Ceccarelli (Dordrecht: Springer, 2007), 296–319. See also: Théodore Olivier, *Théorie géométrique des engrenages: Destinés à transmettre le mouvement de rotation entre deux axes non situés dans un même plan* (Paris: Bachelier, 1842).

Fig. 16 Wooden models of gearings designed by Théodore Olivier for his teaching at the Conservatoire. © Musée des arts et métiers-Cnam, Paris/photos P. Faligot, all rights reserved

their technological applications to new types of gearings,[125] which he presented at the Société d'encouragement. Olivier would actually get very much involved in the Société d'encouragement. He wrote more than 15 reports for the "comité des arts mécaniques" of the society, on various types of innovative devices for railroads, gearings, riffles, machines, or even for rotating the biggest bell of the Metz cathedral, as well as on innovations in mathematical education, especially drawing instruments and geometric models.

After his nomination as a professor of descriptive geometry at the Conservatoire in 1839, Olivier designed a series of about 50 wooden models of gearings for teaching the applications of descriptive geometry by model drawing (see Fig. 16).[126] As for the more fundamental part of his teaching of descriptive geometry, Olivier developed an innovative approach by the methods of rotation, drawdown, and plane shift,[127] and adapted Monge's string models to his own mechanical concerns for ruled surfaces (see Fig. 17).[128]

The movable models he designed allow generating several ruled surfaces by changing the position of generating lines through the motion of an iron frame. These iron and string models were executed by Pixii and sons, a manufacturer of scientific instruments very close to École polytechnique. They were distributed

[125] About the interplay between geometry and technological applications, see, in particular, Francœur's report to a memoir presented by Olivier to the Société d'encouragement in 1829: Louis Benjamin Francoeur, "Rapport sur un Mémoire de M. *Olivier*, relatif à la vis sans fin," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 295 (January 1829): 9–10.

[126] On Olivier's models of gearings, see: Hervé, "Théodore Olivier," 308–12.

[127] Sakarovitch, "Théodore Olivier," 331.

[128] Ruled surfaces play a key role in mechanics because any rigid-body motion (or displacement) is a screw motion. A screw has an axis and, therefore, when a rigid body moves with respect to another body, the locus of all the screw axes is a ruled surface. Olivier's interest in ruled surfaces can be seen as pioneering Plücker's 1869 geometry of straight lines.

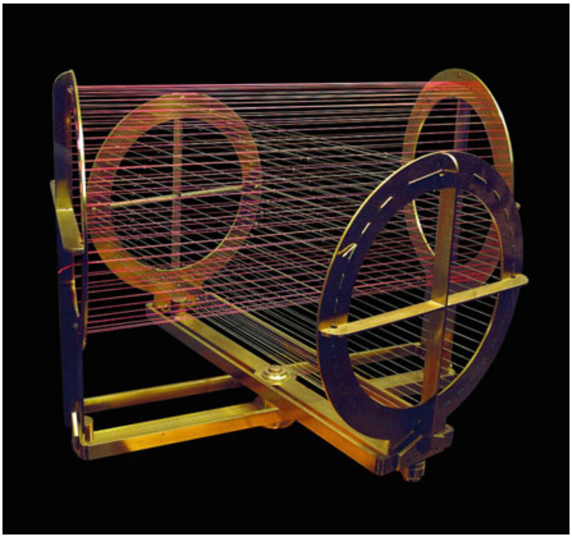

Fig. 17 Théodore Olivier, movable string model of the intersection of two cylinders. © Musée des arts et métiers-Cnam, Paris/photo P. Faligot, all rights reserved

by the Société centrale de produits chimiques,[129] and had a large circulation in engineering schools in Europe and in the USA.[130] Olivier's models fall into two categories.[131] In the first, lines of a determined length generate surfaces and the strings are held taut on a quadrilateral metal frame in which sides are articulated by four parallel hinges. Such is especially the case of the models of hyperbolic paraboloids and of intersections of two cylinders. In a second category of models, the motion results from the variation of the length of the generating lines, which are made of silk strings passing through two metal wires and are attached to lead weights hidden in a wooden box. One of these models allows for turning the combination of a revolution cylinder and one of its tangent planes into the combination of a hyperboloid of one sheet and a hyperbolic hyperboloid, or to a cone and one of its tangent planes. In addition to designing dozens of new mathematical models, Olivier also very much extended the collection of mathematical models, which had been initiated when the Conservatoire had acquired the models designed by Brocchi at École polytechnique.[132]

At the Société d'encouragement, Olivier published several instructions for the organization of the teaching of descriptive geometry for workers.[133] He especially insisted on the differences between training workers and training engineers. While

[129] Société centrale de produits chimiques, *Catalogue général illustré* (Paris: Gauthier-Villars, 1891), 878–83.

[130] Sakarovitch, "Théodore Olivier," 333; Amy Shell-Gellasch, "The Olivier string models at West Point," *Rittenhouse* 17, no. 2 (December 2003): 71–84.

[131] Sakarovitch, "Théodore Olivier," 332; Hervé, "Théodore Olivier," 308–19.

[132] Morin, *Catalogue des collections*, 18–30.

[133] Théodore Olivier, "Instruction pour l'enseignement de la géométrie descriptive dans les écoles d'arts et métiers de Châlons, d'Angers et d'Aix," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 546 (December 1849): 591–96.

descriptive geometry had to be taught as a science in engineering schools, it had to be reduced to a tool when taught to workers. Descriptive geometry, thus, was to be reduced to the "arts of projections" conceived as tools for "solving graphically" practical problems in the workshops, especially surveying relief surfaces as well as fabricating reliefs by the use of the drawings designed by engineers. Surveying required drawing the projections on two orthogonal planes of a relief model, while fabricating reliefs consisted in the reciprocal operation. In contrast to a formal course of geometry, the goal was to learn geometry practically by drawing and manipulating models of an increasing complexity: polyhedrons, plane sections of prisms and pyramids, plane sections of cylinders and cones, intersections of prisms, pyramids, cylinders and cones, the generation of ruled surfaces by the motion of a line, the flattening of developable surfaces on a plane, the construction of tangents and of intersection curves between two surfaces, and helicoids (such as screw-threads). Olivier therefore adapted to the training of workers in the *écoles des arts et métiers* the usual pedagogical methods associated with the teaching of drawing since the eighteenth century: the analytic decomposition/recomposition, the importance of action-learning and practical work, and the central role of models as opposed to textual knowledge and to lectures. For Olivier, the lectures of the professor had indeed to be limited to explaining the graphical methods required to draw special épures, while "graphical work had to be considered as a *manipulation* which does not aim at having the students *copy* drawings but to teach them to construct *exact épures* by using their knowledge and their intelligence."[134]

Olivier even designed a specific model and instrument for the training of workers in descriptive geometry. The 'omnibus' consisted in a box, whose top and bottom were made of cork and could be articulated in order to represent the two planes of projections in descriptive geometry. Four series of cards of various lengths and colors made possible a construction in space, by inserting red cards in the bottom of the box, and representing both the projection of this construction and the projecting lines by cards of three other colors. This instrument, Olivier claimed, "allows the students to touch by the *finger* and the *eye* all the problems relative to points, lines and planes, as well as to *see*, before mobilizing their intelligence by reasoning […] this instrument allows to teach students to *read space* and to switch from projections to relief, and reciprocally."[135]

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[134] Ibid., 595: "Le travail graphique doit être considéré comme une *manipulation* qui a pour but non de faire *copier* des dessins aux élèves, mais de leur apprendre à construire des *épures exactes* en se servant de leur *savoir* et de leur *intelligence*."

[135] Ibid., 596: "Cet instrument permet de faire toucher *du doigt et de l'œil*, aux élèves, tous les problèmes relatifs au point, à la droite et au plan, et de leur faire *voir*, avant d'attaquer leur intelligence par le raisonnement, alors qu'il faut démontrer les solutions des problèmes ; cet instrument a, de plus, l'avantage d'apprendre aux élèves *à lire dans l'espace*, et ainsi de passer des projections au relief, et *vice versa*."

Bardin's Plaster Models

Libre Bardin's use of plaster for designing his own mathematical models highlights a practice of geometry and its applications very different from Olivier's concerns for mechanics and gearings. Bardin's scientific activities were mainly devoted to the applications of descriptive geometry to topography, which constituted the core of his teaching on fortifications at École polytechnique.

Since the sixteenth and seventeenth centuries, the art of fortifications had developed the tradition of using plan-reliefs, i.e. scale models made to visualize building projects or campaigns surrounding fortified locations. From the construction of wooden scale models of cities and fortifications, the practice of plan-reliefs evolved to the fabrication of plaster models of topographies in the eighteenth century.

From the 1830s to the 1860s, Bardin was considered one of the foremost specialists in the application of descriptive geometry to topography.[136] His plaster scale-models of notoriously difficult topographies,[137] such as islands and mountains, were exhibited in various industrial fairs, including the 1855 and 1867 world fairs in Paris,[138] and in London in 1862 (see Fig. 18).[139] His "plan relief stéréotomique" of the Mont Blanc was praised for providing "the geometrical form of the mountain."[140] In contrast with other plan-reliefs, which lacked geometrical precision, most commentators highlighted the interplay between Bardin's theoretical knowledge in geometry and his very practical manual skills for working with plaster:

> Even though relief representation is not new, its fecundity had remained buried and sterile because it lacked applications; until a man, who has been trained to both the exact sciences and to sensing by the use of his eyes and his hands, convinced himself of the usefulness of reliefs for instruction […]. Thanks to the use of relief models, descriptive geometry has become much easier to teach […]. In the hands of M. Bardin, wood, plaster, and carton-pierre are turned into true prodigies of precision and exactness.[141]

136 On the evolutions of the methods and instruments of topographic drawing, see: Laussedat, "Recherches sur les instruments," 225–82.

137 Several of Bardin's plan reliefs are preserved in the Musée des plans reliefs in Paris, such as the one of the island of Port Cros. Others are preserved in the collections of the Conservatoire, such as the ones of the Mont-Cenis or of the landscape surrounding the city of Metz. See: Morin, *Catalogue des collections*, 64–65.

138 Félix Ferri-Pisani, "Cartes topographiques hydrographiques et géographiques," in *Exposition universelle de 1867 à Paris. Rapports du jury international*, vol. 2, ed. Michel Chevalier (Paris: Imprimerie administrative de Paul Dupont, 1868), 587–91.

139 Auguste Daubrée, "Cartes et plans en relief," in *Exposition universelle de Londres de 1862. Rapport des membres de la section française du jury international sur l'ensemble de l'exposition*, vol. 6, ed. Michel Chevalier (Paris: Napoleon Chaix, 1862), 129–33.

140 Jean-Gabriel-Victor de Moléon, ed., *Musée industriel et artistique, ou Description complète de l'Exposition des produits de l'industrie française faite en 1844* (Paris: Société polytechnique, 1844), 123.

141 Ibid., 123–24: "Ainsi la représentation en relief n'est pas neuve, mais, faute d'application elle demeurait enfouie et stérile dans sa fécondité ; il a fallu qu'un homme exercé aux sciences exactes,

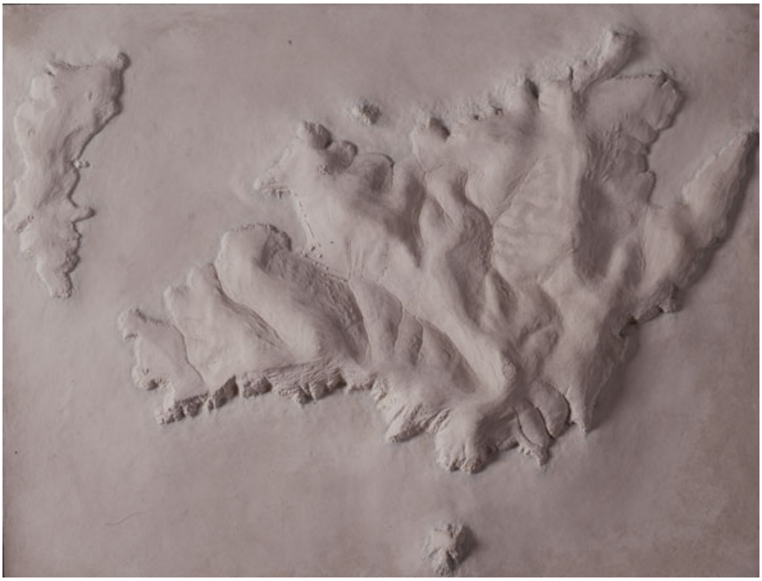


Fig. 18 Libre Bardin, plaster plan-relief of the Island of Port-Cros. © Musée des plans-reliefs (Paris)—Bruno Arrigoni, all rights reserved

Since Bardin's plan reliefs could be molded, and therefore reproduced industrially and quite cheaply, they became widely used for teaching topography.[142] When he became professor at the Conservatoire, Bardin used his skills for modeling plaster to fabricate plaster models of geometric solids that he used as drawing models for teaching descriptive geometry. One of his students, Charles Muret, who would himself become a surveying engineer and a professor at the Institut national agronomique,[143] continued and developed the use of plaster models for both topography and descriptive geometry.[144] Muret designed a collection of about

habitué à sentir par les ʟɪeux et par les mains se convainquit de l'utilité des reliefs dans l'instruction […] Ainsi la géométrie descriptive à l'aide des modèles en relief est devenue bien plus facile à enseigner […]. Le bois, le plâtre et le carton-pierre se transforment, sous la main de M. Bardin, en de vrais prodiges de précision et d'exactitude."

142 Charles Combes and Eugène-Melchior Peligot, eds., "Séance du conseil d'administration du 9 août 1867," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 2e série 14 (1867): 608.

143 Charles Muret, *Topographie, levés ruraux, remembrement,* vol. 2, 3rd ed. (Paris: J.B. Baillière, 1934).

144 Muret's plan reliefs were presented at the 1878 world fair in Paris. See: N.N., *Exposition universelle internationale de 1878. Section française. Deuxième groupe. Classe 16* (Paris: Imprimerie Delalain, 1878), 66. Several plan reliefs designed by Muret, such as of the city of Paris or of the Suez canal, are preserved in the collections of the Conservatoire. See: Conservatoire des arts et métiers, ed., *Catalogue du musée* (Paris: Conservatoire national des arts et métiers, 1953), 141–45.

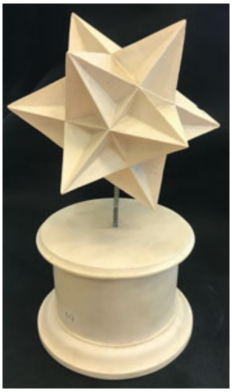


Fig. 19 Charles Muret, plaster model of an icosaedron © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: Frédéric Brechenmacher

600 models for the teaching of geometric drawing (see Fig. 19; see also Fig. 24). From 1865 to 1875, Delagrave edited this collection; a publishing house specialized in textbooks for both secondary and higher education. Muret's models had a very large circulation in both high schools and universities, all over Europe and the USA. The Paris faculty of science especially purchased the whole collection, which would form the seed of the Sorbonne cabinet of mathematics.

Model Drawing in Superior Primary Education

We have seen that a number of former students of École polytechnique had remained faithful to Monge's ideals about the role descriptive geometry and model drawing should play in the alliance between practice and theory. Several of them became involved in various experiments for developing the education of the working class, such as with Francœur and Jomard in the movement of mutual instruction at the beginning of the nineteenth century, and, after the 1820s, with Dupin, Bardin, Olivier and many of their fellow alumni of Polytechnique and Metz in the creation of free public courses of geometry all over the country.

In the 1830s, the French government eventually institutionalized a new system of public instruction for improving the education of children from modest households beyond primary education.[145] Established by the Guizot law of 1833, the "superior primary education" ("enseignement primaire supérieur") established a form of practical education parallel to the general secondary education provided by the lycées.[146] The curriculum of this new system of education especially included geometry and drawing, which were both usually taught by a professor of

[145] On the teaching of drawing in primary schools in France, see: Renaud d'Enfert, Daniel Lagoutte, and Myriam Boyer, *Un art pour tous. Le dessin à l'école de 1800 à nos jours* (Lyon: INRP, 2004). On mathematics in primary schools, see: Renaud d'Enfert, Hélène Gispert, and Josiane Hélayel, *L'enseignement mathématique à l'école primaire, de la Révolution à nos jours. Textes officiels*, vol. 1: *1791–1914* (Paris: INRP, 2003).
[146] Renaud d'Enfert, "Inventer une géométrie pour l'école primaire au XIXe siècle," *Tréma* 22 (2003): 41–49.

mathematics. The issue of challenging the industrial leadership of the British was instrumental in the development of technical education in France in the 1820s–1830s, and the promotion of geometric drawing was considered as a key issue for the construction machine industry.

The mathematician Louis-Benjamin Francœur, who had been both a student and a chef de brigade in the very first promotion of Polytechnique, conceived in 1819 a form of descriptive geometry adapted to mutual instruction: the "dessin linéaire."[147] Linear drawing was designed by Francœur as one of the four branches of primary education, along with reading, writing and arithmetic. It was organized by a progressive, analytic, method, from the drawing of straight lines and the simplest geometric figures to the complex patterns of architecture and eventually the human figure. The analytic method was more generally at the core of mutual instruction in which knowledge was decomposed into a series of "tableaux" ("tables") that were displayed in the classrooms where small groups of 8–10 children were supervised by a "moniteur" ("supervisor"). Francœur's dessin linéaire initially presented 5 tableaux of geometric models. In comparison, 125 tableaux were involved in Jomard's method for teaching reading, and 88 in the one for teaching arithmetic. But the importance of models was increased in the next editions of Francœur's dessin linéaire, with 10 tableaux in 1827, and 16 tableaux in 1832. The development of the diversity of models for the teaching of linear drawing was strongly supported by the Société pour l'instruction élémentaire in 1822 and a large number of textbooks were published after the 1830s along with plates of geometric models (see Fig. 20).[148]

With the Guizot law, the teaching of linear drawing was extended to superior primary education and to the écoles normales primaires established for training the teachers of primary schools. Several alumni of the Polytechnique were involved in designing and promoting the use of geometric models in superior primary instruction, such as with the patterns of cardboard models of polyhedrons published by Maximilien Marie in 1835,[149] and inspired by the models designed in London in 1758 by John-Lodge Cowley.[150]

At the Société d'Encouragement, Olivier strongly supported all pedagogical innovations based on the use of models and instruments. Their novelty was evaluated with the norms of technical and industrial innovations, especially manufacturing cost. For instance, in 1845, Olivier awarded the silver medal of the Société d'encouragement to the folded cardboard models of polyhedrons designed

[147] On linear drawing, see: d'Enfert, *L'enseignement du dessin*, 101–78.

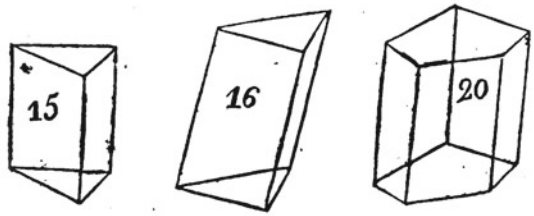
[148] d'Enfert, *L'enseignement du dessin*, 128–42. See especially the exercices of model drawing in: Auguste Bouillon, *Exercices de dessin linéaire contenant un choix très varié de modèles pratiques d'architecture, de marbrerie, de charpente, de menuiserie, de serrurerie, et d'ameublement* (Paris: Hachette, 1847).

[149] François-Charles-Michel Marie, *Géométrie stéréographique, ou reliefs des polyèdres pour faciliter l'étude des corps*, (Paris: Hachette, 1835).

[150] On the history of folding in mathematics, and on Cowley in particular, see: Friedman, *A History of Folding*, 77–80.

Le maître peut, sans frais, exécuter ces modèles en relief, propres à montrer les contours apparents, les arêtes, etc. Ce n'est qu'après avoir acquis l'habitude de ces conceptions, en dessinant sur la planche noire, ayant d'ailleurs sous les yeux les tableaux et les modèles; qu'on peut espérer que les enfants pourront copier de mémoire, sans trop altérer les perspectives.

Cette remarque s'applique aux pyramides, cônes, cylindres, sphères, qui sont dessinés dans les classes suivantes.



14 et 15. *Construisez un prisme triangulaire oblique*, fig. **16**, ou *droit*, fig. **15**.

Le *prisme* est un corps formé de deux polygones égaux et parallèles, dont les sommets semblables sont joints par des arêtes qui toutes sont parallèles et égales entre elles. Telles sont les figures **15**, **16**, etc., jusqu'à **22**. La *hauteur* du prisme est la perpendiculaire aux deux bases qui en mesure la distance; c'est une verticale qui se termine à sa rencontre avec les deux bases horizontales. On dit que le

Fig. 20 Francœur's 2nd tableau. From: Louis-Benjamin Francœur, *Dessin linéaire et arpentage*, 4th ed. (Paris: Bachelier, 1839), 41

by the civil engineer Louis Dupin, not so much because they innovated by integrating written text about the properties of each folded polyhedron on the folded cardboard itself (see Fig. 21),[151] but because they were much cheaper than the wooden models designed by several manufacturer in Paris:[152]

[151] On Dupin's models, see: ibid., 126–30.

[152] See, for instance: Albert Marloye, *Catalogue des principaux appareils d'acoustique et autres objets qui se fabriquent chez Marloye, à Paris, rue de la Harpe* (Paris: Ducessois, 1840), 15. See also the issue of the manufacturing cost as discussed by Olivier when awarding prices to the topographical relief models presented at the national industrial exhibition in 1844: Théodore Olivier, "Cartes

Fig. 21 Louis Dupin's folded models. © Musée des arts et métiers—CNAM/Photo: Aurélien Mole/Mudam Luxembourg, all rights reserved

> Relief models are so useful for teaching geometry that we must promote both the introduction and the continuation of their use in the primary school of art and crafts. Wooden models are pricey […], M. Louis Dupin had the fine idea of constructing a series of cardboard polyhedrons, which can be juxtaposed in order to form a cube.[153]

From 1844 to 1847, Dupin augmented his collection and entrusted their execution to the manufacturer of scientific instruments Molteni and Sigler for "delivering these solids at prices that should facilitate the introduction in schools."[154] The mathematical models and instruments designed for the teaching of geometry were displayed on a regular basis in industrial exhibitions,[155] world fairs, and eventually

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géographiques. Globes terrestres et célestes et terrestres. Cartes en relief. Modèles topographiques en relief. Planétaires," in *Exposition des produits de l'industrie française en 1844. Rapport du jury central*, vol. 2 (Paris: Fain & Thunot, 1844), 521–28. See also Olivier's focus on the issue of the manufacturing cost in regard with innovations in geometrical drawing instruments: Théodore Olivier, "Rapport fait par M. Théod. Olivier, au nom du Comité des arts mécaniques, sur un étui de mathématiques de M. Legey," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 404 (February 1838): 46–48.

[153] Théodore Olivier, "Rapport sur les solides en carton de M. Louis Dupin," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* 487 (January 1845): 10–11: "Les modèles en relief sont si utiles pour l'enseignement de la géométrie, que l'on doit s'efforcer d'en introduire l'usage dans les écoles primaires et d'arts et métiers et de l'y conserver. Les modèles en bois sont coûteux […] M. Louis Dupin a eu l'heureuse idée de construire une série de polyedres en carton, qui, par leur juxtaposition forment un cube."

[154] Théodore Olivier, "Extrait des procès-verbaux des séances du conseil d'administration," *Bulletin de la Société d'Encouragement pour l'Industrie Nationale*, 518 (August 1847): 443.

[155] For an early example, see the report of the 1849 national exhibition in Paris and its discussion of the various innovations developed in Europe for realizing cheap relief maps that may be used for teaching geography in elementary school as well of models for teaching elementary

in special exhibitions such as the gigantic Loan Exhibition of Scientific Apparatus in London in 1876,[156] and the exhibitions of mathematical devices which accompanied the first national and international congresses of mathematicians at the turn of the century. The world fairs, in particular, provided an international space of discussion, comparison, and competition for mathematical tools as for any other industrial innovations.[157] In the 1860s, the reports written by the French delegates to the world fairs highlight a growing sense of the increasing superiority of the models manufactured in Germany, especially in Darmstadt where Schroeder's manufacturing processes allowed to produce wooden models of descriptive geometry for a competitive price thanks to special machine tools and no less than 50 workers.[158]

The Models of Higher Geometry

In the 1860s–1870s, the exhibitions of models and instruments, that had been traditionally associated with technical and primary mathematical education, met with the new models of higher geometry that were designed by prominent mathematicians in the main centers of mathematical academic activity in Europe such as Göttingen, Munich, Cambridge, and Paris.[159]

Even though the models of higher geometry carried strong pedagogical ideals, these ideals were usually very different from the ones associated with the traditional use of models in technical and primary education. To be sure, both the traditional models and the new models of higher geometry were associated with the pedagogical values of visualization and manipulation, i.e. the issue of making use of the eye and the hand in the teaching of mathematics. But while the traditional use of models could not be dissociated from the idea that the eye and the hand had to be trained by the practice of model drawing, models of higher geometry were often designed for universities in which drawing was usually not associated with mathematical education.

geometry: Mathieu, "Globes célestes et terrestres; Machines planétaires; Cartes en relief; Modèles géométriques," in *Rapport du jury central sur les produits de l'agriculture et de l'industrie exposés en 1849*, vol. 2 (Paris: Imprimerie nationale, 1850), 562–66.

156 *Handbook to the Special Loan Collection of Scientific Apparatus 1876*, Piccadily: Chapman and Hall, 1876.

157 See: Ed. Grateau, "Instruments de mathématiques et modèles pour l'enseignement des sciences," in Chevalier, ed., *Exposition universelle de 1867 à Paris*, 521–47.

158 Ibid, 540.

159 Ibid. 541.

Naturalistic Mathematics

Pedagogical ideals were not the only force driving several prominent mathematicians to design models of higher geometry in the 1860s and 1870s. The issue of the classification of cubics and quartics required the careful investigation of the singularities and special configurations that allowed classifying species of surfaces in a naturalistic approach to mathematics.[160] In the 1820s, the classification of conic sections and quadrics had already highlighted the limits of algebraic methods: while analysis had allowed to classify all the surfaces of the second order by the algebraic character of the roots of their characteristic equations, this same method had failed to classify the types of intersections of two quadric surfaces. This issue had fuelled a criticism of the genericity of algebraic methods, which resulted in both the promotion of the geometrical investigation of singularities and the attention to algebraic singularities with the development of specific forms of representations such as invariants, determinants, and matrices.[161]

The geometrical characterization of singularities and special incidence configurations, as well as their combinatoric enumeration, played a key role in the classification of surfaces of order higher than two, especially cubic and quadric surfaces, like Hesse's inflection point configuration for cubic curves, or Schläfli's double six in connection with the 27 lines of a cubic surface (see Figs. 22 and 23). The wooden models designed by Plücker for displaying select features of a certain class of quartic surfaces linked to quadratic line complexes, or the ones constructed

Fig. 22 Model of the 27 lines on a cubic surface. © Collections de l'Institut Henri Poincaré, all rights reserved

[160] Rowe, "Mathematical Models," 5.

[161] On the role of the intersection of conics and quadrics in Cauchy's criticism of the genericity of algebra and ideals of rigor, see: Thomas Hawkins, "Cauchy and the Spectral Theory of Matrices," *Historia Mathematica* 2, no. 1 (1975): 1–20. On the evolution of this issue in algebra in the nineteenth century, see: Frédéric Brechenmacher, "La controverse de 1874 entre Camille Jordan et Leopold Kronecker," *Revue d'histoire des mathématiques* 13 (2007): 187–257.

Fig. 23 Alfred Clebsch diagonal surface. © Collections de l'Institut Henri Poincaré, all rights reserved

by Christian Wiener for representing Clebsch's classification of surfaces, played a role similar to the collections of species or minerals in natural history.[162]

The idea of working with models was especially derived from the traditional use of models and instruments in experimental physics. Plücker himself experimented with rarefied gases, built one of the first gas discharge tubes, and carried out his mathematical models with the assistance of Heinrich Geissler, the inventor of the eponym glass tubes.[163] Klein, who had assisted Plücker in designing his mathematical models, emphasized the connection between these models and Plücker's earlier research in physics as well as the influence of Michael Faraday who, according to Klein, had given Plücker the initial impetus to build models illustrating different types of the surfaces he unveiled as the centerpiece of his new line geometry.[164] It was also thanks to Plücker's reputation as an experimental physicist that his mathematical models had an almost immediate circulation in England.[165] As for the special quartics studied by Kummer in the mid 1860, they were directly associated with the caustics of geometrical optics.[166]

[162] The issue of the classification of conics and cubics was especially brought to the fore in Henri Fehr's reviews on geometric models in the journal *L'enseignement mathématique*. About the several types of classifications materialized by Wiener's models, see: Henri Fehr, "Herm. Wiener. Die Einteilung der ebenen Kurven und Kegel dritter Ordnung in 13 Gattungen," *L'enseignement mathématique* 3 (1901): 310.

[163] Rowe, "Mathematical Models," 5.

[164] Klein, *Gesammelte mathematische Abhandlungen,* vol. 2, 3–7.

[165] Rowe, "Mathematical Models," 6.

[166] Ibid., 8.

These academic ideals contrasted with the ones associated with the traditional use of models and instruments in engineering schools or in industry. Klein, in particular, even though he had been much impressed by Olivier's mathematical models during his trip to Paris in 1870, was chiefly influenced by his two masters, Plücker and Clebsch, whose premature deaths left him with the responsibility of their legacies.[167] A few years later, Klein promoted the use of models in the mathematical laboratory he had founded with Alexander Brill—another former student of Clebsch—at the Munich Technische Hochschule.[168] Even though he carried on with the tradition of action-learning associated with models, as opposed to reading textbooks or assisting plenary lectures, and as providing a direct contact with natural forms, he did not aim at carrying on the tradition of teaching by drawing but was rather inspired by the role of the laboratory as a place of experimentation in physics, which had been especially promoted in Munich by his colleague Carl Linde. Rather than model drawing, it was the construction and the manipulation of mathematical models that Klein promoted as a way to deepen the mathematical training of doctoral students such as Walther Dyck and Hermann Wiener (the son of Christian Wiener).[169]

The naturalistic ideal to "render a great service to geometrical science by calling attention to the concrete shapes of objects, which are too apt, even in the mind of the serious student, to exist only as conceptions very imperfectly realized,"[170] was also the motto of the Cambridge modeling club founded by Arthur Cayley in 1873, and which especially benefited from the contribution of Olaus Henrici,[171] another former student of Clebsch who held a position at University College London where he aimed at developing a 'modern' pedagogical approach to geometry by breaking with both the logico-deductive tradition of Euclid and the algebraic formalism of analytical geometry. Both the classification of geometric surfaces and the material representation of specific mathematical properties aimed at promoting 'observation' in 'pure mathematics,' i.e. a value which had developed in the natural sciences, more precisely in observational sciences, and very much associated

[167] On this issue, and on the first series of Zinc model designed by Klein and his friend Wenker, see: Klein, *Gesammelte mathematische Abhandlungen,* vol. 2, 3.

[168] See: Rowe, "Göttingen Mathematical Tradition," 191; Rowe, "Mathematical Models," 15; Renate Tobies, "Felix Klein in Erlangen und München: ein Beitrag zur Biographie," in *Amphora. Festschrift für Hans Wussing zu seinem 65. Geburtstag*, ed. Sergei S. Demidov, David Rowe, Menso Folkerts, and Christoph J. Scriba (Basel: Birkhäuser, 1992), 751–72.

[169] On Hermann Wiener's use of models in his teaching in Darmstadt, see: Anja Sattelmacher, "Geordnete Verhältnisse. Mathematische Anschauungsmodelle im frühen 20. Jahrhundert," in *Berichte zur Wissenschaftsgeschichte* 36, no. 4: "Bildtatsachen," special issue, vol. 2, ed. Ina Heumann and Axel Hüntelmann (2013): 294–312.

[170] Henry John Stephen Smith, "Geometrical instruments and models," in *The Collected mathematical papers of James Joseph Sylvester*, vol. 2, ed. Henry John Stephen Smith and James Whitbread Lee (Cambridge: Cambridge University Press, 1894), 698–710, here: 699.

[171] June Barrow-Green, "Clebsch took notice of me: Olaus Henrici and surface models," *Oberwolfach Reports* 14, no. "History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences," ed. Jeremy J. Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe (2015): 2788–90.

Fig. 24 Model of two cyclids from the Muret collection. © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: Anne Chauvet

with the emergence of the ideal of objectivity.[172] In topology in particular, and in contrast to the classification of cubics in algebraic geometry:

> […] no complete *corps de doctrine* has yet been formed of the properties of situation of figures […]. We cannot therefore expect to find this part of the science of geometry extensively illustrated by models, or by drawings expressly prepared for the purpose. But any great collection of geometrical objects cannot fail to supply examples of such properties; and what is of more importance, may be expected to suggest entirely new points of view in a branch of inquiry, which, more than almost any other within the range of pure mathematics, is dependent on direct observation.[173]

These naturalistic academic ideals were to be disseminated all over Europe and the U.S.A. with the emergence of semi-industrial manufactures of mathematical models, starting with the editor Ludwig Brill, Alexander Brill's brother, who had inherited the familial printing house in Darmstadt. Klein's own specific philosophy of 'anschauliche Geometrie' has already been the subject of several historiographical studies. Let us simply recall the role played by Klein's discovery in 1870 of an error in a statement on the singularities of the asymptotic curves that lie on a fixed Kummer surface by the observation of a physical model of such a surface made by his friend Albert Wenker.[174] Klein developed the conception that, even though the realization of models usually comes only after algebraic studies with pen and paper, only the material representation of a model can demonstrate the

[172] Lorraine Daston and Peter Galison, *Objectivity* (New York: Zone Books, 2007).
[173] Smith, "Geometrical instruments and models," 700.
[174] See the letter from Klein to Sophus Lie cited in: David E. Rowe, "Klein, Lie, and their early Work on Quartic Surfaces," in S*erva di due padroni: Saggi di Storia della Matematica in onore di* Umberto *Bottazzini,* ed. Alberto Cogliati (Milano: Egea, 2019), 189–198. See also: Rowe, "On Franco-German Relations," 21–36.

very existence of a geometrical object and impress its 'true character' in the mind. As we have seen in the introduction of this paper, this idea of 'impressing the mind' by a direct contact with nature was already crucial to Rousseau's philosophy of education, but, in contrast to Rousseau, Klein did not associate it with the practice of drawing but with observation:

> There is an essential [*eigentliche*] geometry, which does not only mean to be, as the investigations discussed in the text are, a visualized [*veranschaulichte*] form of abstract investigations. Here it is the task to grasp the spatial figures in their full figurative reality [*gestaltliche Wirklichkeit*], and (which is the mathematical side) to understand the relations valid for them as evident consequences of the principles of spatial intuition [*Anschauung*]. For this geometry, a model—be it realized and observed or only vividly imagined—is not a means to an end but the thing itself.[175]

Visualization thus played a key role in the 'anschauliche Geometrie,' which Klein associated with a naturalistic philosophy of mathematical objects as both real objects and witnesses of the very nature of the human mind. Accordingly, Klein developed a pedagogical practice of models disconnected from drawing, even though the "visualization" allowed by models did require a preliminary training of the eye and the hand. Significantly, the creation of Klein's special seminar and laboratory of mathematics in Munich was made possible because, unlike most other such institutions, the Munich Technical Hochschule trained not only engineers and architects but also teaching candidates: this situation provided Klein with students who had already been trained in geometric drawing, as well as with the opportunity to disconnect his *anschauliche* approach to geometry with the practice of drawing, since his seminar was limited to the students pursuing the teaching candidates program.[176]

The Darboux-Caron Wooden Models

In contrast with the use of models promoted by Klein, the development of models of higher geometry in France carried on the tradition of model drawing. It actually resulted from an importation in the University of Paris of pedagogical practices developed in technical and primary education. Gaston Darboux, who held the chair of higher geometry at the Sorbonne, played a central role in this evolution. In the

[175] Felix Klein, *Vergleichende Betrachtungen über neuere geometrische Forschungen* (Erlangen: Andreas Deichert, 1872), 42: "Es gibt eine eigentliche Geometrie, die nicht, wie die im Texte besprochenen Untersuchungen, nur eine veranschaulichte Form abstracterer Untersuchungen sein will. In ihr gilt es, die räumlichen Figuren nach ihrer vollen gestaltlichen Wirklichkeit aufzufassen und (was die mathematische Seite ist) die für sie geltenden Beziehungen als evidente Folgen der Grundsätze räumlicher Anschauung zu verstehen. Ein Modell – mag es nun ausgeführt und angeschaut oder nur lebhaft vorgestellt sein – ist für diese Geometrie nicht ein Mittel zum Zwecke sondern die Sache selbst." English translation from Mehrtens, "Mathematical models," 289. See also: Felix Klein, "A comparative review of recent research in geometry," *Bulletin of the New York Mathematical Society* 2, no. 10 (1893): 244.
[176] Tobies, "Felix Klein," 751–72.

Fig. 25 The cabinet of mathematics of the Sorbonne (before 1914). Photo Ch. Barenne, Paris. ca. 1914

early 1870s, he promoted the development of the mathematical cabinet of the Paris faculty of science, whose collection of models had been initiated with the acquisition of the Muret collection (see Fig. 25).

Darboux also attentively followed the innovations developed abroad in the design of new models. In the *Bulletin des sciences mathématiques et astronomiques,* a review journal he managed since 1868,[177] he especially reviewed the models designed by Brill and Klein in Munich (see: Fig. 26):

> One has often wondered whether drawings and models are useful for mathematical education. […]. Whatever the opinion one may have on this issue […], everyone will agree that models provide a lively and striking ingredient for both students and professors, models allow displaying the results obtained after painful computations, or arduous discussions, in a real, concrete, and elegant form.[178]

[177] On Darboux's editorial work in the *Bulletin*, see: Barnabé Croizat, "Gaston Darboux: naissance d'un mathématicien, genèse d'un professeur, chronique d'un rédacteur," (Phd diss., University of Lille, 2016).

[178] Gaston Darboux, "Comptes rendus et analyses," *Bulletin des sciences mathématiques et astronomiques* 2, no. 6 (1882): 5–14, here: 5-6: "Souvent on s'est demandé s'il était utile

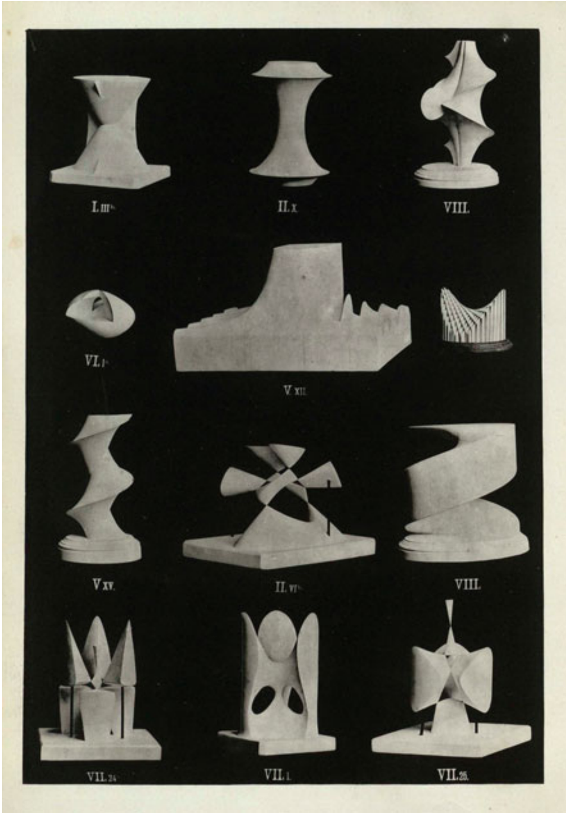

Fig. 26 Photos of models at the inner cover of Ludwig Brill, ed., *Catalog mathematischer Modelle für den höheren mathematischen Unterricht* (Darmstadt: L. Brill, 1881)

Yet, in contrast to most other prominent European mathematicians, Darboux did not only promote the value of vizualization of models but also carried on the traditional practice of model drawing. His lectures at the Paris faculty of science were accompanied with practical drawing activities in the tradition of the pedagogical methods developed by Monge at the Polytechnique and which, as we have seen, had broadly circulated in both technical and primary education in the nineteenth

d'employer des dessins et des modèles dans l'enseignement mathématique […] Et pourtant toute personne, quelle que soit son opinion sur la question posée précédemment, voudra bien convenir que le modèle fournit non seulement à l'élève, mais aussi au professeur, un élément plein de vie, saisissant, alors qu'après un calcul pénible ou après une discussion ardue le résultat peut être présenté sous une forme réelle, concrète et élégante." See also: Gaston Darboux, "Revue bibliographique," *Bulletin des sciences mathématiques et astronomiques* 8 (1875): 7–17.

century, but not yet in the university system.[179] The drawing activities were supervised by Joseph Caron, who had been appointed director of graphical works at École Normale Supérieure in 1872, a position identical to the one Bardin used to hold at École polytechnique.

Darboux has often been presented as the personification of the shift that occurred in the figure of the mathematician in France at the turn of the twentieth century, from the 'ingénieurs savants' trained at École polytechnique to the university professors trained at École normale. Darboux was indeed one of the first prominent mathematicians who favored École normale over the Polytechnique after having been ranked first in the competitive exams of both schools in 1861. While such a choice was very uncommon in the 1860s, it would become almost obvious for aspiring mathematicians in the 1880s. But Darboux's association with Caron for importing in the university the pedagogical methods developed at the Polytechnique also highlights a form of continuity in the evolutions of mathematics in France in the late nineteenth century.

In the tradition of Brocchi, Bardin, Olivier, and many others, Caron designed material models for the practical drawing activities associated with Darboux's lectures on curves and surfaces (see Fig. 27).[180] From 1872 to 1915, he supplied the Cabinet de mathématiques of the Sorbonne with about a hundred models, mainly made of wood.[181] He also published several textbooks of descriptive geometry for the candidates preparing for the competitive exams of the grandes écoles, such as École polytechnique and École normale supérieure.[182]

But while the use of models by Monge's followers had focused on either the basic elements of descriptive geometry or on its applications to the engineering sciences, Caron's wooden models aimed at representing the much more theoretical configurations presented in Darboux's lectures, such as a Kummer surface with twelve real double points, a rational algebraic surface of degree eight generated by the plane section of a cylinder rolling on another cylinder, the envelope of the normals for a Plücker conoid, minimal surfaces, etc. Even though they carried on the tradition of model drawing, Caron's series of wooden models of higher geometry thus broke up with the issue of the alliance between theory and applications, which, as we have seen, had been very much associated with the use of models at Polytechnique and, more generally, in technical education (see Fig. 28).

[179] At the time when Poncelet held the chair of applied mechanics at the Sorbonne, the latter could do nothing more than display a collection of models during his plenary lectures, since the university did not have any laboratory or other facility for practical work and model drawing. See: Paul Appel, "L'enseignement scientifique à l'Université de Paris," *L'enseignement mathématique*, 8 (1906): 337–42. The practical orientation of Poncelet's course at the Sorbonne was scarcely tolerated. See: Chatzis, "Les cours de mécanique appliquée," 128.

[180] Gaston Darboux, *Leçons sur la théorie générale des surfaces et les applications géométriques du calcul infinitésimal*, 1st ed. (Paris: Gauthier-Villars, 1887–1896).

[181] Apéry, "Caron's Wooden Mathematical Models," 38–48.

[182] See Joseph Caron, *Cours de géométrie descriptive. À l'usage des classes de mathématiques spéciales* (Paris: E. Foucart, 1883).

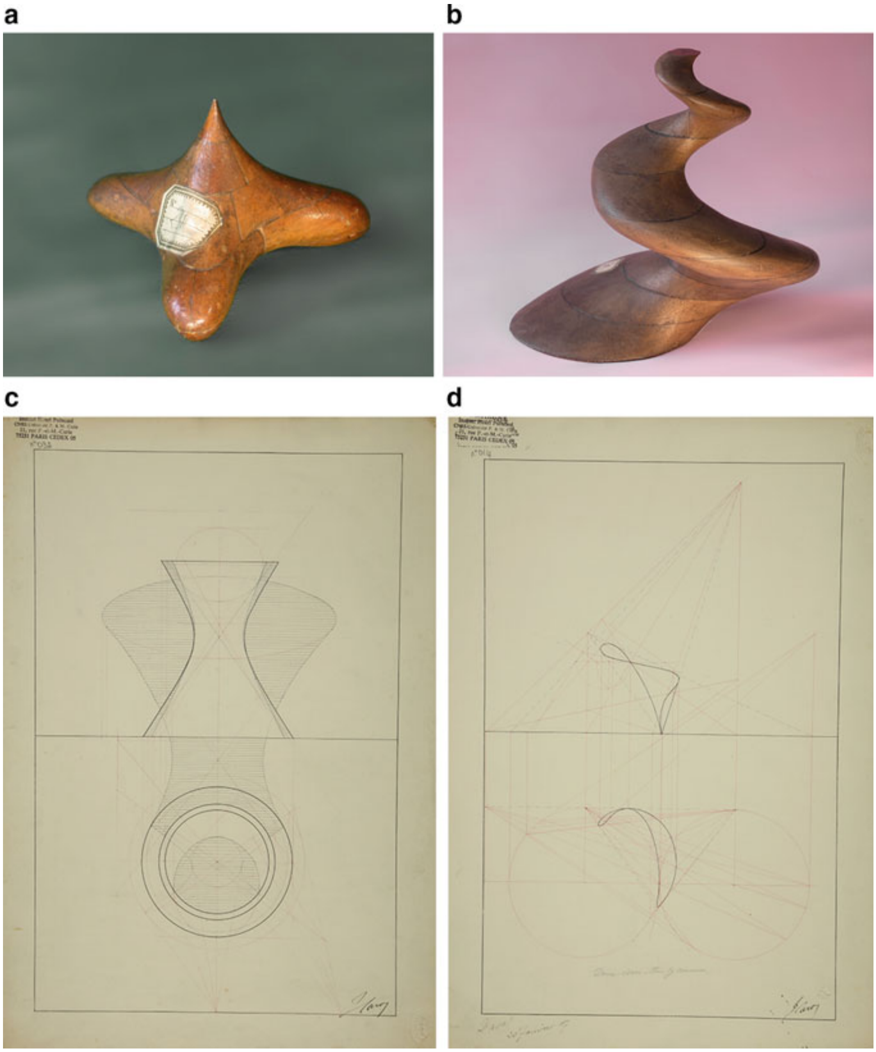

Fig. 27 **a** Quartic surface with a peak by Joseph Caron. © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: François Apéry. **b** Spiral surface by Joseph Caron. © Collections de l'Institut Henri Poincaré, all rights reserved. Photo: François Apéry. **c** Épure of a hyperboloid, with a work on shadows, by Joseph Caron. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°032. **d** Épure by a student of Joseph Caron in 1907: two cones with a common tangent plane. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°014

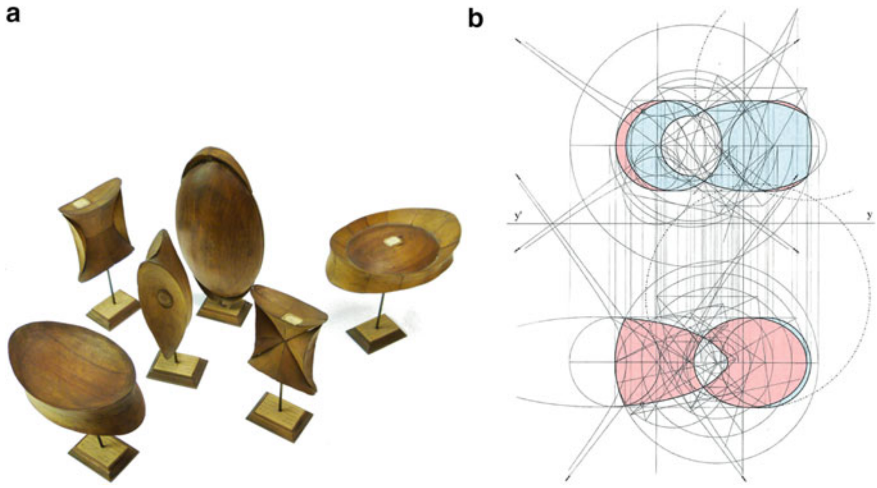

Fig. 28 **a** Six wooden models of normal surfaces by Joseph Caron. © Collections de l'Institut Henri Poincaré, all rights reserved. Photo © Anne Chauvet. **b** Épure of the common volume between two toruses by Joseph Caron at École normale supérieure in 1897. © Collections de l'Institut Henri Poincaré, all rights reserved. This épure is especially famous for having been considered as especially difficult by the mathematician Henri Lebesgue, who attended Caron's lectures in 1897. See: François Apéry, "La collection," in *Objets mathématiques*, ed. Cédric Villani et Jean-Philippe Uzan (Paris: CNRS éditions, 2017), 10–31, here 15

As with the models of higher geometry promoted by Klein in Germany, the ones of Darboux and Caron proceeded from an autonomization of mathematics as a discipline in the context of the development of higher scientific education. Even though Klein and Darboux were both active advocates of the interplay between theory and applications, this interplay did not take the same meaning in universities, even German technical universities, as the one which had been promoted in École polytechnique or in the Conservatoire des arts et métiers. Its focus was on the interplay between mathematics and other academic scientific disciplines rather than aiming at a direct usefulness for engineering sciences or the industry.

Darboux's course of higher geometry at the Sorbonne was very much oriented to fundamental applications to mechanics and optics. Several models designed by Caron display configurations of cinematic geometry and mechanics, such as space curves generated by the motion of a cylinder on a plane. Another series of eight models made between 1912 and 1914 were devoted to the caustics generated by a wave front in optics.[183] In the early 1890s, Darboux himself got involved in the conception of a drawing instrument based on geometrical notions but designed for experimental activities in mechanics. The herpolhodographer, designed with

183 Apéry, "Caron's Wooden Mathematical Models," 44–47.

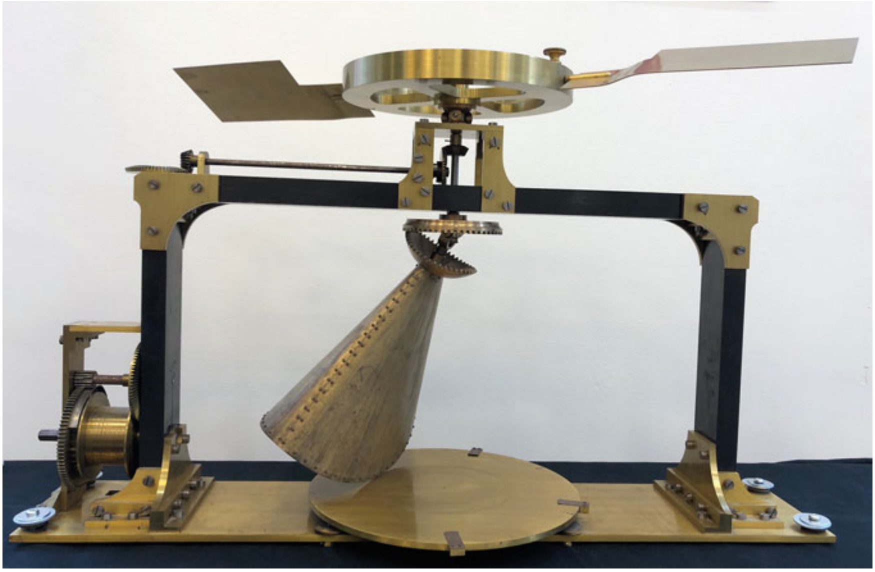

Fig. 29 Herpolodographer of Gaston Darboux and Gabriel Koenigs. © Collections École polytechnique, Palaiseau, all rights reserved

Gabriel Koenigs, made the tracing of herpolodies possible, i.e. space curves generated by the rotation of a rigid body around its center of gravity (see Fig. 29). It was fabricated by the manufacturer Château Père et fils in 1900 and presented to the world fair in Paris that hosted the second International Congress of Mathematicians.

While the traditional mathematical models and instruments had been displayed in industrial exhibitions and world fairs since the 1840s, exhibitions of models of higher geometry participated in exhibitions of scientific instruments and equipment, such as the major international exhibition held at the South Kensington Museum in 1876,[184] as well as to the emergence of congresses devoted specifically to mathematics, such as the conferences of German mathematicians in Munich in 1893 and in Hamburg in 1902, the international conference organized after the Chicago World fair in 1893, and the international congress of mathematicians in

[184] Henry John Stephen Smith, "Geometrical Instruments and Models," in *The Collected Mathematical Papers of Henry John Stephen Smith,* vol. 2, ed. James Whitbread Lee Glaisher (Oxford: Clarendon Press, 1894), 698–710.

Heidelberg in 1904.[185] Collections of models therefore participated in the emergence of both national and international communities of mathematicians and in the shaping of their public image.[186] Significantly, the founding congress of an association of professors of mathematics, organized by David Eugene Smith in New York in 1904, displayed both a collection of models and a collection of photographs and portraits of famous mathematicians.[187] By contrast, École polytechnique never purchased any of the models of higher mathematics manufactured at the turn of the century: since mathematical models were associated with model drawing at the Polytechnique, the more elementary models of descriptive geometry were undoubtedly more relevant than the ones of higher mathematics, especially since geometric drawing was a part of the elementary training of the students and was not anymore an issue in the more advanced courses of analysis, geometry, or mechanics.[188]

Models and the 1902 Educational Reform in France

We have seen that the development of collections of models of higher geometry participated in the much larger phenomenon of the autonomization of mathematics as an academic discipline, in contrast to the broad spectrum covered by the 'mathematical sciences' of the first part of the nineteenth century. The emergence of a market for model manufacturers was, in particular, a consequence of both the development of higher education in Europe and of the increasing role mathematics played in both general and technical education. At the turn of the twentieth century, several European nations initiated large educational reforms that aimed at promoting the links between pure and applied sciences in a context of fierce industrial, economical and military competition. Darboux and Klein played a key

[185] Henri Fehr, "Le 3^e^ Congrès international des Mathématiciens; Heidelberg, 1904. Les expositions de bibliographie et de modèles et instruments," *L'enseignement mathématique* 6 (1904): 476–81. On models in the ICMEs and ICMIS, see: Gert Schubring, "Historical comments on the use of technology and devices in the ICMEs and ICMI," *ZDM Mathematics Education* 42 (2010): 5–9.

[186] Ulf Hashagen, "Mathematics on Display: Mathematical Models in Fin de Siècle Scientific Culture," in *Oberwolfach Reports* 14, no. 4: "History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences," ed. Jeremy J. Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe (2015): 2838–41.

[187] N.N., "Association des professeurs de mathématiques des Etats Moyens et du Maryland," *L'enseignement mathématique* 6 (1904): 63–64.

[188] At the turn of the twentieth century, descriptive geometry was taught in the preparatory classes to the grandes écoles in the Lycées, i.e. before entering the Polytechnique. See: C. Roubaudy, *Traité de géométrie descriptive à l'usage des élèves des classes de mathématiques spéciales et des candidats à l'École polytechnique* (Paris: Masson, 1916).

role in these reforms.[189] Both aimed at developing the connections between general and technical education and both promoted action learning in the teaching of mathematics, especially by the use of models, in contrast to the tradition of Euclid's elements.[190]

In France, the 1902 reform played a crucial role in the development of scientific education in the lycées, i.e. the system of general secondary education, which, until then, had been dominated by the humanities. In contrast to primary and technical education, secondary education had maintained the teaching of imitation drawing in the tradition of the Beaux-Arts, with a focus on the model of the human figure. In 1852, the distinction between a scientific section and a literary section in the lycées had allowed the introduction of linear drawing,[191] and the role of linear drawing in the scientific sections of the lycées had been strengthened in the 1880, but it had nevertheless remained within the scope of the teaching of imitation drawing until 1902 when it was incorporated into the teaching of mathematics.[192]

University professors who aimed at adapting secondary education to the 'modern world' conducted the 1902 reform. But, in contrast with what had happened decades before in superior primary education and technical education, the development of a general scientific education was not legitimated solely by the usefulness of the applications of sciences. The reformers aimed not only at training "practical and useful men" but also at founding a "new humanism" in which "scientific humanities" would be no less involved than the literary humanities in the "formation de l'esprit" ("formation of the human spirit").[193]

As the president of the commission for the revision of the programs of mathematics, Darboux promoted the activity of the students, the "experimental method,"

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[189] In Germany, the "Meraner Lehrplan" presented in 1905 at the meeting of the Deutschen Mathematiker-Vereinigung in Meran was a new syllabus adopting some of the basic points of Klein's reform movement. It proposed approaching geometry via intuitive geometry and especially promoted the use of models for strengthening spatial intuition, the consideration of geometrical configurations as dynamic objects, making room for applications, and in connection with practical activities of drawing and measuring with the use of straightedge and compass, aiming especially at a coordination of planimetry and stereometry. See: August Gutzmer, "Bericht betreffend den Unterricht in der Mathematik an den neunklassigen höheren Lehranstalten," *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* 36 (1905): 543–53; Felix Klein, *Vorträge über den mathematischen Unterricht*, vol. 1 (Leipzig: Teubner, 1906), 208–19.

[190] This evolution was promoted at an international level by a great many contributors to *L'enseignement mathématique*. See, for example: "Conférence sur l'enseignement scientifique en Allemagne," *L'enseignement mathématique* 12 (1910): 387–93.

[191] d'Enfert, *L'enseignement du dessin*, 172.

[192] Renaud d'Enfert, "Entre mathématiques et technologie: l'enseignement du dessin géométrique dans le primaire et le secondaire (France,1880-début XXe siècle)," *Revista de História da Educação Matemática* 2, no. 2: "HISTEMAT," special issue (2016): 39–55.

[193] Bruno Belhoste, "L'enseignement secondaire français et les sciences au début du XX^{e} siècle. La réforme de 1902 des plans d'études et des programmes," *Revue d'histoire des sciences* 43, no. 4 (1990): 371–400; Bruno Belhoste, Hélène Gispert, and Nicole Hulin, eds., *Les sciences au lycée: Un siècle de réformes des mathématiques et de la physique en France et à l'étranger* (Paris: Vuibert-INRP, 1996).

and a focus on "concrete problems" as opposed to the abstract and logical reasoning of the traditional framework of Euclid's geometry.[194] In addition to transferring to mathematical education several ideals of the natural sciences, such as "observations," "experimentation," and "classification," the reform promoted the adaptation to general education of the pedagogical methods of primary and technical education, which aimed at rendering mathematics more accessible to more students. Learning by drawing was especially considered as one of the best methods to promote the activity of the students, "with the use of collections of models and of elementary instruments."[195] It aimed at developing the student's intuition of the geometrical space by experimenting with its "reality," and thereby developing a "lively perception" of the theorems of geometry as well as of their applications and industrial potentialities.[196] The use of plaster and string models was, in particular, associated with several pedagogical values such as the one of motivation, by "making geometry more lively and interesting," the valorization of manual skills, as a counterweight to "purely verbal definitions," as well as the capacity to "judge the usefulness" of theorems by experimental activities.[197]

The goal of the 1902 reform in transferring to general education the pedagogy of model drawing developed in technical education is especially highlighted by the professional trajectory of Célestin Roubaudi at the turn of the twentieth century (see Figs. 30, 31 and 32). Roubaudi had been trained at the École normale spéciale of Cluny, a school established in 1865 for training the teachers of technical schools and which stressed the important role of geometric drawing. After having passed the special competitive exam for teaching mathematics in technical schools, Roubaudi became professor of descriptive geometry at Cluny between 1880 and 1891, when this special school was cancelled. After the 1902 reform, Roubaudi moved from technical to general education by teaching descriptive geometry to students who prepared for the competitive exams of École polytechnique and École normale supérieure at the lycée Saint Louis and the lycée Louis-Le-Grand in Paris. He published several books on the teaching of geometric drawing in both general secondary education and in the grandes écoles and came to be considered as the one of the best specialists in descriptive geometry. After 1909, Roubaudi succeeded

194 N.N., "Modifications apportées au plan d'études des lycées et collèges de garçons," *L'enseignement mathématique* 7 (1905): 491–97.

195 Ibid., 495: "une collections de modèles et d'appareils simples". For an example of a textbook published in line with the 1902 reform and promoting the connection between geometry, drawing, and motion, see: Carlo Bourlet, *Cours abrégé de géométrie* (Paris: Hachette & Cie, 1907).

196 N.N., "Modifications apportées", 495. Among the many discourses promoting this evolution of mathematical education, see, in particular: Louis Crelier, "Le dessin de projection dans l'enseignement secondaire," *L'enseignement mathématique* 6 (1904): 300–4. See also: Christian Beyel, "L'enseignement de la géométrie descriptive dans les écoles moyennes," *L'enseignement mathématique* 3 (1901): 431–36.

197 N.N., "Modifications apportées", 493–4. See also: "Circulaire, adressée par M. Le Vice-Recteur de l'Académie de Paris à MM. les Inspecteurs d'Académie, Proviseurs, Principaux et Professeurs de Mathématiques et de Physique du ressort," *L'enseignement mathématique* 9 (1907): 231–34.

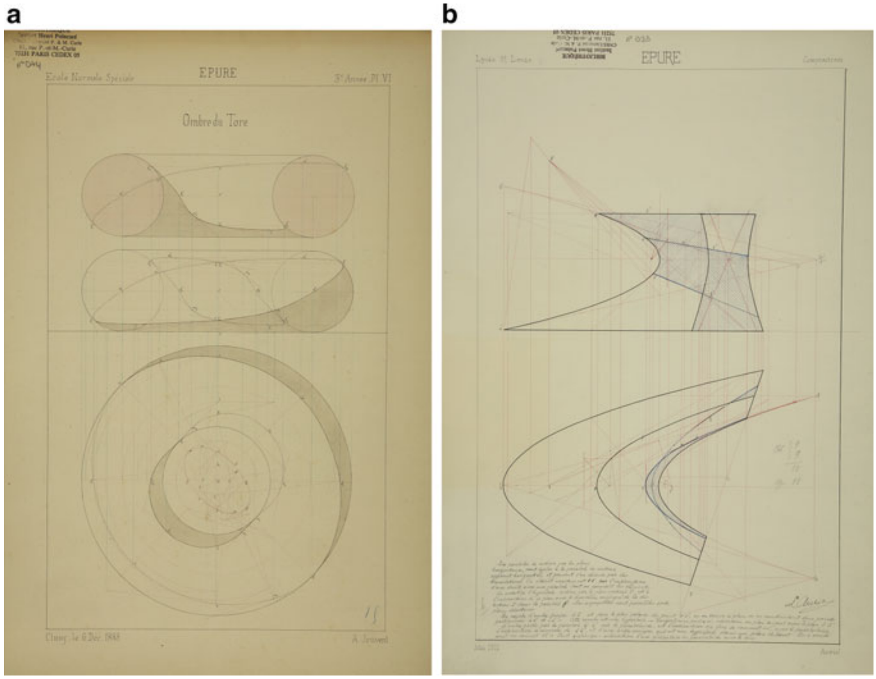

Fig. 30 **a** Épure of the shadow of a torus by Jouvent, a student of Célestin Roubaudi at the École normale spéciale de Cluny in 1888. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°074. **b** Épure by a student of Roubaudi at the lycée Saint Louis in Paris in 1911: intersection of a paraboloid and a cone. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°025

Joseph Caron as the director of graphical works at École normale supérieure and thus participated to the training of French elite mathematicians.

Even though the 1902 reform focused on secondary education, it also resulted in setting new goals for teacher training and thus impacted the universities. The agrégation of mathematics, i.e. the selective competitive exam one had to pass to become a professor in the lycées, was adapted to the reform in 1904. Faculties of science were encouraged to promote experimental activities by creating "laboratories of mathematics," "furnished with models and instruments as numerous and as diverse as possible."[198] Mathematical models thus participated to the hybridation

[198] N.N., "Les études générales des Mathématiques pures et appliquées et de la Physique," *L'enseignement mathématique* 10 (1908): 11–18, here 13 and 16: "laboratoires de mathématiques," and "Nous recommendons aussi des collections de modèles mathématiques […]. L'étendue de ces installations devrait être comprise à peu près […]."

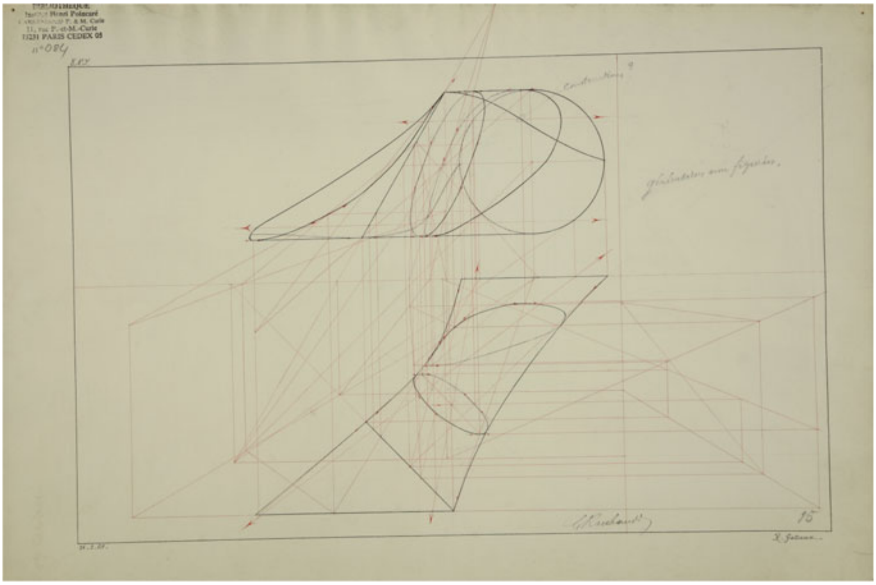

Fig. 31 Épure of the intersection of a hyberboloid and a cylinder by René Gateaux, a student of Célestin Roubaudi at École normale supérieure in 1908. Considered as one of the most promising mathematician of his generation, René Gateaux was killed in action during the first months of World War I. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°084

of two places of knowledge, the university library, a traditional place of mathematical practice, and the laboratory, which had become one the most emblematic places of both scientific and industrial activity in the century.[199]

In Paris, the mathematicians Jules Tannery and Émile Borel, who had both been strong supporters of the 1902 reform,[200] created the "laboratory of mathematics education" of the École normale supérieure with financial support of the faculty of science, secured by its dean, the mathematician Paul Appel.[201] It is likely that

[199] For the historiography of space in science studies see e.g.: Michel de Certeau, *The practice of everyday life*, trans. Steven Rendall (Berkeley: University of California Press, 1984); Owen Hannaway, "Laboratory Design and the Aim of Science: Andreas Libavius versus Tycho Brahe," *Isis* 77 (1986): 585–610; Adi Ophir and Steven Shapin, "The place of knowledge: a methodological survey," *Science in Context* 4, no. 1 (Spring 1991): 3–21; David N. Livingstone, "The spaces of knowledge: contributions towards a historical geography of science," *Environment and Planning D: Society and Space* 13, no. 1 (1995): 5–34; David Aubin, Charlotte Bigg, and Otto H. Sibum, eds., *The Heavens on Earth: Observatories and Astronomy in Nineteenth-Century Science and Culture* (Durham: Duke University Press, 2010).

[200] Émile Borel, "Les exercices pratiques de mathématiques dans l'enseignement secondaire," *Revue générale des sciences pures et appliquées* 15 (1904): 431–40.

[201] On laboratory of mathematics, especially in Italy, see: Livia Giacardi, "The Emergence of the Idea of the Mathematics Laboratory in the Early Twentieth Century," in "*Dig where You Stand"*

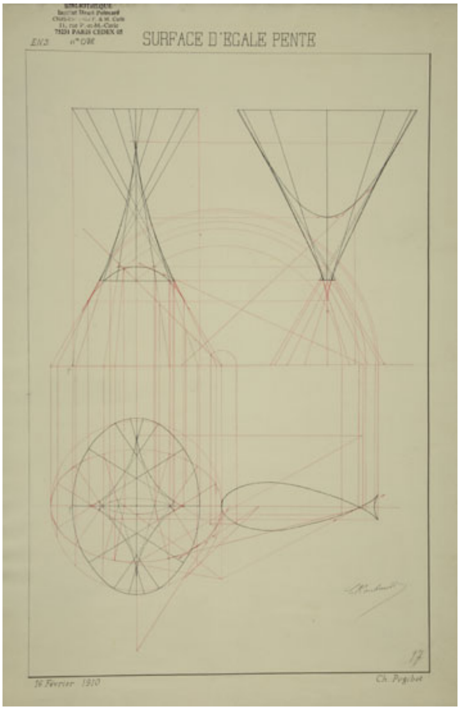

Fig. 32 Épure of a surface of constant slope by a student of Roubaudi at the École normale supérieure in 1910. The issue of drawing a surface of constant slope highlights the autonomization of mathematics as an academic discipline in regard with the issues associated with geometric drawing in elementary or technical schools. © Collections de l'Institut Henri Poincaré, all rights reserved, épure n°088

this laboratory was inspired by the evolution of the preparation to the German certificate of capacity for teaching in superior secondary education which, after 1901, included seminars and, on the model of the laboratory established by Klein in Munich in the 1870s, aimed at promoting both the activity of trainee teachers and the relationships between pure and applied mathematics, especially by the fabrication of models and the manipulation of instruments.[202]

The École normale laboratory aimed at training future teachers: models in wood, cardboard, or wire and cork were conceived and built for teaching geometry and mechanics. The didactic uses of other instruments such as mechanical linkages, pantographs, inversors, calculating machines, and instruments for geodesy and land surveying were also taught. The establishment of this laboratory thus participated to the expansion of the collection of the mathematical cabinet of the

2: Proceedings of the Second International Conference on the History of Mathematics Education, ed. Kristín Bjarnadóttir, Fulvia Furinghetti, José Matos, and Gert Schubring (Lisbon: UIED, 2011), 203–25; Livia Giacardi, "The School as a 'Laboratory.' Giovanni Vailati and the Project for the Reform of the Teaching of Mathematics in Italy," *International Journal for the History of Mathematics Education* 4, no. 1 (2009): 5–280.

[202] Friedrich Pietzker, "L'enseignement universitaire et l'instruction de maîtres des Gymnases," *L'enseignement mathématique* 3 (1901): 13–25. See also: Henri Fehr, "L'enseignement des mathématiques supérieures à Iéna," *L'enseignement mathématique* 3 (1901): 126.

Sorbonne, through both the local production of new models and regular acquisitions from the catalogs of German manufacturers such as Brill/Schilling. The local production of models was nourished by the creation of a woodcraft workshop at the École normale, where the students who prepared for the agrégation of mathematics had to practice woodcraft on a weekly basis under the supervision of a craftsman. This training in the handling of "the saw, the plane and the jointer plane" provided the students with the opportunity to design new models for the teaching of mathematics. As one of them, the later mathematician Albert Châtelet, phrased it in 1909:

> […] it is very useful [for teachers] to be aware of the skills required to master the design of small wooden models as well as to be able, when needed, to conduct the work of a craftsman for the reproduction of a model […]. A few collections of mathematical models are available in the market in France but they are mostly intended for primary education. Teachers, therefore, have to either design themselves the devices they would like to use in the classroom or to conduct their fabrication.[203]

The students were especially inspired by the new pedagogical practices developed by the French society of physics for promoting experimental education, as well as by the collections of instruments of the laboratory of physical mechanics. This laboratory had been created at the Sorbonne at the end of the nineteenth century for extending from geometry to applied mechanics the pedagogical method promoted by the tandem Darboux/Caron.[204] But the students also developed new practices specific to mathematical education, by designing "actual duplicates of proofs, by the use of figures in space instead of drawings on the blackboard."[205] Such models aimed at visualizing both traditional methods, such as the computation of the volume of a polyhedron by its decomposition into elementary polyhedrons, or the new concepts recently introduced in the lycées such as isometries, displacements, dilatations, and inversions: "motion cannot be represented on the blackboard."[206]

> Finally, we have a much smaller number of models associated with the geometry of the 5th book [of Euclid]. One of the most serious difficulties for beginners is to 'see' what is represented by the more or less rough figures in perspective used for illustrating the main proofs of the 5th book. This difficulty would be radically diminished if, before drawing a figure

[203] Albert Châtelet, "Le laboratoire d'enseignement mathématique de l'École Normale Supérieure de Paris," *L'Enseignement mathématique* 11 (1909): 206–10, here: 207-8: "D'autre part, il lui serait également très utile de se rendre compte des difficultés à surmonter pour la confection d un petit modèle en bois, et, de pouvoir au besoin diriger le travail d'un ouvrier pour la reproduction du modèle. […] On trouve actuellement dans le commerce, en France, quelques collections de modèles mathématiques mais destinées surtout à l'Enseignement Primaire. Les Professeurs sont donc encore obligés de faire confectionner sur place ou de confectionner eux-mêmes les appareils qu'ils désireraient utiliser dans leur classe."

[204] See Paul Appel, "L'enseignement scientifique à l'université de Paris," *L'Enseignement mathématique* 8 (1906): 337–43.

[205] Châtelet, "Laboratoire d'enseignement," 208.

[206] Ibid., 209.

> on the blackboard, the teacher showed the true figure in space to his students—a figure on the blackboard being basically nothing more than a diagram whose nature is more algebraic than geometric. For doing so, no more is required than a few cork slides, some wire and a little ingenuity. […] We do not have any model for descriptive geometry […].[207]

The Golden Age of Mathematical Models in View of the Decline of Model Drawing

As is illustrated by the absence of models of descriptive geometry at the École normale laboratory, the use of models in teacher training participated to the autonomization of mathematics as a specific teaching discipline. This practice of models broke with both the traditional association of mathematics with Euclid geometry in general education and with the intimate relationship between descriptive geometry and applications in technical education. It is in this context that the use of mathematical models in the teaching of mathematics began to be truly disconnected from the practice of drawing in France and that models came to be considered as a tool of vizualization, complementary to the figures drawn on the blackboard, rather than as a way to educate the hand and the eye.

It is also in this context that collections of models of both elementary and higher mathematics were established and developed in a great number of faculties of science and in the lycées. But it is difficult to assess whether these collections were actually used by teachers and mathematicians beyond specific areas such as the École normale laboratory, the Darboux/Caron lectures at the Sorbonne, as well as primary and technical education where the use of models had been established decades before,[208] and where the links between the teaching of mathematics and drawing were still very vivid. Historical sources on the actual use of models of higher mathematics are scarce. The overviews of both local and national pedagogical methods published in the journal *L'Enseignement mathématiques* usually promote the use of models and instruments in accordance with the progressive editorial line of this journal. But they often come with tempered criticisms about the practical difficulties associated with models, described as cumbersome, fragile,

[207] Ibid., 209: "Enfin nous avons en beaucoup plus petit nombre des modèles relatifs à la géométrie du Ve livre. Une des difficultés les plus sérieuses pour les commençants est en effet de 'voir' ce que représentent les figures de perspective plus ou moins grossières qui servent à illustrer les principales démonstrations du Ve livre. Cette difficulté serait bien diminuée si, avant de faire une figure au tableau—figure qui n'est au fond qu'un schéma plus algébrique que géométrique—le professeur montrait aux élèves la figure elle-même de l'espace. Pour cela il suffit de quelques plaques de liège, quelques fils de fer et d'un peu d'ingéniosité. […] Nous n'avons encore aucun modèle pour la géométrie descriptive […]."

[208] On the continuation of the practice of model drawing in technical education in France, see: "Conférences sur l'enseignement technique moyen en France," *L'enseignement mathématique* 12 (1910): 393–412.

costly or even locked out in a closet…[209] A few contributors developed more in depth criticisms about whether models of higher mathematics were actually helpful for visualizing mathematical properties: since these models often display singular configurations rather than a global point of view, they usually required from the observer an important preliminary knowledge in mathematics.

More importantly, "in education, innovations do not get a foothold overnight" as a contributor to *L'Enseignement mathématique* stressed it in 1914, "and thus the use of models and instruments has not yet become very common."[210] To be sure, evolutions of official national programs of instruction do not guarantee the local evolution of the actual practices of teachers. Especially in the case of a reform such as the one of 1902 in which the official national programs of instruction were designed by a few university professors with little consultation of secondary school teachers. The traditional opposition in France between primary and secondary education, technical and general education, grandes écoles and universities was another obstacle for the adaptation in the secondary and general education of pedagogical methods developed in the primary and technical education.

The 1902 reform had withdrawn geometric drawing from the scope of the teaching of drawing and attributed it to the teaching of mathematics. As a result, the traditional technical dimension of linear drawing was marginalized in the lycées, while it remained at the core of primary and technical education. Thus, the reform eventually increased the opposition between the geometric drawing taught by draughtsmen, often architects or engineers, with a focus on its applications and through a large and diversified use of models, and the geometric drawing taught by professors of mathematics as an auxiliary to geometry promoted as an "instrument of culture" in general education.[211] While the alliance between theory and application was at the core of the teaching of geometric drawing promoted by Monge and his followers, the turn of the century saw the growing autonomization of technical, or industrial, drawing from geometric drawing,[212] and especially from descriptive geometry.[213]

In *L'Enseignement mathématique*, several secondary school teachers opposed the value of visualization and manipulation associated with models to the traditional ideal of rigor associated with mathematics in general education.[214] While

[209] See: Meunier, "Reliefs à pièces mobiles," 167; Henri Vuibert, *Les Anaglyphes géométriques* (Paris: Vuibert, 1912), 8.

[210] Staeckel, "La préparation mathématique," 320: "[…] l'usage des modèles et des appareils n'est encore que fort peu répandu."

[211] d'Enfert, "Entre mathématiques et technologie," 46.

[212] Crelier, "Le dessin de projection," 300.

[213] S. May, "De la concordance entre le dessin technique et la géométrie descriptive," *L'enseignement mathématique* 14 (1912): 53–57; Louis Kollros, "Le dessin linéaire et la géométrie descriptive dans les écoles réales," *L'enseignement mathématique* 14 (1912): 63–64. Staeckel, "La préparation mathématique," 321.

[214] On a comparison between the new and the ancient pedagogical methods, see: J.-P. Dumur, "Les mathématiques pratiques dans les 'Public Schools,'" *L'enseignement mathématique* 16 (1914): 148–49.

they did not reject entirely models and instruments, they pleaded for limiting their use to the primary and elementary schools since no manipulation or visual demonstration should challenge the rigor of a mathematical proof on the blackboard.[215] Even though collections of models participated in shaping the place of mathematics, by the hybridation of libraries and laboratories, as well as the persona of mathematicians, by public exhibitions, a tension arose with a more ancient symbolic attribute of the professor of mathematics: the blackboard.

Even in the reformist camp, new devices of visualization such as projection devices, cinema, photogrammetry, and stereoscopy quickly challenged the value of modernity associated with models. Stereoscopy, in particular, made the visualization of relief with plane pictures possible. The mathematical principles of stereoscopic photography had been laid in the 1850s and photographs had been used since then as a form of visualization complementary to the use of models, as is exemplified by the two stereoscopic photographs of the first model of the 27 lines on a cubic surfaces that were shot very soon after the model had been designed by Wiener in 1868.[216] Stereoscopy would become more and more popular after 1905 and, because stereoscopic plates were cheaper and less cumbersome than actual models, they tended to be seen as a 'modern' alternative to collections of models.[217] Challenged by new techniques of visualization, models tended to be reduced to manipulation. But manipulation in mathematics was often more effectively performed by the actual construction of models by students,[218] than by the use of preexisting collections, which soon collected dust in forsaken closets.[219]

Open Questions: Models, Mathematical Modelization, and the Graphical Method

The golden age of mathematical models at the turn of the twentieth century coincided with a decline of the traditional pedagogical practice of model drawing in the teaching of mathematics. The advent of large collections of models of higher mathematics all over Europe and the U.S.A. therefore carried with it the onset of

[215] Meunier, "Reliefs à pièces mobiles," 167; Staeckel, "La préparation mathématique," 320–21.

[216] Christian Wiener, *Stereoscopische Photographien des Modelles einer Fläche dritter Ordnung mit 27 reellen Geraden: Mit erläuterndem Texte* (Leipzig: Teubner, 1869).

[217] Henri Fehr and G. Stiner, "Vues stéréoscopiques pour l'enseignement de la Géométrie," *L'enseignement mathématique* 8 (1906): 385–90; Henri Fehr, "Le stéréoscope et ses applications scientifiques," *L'enseignement mathématique* 9 (1907): 142–46; Charles Perregaux and Adolphe Weber, *Le relief en Géométrie par les couleurs complémentaires: 50 planches de stéréométrie et de géométrie descriptive* (Bienne: E. Magron, 1916).

[218] Vuibert, *Les Anaglyphes*, 8.

[219] When he discovered it in 1934, Man Ray referred to the collection at the Institut Henri Poincaré as collecting dust. See: Man Ray, "Notes sur les Équations shakespeariennes, To be continued unnoticed, Some Papers by Man Ray in connection with his exposition," in *Catalog of Man Ray's exhibition at the Copley galler*y (Beverly Hills: Copley gallery, 1948), 7–9. Man Ray, *Autoportrait* (Paris: Laffont, 1964): 314.

obsolescence, the function of models reduced to visualization and manipulation. Both the grandeur and the decadence of models have therefore to be analyzed in view of the long-term relationship between mathematics and drawing.

This relationship especially raises open historical questions about the role that may have been played by models in the emergence of mathematical modelization.[220] The history of modelization has tended to focus on theoretical developments in mathematics and neighboring sciences such as mechanics and physics. Even though historians have investigated several practices of visualization, of writing, and of computations, research in the history of mathematics has often laid the emphasis on practices of visualizations associated with academic publications, while the palette of model drawing techniques and devices encapsulated in the mathematics of the engineers in the nineteenth century have rather been associated with the history of technology.[221] Significantly, drawing has often been considered as a burden in the training of prominent mathematicians such as Camille Jordan and Henri Poincaré, who failed to rank first when they graduated from École polytechnique because of their bad grades in drawing. Yet, geometric drawing may be considered retrospectively as one of the roots of mathematical modelization, because of both its ubiquitous use in technology and its intimate relationship with mathematical education and academic publications.

We shall especially argue that descriptive geometry played an exemplary role for innovative graphical methods and visualization devices throughout the nineteenth century. This role calls for reassessing the usual historiographical description of Monge's descriptive geometry as a transitional discipline, understood as both the ultimate perfecting of previous graphical techniques and the "last stage of a tradition that is losing momentum,"[222] while algebra and analysis would become increasingly important in the training of engineers. It especially raises new questions about the history of descriptive geometry in the nineteenth century from the perspective of the evolution of the graphical methods associated with it.

An important issue that calls for further investigation is the role played by model drawing in the development of the very techniques of visualization that would eventually render both models and mathematical drawing obsolete. In the nineteenth century, several forms of mathematical visualization were developed without being subjected to any reflexive discourses or theoretical developments. Quite often, these forms of representations were not considered as mathematical objects, or methods, for decades, and could not be dissociated from specific, and

[220] Moritz Epple, "A plea for Actor's Categories: On Mathematical Models, Analogies, Interpretations, and Images in the 19th Century," *Oberwolfach Reports* 14, no. 4: "History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences," ed. Jeremy J. Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe (2015): 2773–79.

[221] Yves Deforge, *Le graphisme technique: son histoire et son enseignement* (Seyssel: Champ Vallon, 1981).

[222] Sakarovitch, "Gaspard Monge," 240; Belhoste et al., "Les exercices," 109.

often tacit, cultural practices.[223] By contrast, we have seen that model drawing had been formalized early on in the eighteenth century, with the interplay of a mathematical theory, descriptive geometry, and its applications. We have seen also that drawing was at the core of the mathematical training of the polytechnicians who, in the nineteenth century, were active in all the branches of the mathematical sciences and involved in both academic and engineering activities.

Geometric drawing provided these polytechnicians a model for designing various new forms of visualization, which would eventually fall under the designation of 'graphical method' at the turn of the twentieth century.[224] Several alumni of Polytechnique especially supported the emergence of photography, which they considered as an improvement of the épures of descriptive geometry. When he committed himself to convince the French government to fund the daguerreotype, François Arago, an illustrious alumnus and professor of École polytechnique, contrasted the precision and fastness of Louis Daguerre's innovation with the épures drawn by polytechnicians during the campaign of Egypt:

> When looking at the first tableaux that M. Daguerre exhibited to the public, everyone thought about the immense advantage that such an exact and swift means of reproduction would have provided during the campaign of Egypt; everyone was struck by the reflection that, should photography had been known in 1798, the faithful picture of so many iconic tableaux would not have been lost for the scholarly world […]. Had the Institute of Egypt been furnished with two or three of M. Daguerre's devices, […] vast areas of the fictional or conventional hieroglyphs that are represented in several plates of its celebrated masterpiece [i.e. *L'expédition d'Égypte*] would have been replaced by real hieroglyphs; and their design would have surpassed in accuracy, and local color the works of the most skilled painters; and photographic images, the formation of which is submitted to the rules of Geometry, would have allowed to reassemble, with only a small set of data, the exact dimensions of the highest and most inaccessible parts of the ancient monuments.[225]

[223] For a case study on the algebraic cultures associated with the use of the tabular representation of matrices, see: Frédéric Brechenmacher, "Une histoire de l'universalité des matrices mathématiques," *Revue de Synthèse* 131, no. 4 (2010): 569–603. On the analytical representation of substitutions, see: Frédéric Brechenmacher, "Self-portraits with Évariste Galois (and the shadow of Camille Jordan)," *Revue d'histoire des mathématiques* 17, no. 2 (2011): 271–369.

[224] Dominique Tournès, "Mathematics of Nomography,“ in *Mathematik und Anwendungen*, ed. Michael Fothe, Michael Schmitz, Birgit Skorsetz, and Renate Tobies (Bad Berka: Thillm, 2014), 26–32; Dominique Tournès, "Une discipline à la croisée de savoirs et d'intérêts multiples: la nomographie,“ in *Circulation Transmission Héritage, Actes du XVIIIe colloque inter-IREM*, ed. Pierre Ageron and Évelyne Barbin (Caen: Université de Caen Basse-Normandie, 2011), 415–48; Dominique Tournès, "Pour une histoire du calcul graphique," *Revue d'histoire des mathématiques* 6, no. 1 (2000): 127–61.

[225] François Arago, *Rapport de M. Arago sur le Daguerréotype, lu à la séance de la Chambre des Députés, le 3 juillet 1839* (Paris: Bachelier, 1839), 25–31: "A l'inspection de plusieurs des tableaux qui ont passé sous vos yeux, chacun songera à l'immense parti qu'on aurait tiré pendant l'expédition d'Égypte, d'un moyen de reproduction si exact et si prompt ; chacun sera frappé de cette réflexion, que si la photographie avait été connue en 1798, nous aurions aujourd'hui des images fidèles d'un bon nombre de tableaux emblématiques […] Munissez l'institut d'Égypte de deux ou trois appareils de M. Daguerre […] […] de vastes étendues d hiéroglyphes réels iront

As is illustrated by Arago's early use of the daguerreotype to shoot pictures of the moon, the issue of providing a precise mathematical visualization of inaccessible areas was an early and important application of photography, which fuelled several innovations, such as photogrammetry and metrophotography. These innovations raised new mathematical problems, such as of the rectification of the photographs shot from aerostats.[226] These problems were associated with important issues in both civil and military topography, as is illustrated by the siege of Sévastopol in 1854–1855, when British and French photographers made use of aerostats for scouting the fortifications of the Russians. After Sévastopol had fallen, the colonel Langlois was put in charge of painting a panorama of the siege. A former student of the Polytechnique who had become a painter and had specialized in the painting of military scenes, Jean Charles Langlois surveyed the topography of the scene by making use of both the drawing techniques of descriptive geometry and photography: "he surveyed the map of the scene and the positions of the armies from the top of the Malakoff tower […] by the use of photographic devices and thus applied, for the first time, photography to surveying panoramic maps."[227]

Panoramas were a specific form of geometric visualization based on conic, spherical, or cylindrical perspectives. The issue of surveying panoramic maps of both the topography and of the geological nature of mountains gave rise to the development of the field of topophotography in the late 1850s, in which several former students of the Polytechnique and of the Metz application school where involved, such as the geologist Aimé Civiale. Again, the rectification of photographs, as well as their use for measurement in topography, raised difficult mathematical issues, which were tackled in academic publications in the *Comptes rendus de l'Académie des sciences*. It is in this context that Libre Bardin designed the plans-reliefs that would eventually lead him to fabricate plaster mathematical models. As a matter of fact, Bardin made use of photography for surveying the Mont-Blanc,[228] as well as for exploring innovating forms of mathematical visualization, such as radiant panoramas, which consisted in the mathematical anamorphosis of a whole panoramic view on a plane surface (see Fig. 33).

Radiant panoramas allow a direct visualization of all the angles between any vertical plane and any point of the panorama: concentric circles are drawn around

remplacer des hiéroglyphes fictifs ou de pure convention; et les dessins surpasseront partout en fidélité, en couleur locale, les couvres des plus habiles peintres; et les images photographiques étant soumises dans leur formation aux règles de la géométrie, permettront, à l'aide d'un petit nombre de données de remonter aux dimensions exactes des parties les plus, élevées les plus inaccessibles des édifices."

226 Laussédat, "Recherches sur les instruments," 241.

227 Germain Bapst, *Essai sur l'histoire des Panoramas et des Dioramas: Extrait des Rapports du Jury international de l'Exposition universelle de 1889* (Paris: G. Masson, 1889), 25: "[…] le colonel Langlois […] leva du haut de la tour Malakoff, au moyen d'appareils photographiques, les plans des positions occupées par les armées et appliqua ainsi pour la première fois, comme nous l'avons déjà vu, la photographie à la levée des plans panoramiques."

228 Laussédat, "Recherches sur les instruments," 251.

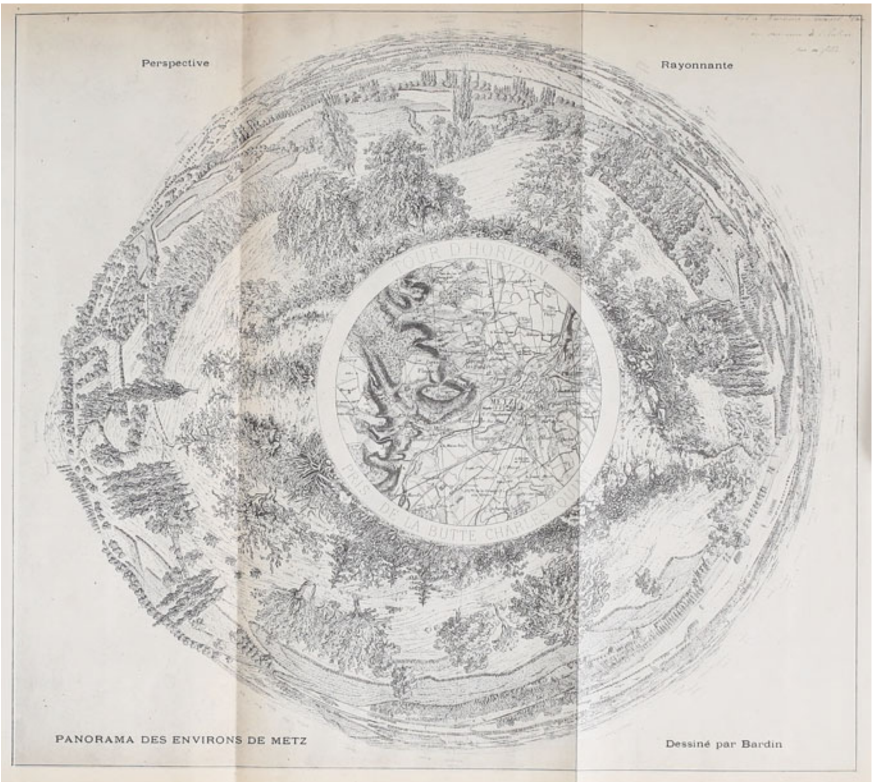

Fig. 33 Libre Bardin's radiant panoramas of the environs of Metz. From Aimé Laussédat, *Recherches sur les instruments, les méthodes et le dessin topographiques* (Paris: Gauthier-Villars, 1901), plate IV

the center of perspective, each circle representing the points of a same angular height.[229] This direct and simple visualization of angles was considered as especially helpful for surveying and leveling. Thus radiant panoramas were considered as providing a solution to the mathematical problem of photography. Further, Bardin's radiant panoramas highlight, once again the role played by model drawing in graphical innovations: the panorama of the environs of Metz was constructed by the mathematical transformation of a preexisting developed cylinder panorama designed by one of the draughtsmen involved in the teaching of mathematical drawing at the Metz school.

Because mathematical drawing played a key role in the engineering sciences, innovative graphical techniques of visualization were rather evaluated with the

[229] The development of radiant panoramas had especially been initiated by the panographer designed in 1827 by the geographer and mathematician Louis Puissant.

criteria of industry than of the academy. The criteria of precision, effectiveness, and production cost were especially favored over the one of conceptual novelty. These criteria, which, as we have seen, Olivier applied when evaluating mathematical models, were actually applied to all graphical techniques. In his report on a new drawing machine for reproducing, enlarging, or reducing any épure, Olivier claimed that, even though there was nothing new in the design of this camera obscura, its realization was nevertheless innovative since it allowed to "save time" with no loss of "mathematical exactness":

> […] inventions rarely show new principles, most of the time a truly new invention is based on a new way to materialize known principles; it is often a new modality that provides the effective simplification of a mechanism that used to be too complicated and costly; it is more importantly the achievement of a simpler machine, a machine that can be used with more speed and more security, and which can be delivered to the industry for a cheaper price.[230]

In turn, these criteria of precision, effectiveness and simplicity gave rise to a new approach to geometric constructions, such as with Émile Lemoine's geometrography.[231] In many ways, mathematical models can be considered as falling in the more general category of graphical methods for a large part of the nineteenth century. Investigating further this more general context would allow us to understand more precisely the emergence of collections of models of higher mathematics after the 1860s, which broke with the tradition of model drawing in a process of autonomization of the forms of visualization specific to academic mathematics. This prowess was not limited to material models and went along with theoretical developments on the mathematical properties of forms of visualization. It therefore played a role in the emergence of the concept of mathematical modelization.

For this reason, the evolution of mathematical models in the nineteenth century should not be reduced to a unique path, from applied, or engineering mathematics, to academic mathematics. The collections of models of higher mathematics are only one of the many forms of evolution of the variety of graphical techniques designed in the nineteenth century. A striking example is provided by Etienne Jules Marey's "méthode graphique."[232] While Marey is best remembered today for his

[230] Théodore Olivier, "Rapport fait par M. Théodore Olivier, au nom du comité des arts mécaniques, sur une machine à dessiner présentée par M. Grillet," *Bulletin de la société d'encouragement pour l'industrie nationale* 488 (February 1845): 49–52, here: 49–50: "[…] dans les inventions, les principes nouveaux, sont rares, et presque toujours ce qui constitue une invention qui doit être considérée comme réellement nouvelle, c est une nouvelle manière de matérialiser un principe connu; c'est souvent un mode nouveau, qui simplifie d'une manière heureuse un mécanisnie trop compliqué et trop coûteux; c'est surtout arriver à une machine plus simple que celles connues, et que l on puisse faire fonctionner avec plus de rapidité et plus de sûreté, et dont les produits puissent être livrés à l'industrie à un prix moins élevé."

[231] Émile Lemoine, "La géométrographie ou l'art des constructions géométriques," *Association française pour l'avancement des sciences* 21 (1892): 36–100.

[232] Marey developed the idea of the superiority of the graphical method through its application to the investigation of blood circulation. See: Étienne-Jules Marey, *Du mouvement dans les fonctions de la vie*, *Leçons faites au Collège de France* (Paris: Germer Baillière, 1868).

motion pictures of chronophotography, and often celebrated as a forerunner of cinema, his main aim was to develop a mathematical description of motion by the use of what he designated as "photographic épures." He eventually named his approach "the graphical method," subsuming all graphical techniques and instruments, from drawings to photographs or even the visualization of timelines in textbooks of history:

> The graphical method has driven progresses in almost all the branches of science and, for this reason, has benefited considerable development recently. Arduous statistics have given way to tables in which the inflexions of a curve throw light on all the phases of a phenomenon. Moreover, tracing devices can draw automatically the curve of either physical or physiological phenomena which could not be observed directly because of their speediness, slowness, or weakness. Yet, the inscription of phenomena in the form of curves may sometimes prove defective; a more powerful method has been created: Chronophotography.[233]

Photography, Marey claimed, "is increasingly replacing drawings, maps, and relief figures" (i.e. models).[234] Aiming at developing a mathematical representation of motion in space through photography, he started with the investigation of the mechanical motions of the basic elements of geometry, i.e. the point and the line. His first chronophotographical épures were devoted to generating ruled surfaces, such as a cylinder, a hyperboloïd and a cone, by the motion a single string, with an explicit reference to Olivier's mechanical string models and their use for the teaching of descriptive geometry at the Conservatoire (see Fig. 34).

For the investigation of more complicated motions, such as the one of a runner, the surface was then reduced 'geometrically' to a series of points and lines that allowed to superpose several photographs in what constituted an 'épure' of 'geometric chronophotography.' (see Fig. 35) Recall that the concept of 'épure' is underpinned by a process of 'reduction' in the mathematization, or the modelization, of a phenomenon.

The emergence of the graphical method can be seen as an evolution of mathematical models different from the emergence of models of higher mathematics. Both broke with the tradition of model drawing but not for the same reason. On the one hand, the practice of drawing had never been a legitimate activity for teaching mathematics in universities and, as we have seen, the use of mathematical models traditionally aimed at developing action-learning pedagogical methods in opposition with reading texts or attending lectures. Often presented as substitutes for direct contact with nature, the knowledge associated with mathematical models was opposed to the one of professors, and had even fuelled criticisms about the pedantry of academic knowledge. On the other hand, drawing was considered as obsolete because slower and less precise than new graphical devices.

As aforementioned, the graphical method subsumed a great variety of visualization techniques and instruments, including the ones developed in statistics, such

[233] Étienne-Jules Marey, *Le mouvement* (Paris: G. Masson, 1894), Avant-propos. English translation: Étienne-Jules Marey, *Movement* (London: Heinemann, 1895), Preface (translation modified).
[234] *Marey, Le mouvement,* 18.

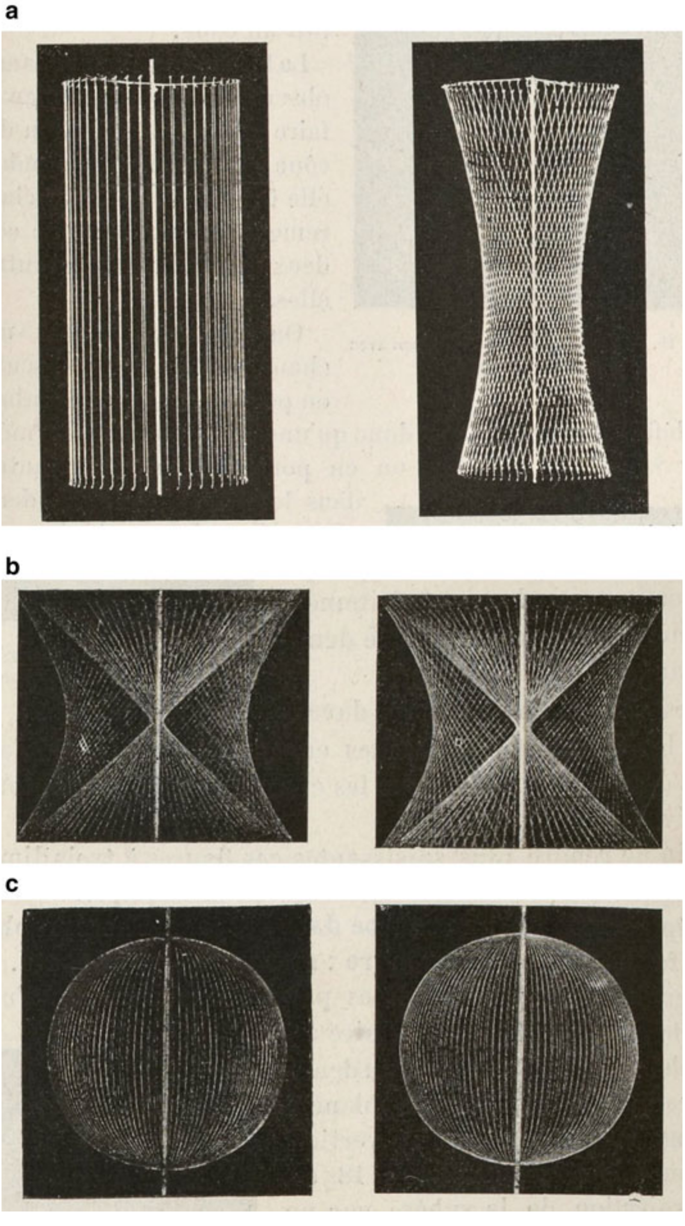

Fig. 34 **a** Cylinder and hyperboloid generated by the rotation of a white string, Étienne-Jules Marey, *Le mouvement* (Paris: G. Masson, 1894), 25, Figs. 14, 15. © Bibliothèque nationale de France. **b** Sphere generated by the rotation of a half-ring of white string. From ibid., 28, Fig. 19. © Bibliothèque nationale de France, all rights reserved. **c** Hyperboloid and its asymptotic cone. From ibid., 28, Fig. 20 © Bibliothèque nationale de France, all rights reserved

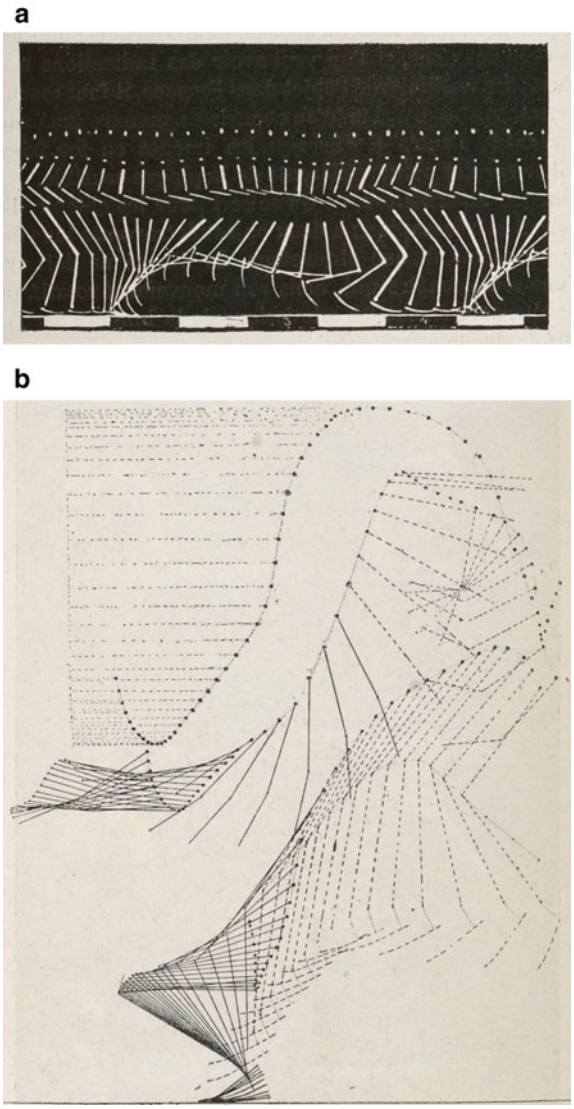

Fig. 35 **a** Pictures of a runner reduced to shiny lines (geometric chronophotography). From Étienne-Jules Marey, *Le mouvement* (Paris: G. Masson, 1894), 61. © Bibliothèque nationale de France, all rights reserved. **b** Photographic épure of a jumper. From ibid., 138

as with the choropleth map designed by Dupin in 1826 for representing the distribution of illiteracy in France,[235] or in the field of graphical statics,[236] geodesy,[237] the promotion of function graphs and their use for approximating the roots of algebraic equations,[238] and the methods of graphical calculation that would give rise to a specific mathematical theory: nomography.[239] When Olivier reviewed Léon Lalanne's pioneering "abaques" of graphical computation at the Société d'encouragement, it was plain to him that the contour lines used in topographic maps for representing reliefs were a major source of inspiration for the graphical layout designed by Lalanne (see Fig. 36).[240] The specificity of Lalanne's method for graphical calculation was that it displayed only straight lines, and was therefore based on the transformation of the curves of several functions. For Olivier, this approach was inspired by a fundamental principle of descriptive geometry, i.e. the transformation of a surface into a simpler one, and of systems of lines in space into systems of lines on a plane.

As with the models of higher mathematics, the graphical method aimed at enhancing visualization, and therefore precision, but it rather focused on an ideal of 'clarity' in the representation than on the ideal of objectivity associated with academic sciences. Clarity was a necessary preliminary to effectiveness; it required not only simplicity but more importantly to make the 'choice' of what should be simplified, and therefore carried on the main value traditionally associated with the teaching of drawing: training the eye for improving the capacity of judgment. This traditional value was still emphasized by Carlo Bourlet, in his inaugural lecture as the new professor of descriptive geometry at the Conservatoire in 1906:

> One should never forget that the unique purpose of descriptive geometry is to represent pieces of stones, of woods, of machines, and architectural details with the precision and clarity required for any effective achievement. The artisan to whom the drawing will be transmitted needs to recognize at first glance the form and the details of the piece he has

[235] Gilles Palsky, "Connections and Exchanges in European Thematic Cartography. The Case of XIXth Century Choropleth Maps," *Belgeo* 3, no. 4 (2008): 413–26.

[236] Konstantinos Chatzis, "La réception de la statique graphique en France durant le dernier tiers du XIXe siècle," *Revue d'histoire des mathématiques* 10, no. 1 (2004): 7–43.

[237] Martina Schiavon, *Itinéraires de la précision. Géodésiens, artilleurs, savants et fabricants d'instruments de précision en France, 1870–1930* (Nancy: Presses universitaires de Nancy, 2014).

[238] See in particular the report devoted to the graphical method of Carl de Ott, a professor of descriptive geometry in Prague, by: Jules Morin, "De l'utilité de l'application de la géométrie aux calculs algébriques," *Bulletin de la société d'encouragement pour l'industrie nationale*, 502 (April 1846): 447–84; 714–15.

[239] As with model drawing, the main actors of the development of nomography were engineers trained at the Polytechnique and aimed at providing a mathematical formulation to key issues in engineering science such as cuttings and embankments. See: Dominique Tournès, "Mathematics of Engineers: Elements for a New History of Numerical Analysis," *Proceedings of the International Congress of Mathematicians* 4 (2014): 1255–73.

[240] Théodore Olivier, "Rapport fait par M. Théodore Olivier, au nom du comité des arts mécaniques, sur un abaque ou compteur universel de M. Léon Lalanne," *Bulletin de la société d'encouragement pour l'industrie nationale*, 502 (April 1846): 161–62.

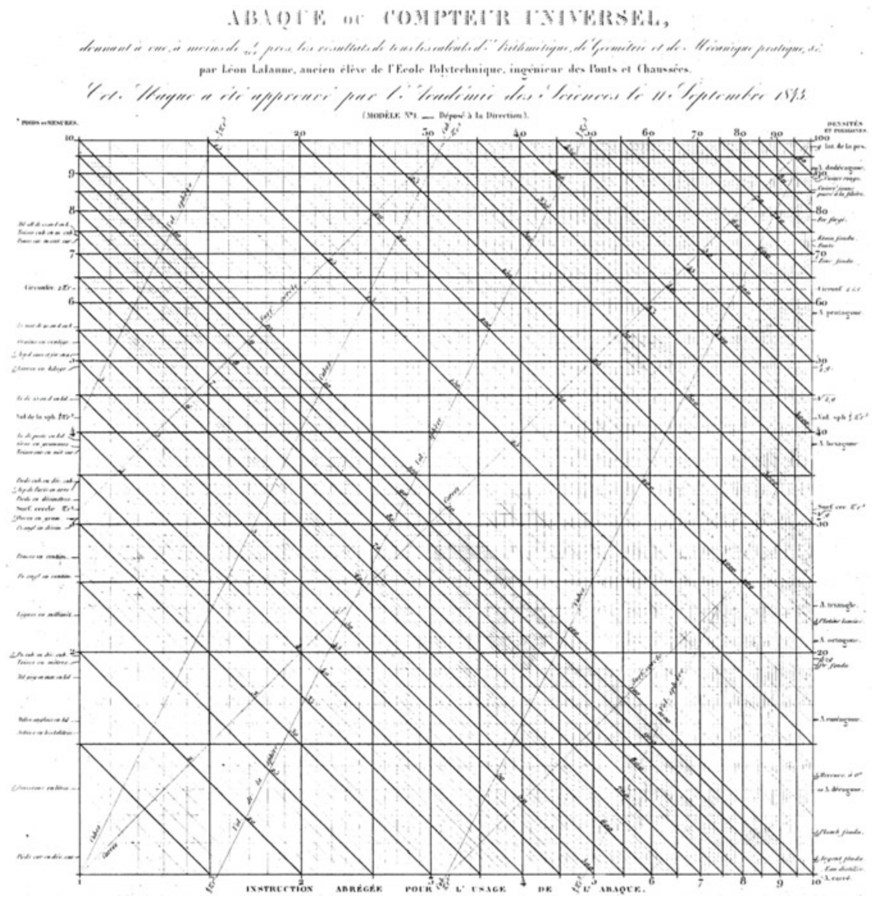

Fig. 36 Léon Lalanne's abaque. From Théodore Olivier, "Rapport fait par M. Théodore Olivier, au nom du comité des arts mécaniques, sur un abaque ou compteur universel de M. Léon Lalanne," *Bulletin de la société d'encouragement pour l'industrie nationale* 502 (April 1846): 161

to fabricate. The role—and I shall even say the *duty*—of the draughtsman is to represent objects in a simple manner. He cannot choose randomly between projection planes, or even modes of representation, but he has to be very judicious in his choices. He has to *make see* and therefore being able to *see by himself*.[241]

[241] Carlo Bourlet, "La géométrie descriptive au conservatoire des arts et métiers de Paris," *L'enseignement mathématique* 9 (1907): 89–93, 91–92: "Il ne faut pas, en effet, oublier que le but unique de la Géométrie descriptive et de la Stéréotomie est de représenter des morceaux de pierre, des pièces de bois, des organes de machines, des détails et ensembles architecturaux d'une façon claire et précise qui en permette l'exécution. L'artisan, auquel on transmettra le dessin, doit pouvoir, d'un premier coup d'œil, connaître la forme et les détails de la pièce qu'il est chargé d'exécuter. Le rôle—je dirai plus—le devoir du dessinateur est donc de présenter ces objets d'une

Reassessing the history of mathematical models in view of the development of the graphical method calls especially for further investigation on the interplay between models and instruments.[242] The origin of the graphical method was indeed usually attributed to the device designed by Poncelet and Morin for tracing automatically the altitude of a body in free fall.[243] As most proponents of descriptive geometry and model drawing, Poncelet did not separate the use of models from the one of instruments.[244] Recall that already at the creation of École polytechnique, Monge's cabinet of models as well as the practice of drawing in the 'petites salles' was considered as an adaptation to mathematical education of the practice of experimenting with instruments in chemistry laboratories. Moreover, Monge and his pupils often studied geometrical problems closely connected with experimental physics, especially geometrical optics.

Investigating further the role models and instruments played in the interactions between mathematics and experimental sciences would allow us to shed new light on the emergence of models of higher mathematics. As we have seen, these models were often associated with a naturalistic approach to the sciences of surfaces, which involved not only geometry but also mechanics and optics. As a matter of fact, several of the earliest models of Monge's cabinet were used not only in the teaching of mathematics, but also of physics, and chemistry, such as with cardboard crystallographical models. This versatility of models is also exemplified by the plaster models designed by Augustin Fresnel for his work on the theory of light.[245] Ampère's 1815 theory of the internal organization of molecules is another typical example of the interplay between crystallography, chemical combinations,

manière simple. Il doit, non pas s'imposer au hasard des plans de projection, ni même un mode de représentation, mais les choisir judicieusement pour atteindre le maximum d'effet utile. Il doit faire voir; et pour cela, il faut d'abord qu'il voie lui-même."

242 The development of nomography provides an example of graphical method in which representations cannot be dissociated from instruments: Lalanne's "abaques" were both graphical tables and computing devices. See: Dominique Tournès, "Construire pour calculer," in *Les constructions mathématiques avec des instruments et des gestes*, ed. Évelyne Barbin (Paris: Ellipses, 2014), 265–96.

243 Marey, *Du mouvement dans les fonctions de la vie*, 107; Gustave Le Bon, "La méthode graphique et les appareils enregistreurs. Leurs applications aux sciences physiques, mathématiques et biologiques," in *Études sur l'exposition universelle de 1878*, ed. Eugène Lacroix (Paris: Librairie scientifique, industrielle et agricole, 1878), 7:329–432.

244 Olivier was an especially strong proponent of the use of instruments in the teaching of geometry and particularly promoted the inventors of new tracing devices. Among the several reports he devoted to this issue at the Société d'encouragement, see the powerful plea for instruments by which he concluded a report on folding procedures, Théodore Olivier, "Rapport sur une nouvelle méthode de géométrie pratique, sans instruments, de M. Martin Chatelain," *Bulletin de la société d'encouragement pour l'industrie nationale,* 544 (October 1849): 481–85.

245 Letter from Roquet to Barré de Saint Venant, "Le remercie pour le don au lycée de modèles en plâtre (recherches de Monge et de Fresnel)," July 3, 1863, Archives of École polytechnique. Fonds Barré de Saint Venant.

and geometry,[246] while Louis Poinsot's theory of order highlights another kind of interplay between polyhedrons, mechanics, algebra, and number theory, which turned to be instrumental to the development of both group theory and topology.[247]

Both the design and the use of crystallographical models required specific instruments, i.e. goniometers, for measuring angles between crystal faces. Goniometers quickly evolved from mechanical devices into optical devices, and turned out to be instrumental for the intimate connections between crystallography, geometry, mechanics, and optics in Fresnel's wave theory of light in the 1820s, which, in turn, resulted in the design of what may be considered as the first models of higher mathematics, i.e. Fresnel wave surfaces made of plaster, to which Kummer would provide a generalization with his research on quartics in the 1860s. In the experimental sciences the design of models involved a close collaboration of scholars and manufacturers of instruments, such as with Fresnel and the optician Jean-Baptiste Soleil in Paris, for designing both instruments and models of wave surfaces,[248] and with the physicist Gustave Magnus and the draughtsman Ferdinand Engel in Germany, whose models of Fresnel wave surface of crystals were exhibited at the world fairs of London and Paris in 1851 and 1855, before being commercialized in Germany and in the USA. Plaster models of Fresnel wave surfaces can already be found in the catalogues of the Parisian manufacturers Soleil/Duboscq and Hoffman in the 1840s, accompanied with specimens of crystals, wooden models of crystallographic polyhedrons, goniometers as well as artifacts for visualizing surfaces by luminous projections.'

Conclusions

Over the course of the eighteenth and nineteenth centuries, the history of mathematical models in France cannot be dissociated from the one of model drawing in mathematical education. The specificity of the French educational system was mainly due to the centuries-long trend towards centralization, which culminated during the French Revolution with the creation of several central and national institutions such as École polytechnique, École normale and the Conservatoire national des arts et métiers. These 'grandes écoles' were created in opposition to

[246] André-Marie Ampère, "Lettre de M. Ampère à M. le Comte Berthollet, sur la détermination des proportions dans lesquelles les corps se combinent, d'après le nombre et la disposition respective des molécules dont leurs parties intégrantes sont composées," *Journal des mines* 37, no. 217 (January 1815): 5–40.

[247] Jenny Boucard, "Louis Poinsot et la théorie de l'ordre: un chaînon manquant entre Gauss et Galois?" *Revue d'histoire des mathématiques* 17 (2011): 41–138; Frédéric Brechenmacher, "The theory of order, a specific *nineteenth* century model of scientificity," *Oberwolfach Reports* 14, no. 4: "History of Mathematics:Models and Visualization in the Mathematical and Physical Sciences," ed. Jeremy J. Gray, Ulf Hashagen, Tinne Hoff Kjeldsen, and David E. Rowe (2015): 2808–11.

[248] Letter to Barré de Saint–Venant, "Remerciements pour le modèle (surface d'onde de Fresnel) que Saint Venant a offert à la société philomatique," 1863, Archives of École polytechnique. Fonds Barré de Saint–Venant.

the traditional universities. In contrast to the pedagogical method of plenary lectures, they aimed at promoting the activity of the students through the practice of science experiments and geometric drawing. École polytechnique, in particular, continued the long tradition of apprenticeship and companionship in the training of engineers. But in contrast with the royal engineering schools of the eighteenth century, Polytechnique articulated the practice of geometric drawing with theoretical lectures. It is in this specific contest that Monge's descriptive geometry fully blossomed as a new branch of the mathematical sciences.

On the one hand, this new science carried on the traditional idea that teaching geometry to engineers required to 'educate the hand and the eye' through model drawing. Models were thus considered as substitutes for natural forms and supported pedagogical methods that promoted action learning, relegated the role of the teachers to the one of supervisors, or even praised the mutual instruction of students by students. The very epistemological essence of this activity was the transmission of a non-textual form of geometric knowledge, one which required practical work and could not be subsumed to reading texts or attending lectures: drawing was knowing. But on the other hand, the practice of drawing was articulated to theoretical lectures in the most advanced sciences of the time, especially analysis. The teaching of Monge's descriptive geometry was organized by the process of decomposition/recomposition at the core of the "esprit d'analyse." It followed a progression from the simple to the complex: the students had first to copy geometric figures and their intersections, i.e. models of two dimensions, in order to acquire exactness of eye, before passing to three dimensional geometric models, and eventually to the natural models of topographical landscapes, buildings, or technological devices. When their training was completed, students were supposed to be able to decompose a complex figure into a series of simple elements, corresponding to the models they had been trained with, and to recompose a complex drawing from its elementary parts.

Even though the instruction plan devised initially by Monge in 1794 was quickly challenged by the increasing role attributed to analysis at the expense of descriptive geometry, the practice of model drawing continued to play an important role over the course of the nineteenth century. Because of the central role played by the École polytechnique in the emergence of a national educational system, this pedagogical approach to the teaching of geometry spread to the other institutions of technical education that were created in the first decades of the nineteenth century, starting with the drawing school created by the Conservatoire national des arts et métiers, the movement for mutual instruction, and eventually with the institutionalization of the national system of superior primary education in 1833.

For Gaspard Monge, descriptive geometry embodied the "esprit d'analyse," not only in the sense that its teaching could be organized from the simple to the complex, but also because it provided a heuristic 'method for finding the truth.' The important role played by model drawing at École polytechnique participated in a more general plan for articulating practice and theory. In the first half of the nineteenth century, the legacy of Monge's ideals about the role descriptive geometry and model drawing should play in the alliance between practice and theory remained especially vivid in the school for artillery and military engineering applications at Metz. The followers of Monge who graduated from the Metz school were strong proponents of industrialization. Along the line of the Saint-Simonian philosophy, they considered that industrial prosperity required putting the 'useful innovations' made by scholars in the service of the nation by increasing the instruction of the industrial class. In doing so, they participated in spreading the pedagogical practices developed at the Polytechnique, especially model drawing, as well as the ideals the school had inherited from the Enlightenment. These engineers and artillery officers often associated the teaching of geometry by model drawing with moral issues, especially the value of discipline and the taste for work well done. The diffusion of geometric drawing in primary and technical education in the 1830s therefore participated in both the conservative political agenda of the constitutional monarchy and in an ideal of emancipation through education in mathematics.

Several important innovations in the design of geometric models were made in this context in the 1840s and 1850s. The diversity of these innovations highlights the variety of the public and issues associated with the teaching of geometric model drawing. Olivier's movable string models were designed for his teaching of descriptive geometry at the Conservatoire national des arts et métiers, with a view to applications in the drawing of gearings. By contrast, Olivier designed a simple and cheaper cardboard model, the 'omnibus,' for raising the elementary mathematical instruction of the greatest number of children. Bardin's plaster models emerged from his research in topography and inherited from the practice of designing plans-reliefs in the arts of fortification and topography. The novelty of these innovations was evaluated with the norms of technical and industrial innovation, especially manufacturing cost, in various local industrial fairs, on the national scene of the Société d'encouragement pour l'industrie nationale, and on the international setting of the world fairs.

The emergence of models in higher mathematics in the 1860s broke with the long tradition of model drawing. Even though the models of higher geometry carried strong pedagogical ideals, these ideals were usually very different from the ones associated with the models designed for technical and primary education. To be sure, both the traditional drawing models and the new models of higher geometry were associated with the pedagogical values of visualization and manipulation, i.e. the issue of making use of the eye and the hand in the teaching of mathematics. But while the traditional use of models could not be dissociated from the idea that the teaching of geometry required first to train the eye and the hand by the practice of model drawing, the models of higher geometry were often designed for universities in which drawing was usually not associated with mathematical education. The idea of working with models in the universities was rather derived from the use of models and instruments in experimental physics. Both the classification of geometric surfaces and the material representation of specific mathematical properties and singularities aimed at promoting observation in pure mathematics, i.e. a value that had developed in observational sciences. These academic ideals contrasted with the ones associated with the traditional use of models and instruments in engineering schools or in industry. The key role devoted to models in Klein's approach to geometry was based on the traditional idea, already much valued in Rousseau's philosophy of education, that only the material representation of a model can impress the 'true character' of a geometric object in the mind. Even so, Klein did not associate this ideal with the practice of drawing but with the one of observation and with a naturalistic philosophy of mathematical objects as both real objects and witnesses to the very nature of the human mind.

To be sure, the rupture between the new models of higher geometry and model drawing did not happen overnight. On the contrary, Gaston Darboux attempted to introduce in the general education of the lycées and universities the pedagogical practices of model drawing that had developed in technical and primary education. Yet, the series of wooden models designed by Joseph Caron for Darboux's lectures on higher geometry nevertheless participated in the shaping of mathematics as an academic discipline in the context of the development of higher scientific education. Even though Klein and Darboux were both active advocates of the interplay between theory and application, this interplay did not take on the same meaning in universities as had been promoted by Monge and his followers. Its focus was on the interplay between mathematics and other academic scientific disciplines, rather than at aiming at a direct usefulness for engineering sciences or the industry. The development of collections of models of higher geometry participated in the much larger phenomenon of the autonomization of mathematics as an academic discipline, in contrast to the broad spectrum covered by the mathematical sciences in the first part of the nineteenth century. The emergence of a market for model manufacturers was, in particular, a consequence of both the development of higher education in Europe and of the increasing role mathematics played in both general and technical education.

In addition to transferring to mathematical education several ideals from the natural sciences, such as observation, experimentation, and classification, the French educational reform of 1902 attempted to promote the adaptation to general education of the pedagogical methods of primary and technical education, such as the use of models. But the use of models in the lycées aimed mostly at rendering mathematics more accessible to more students. It broke with both the traditional association of mathematics with Euclidian geometry in general education and with the intimate relationship between geometry and application in technical education. It is in this context that the collections of models of both elementary and higher mathematics were established and developed in a great number of faculties of science and the lycées. But it is also in this context that the use of mathematical models in the teaching of mathematics began to be truly disconnected from the practice of drawing in France and that models came to be considered as a tool of visualization, complementary to the figures drawn on the blackboard, rather than as a way to educate the hand and the eye. This evolution is especially exemplified by the absence of models of descriptive geometry in the collection that was set up for teacher training at École normale supérieure. The use of models in general education participated to the autonomization of both mathematics as a specific teaching discipline and of geometric drawing as specific to technical education.

The decline of the production and the use of mathematical models after World War I have often been seen as a consequence of the evolution of mathematics, such as with Herbert Mehrtens' claim that models had a place neither in modernism nor in the traditions of counter-modernism within mathematics.[249] But the discussions on the fading golden age of models have usually focused on the collections of models of higher geometry and even more precisely on the issue of the influence of Klein's *anschauliche* approach to mathematics, especially with regard to both formalism and intuitionism. Yet, in view on the more ancient tradition of model drawing for teaching mathematics, the increasing autonomization of mathematics with regard to drawing at the turn of the twentieth century was a major cause for the decline of geometric model in the following decades. Another important aspect is that the golden age of models of higher mathematics rose and fell during the time of the emergence of the figure of the mathematician as a university professor. Models of higher mathematics were designed or ordered by professors, while mathematical models had been traditionally challenging the role of professors in the teaching of mathematics and promoting pedagogical approaches to mathematics such as action learning and companionship. While models had usually been considered as substitutes to natural forms, their decline coincided with the increasing role of textual knowledge and lectures in the teaching of mathematics.

The golden age of mathematical models at the turn of the twentieth century coincided with a decline of the traditional pedagogical practice of model drawing in the teaching of mathematics. The advent of large collections of models of higher mathematics all over Europe and the U.S.A. therefore carried within it the forthcoming obsolescence of models, the function of which was reduced to the one of visualization and manipulation. Both the grandeur and the decadence of models have therefore to be analyzed in view of the long-term relationship between mathematics and drawing. This relationship especially raises open historical questions about the role that may have been played by models in the emergence of mathematical modelization. Geometric drawing may indeed be considered retrospectively as one of the roots of mathematical modelization, because of the model role it played for the development of the graphical methods and visualization devices that would eventually render both models and mathematical drawing obsolete. Model drawing especially carried on an ideal of clarity in the representation, in contrast to the ideal of objectivity associated with academic sciences and the models of higher mathematics. This ideal was especially consubstantial to the concept of épure, which implied the making of choices, and to the main pedagogical value associated with drawing: training the eye for improving the capacity of judgment. Its transfer to other graphical methods implied new issues in the formalization of the choice, or judgment, of what should be simplified in a representation, and these issues could not be dissociated from the instruments used for observation, experiments, as well as for tracing and representing. Reassessing the history of mathematical models in view of the development of the graphical method therefore calls for further investigations on the manifold links of mathematical models to the history of instruments, experiments, the natural sciences and the variety of graphical devices developed in the late nineteenth century between pedagogical, instrumental and research goals. The emergence of models of higher mathematics in the 1860s is only of the many lines of development of the traditional association between geometry, drawing, and models.

[249] Mehrtens, "Mathematical Models," 293.