Chapter 15
A General Theory of Inference

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Abstract This is an unpublished article from Popper's Nachlass.

Editorial notes: The source manuscript is from KPS Box 12, Folder 10. It is marked "*A General Theory of Inference* BY K.R. Popper", and consists of 54 handwritten pages, numbered 1 to 52, and an additional sheet numbered 39A and 39B. There is a mark on page 39 with an instruction where to insert pages 39A/B. In footnote 2 Popper mentions that he has not yet published anything concerning the problem of characterising valid inferences in general apart from "Why are the Calculuses of Logic and Arithmetic Applicable to Reality" (Popper, 1946c). This places the manuscript in the second half of 1946, probably before the meeting with Bernays in December 1946. The article could also be seen as a precursor to "Logic without Assumptions" (Popper, 1947b; this volume, Chapter 2) where Popper also discusses different notions of translation in the context of Tarski's concept of logical consequence.

1. Distinction Between General and Special Theories of Inference

1 Since Aristotle, logicians have been concerned, practically without exception, with the problem of the various special theories of inference, as opposed to those of a general theory of inference. They developed rules of inference for special languages (or calculi), such as the language of categorical propositions, for which Aristotle constructed a set of rules of inference, the so-called valid moods of the syllogism; or the more developed language of categorical propositions, usually called "calculus of classes", for which Boole and his successors developed an exhaustive theory of inference; or the language of hypothetical and disjunctive propositions, for which the Stoics constructed the set of rules called "modus ponendo ponens", "modus tollendo tollens", etc; or the more developed form of this language, usually called "calculus of propositions", for which Frege and Russell gave an exhaustive theory of inference.

Most of the contemporary work in logic is confined to the closer investigation of
2 these | languages and their rules of inference, or to the construction of new languages (or calculi), or to the investigation of the relations between these calculi.

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The fundamental problem of a general theory of inference is altogether different. It is not the problem of finding valid rules of inference for some language or other, but the problem of characterising valid inferences in general, for example, by giving a general and adequate definition of the term "valid inference".

If this can be done, then many problems might be solved which otherwise must remain insoluble. The most important of these problems is that of proving the validity of those rules of inference which are assumed as fundamental or axiomatic by the various special theories of inference.

I shall briefly comment on the fundamental nature of this problem.

3 The special theories of inference cannot do more than analyse | intuitive inference. They can give a number of general rules corresponding to some of the inferences in the language in question which we intuitively recognized as valid. They can reduce these rules to a small number of rules which can be accepted as valid on intuitive grounds, and which may be described as the *axiomatic rules of inference* for the language in question. They can combine these in such a way that rules emerge which are too complicated to be judged by intuition but can, nevertheless, be shown to be valid, if only we assume the axiomatic rules to be valid. Beyond this, the special theories cannot go.

But the general theory might go further. It might be able of being applied to the various systems of axiomatic rules, and to establish that they are indeed valid rules of inference. In this way, the various special theories of inference would be reduced to applications of the general theory of inference. In establishing the validity of the various axiomatic rules, the general theory would further reduce the intuitive element in logic. But this is obviously the task of logic – to give definite rules to which we may refer instead of having to appeal to logical intuition, i.e., to the intuitive conviction that a certain inference is valid.

4 | It must be admitted, of course, that the general characterisation or definition of a valid inference may retain intuitive elements. But these elements may be of a very different character from that "logical intuition", as we have called it, which logic attempts to replace by rules. Besides, the adequacy of any definition may be discussed in various ways, for example, with respect to the capacity of the definition to establish,
5 without any further | recourse to logical intuition, those rules of inference which have been long accepted as valid.

2. Contributions to the Problem

The first logician who, to my knowledge, conceived the idea of a general theory of inference was Carnap who devoted part of his book *Logical Syntax of Language*[a] (1934 and 1937) to what he called "General Syntax". Carnap himself is now convinced (owing to Tarski's criticism) that this attempt was unsuccessful (although it raised many important problems). A second attempt was made by Tarski, in his paper *On*

[a] Carnap (1934a, 1937).

6 *the Concept of Logical Consequence*[1]. Further | contributions were made by Carnap in his book *Introduction to Semantics*[b] (1942; see especially the reference, on p. vii, to the distinction between logical and descriptive signs). – Since 1937 I made use of Tarski's ideas in my lecture courses on logic[2]; more especially, I attempted (a) to apply Tarski's general concept of logical inference to the problem of establishing the validity of special rules of inference, (b) to analyse the distinction between logical and descriptive signs and the dependence of the general concept of inference on this distinction, (c) to analyse the relations between the "meaning" of logical signs and
7 the "meaning" of logical inference. I have convinced myself, in these lectures, | that a consideration of the practical value of inference is a useful heuristic starting point for the discussion, and that, from this starting point, we can, in stages, proceed to Tarski's definition; and further, to a statement and to a criticism of this definition, and to a re-statement of what I consider the fundamental problem of General Logic. This will be my procedure in the next three sections (section 3 & 4). In section 5 I shall give some alternative formulations of Tarski's definition, preparatory to a solution of the problem which will be sketched in section 6.

3. Steps towards Tarski's Definition of Logical Consequence

8 | It is an empirical fact that most of us "know how"[3] to draw inferences. Apparently, we learn this very early, and without being aware of it. This may suggest to us that drawing inferences – or "putting two and two together", as this activity is called colloquially – is of some practical value to us. Why? I suggest, because

(1) Valid inferences are such that, if the premises are all true statements of facts, the conclusion must be true as well.

In other words, if we have reliable information, and "know how" to draw inferences, we can rely on the secondary information thus obtained. For example, we may know from George that he sailed by the "Aquitania" and then read in the paper that the "Aquitania" arrived and that all on board are well. Putting two and two together, we

[1] ⟨Tarski, 1936b⟩, *Ueber den Begriff der logischen Folgerung*, published in *Actes du Congrès internationale de philosophie scientifique, fasc. VII.* (Paris, 1936). It may be noted that this important paper is missing in Z. Jordan's ⟨1945⟩ Bibliography in *Polish Science and Learning*, No. 6 (Oxford 1945).

[2] I have, however, nothing published on this problem, except some remarks in my paper ⟨Popper, 1946c⟩ "Why are the ⟨Calculuses⟩ of Logic and Arithmetic Applicable to Reality?" in the *Proceedings of the Aristotelian Society* (Supplementary Volume, 1946).

[3] G. Ryle has drawn attention to the fact that we often "know how" to do a certain thing, as opposed to the explicit knowledge, the "knowing that" a certain thing can be, or must be, done in a certain way. Cp. his Presidential Address to the Aristotelian Society (⟨Ryle, 1945⟩), and my above mentioned paper.

[b] Carnap (1942).

shall draw the conclusion "George arrived and is well"; and provided our original information was true, we may rely on the conclusion to be a true statement of fact.

We take (1) as the first step towards a definition of valid inference. It leads
9 immediately to a | formulation (2) which has been used, in some form or other, by many logicians since Aristotle:

(2) An alleged inference is certainly invalid if the premises are all true and the conclusion false.

We can express (1) and (2) also by saying (with Aristotle):

(3) If an inference is valid, then the second of the following four possible combinations cannot occur: (1) The premises are all true, and the conclusion is true; (2) the premises are all true and the conclusion is false; (3) the premises are not all true and the conclusion is true; (4) the premises are not all true, and the conclusion is false. Combinations (1), (3), and (4) can occur, that is to say, only the second combination is incompatible with validity.

All this would be of little value if we could not add

(4) The validity of an inference can be known without information about the truth or falsity of the premises and the conclusion.

10 That is to say, we need not have any | direct information concerning the conclusion: information about the truth of the premises and knowledge concerning the validity of the inferences is enough, and can be obtained without consideration of the conclusion. On the other hand, we shall say that

(5) The property of transmitting unfailingly the truth of the premises (*if* they are true) to the conclusion is, practically and theoretically, the most important characteristic of a valid inference.

If we consider (5), it is clear that such a term as "unfailingly" or "necessarily" is unsatisfactory. It can be replaced, however, by reference to what we may call the "logical form" of an inference (a term which, in its turn, needs further analysis):

(6) An inference is valid if its logical form is such that no other inference *of the same logical form* can have true premises and a false conclusion.

11 | If we have an inference under consideration, not knowing whether it is valid, then we can try to construct an inference *of the same logical form*, but with true premises and a false conclusion. Such an (obviously invalid) inference with true premises and a false conclusion we call a "counter example" of the inference in question. (If the original inference has true premises and a false conclusion, then it is its own counter-example.) Using this term counter-example, we can now say:

(7) An inference is valid if and only if no counter example of it exists.

This is a reasonably satisfactory general definition, ⟨and⟩ although it will have to be amended in many ways, it can be used for proving the invalidity (and even
12 the validity) of various special rules of | inference. It is based on the technical

term “counter-example” which, in turn, is based on the concepts “true”, “false” (or “non-true”), and “logical form”.

Of these two concepts we shall take the concept “true” for granted (in the sense of the correspondence theory, i.e., of the phrase: “A statement *s* is true if and only if it corresponds to the facts”). We can do this because of the famous analysis of this concept by Tarski; only this analysis made a general theory of inference possible. Nevertheless, we shall show, later, that we can eliminate this concept, if we use, with Carnap and Tarski, the name “Semantics” for a theory which makes use of the term “truth” (and related terms, such as “*X* is the name of *y*” or “*X* designates *y*”), and if
13 we use, with Carnap, the name “Syntax” | for a theory which considers the logical form of expressions without using terms characteristic of semantics, then we can say that we shall first construct a semantical concept of valid inference, and later a syntactical one. (Tarski’s concept of inference belongs to Semantics.)

The second concept mentioned above, that of the “logical form” of an inference, can be replaced by that of a “logical skeleton” (of a statement, or of a class of statements, or of an argument or inference). This term, in its turn, is based on the classification of the signs of the language under consideration into two classes: the *logical signs* or, as I prefer to call them, the *formative signs* on the one hand, and the *descriptive signs* on the other.

14 | This distinction is one which all logicians since Aristotle have made; but it was first explicitly discussed by Carnap (in his “Logical Syntax”). Logical or, as we shall say, *formative* signs are, for example, Aristotle’s affirmative and negative copulas, “is” or “are”, and “is not” or “are not”; further the words “all” and “some”. Other examples are the words “if . . . then . . .”, “or”, “neither . . . nor . . .”, “and”, “it is not the case that . . .”, etc. Descriptive signs are, for example, the words “cow”, “justice”, “the 1st of November 1946”, etc. The problem whether we can give a more general explanation of this distinction rather than an explanation based on examples will be discussed below.

15 | Once the distinction between logical and descriptive signs is given, it is easy to explain the term “logical skeleton”. We obtain the logical skeleton of a sentence, or a set of sentences, or an argument, by deleting all descriptive constants (and only these), taking care, however, to indicate *repetitions* of the descriptive constants. (This can be best achieved by the use of variables: we put variables in the empty places left by the elimination of the descriptive constants, and use the same variable wherever the same descriptive term has been eliminated, and otherwise different variables.)

16 | Considering this explanation, our definition (7) is only a shorter way of saying:

(8) An inference is valid if, and only if, an inference with the same logical skeleton, and with true premises and a false conclusion, does not exist.

4. Tarski’s Definition of Logical Consequence

We are now prepared to take the last step which leads us to Tarski’s final concept of logical consequence or of an inference.

A definition such as (7) or (8) is, as Tarski points out, not entirely satisfactory;
17 although its intentions are correct, it does not completely answer these | intentions, if
we accept it as a definition for inference *in general*, i.e., inference in any language. The simple reason is that some languages may not possess those descriptive terms which are needed for establishing a counter example. Take an invalid inference such as

All men are mortal.
All Greeks are mortal.
———————————
All Greeks are men.

In order to establish a counter example, we clearly need, at the very least, either (a) one other term, say "*t*" ("dogs" for example), which satisfies the following condition:
18 "All *t* are mortal" is true, "All *t* are men" is false, | or else (b) one other term, say "*u*"
("plants", for example), which satisfies the conditions: "All *u* are mortal" is true and "All Greeks are *u*" is false. Now it is easy to construct a language which does not contain such terms; for example, we can take Aristotle's formative signs and all the rules of Aristotelian logic, and add to these the terms "men", "mortal", and "Greeks"; or we could add even the terms "Greeks", "Persians", "Egyptians", etc., and the name of any property shared by all these. In both these languages, no counter-example
19 exists to show the invalidity of the inference | mentioned. Accordingly, the inference
would be valid in the sense of our definition (7) and (8), which is certainly against our intentions.

In order to avoid this unwanted consequence of definitions of the type (7) or (8), Tarski introduces the idea of a *model of a logical skeleton*. (The idea is taken from the postulational technique: we may say, for example, that light-rays (on Earth) and Fadenkreuze ⟨crosshairs⟩ are a model of, or satisfy, the postulate systems of Euclidean
20 Geometry, if we put light-rays in place | of what Euclid meant by "straight lines", and
Fadenkreuze ⟨crosshairs⟩ for whatever Euclid meant by "points".) By a model, Tarski means a set of things, properties, relations etc. (*not* their names); and he says that a model satisfies a logical skeleton, or that a model is a model *of* a logical skeleton, if and only if, the skeleton becomes true after substitution of appropriate names of these things (which names are to be added, if necessary, to the vocabulary of the
21 language in question). In | this sense, the set consisting of the class of all Greeks and
the class of all mortals (of the classes themselves, and not of their names) satisfies, or is a model of, the skeleton "All *x* are *y*", or the set consisting of Socrates, the class of teachers (of somebody) and Alcibiades (of the men themselves and the relation and
22 not of their names) satisfies, or is a model of, | the logical skeleton "*a* is one of the
R of *b*". The idea that a model *satisfies* (or *fulfils*) a skeleton, or it is a model *of* a skeleton, is clearly very closely related to the idea of truth.

With the help of this idea of a model, we can now formulate Tarski's definition of logical consequence or of valid inference:

(9) An inference is valid if every model satisfying the skeleton of the premises is a model satisfying the skeleton of the conclusion.

5. Criticism of Tarski's Definition. Present State of the Problem

23 | Tarski himself has given an excellent criticism of his definition. He points out that
the definition is dependent upon the distinction between formative (or logical) and
descriptive signs, and that we cannot be sure whether this distinction is not somewhat
arbitrary. Surely, the examples given above are innocent enough. But if we realize
that we are inclined to take such mathematical concepts as ">" ("greater than") or
"=" ("equals") as formative while we would hardly doubt that "taller than" or "of
24 equal height" are descriptive, then we may | begin to doubt whether the distinction
is not somewhat arbitrary. But if this is so, then there may be a number of different
classifications of the signs of a language into formative and descriptive possible, and
to each of these would correspond a different concept of valid inference.

Tarski's ultimate attitude, accordingly, is rather sceptical. He seems to think that
25 the element of arbitrariness in the | distinction between formative and descriptive
signs cannot be eliminated, and that the concept of inference, since it depends on this
distinction, has a similarly arbitrary character.

The situation can be also described in this way. All logic, and especially the general
theory of inference, attempts to replace our logical intuition by definite rules. We now
26 find that what we have | achieved is only this: we have replaced the logical intuition
of the validity of inferences by the intuition of the (formative or descriptive) character
of the signs of the language in question.

I believe that this is, indeed, an achievement. It is an achievement because in many cases – say, in Aristotle's or Boole's or Frege's and Russell's systems – the distinction between formative and descriptive signs is fairly obvious, and accordingly Tarski's definition can be applied with considerable success. But that we have not really solved the problem we started to solve seems obvious enough.

27 | Carnap seems to take a more optimistic view than Tarski. He points out,
very forcefully, the devastating consequences of Tarski's scepticism. One of these
consequences is that we could no longer maintain the distinction between logically
true and factually true statements; this distinction which has been widely used by
philosophers ("analytic" or "tautological" versus "synthetic" propositions), Carnap
considers (rightly, without doubt) as "indispensable for the logical analysis of science"[4].
28 Yet | Carnap admits[5] that "no satisfactory precise definition . . . is known" of formative
(or descriptive) signs.

From what has been said, it seems clear that the problem (a) of defining valid inference without assuming a classification of the signs of the languages under consideration into formative and descriptive and/or (b) of giving a satisfactory definition of formative and descriptive signs, is perhaps the most urgent and fundamental unsolved problem of general logic.

29 | I may say here that I was inclined for a long time to consider these problems as
insoluble. One of the reasons was, *undoubtedly*, the influence of Tarski's authority,
and the fact that there is, *undoubtedly*, something naïve in the belief that the signs

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[4] Carnap ⟨1942⟩, Introduction to Semantics, p. vii.

[5] *Ibid*.

of all languages must be capable of a neat division into two parts. Tarski's criticism of this naïveté is in any case a major logico-philosophical discovery, whether or not
30 this division can be in some way or other justified; and | it seems difficult, in the face of such a discovery, to uphold the main tenets of the view criticized, without laying oneself open to the charge of dogmatic and wishful thinking.

In spite of all that I am now inclined to consider that Carnap's optimism is justified, although in a somewhat modified and restricted way, and that the fundamental problem of general logic is solvable to a very considerable extent. In what follows I shall give an outline of what I believe is a way to a solution.

6. Reformulation of Tarski's Definition

31 | As a first step towards a solution, I shall re-state Tarski's definition in a slightly different form, viz., by replacing the idea of a model by the practically equivalent idea of a form-preserving interpretation.

Let us call the language system for which we wish to give a general definition of valid inference the language-system S_1. Since we intend to give a general definition, we must not assume anything about this language, apart from the fact that it is a language in the logical sense: we assume that it is not only an expressive but also a
32 descriptive language, | that is to say, capable of descriptions which are true or false. Such descriptions are *statements* or classes of statements. (We shall use lower case italics from the beginning of the alphabet as (variable) names for statements, and upper case italics as names of classes of statements. "$a \in A$" should be read "The statement a is an element of the statements class A".) We thus assume that S_1 is a meaningful language, that we understand it as we do, say, English.

We further assume that we have other languages S_2, S_3, etc., about which we
33 make the same assumption. We further assume that all the statements of | S_1 can be translated into the statements of these other languages (but not necessarily *vice versa*: the other languages may be richer than S_1, i.e., contain statements which cannot be translated into S_1).

Besides the normal translations of S_1 into the other languages we shall speak of *interpretations* – of S_1 in S_2, or in S_3, etc. An interpretation is a kind of mock-translation, or a generalized translation. It is based on some set of rules which correlate to certain signs, or sets of signs of S_1, some signs, or sets of signs, in another language, say in S_2. All such rules of interpretation must satisfy the following general principles:
34 (1) The | rules must be such that to every sentence of S_1, a sentence in S_2 is correlated. (2) Within the limits drawn by (1), certain signs (or groups of signs, i.e. phrases) of S_1 may be singled out as "meaningless" in this interpretation, that is to say, they are not correlated to any signs in S_2. (3) All other signs, or a group of signs (phrases) are to be correlated to signs, or groups of signs, in S_2.

It is clear from this description that an interpretation may correlate true statements to false statements, and *vice versa*. We may, for example, interpret the word "All" of
35 S_1, wherever it occurs, by a word of S_2 that means | "some", or "most", or "30%";

or we may treat it as "meaningless", i.e., as redundant. And we can interpret all descriptive terms in whatever way we like. It is clear, furthermore, that if a series of statements which constitutes a valid inference in S_1 is interpreted in this way in S_2, then the correlated series of statements in S_2 will as a rule no longer constitute a valid inference. (This is obvious from the possibility of interpreting "all" as "some".)

We now introduce the concept of an interpretation which *preserves* a certain sign or a class of signs.

(10)[c] If in an interpretation a sign or class of signs is always normally translated,
36 i.e., taken in its proper meaning, then | we say of an interpretation that it preserves this group of signs.

Now we shall again assume, as in Tarski's definition, that the signs of S_1 are classified into two groups, the formative and descriptive signs; and we call an interpretation which preserves all the formative signs a "form-preserving interpretation". With the help of this concept we can restate Tarski's definition of inference in this way:

(11) An inference in S_1 is valid if and only if the following holds: if, in some form-preserving interpretation, all the premises are true, then the conclusion is also true.

37 | Since we here speak of all interpretations – not, perhaps, of interpretations confined to the particular language S_2 which may happen to be as poor as S_1 – the same generality is achieved as in Tarski's definition (as opposed to the definitions (7) or (8)). For if a model exists which satisfies the conclusion but not the premises, then a language can be constructed which contains the appropriate names, and in this language, accordingly, a form-preserving interpretation of S_1 can be given in which all the premises of the inference in question are true and the conclusion ⟨is⟩ false.

We can, if we like, call such a form-preserving interpretation a counter-example
38 (of the alleged inference); and if we | re-define our term counter-example, then we can also define a valid inference by a formula identical with (7), only that the term "counter-example" now means "a form-preserving interpretation that correlates to all premises true sentences and to the conclusion a false sentence". We thus get

(12) An inference is valid if no counter example exists.

Tarski's criticism of his concept of inference remains, of course, completely in force, with all its consequences, for the concept of a form-preserving interpretation
39 is in precisely the same way dependent upon the | division of the signs of S_1 into formative and descriptive signs, as was the case in the original distinction.

39A | However, certain minor advantages arise. We can, for example, give with the help of our method a simple and convincing definition of those statements which are logically true:

(13) A statement a of S_1 is logically true if and only if all form-preserving interpretations of a are true. (The corresponding definition with the help of the idea of a "model" would be: "A statement a of S_1 is logically true if its logical skeleton is

[c] Number added.

satisfied by all models." But this would not do – the models may not fit. We would have to say "by all fitting models", and explain in general what a fitting model is, – not a very easy task.)

39B | Since Carnap rightly emphasises the link between our problem and that of a general characterisation of logically true statements of a language, it is of some advantage if we can show this link clearly. The obvious link between (11) and (13) and the fact that they equally use the idea of form-preserving interpretations, does not need to be further stressed.

39cont. | We now proceed to construct a further alternative and equivalent definition. Let us assume that "I_k", or "I_l", or "I_m", etc., are names of certain form-conservative interpretations. (We do not assume that the set of these interpretations is denumerable; and in general, it will not be denumerable.) In each of these form-conservative interpretations, a certain class of statements of S_1 will be correlated only with *true*
40 statements of the language in which the statements of S are interpreted. | Let us denote
the class of all those statements of S_1 which are, in the interpretation I_k, correlated with true statements, by the sign "M_k". It is clear that, although all statements of S_1 which belong to M_k are true in the interpretation I_k, they will not be, in general, true in the language S_1, or in other interpretations, except those statements which are logically true. These will be true in every form-conservative interpretation, that is to say, they will be in every of the classes M_k, M_l, M_m, etc, which are correlated with classes of true statements in the interpretation I_k, I_l, I_m, etc.; in other words, the logically true statements will be those which belong to all these classes.

I shall call the classes M_k, M_l, etc., "fundamental classes". The class of all
41 fundamental classes will be denoted by "*Fd*". Thus "$M_k \in Fd$" | means "M_k is a
fundamental class". From what has been said it is clear that the class *Fd* is a class whose elements are the various classes of all those statements of S_1 which are true in the various form-preserving interpretations. We now can define

(14) The class of logically true statements of S_1 is the product of the class of fundamental classes.

or

(15) The statement a is logically true (or in signs: $a \in Lt$) if and only if a is an element of every fundamental class; or in signs (I use the notation of Principia Mathematica) $a \in Lt \equiv M \in Fd \supset a \in M$.

Similarly, we can define again the valid inferences in S_1:

(16) An inference in S_1 with the premises A and the conclusion b is valid (in signs, $A \rightarrow b$) if and only if the following hold: if all the premises in A belong to some fundamental class, then the conclusion b belongs to the same fundamental class; in signs: $A \rightarrow b \equiv M \in Fd \supset (A \subset M \supset b \in M)$.

42 | All these definitions, just as in Tarski's, are semantical (they make use of the concept of truth), and they all are, like Tarski's, dependent on that crucial distinction between formative and descriptive signs.

7. Relativization of the Concept

The solution of our fundamental problem does not, I believe, lie in the more or less
dogmatic assertion that there are signs in every language which are (as it were) “by
43 nature” | formative. It lies, rather, in admitting the relativity of Tarski’s concept of
inference, as stressed in Tarski’s sceptical criticism of his definition, and in facing
even the most unpleasant and radical consequences of the situation thus created. This
opens the way to constructing a theory which, I believe, can satisfy those who, like
44 Carnap, find it necessary to | demand an absolute distinction between formative and
descriptive signs.

We shall start with the frank admission that the signs of most languages, and
surely of all naturally grown languages, cannot be neatly divided into formative and
descriptive signs, and there may be even languages which do not contain any separate
formative signs. (This should be obvious enough, considering that in practically all
naturally grown languages, and even in many languages constructed by logicians,
45 there are statements constructed of descriptive signs only; take, for | example, the
statement “I won” in English, or even a compound of three component statements like
“*veni, vidi, vici*” in Latin. On the other hand, there are certainly languages, constructed
by logicians – for example, that of Principia Mathematica – which do not contain any
descriptive signs.

In view of this situation, and of our aim to treat the problem in a completely general
way, we shall give up the assumption that we have, as it were, inside information
46 about the nature and classification of the signs of the languages under | consideration;
and we shall, later on, develop a method which does determine the formative signs
“from without”, as it were, or by the way in which they affect the general properties of
the language. (This is a very vague way of stating our programme, but it will become
more definite in what follows.)

Having given up the idea that we know how to classify the signs of a language
S_1 into formative and descriptive signs, we shall now start with the (provisional)
assumption that any classification is as good as any other, or, in other words, that we
47 can classify | the signs of S_1 in as many ways as we like into formative and descriptive
signs. We can start by making first a division in which all signs are descriptive and
none formative; we can make a second division by picking out, quite arbitrarily,
one sign and classify it as formative; a third division may pick out another sign
as formative; a fourth division may pick out perhaps two or three; and so on, until
we obtain, ultimately, a division in which all signs are put into the class labelled
48 | “formative signs”[6]. We shall denote the first division – the one in which the class of
formative signs is empty – by the name “D_0”, and the last by the name “D_z”; and
unspecified divisions will be indicated by “D_m”, “D_n”, “D_p”, “D_q”, etc.

Now to each of these divisions – say the division D_n – there will correspond a set
49 of form-conservative interpretations; form-conservative in the sense that | they are

[6] In general, the number n of different signs will be finite, and therefore the number of different classifications will be also finite, viz. 2^n. But our considerations are not dependent on the assumption that the number of different signs is finite, or the set of different classifications denumerable.

conservative with respect to the class of signs labelled in this division as “formative”. Accordingly, to each division D_n there will correspond a class Fd of fundamental classes M_k (where M_k is the class of all statements of S_1 which become true in the interpretation I_k). In order to indicate that the class Fd corresponds to a certain division D_n, we shall now replace “*Fd*” by “Fd_n”. Of a class M which is an element of Fd_n we shall say that it is an n-fundamental class. (We observe at once that, since
50 the interpretations corresponding to D_z are | simply the normal translations, Fd_z can have only *one* element M, viz., the class of statements which are true in S_1. On the other hand, the elements of Fd_0 are *all* the various statement classes of S_1.[7])

With the help of the concept “Fd_n”, we can now restate our definitions (15) and (16) in a way which makes their relativity, i.e., their dependence on an arbitrary division D_n, explicit:

(17) The statement a is logically true on the basis of the division D_n (or in signs,
51 $a \in Lt_n$) if and | only if a is an element of every class M which is n-fundamental; or in signs: $a \in Lt_n \equiv M \in Fd_n \supset a \in M$.

(18) An inference with the premises A and the conclusion b is valid on the basis of the division D_n (in signs $A \underset{n}{\rightarrow} b$) if and only if the following holds: if all the premises in A belong to some n-fundamental class, then the conclusion b belongs to the same n-fundamental class; in signs: $A \underset{n}{\rightarrow} b \equiv M \in Fd_n \supset (A \subset M \supset b \in M)$.

This explicit recognition of the relativity of Tarski’s concept of inference may now be used for a solution of our fundamental problem.

8. A Method of an Absolute Characterization of Formative Signs

52 | On the basis of our explicitly relative concept of inference, we shall now give a general characterization of signs of a language S_1 which we shall call “absolute formative signs”. (They must not be confused with Carnap’s “absolute concepts”.) *They are those signs whose meaning or function in D_n of S_1 can be characterized completely on the basis of our relative concept of inference.*

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[7] That is to say, if the number of statements in S_1 is infinite, Fd_0 is a non-denumerable set.

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