

Chapter 6

Vibration Response-Based Damage Detection



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Abstract This chapter aimed to present different data driven Vibration-Based Methods (VBM) for Structural Health Monitoring (SHM). This family of methods, widely used for engineering applications, present several advantages for damage identification applications. First, VBMs provide continuous information on the health state of the structure at a global level without the need to access the damaged elements and to know their location. Furthermore, damage can be identified using the dynamic response of the structure measured by sensors non-necessarily located in the proximity of damage and without any prior knowledge about the damage location. By principle, VBMs can identify damage related to changes in the dynamic properties of structures, such as stiffness variations due to modifications in the connections between structural elements, or changes in geometric and material properties. A classification of different VBMs was presented in this chapter. Furthermore, several case studies were presented to demonstrate the potential of these methods.

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6.1 Introduction

A broad class of damage detection methods, the so-called vibration-based methods, rely on dynamic data acquired by sensors positioned over the investigated structure. The objective of this review, which is not intended to be comprehensive, is to describe the use and goals of these methods that allow assessing structural damage conditions related to changes in the dynamic behaviour of the structure, e.g., damages that cause losses of stiffness. Furthermore, vibration-based methods are used to detect damage at a global level, using sensors not necessarily located in the proximity of the damage. The flowchart describing a general vibration-based strategy is shown in Fig. 6.1.

The identification of system parameters through vibration analyses and modal techniques has seen an active and ongoing interest for the last three decades (Fritzen et al. 1998; Fritzen 1986). The evolution of this field of research (Kerschen et al. 2005) and the move towards self-diagnosis (Fritzen and Kraemer 2009) has received further attention. The choice, distribution and placement of sensors, inextricable related to the value that can be obtained from them has been looked into in detail as well (Papadimitriou 2004; Papadimitriou et al. 2001; Papadimitriou and Lombaert 2012). Output-only identification (Basseville et al. 1993; Basseville et al. 2001) and sub-space type approach (Basseville et al. 2004; Mevel et al. 1999), damage parameter identification (Pardo De Vera and Güemes 1998; Görl and Link 2001) enabling the connection between models and reality (Alvin et al. 2003), including nonlinear aspects (Farrar et al. 2007) have been proposed. This development had also gone hand in hand with typical evolution of technology (Güemes et al. 2001). Despite this strong interest around the turn of the twenty-first century, there are several gaps, especially around the demonstrative examples of relevant aspects of aerospace monitoring that are yet to be addressed.

Depending on number and location of the recorded responses, different levels of refinement in the identification of damage can be achieved, namely detection, localization and assessment (Rytter 1993). Modal frequencies that can be used to

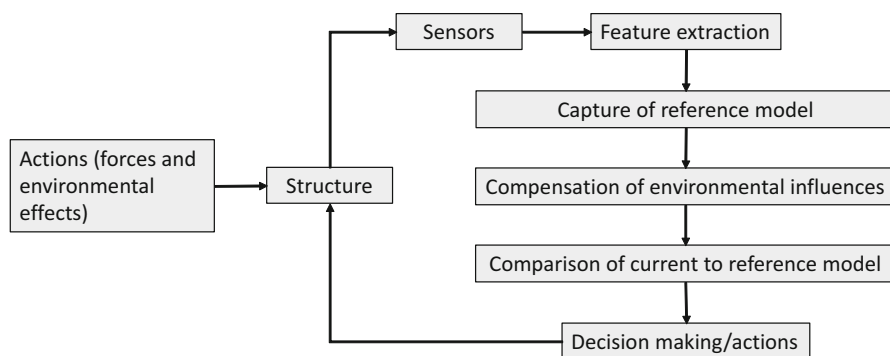


Fig. 6.1 General flowchart for vibration-based SHM methods

detect damage can, in principle, be identified using the response measured at a single location, provided all the modes contribute to that response. Damage detection, i.e., the identification of the existence of damage, might be carried out using a single sensor able to capture significant features of the structural response, e.g., the natural frequencies. The identification of damage location requires more distributed measurements and thereby a higher number of responses. Damage assessment, i.e., the identification of type and severity of damage, requires a numerical model of the structure that maps the dynamic records to different damage types and scenarios. The calibration of a such and the results it can provide depends on the number of responses available for its updating.

In literature, several methods for vibration-based damage identification have been proposed and different ways can be used for their classification. A traditional distinction is between model-based methods that make use of a numerical (e.g., finite element) model of the structure for the identification of damage and response-based methods that only use the experimental measured response to forced or ambient vibrations. In model-based methods, damage is identified through a procedure of model updating aimed at modifying the parameter of the model to achieve a better agreement with the experimental response. The most common model updating techniques include optimal matrix update, sensitivity-based and eigenstructure assignment methods (Doebling et al. 1996; Imregun and Visser 1991; Mottershead and Friswell 1993). The quality of the model updating is strictly related to the uncertainty in the model parameters and depends on the class of mathematical model chosen, on the measurement errors, excitation bandwidth and on the number and location of sensors (Limongelli and Giordano 2020; Papadimitriou et al. 2000).

The choice of the type of sensors is driven by the frequency range of interest. Accelerations are commonly measured and used directly to extract damage features or integrated to obtain velocities and displacements. In the latter case, a proper pre-processing is needed before performing the integration and information about residual displacements may be lost in the integration. Strain measurements are a further choice but provide local rather than global information and require the deployment of a higher number of sensors.

This chapter is limited to response-based methods, i.e., output-only. In fact, measuring the input excitation in aerospace applications may be challenging. Model-based methods have usually a considerable computational cost due to the need to update the parameters of the model which might involve iterative optimization procedures. This makes them less suitable for real-time applications. Conversely, response-based methods are not suitable to provide both the type and the severity of the damage, whereby model-based methods become a useful option.

Several vibration-based methods that rely only on recorded responses have been proposed in the literature. These methods can be classified in several ways, concerning the definition of the Damage Sensitive Features (DSFs).

One of the most common classifications concerns the domain where DSFs are retrieved, namely, frequency, time, and time-frequency domain. The oldest and most popular methods operate in the frequency domain. Among these, Frequency Response Functions (FRFs) and modal parameters are the most employed. Natural

frequencies have been used in the first studies to detect damage (Cawley and Adams 1979; Ostachowicz and Krawczuk 1990). Generally, natural frequencies are not suitable to detect small damage and to localize damage due to their sensitivity to environmental changes (Peeters and De Roeck 2001). More advanced methods based on natural frequencies have been developed (Chang and Chen 2005; Li et al. 2020; Xiang and Liang 2012). Several frequency-domain methods are based on the use of modal or operational parameters (Limongelli 2019), mainly combinations of frequencies, modal shapes, and their derivatives (Abdel Wahab and De Roeck 1999; Pandey et al. 1991; Ratcliffe 1997; Stubbs et al. 1992) or in terms of operational shapes retrieved from frequency response functions (Dutta and Talukdar 2004; Giordano and Limongelli 2020; Limongelli 2010; Pai and Jin 2000; Parloo et al. 2003; Zhang et al. 2013).

Conversely, time-domain methods operate directly on the recorded time series, using separation techniques, filtering, and training time series models. In turn, these methods may be divided into parametric and non-parametric approaches. The first describes the underlying stochastic process using autoregressive and/or moving average parameters. Furthermore, non-parametric approaches involve the direct estimation of features, such as time spectra and covariance, of the collected time series, which are then employed as DSFs.

Recently, time-frequency methods have also been employed for structural and damage identification, relying on two-dimensional (i.e., time-frequency or time-scale) distributions which highlight the presence of features in the collected signal.

Another common classification is based on the applicability of the methods to either time-invariant or variant scenarios. The former concern cases where the structural dynamic features and the external conditions (i.e., loads and environment) do not vary over time, whereas the latter allows the identification of damage during the occurrence of structural modifications, such as change of boundary conditions, or environmental variations.

In this chapter, a survey of common response-based methods (schematized in Fig. 6.2) for damage identification was provided. This non exhaustive survey is intended to give an insight into the most used methods for SHM for aerospace engineering, providing recent trends in research.

6.2 The Rationale of Vibration-Based Methods

Vibration-based methods for damage identification are based on the premise that a change of stiffness induces changes in modal and operational parameters that can be thereby used as damage features. Environmental and operational effects as well may produce variations of these features and must be carefully accounted for as will be illustrated in Sect. 6.3.

The equation of motion of an n degree of freedom structure subjected to an excitation, $F(t)$, can be written as:

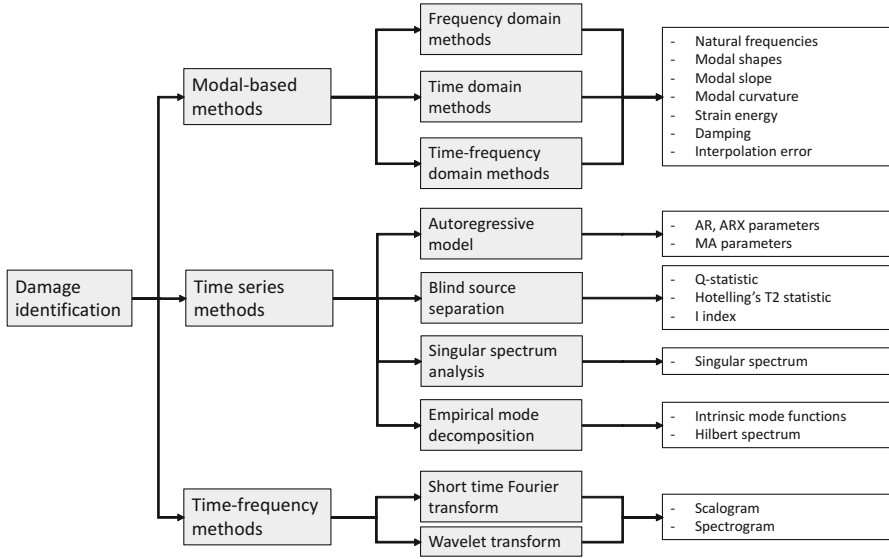


Fig. 6.2 Classification of vibration-based SHM methods

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(t) \quad (6.1)$$

where $\mathbf{X}(t)$ is the $(n \times 1)$ vector of displacement coordinates while \mathbf{M} , \mathbf{K} and \mathbf{C} are the $(n \times n)$ mass, stiffness and viscous damping matrices of the system, $\mathbf{F}(t)$ is the $(n \times 1)$ excitation vector. The natural frequencies can be estimated starting from the equation of free vibrations for undamped systems, as follows:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = 0 \quad (6.2)$$

By introducing the change of coordinates $\mathbf{X}(t) = \mathbf{w}_i q_i(t)$ and the general solution $q_i(t) = A_i \cos \omega_i t + B_i \sin \omega_i t$, a matrix eigenvalue problem is obtained:

$$\mathbf{K}\mathbf{w}_i = \omega_i^2 \mathbf{M}\mathbf{w}_i \quad (6.3)$$

The n eigenvalues and eigenvectors of this problem are respectively the squares of the circular frequencies ω_i^2 and the mode shape vectors ϕ_i of the n degrees of freedom system. The natural frequencies f_i are related to the circular frequencies by means of the relation $f_i = \omega_i/2\pi$. The mode shapes are defined in arbitrary scale. It can be shown that for the real, symmetric, positive definite mass and stiffness matrices, the eigenvalues are real and positive.

6.3 Environmental and Operational Influences

The basic objective of vibration based SHM is to ascertain if damage is present or not based on measured dynamic characteristics of the system to be monitored. The damage can alter the stiffness, the mass or the energy dissipation of a structure and in this way change its basic dynamic characteristics. These characteristics (natural frequencies, vibrational modes, damping) could be strongly influenced by the environmental and operational conditions. This fact is noticed by many authors, see (Balmès et al. 2008; Giraldo et al. 2006; Lysgaard et al. 2021; Sohn 2007; Sohn et al. 2001; Wah et al. 2017), and different approaches are proposed to consider this problem. During the operation of a structure, or during the tests of a structure for damage, measurements are stored, and the selected features are extracted from data. These features then have to be compared with the features corresponded to the intact structure. If, however, the environmental conditions (temperature, humidity, wind) of the measured features are different from the reference ones, a false alarm for a damage could arise, or the damage could be masked. In reference (Manoach et al. 2012a), for example, it was shown that if the temperature of the tested beam is different from the reference one, the selected feature (Poincaré map) extracted from the measured forced response of the tested beam cannot locate the existing damage. The temperature changes are the most important environmental condition which has to be taken into account in SHM process. Thermal loads introduce stresses due to thermal expansion, which lead to changes in the modal properties. Additional important environmental and operational conditions could be wind-induced variation effect, boundary conditions, humidity (Sohn 2007).

In this sense, a key factor for SHM is to define a reference state for the expected systems or structures. The reference signals which are used to detect a potential damage in a structure should be recorded considering all possible environmental and operational factors. In many cases, however, these conditions cannot be repeated in the tested structure. That is why, several approaches to overcome the problem with the different conditions have been proposed in the literature. A possible way is to use multivariate statistical tools (so called Principal Component Analysis (PCA)) to extract features that are sensitive to damages but less sensitive to the effects of the changing environmental conditions (Manson 2002; Yan et al. 2005a).

Sohn et al. (2001) proposed an auto-associative neural network to perform NonLinear Principal Component Analysis (NLPCA). The authors have proposed to perform peculiar training of the NLPCA using the auto-associative neural network to extract the dependency of the damage sensitivity features on the unmeasured environmental conditions. Another approach (Reynders et al. 2014) was to use Kernel PCA to create nonlinear output-only model of the undamaged structure to be used as baseline. In Reference (Yan et al. 2005b), a method is proposed which introduces the concept of two maximal and minimal environmental conditions as baseline so as to remove the need to use features from a wide range of environmental conditions as baseline.

Considering all these studies, it could be concluded that the proper definition of the reference state of the intact structure and taking into account the environmental and operational conditions could be crucial for the correct and reliable SHM.

6.4 Modal-Based Methods and Damage Features

Modal parameters, i.e., modal shapes, natural frequencies and modal damping are among the most used DSFs in SHM applications due to their direct physical interpretation. Several studies have been presented in the scientific literature aimed at detecting, localizing or even quantifying damage, using the modal parameters or quantities derived from them.

Different techniques have been proposed to identify modal parameters based on structural vibrations (namely, Operational Modal Analysis (OMA) techniques), which operate in the frequency, time, or joint time-frequency domain.

In the first case, the Fourier transform is applied to the recorded signal and the modal parameters are identified by analysing resonance peaks in the spectrum of the structural response. The most used methods in this context are the basic frequency domain approach and Frequency Domain Decomposition (FDD) (Brincker and Ventura 2015). As concerns time-domain approaches, free decay responses are generally employed upon obtaining them through correlation or random decrement functions. The Ibrahim Time-Domain (ITD), the stochastic subspace identification and the Eigensystem Realization Algorithm (ERA) are well-known methods (Brincker and Ventura 2015). However, both time and frequency approaches are based on strict assumptions about the nature of exciting input and structural behaviour. Firstly, the excitation must be stationary and have a wide frequency band. Furthermore, structural behaviour should be linear and not vary over time. To relax the assumptions on the nature of the input and the non-variability of structural conditions, several methods have been proposed based on time-frequency and Time-Scale Representations (TFRs) generally obtained using the Short-Time Fourier Transform (STFT) and the Wavelet Transform (WT). For modal identification, simple ridge extraction (Delprat et al. 1992; Rankine et al. 2007) or other methods, such as the clustered filter bank approach (CFB) (Quqa et al. 2020) or the Modal Assurance Distribution (MAD) (Quqa et al. 2021), are employed.

6.4.1 *Natural Frequencies*

Natural frequencies are amongst the first DSFs employed in SHM applications due to their ease of evaluation. Nevertheless, several studies confirmed that damage identification methods based solely on natural frequencies may not be reliable if applied to complex structures or if the damaged entity is modest (Güemes et al. 2020). Generally, damage is a local phenomenon, whereas natural frequency is a

global feature not appropriate to identify the damage location except for simple applications. There are however attempts to localize damage only based on the information about the natural frequencies. Such methods can be found in the works of Cawley and Adams (1979) and Ostachowicz and Krawczuk (1990). Herein, the authors used the frequency shifts to detect damage.

A more advanced method to detect and localize damage was developed recently in Li et al. (2020). Particularly, the authors have studied the damage-induced Relative Natural Frequency Change (RNFC) curves:

$$\Delta\omega_{ij} = \frac{\omega_j - \omega_{ij}^*}{\omega_j} \quad (6.4)$$

where ω_j is the j -th natural frequency of the intact beam and ω_{ij}^* is the j -th natural frequency of the damaged beam with the damage at the i -th sensor location. Lin and Cheng (2008) proposed an index based on the natural frequencies and used it to determine the depths of two cracks in a monodimensional structural element. Chang and Chen (2005) and Xiang and Liang (2012) presented a two-step multiple damage detection method that estimates the natural frequency-based defect severity upon localizing the damage through the mode shapes.

6.4.2 Mode Shapes

The simplest and easiest (after natural frequency method) method which uses the changes in the mode shapes is the so-called modal displacement method (Allemang and Brown 1982; Farrar and James 1997; Farrar et al. 1994). The Modal Displacement (MD) method is based on the difference in the measured modal displacements of damaged and healthy structures. The maximal absolute value of the differences between the modal displacements of a beam could be representative of the damage location. The maximal absolute value of the differences between the modal displacements at location $i = 1, \dots, N_{\text{nodes}}$ is computed as follows:

$$\Delta w_i = \sum_{j=1}^{N_{\text{modes}}} \left| w_{ij}^* - w_{ij} \right| \quad (6.5)$$

where N_{nodes} is the number of nodes used to discretize the beam, N_{modes} is the number of identified modes, w_{ij} is the displacement of the j -th mode at the i -th node of the intact beam and w_{ij}^* is the same for the damaged beam.

Based on the mode shapes, several techniques have been developed over the years to improve the performance of the modal displacement method. An example of such method is the so-called Modal Assurance Criterion (MAC) method suggested by Allemang and Brown (1982), which detects mode shape changes over the entire structure by taking advantage of the orthogonality of the normal modes. Kim et al.

(1992) furthered the MAC in the development of the Coordinate Modal Assurance Criterion (COMAC) method, which uses modal node displacements to detect and locate damage. An example of successful usage of MAC to correlate modes of undamaged space shuttle orbiter body flap with a damaged flap is the work (West 1984).

6.4.3 Modal Slope

Another simple method for damage detection is based on the modal slopes (i.e., the first derivative of modal shapes). The absolute differences between the squared modal slopes of the damaged and healthy structures at location $i = 1, \dots, N_{\text{nodes}}$ is computed as follows:

$$\Delta W_i'^2 = \sum_{j=1}^{N_{\text{modes}}} \left| w_{ij}'^2 - w_{ij}'^2 \right| \quad (6.6)$$

where N_{modes} is the number of identified modes. Sign “prime” in the equation denotes derivative with respect to the space variable. Damage is located at the location where the damage index $\Delta W_i'^2$ reaches the maximum value. This method is more often used considering only the first mode, i.e., $N_{\text{modes}} = 1$. The method frequently gives additional, false peaks and it is not widely used for real applications (Manoach et al. 2016a).

6.4.4 Modal Curvature

A popular modal-based approach is the Modal Curvatures method (MC) (Pandey et al. 1991), which shows a better sensitivity to damage than the aforementioned DSFs. In this case, the damage indicator is defined at location $i = 1, \dots, N_{\text{nodes}}$ by the following expression:

$$\Delta W_i'' = \sum_{j=1}^{N_{\text{modes}}} \left| w_{ij}'' - w_{ij}'' \right| \quad (6.7)$$

MC-based methods are generally more reliable than the MD-based approaches, especially if curvatures are directly measured from the strain mode shape (Deraemaeker et al. 2006). The authors of (Pandey and Biswas 1994) noticed that MC-based damage detection generally leads to a curve which has a prominent peak at the damage location, but may also manifest smaller outliers, generating issues in practical applications. Ciambella and Vestroni (2015) have managed to avoid this disadvantage. By using a perturbative solution, they have proved that the modal

curvatures do not provide information on the damage location, if not properly processed. The authors thus introduced a novel filtering procedure for MCs, which leads to effective damage localization considering only one mode.

A modification of the modal curvature method is the Modal Curvatures Square Method (MCSM) (Deraemaeker et al. 2006), which can be used to enhance the differences in the curvatures of the intact and damaged structures, i.e.

$$\Delta W_i'^2 = \sum_{j=1}^{N_{modes}} \left| w_{ij}'^2 - w_{ij}''^2 \right| \quad (6.8)$$

6.4.5 Strain Energy

The Modal Strain Energy method (MSE) is one of the most effective modal-based methods. It was suggested in Stubbs et al. (1995) and Stubbs and Kim (1996). Primarily developed for addressing strain-based theories for Euler-Bernoulli beams, the method has been extended to portal blocks, sway frames, cables and complex systems as a whole in the later stages. This was pioneered by the exemplary work by Cornwell et al. (1999).

Numerical evaluations and experimental test beds have evidenced the method applicability to deep (Timoshenko) beams where bending and shear deformation contributes to the strain energy. Considering the i -th element of the beam, the j -th mode strain energy for the healthy and damage structure are obtained as:

$$U_{ij} = \frac{1}{2} \int_{a_i}^{a_{i+1}} \left\{ (EI)_i \left(\frac{\partial^2 w_j}{\partial x^2} \right)^2 + k \frac{(Ebh)_i}{2(1+\nu)} \left(\frac{\partial w_j}{\partial x} - \psi_j \right)^2 \right\} dx;$$

$$U_{ij}^* = \frac{1}{2} \int_{a_i}^{a_{i+1}} \left\{ (EI)_i^* \left(\frac{\partial^2 w_j^*}{\partial x^2} \right)^2 + k \frac{(Ebh)_i^*}{2(1+\nu)} \left(\frac{\partial w_j^*}{\partial x} - \psi_j^* \right)^2 \right\} dx, \quad (6.9)$$

and

$$U_j = \sum_{i=1}^n U_{ij}, U_j^* = \sum_{i=1}^n U_{ij}^*$$

where j is the mode index, U_{ij} is the strain energy of j -th mode for the i -th element. The variables for the damaged beam are denoted by ‘*’. Angles ψ_i and ψ_i^* denote the rotation of the cross-section of the i -th elements for the intact and damaged beam, respectively.

In the above-mentioned articles, it was shown that the strain energy ratio

$$\frac{(EI)_i^*}{(EI)_i} = \frac{\frac{J_{ij}}{U_j}}{\frac{J_{ij}^*}{U_j^*}} = \frac{\gamma_i^*}{\gamma_i} = f_i$$

$$\gamma_i^* = J_{ij} U_j^*; \gamma_i = J_{ij}^* U_j \quad (6.10)$$

where

$$J_{ij}^* = \int_{a_i}^{a_{i+1}} \left\{ \left(\frac{\partial^2 w_j^*}{\partial x^2} \right)^2 + k \frac{6}{h^2(1+\nu)} \left(\frac{\partial w_j^*}{\partial x} - \psi_j^* \right)^2 \right\} dx$$

$$J_{ij} = \int_{a_i}^{a_{i+1}} \left\{ \left(\frac{\partial^2 w_j}{\partial x^2} \right)^2 + k \frac{6}{h^2(1+\nu)} \left(\frac{\partial w_j}{\partial x} - \psi_j \right)^2 \right\} dx \quad (6.11)$$

can successfully indicate damages.

In addition to the localization, the extent of damage is also indicated by the strain energy ratio. This ratio tends to 1 for the undamaged regions and it becomes higher when damage occurs.

Detection of fault is incentivized by the numerical value of strain energy. Considering the neighborhood of damage, strain energy can prove to be a vital indicator using a damage index. This index f_i is an excitation frequency dependent parameter that can be related to the cumulative strain energy ratio as:

$$f_m = \frac{1}{N} \sum_{j=1}^N f_{j_m} \quad (6.12)$$

An improved version of the method described by Stubbs and Kim (1996) is presented here. Complications might arise when the angular rotation has to be determined directly. In Manoach et al. (2017), the authors have critically investigated the second term that evolves from the expression of strain energy (as a direct resultant of shear stress). Comparisons using both the formulas have also been carried out. In some cases, consideration of the full strain energy could essentially improve fault detection and its extent.

6.4.6 Damping

Modal damping has also been proposed as a possible feature for damage detection since the presence of a crack in a cross-section of a thin-walled structure will increase internal friction, which in turn will increase the value of damping (Manoach et al. 2016b, 2012b; Yamaguchi et al. 2013).

This possibility was firstly highlighted by Williams and Salawu (1997) and Farrar et al. (1994) who both concluded that measuring modal damping from vibration data

produced large deviations that significantly decreases damping as a reliable damage indicator.

Kyriazoglou et al. (2004) explored the use of the specific damping capacity SDC for damage detection and localization in composite laminates:

$$SDC = \frac{\Delta U}{U} \quad (6.13)$$

where ΔU is the energy dissipated in one cycle and U is the total energy stored in that cycle. In this work, the change in natural frequencies or flexural modulus was used as a damage indicator in glass-fiber-reinforced plastic laminates; however, the authors had concluded that SDC is more sensitive to damage.

6.4.7 Interpolation Error

The interpolation method (Limongelli 2010) is based on the use of cubic spline functions to interpolate the deflected shapes estimated from dynamic records. One of the main advantages of this method is that it does not require the estimation of the modal curvatures to obtain the damage indices. The interpolation is carried out at the i -th location considering all the measured components of the deflected shapes $w_1, w_2, \dots, w_{i-1}, w_{i+1}, \dots, w_n$ except w_i . The interpolation error is defined as follows:

$$\begin{aligned} E_{k,i} &= \left| \left(c_{0,k,i} + c_{1,k,i}(z_i - z_{i-1}) + c_{2,k,i}(z_i - z_{i-1})^2 + c_{3,k,i}(z_i - z_{i-1})^3 \right) - w_{k,i} \right| \\ &= |\hat{w}_{k,i} - w_{k,i}| \end{aligned} \quad (6.14)$$

The coefficients $c_{0,i}$, $c_{1,i}$, $c_{2,i}$, and $c_{3,i}$ of the cubic spline function are computed imposing interpolation and continuity conditions at all the instrumented locations. Due to the so-called ‘Gibbs’ phenomenon for splines functions, a sharp increase of the interpolation error arises at the locations with curvature discontinuities, thus allowing to locate damage. Refer to (Limongelli 2003) for additional information on the interpolation procedure. The interpolation error method can be applied using both modal and operational shapes since the curvature discontinuity affects both types of deformed shapes (Giordano and Limongelli 2020). To increase the sensitivity of the damage feature and increase the difference between its value at the damaged and undamaged locations, the interpolation error is computed combining the contribution of all the n_{shapes} modal or operational shapes as follows:

$$E_i = \sqrt{\sum_{k=1}^{n_{shapes}} |\hat{w}_{k,i} - w_{k,i}|^2} \quad (6.15)$$

Changes of interpolation error between two different states (reference and potentially damaged) highlight the onset of a curvature discontinuity. Hence, the difference of interpolation error δE_i defined in Eq. (6.16) is assumed as the damage feature at location i .

$$\delta E_i = E_i^D - E_i^U \quad (6.16)$$

The extension of this method to two-dimensional structures has been proposed in Limongelli (2017), and it is referred to as the surface interpolation method (SIM). The SIM is based on the use of a bi-cubic spline interpolating function, which can be thought of as a surface constructed from sets of cubic spline functions. A comprehensive and rigorous description of the SIM can be found in Limongelli (2019).

The readers who are interested to see more references for different modal-based methods for damage detections can turn to these excellent review articles: Doebling et al. (1996); Maia et al. (2003) and Montalvão et al. (2006).

6.5 Time Series Methods

6.5.1 Autoregressive Parameters

Autoregressive (AR) models are widely used for time series modelling (Gul and Necati Catbas 2009; Nair et al. 2006; Sohn and Farrar 2001; Worden et al. 2002; Yao and Pakzad 2012), due to the simple procedure required for the identification of an underlying model, performed through linear least-squares minimization which involves low computational effort and model uncertainties (Datteo et al. 2018).

AR models were developed in econometrics to describe time-varying processes, representing the output variable as a linear combination of its own previous samples plus an error term. In particular, each sample $x[t]$ of a collected signal is expressed as:

$$x[t] = c + \sum_{i=1}^p \varphi_i x[t-1] + \varepsilon[t] \quad (6.17)$$

where φ_i are the AR parameters, p is the model order, i.e., the number of lags considered in the description of each sample of the output variable, c is a constant and $\varepsilon[t]$ is the error term. The order is generally not known a priori, requiring specific techniques to find the optimal value. To this aim, the Aikake Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are widely employed (Ljung 1998) in research.

In the last decades, several types of autoregressive models have been investigated for SHM. Sohn et al. (2000) studied features based on the analysis of residuals, generating control charts (X-Chart, S-Chart, EWMA) where outliers can be easily

identified. Yao and Pakzad (2012) compared different pattern recognition algorithms using AR models and introduced the Cosh spectral distance as a new damage feature.

Despite the growing use in SHM applications, AR models are only-pole functions and spurious poles are usually introduced to represent the response of complex systems, which also depend on the zeros of the FRF of the physical model (Yao and Pakzad 2012). Moreover, the use of these models in the field of structural dynamics is based on the assumption that the force that excites the system is a random Gaussian process, which is generally acceptable only when analysing long enough time series.

Other advanced models have been used for better describing the dynamic behaviour of complex and time-varying systems. AutoRegressive Moving Average (ARMA) has been successfully employed to describe the response of linear systems subjected to random excitation (Datteo et al. 2018), employing however a nonlinear least-squares approach to identify the moving average coefficients, which involves high computational cost and convergence issues. Sohn and Farrar (2001) proposed a two-stage prediction model, i.e., combining auto-regressive and AutoRegressive with eXogenous inputs (ARX) techniques, consisting of a linearized version of ARMA, to address these problems. The ARX model was also used by Roy et al. (2015), who provided a mathematical formulation relating the variations in ARX coefficients with the stiffness of the structure, which cannot be directly provided in the case of a pure AR model.

Due to the univariate nature of AR, ARX and ARMA models, they are usually employed for damage detection, performing novelty detection on single data channels considered individually. To the aim of damage localization, multivariate extensions of the aforementioned models have been presented, including multivariate autoregressive (MAR) models (Achilli et al. 2020) and vector autoregressive moving average (VARMA) (Entezami and Shariatmadar 2019; Mosavi et al. 2012). In such models, each sample of a given time series (representing the signal collected by a given acquisition channel) is described using also the p past samples of other signals, collected at different locations. Recently, nonlinear extensions of time series methods, such as the nonlinear autoregressive models with exogenous variables (NLARX) have been presented to deal with systems that show strongly nonlinear behavior. Hu and Kaloop used this model combined with wavelet neural networks to identify the thermal response of a bridge (Hu and Kaloop 2015).

Time-dependent extensions of ARMA models were also proposed to deal with time-varying systems and nonstationary excitation (Poulimenos and Fassois 2006; Spiridonakos and Fassois 2009; Spiridonakos and Fassois 2012). The class of functional series time-dependent autoregressive moving average (FS-TARMA) were surveyed in (Spiridonakos and Fassois 2014), where a comparative assessment of the main modelling and identification literature methods is presented.

Similar to the use of ARX is the idea behind the use of Kalman Filtering (Seibold et al. 1996). A reference multi-input multi-output state space model is identified and produces residuals when used on new measurements. The residual is used as a damage feature.

A dynamic input reconfiguration architecture based on the input control redundancy available in the system can be achieved by the use of Nullspace-based Subspace Fault Detection (NSFD) (Varga 2011). Using both ultrasonic and vibration-based studies, methods based on NSFD has shown promise towards SHM of aerospace structures. Narrowband burst signals have shown wide applicability to excite systems in order to reduce the impact of multimodal wave dispersion and propagation. However, with the NSFD method being extremely adaptable, simulated sensor signals are obtained from narrowband, broadband and more specifically, random excitations for fault detection based SHM (Peni et al. 2018; Varga 2011). As multiple applications of NSFD have been researched over the years, notable among them involve the methods based on dynamic nullspace computed from the linear time-varying model of the system. Cases where no uncertainty is considered invoke the possibility of influencing the nominal control loop with the most advanced Fault Tolerant Control (FTC) techniques (Shokravi et al. 2020). However, environmental impact studies necessitate a paradigm shift towards enabling robust active methods such as linear parameter-varying, switching, and gain-scheduled certifiable algorithms.

6.5.2 *Intrinsic Mode Function and Hilbert Spectrum*

Damage detection of structural elements has received considerable attention with the introduction of Empirical Mode Decomposition (EMD) (Huang et al. 1998; Kunwar et al. 2013; Li et al. 2007; Peng et al. 2005; Xu and Chen 2004; Yang et al. 2004). Associated with EMD is the energy Damage Index (DI) that differentiates the damaged state of the system from its pristine state. The EMD method decomposes a signal into Intrinsic Mode Functions (IMFs) that satisfies the following conditions: (i) it is a well-behaved mono-component function and (ii) the average values of the envelopes defined by the local extrema are zero. A stalwart in the signal decomposition genre, EMD decomposes any given signal into its corresponding IMFs, even under nonstationary environments, in an adaptive manner, without pre-selecting any basis (Huang et al. 1998; Xu and Chen 2004). In modal identification theory, for MDOF systems, modes of vibration can be obtained based on the free-vibration responses.

IMFs are extracted through *sifting*, a technique rigorously discussed in supporting literature. Suppose a signal $x(t)$ is to be decomposed. The running mean of the envelope is subtracted which produces the difference $x(t) - m_1 = h_1$ to obtain primary IMF, containing the shortest period component of the signal. The component h_1 is now examined if it satisfies the conditions sufficient for an IMF. In case of discrepancies, sifting continues in successive iterations until the conditions for an IMF are addressed. Sifting decomposes the data into n IMFs, C_i and a residue r_n —a statistical constant or a mean trend. Mathematically, the procedure can be expressed as:

$$X(t) = \sum_{i=1}^n C_i + r_n \quad (6.18)$$

where $X(t)$ represents an acceleration response for any n -DOF system. Localization of damage is carried out by employing the DI based on the energy of the first IMF of the signal. The vibration response extracted from the health system is first acquired and filtered to retain only the first natural frequency within the data. The first IMF is then extracted through EMD to obtain the energy for each sensor as:

$$E = \int_0^{t_0} (IMF)^2 dt \quad (6.19)$$

Finally, the DI for each sensor is given by the following expression:

$$DI = \left| \frac{E_{Healthy} - E_{Damaged}}{E_{Healthy}} \right| \times 100 \quad (6.20)$$

High index values may be associated with proximity to the damage in a monitored system. However, a key drawback of this approach is the requirement of measured responses at all floor levels. To overcome these difficulties, recently developed damage detection algorithms premised on decorrelating the signals using orthogonal transformations are utilized.

6.5.3 Signal Components

The method of separating signal constituents from a mixture without prior information about the sources is known as Blind Source Separation (BSS) (Belouchrani et al. 1997; Sadhu and Hazra 2015; Zang et al. 2004). A signal decomposition tool, BSS has not only been used in acoustics, but also for structural modal identification and damage detection in built infrastructures (Hazra et al. 2012). BSS estimates primary source signals from the vibration output of monitored systems infested with noise. The estimation, mostly carried out on output signals, primarily identifies the mixing system first and then estimates the source signals implicitly by some suitable optimization procedure (Hazra et al. 2012). In addition to its utility as a system identification tool, BSS uses its second-order statistics such as auto-correlation functions (Antoni 2005) or higher-order statistics like non-Gaussianity (Belouchrani et al. 1997) for structural damage detection. Independent Component Analysis (ICA)—a form of BSS-0 has been extensively used for identifying damage patterns of dynamical systems (Hyvärinen and Oja 2000; Sadhu and Hazra 2015). However, it was later observed that ICA suffered from some performance issues such as the presence of higher structural damping and measurement noise. The use of auto-correlation functions prompted the utilization of Second-Order Blind Identification

(SOBI)—pivoted around windowing of the vibration datasets—that proved insensitive towards lower structural damage levels.

Considering an MDOF structure subjected to Gaussian broadband excitation, $F(t)$, the equation of motion can be expressed as:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(t) \quad (6.21)$$

where $\mathbf{X}(t)$ is the vector of displacement coordinates while \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping and stiffness matrices of the system, respectively. The solution to Eq. (6.21) can be written according to:

$$\mathbf{X}(n) = \mathbf{A}\mathbf{S}(n) \quad (6.22)$$

In the above equation, \mathbf{X} is the measurement matrix, the matrix of the corresponding modal coordinates is represented by \mathbf{S} and \mathbf{A} is the modal transformation matrix with n number of the total datapoints in the sample space. Furthermore, it has been observed that the columns of the mixing matrix \mathbf{A} are linearly independent that corresponds to the vibratory structural modes. SOBI evaluates two covariance matrices at time lags 0 ($\mathbf{R}_X(0)$) and p ($\mathbf{R}_X(p)$) respectively, and perform joint diagonalization to estimate the unknown mixing matrix, \mathbf{A} . Mathematically, one can write:

$$\left. \begin{aligned} \mathbf{R}_X(0) &= E[\mathbf{X}(n)\mathbf{X}(n)^T] = \mathbf{A}\mathbf{R}_S(0)\mathbf{A}^T \\ \mathbf{R}_X(p) &= E[\mathbf{X}(n)\mathbf{X}(n-p)^T] = \mathbf{A}\mathbf{R}_S(p)\mathbf{A}^T \end{aligned} \right\} \quad (6.23)$$

where $\mathbf{R}_S(p) = E[\mathbf{S}(n)\mathbf{S}(n-p)^T] = \mathbf{I}$. Typically, SOBI encompasses three key steps that are addressed sequentially: whitening orthogonalization and unitary transformation. As the data predominantly evolves from a zero-mean process, the first step towards whitening is carried out by diagonalizing $\mathbf{R}_X(0)$ as: $\mathbf{R}_X(0) = \mathbf{V}_X\boldsymbol{\lambda}_X\mathbf{V}_X^T$, with \mathbf{V}_X and $\boldsymbol{\lambda}_X$ being the principal components of eigenspace. The whitened signals are then obtained from the following equation with \mathbf{Q} idealized as the whitening matrix:

$$\bar{\mathbf{X}}(n) = \boldsymbol{\lambda}_X^{-1/2}\mathbf{V}_X^T\mathbf{X}(n) = \mathbf{Q}\mathbf{X}(n) \quad (6.24)$$

The correlation between the measured responses is removed using whitening, given by the relation: $\mathbf{R}_{\bar{X}}(0) = E[\bar{\mathbf{X}}(n)\bar{\mathbf{X}}(n)^T] = \mathbf{I}$, obtaining the following expressions:

$$\mathbf{R}_{\bar{X}}(p) = \frac{1}{N} \left[\sum_{n=1}^N \bar{\mathbf{X}}(n)\bar{\mathbf{X}}(n-p)^T \right] = \mathbf{Q}\mathbf{A}\mathbf{R}_S(p)\mathbf{A}^T\mathbf{Q}^T \quad (6.25)$$

From the above equation, it can be understood that both the whitened covariance matrix and the matrix product $\mathbf{Q}\mathbf{A}$ (a unitary matrix) can be numerically diagonalized. The EVD of the matrix $\bar{\mathbf{R}}_X(p)$ satisfies the relation, $\mathbf{V}_{\bar{X}}\mathbf{R}_{\bar{X}}(p)\mathbf{V}_{\bar{X}}^T = \boldsymbol{\lambda}_{\bar{X}}$. As the

diagonal matrix $\lambda_{\bar{X}}$ has distinct eigenvalues, the mixing matrix can be estimated, according to:

$$\hat{\mathbf{A}} = \mathbf{Q}^{-1} \mathbf{V}_{\bar{X}} = \mathbf{V}_X \lambda_X \mathbf{V}_X^T \quad (6.26)$$

SOBI carries out an approximate joint diagonalization approach based on Givens rotation technique to find the unitary matrix $\mathbf{Q}\mathbf{A}$ that diagonalizes the whitened covariance matrix $\bar{\mathbf{R}}_x(p)$ at one or several non-zero time lags. The problem then transforms into finding a minimum performance index \mathcal{J} such that the unitary diagonalization satisfies the relation $\mathbf{D} = \mathbf{V}^T \tilde{\mathbf{R}}_{\bar{X}}(p) \mathbf{V}$. Therefore:

$$\mathcal{J}(\mathbf{V}, p) = \sum_p \sum_{1 \leq i \neq j \leq n_s} \left\| \mathbf{D}_{ij}^p \right\|^2 \quad (6.27)$$

where, \mathbf{V} is the unitary matrix and also the joint approximate diagonalizer for all p -shifted covariance matrices $\tilde{\mathbf{R}}_{\bar{X}}(p)$. Once the mixing matrix is obtained, the estimated sources can now be evaluated, which was the primary objective of carrying out SOBI:

$$\hat{\mathbf{S}}(n) = \hat{\mathbf{A}}^{-1} \mathbf{X}(n) \quad (6.28)$$

The matrix $\hat{\mathbf{S}}(n)$ contains the modal responses from which the modal constituents (i.e., natural frequencies and damping ratios) can be obtained. Once the modal responses are obtained from SOBI, the instant and location of damage need to be identified. However, it is a well-understood fact that the computational efficiency of the entire process is significantly escalated due to the simultaneous algebraic operations that take place in windows and permutation ambiguity (Cichocki and Amari 2002). To overcome these impediments, comparatively recent trends in structural damage detection are explored such as singular spectrum analysis incorporated in a recursive first-order perturbative framework.

Structural damage manifests in the form of alteration of inherent system properties that form the implementation of BSS as a damage detection tool. Time series models with pre-selected model orders are used to characterize the sources that are obtained from the past observations and then the future measurement sources are accordingly predicted (Zang et al. 2004). The newer measurements contain the information about the damaged state of the structure and are compared against the baseline data pre-recorded from the pristine state of the system, which enables the estimation of damage adaptively.

6.5.4 Damage Indices Based on Extracted Features

PCA is a methodical dynamic exploratory analysis that yields uncorrelated data on an orthogonal basis derived from the physical dataset (De Boe and Golival 2003;

De Oliveira and Inman 2015; Gharibnezhad et al. 2015; Hot et al. 2012; Jolliffe 1986; Kerschen et al. 2005; Koh et al. 2005; Lee et al. 2005; Li et al. 2000; Lovera et al. 2000; Misra et al. 2002; Mujica et al. 2014; Nguyen and Golinval 2010; Richman 1986; Tipping and Bishop 1999; Yan et al. 2005a). In recent years, PCA has been explored for dimensionality reduction, to account for operational and environmental variability and so on (De Oliveira and Inman 2015; Gharibnezhad et al. 2015; Lee et al. 2005; Li et al. 2000; Nguyen and Golinval 2010; Yan et al. 2005b). PCA detects the presence and location of structural damage along with its extent from vibration-based outputs. PCA, however, requires a baseline model from the healthy structure and compares it with the monitored state to identify the presence of damage through statistical DIs such as T^2 , Q , I^2 , and ϕ ; instances of which, are replete in literature (Jolliffe 1986).

PCA, a method to orthogonally project data onto a lower-dimensional space, ensures that variance of the projected data is maximized. A lower-dimensional uncorrelated dataset reveals relevance which can be obtained using eigenvalue decomposition on the covariance matrix obtained from the physical responses. It is closely related to the method of Proper Orthogonal Decomposition (POD) (Feeny 2002; Feeny and Liang 2003; Han and Feeny 2003; Kappagantu and Feeny 2000a, 2000b; Kerschen and Golinval 2003). Consider a multivariate dataset X obtained from sensor responses. PCA seeks an orthogonal transformation of the form: $T = PX$ where T represents the space of transformed variables that are decorrelated in the orthogonal space. So, PCA can be thought of as a constrained optimization problem where the objective is to diagonalize TT^T subject to the constraint $PP^T = I$. Mathematically, it translates to defining an objective function O as under:

$$O = TT^T - \Lambda(PP^T - I) \quad (6.29)$$

The optimal transformation is obtained by setting the partial derivative of O to zero as follows:

$$\frac{\partial O}{\partial P^T} = 0 \Rightarrow (X^T X)P = \Lambda P \quad (6.30)$$

This decomposition produces the Principal Orthogonal Values (POVs) (the eigen value matrices) and Proper Orthogonal Matrices (POMs) (eigen vector matrices) (Feeny and Liang 2003; Kappagantu and Feeny 2000b; Kerschen et al. 2005). The POVs maps the relative significance of each POM in the response. The uncorrelated new set of variables produced are known as the principal orthogonal components (POCs) (Feeny and Liang 2003). Potentially, the uncorrelated newer set of variables have improved lower dimensional information that aids in faster structural damage detection process. As these projections are sometimes not enough by themselves, it is necessary to use certain statistical parameters that are considered as DIs. In this context, a few DIs and their utility are described next.

Q-statistic (SPE index) represents the variability of the data projection within the residual subspace. Considering the i -th row of the matrix \mathbf{E} , the *Q-statistic* for each experimental observation is defined as:

$$Q_i = e_i e_i^T = x_i (\mathbf{I} - \mathbf{P} \mathbf{P}^T) x_i^T \quad (6.31)$$

Hotelling's T^2 -statistic (D index) checks the variability of the projected data in the new space of the PCs using score matrix \mathbf{T} . The concept of Euclidean distance comes into effect by utilizing the covariance matrix \mathbf{C}_X as the normalization factor. For the i -th sample, the DI is expressed according to:

$$\begin{aligned} T_i^2 &= \sum_{j=1}^r \frac{t_{sij}^2}{\lambda_j} = t_{si} \mathbf{\Lambda}^{-1} t_{si}^T \\ &= x_i \mathbf{P} \mathbf{\Lambda}^{-1} \mathbf{P}^T x_i^T \end{aligned} \quad (6.32)$$

In the above equation, t_{si} is the i -th row vector of the matrix \mathbf{T} , defined as the projection of the experiment x_i onto the new space, related by the expression, $t_{si} = x_i \mathbf{P}$.

Combined index (ϕ index) is essentially a blend of the *Q*-index and the T^2 index for merging information into a single value. The mathematical expression defining this DI is given as:

$$\begin{aligned} \phi \text{ index} &= Q \text{ index} + T^2 \text{ index} \\ &= x^T \mathbf{M}_{\phi} x \\ &= x^T (\mathbf{I} - \mathbf{P} \mathbf{P}^T + \mathbf{P} \mathbf{\Lambda}^{-1} \mathbf{P}^T) x \end{aligned} \quad (6.33)$$

I index has its very roots in its utilization for clinical studies and mainly used for meta-analysis, accounting for a percentage of heterogeneity. The *I* index provides variation in study outcomes between experimental trials and can be mathematically defined as:

$$I \text{ index} = x^T \mathbf{M}_I x \quad (6.34)$$

6.5.5 Singular Spectrum Analysis (SSA)

An extension of the application of EigenValue Decomposition (EVD), (which is the crux of PCA) to elicit Principal Components (PCs), using the inputs from a single dataset, arranged in a Hankel structure, referred to as Singular Spectral Analysis

(SSA), is also gaining popularity in recent times (Carniel et al. 2006; Chao and Loh 2014; Elsner and Tsonis 1996; Groth and Ghil 2015; Hassani 2010; Hassani et al. 2011; Kilundu et al. 2011; Lakshmi et al. 2017; Liu et al. 2014; Murotani and Sugihara 2005).

In the strictest sense, prior assumptions regarding stationarity, normality or linearity of a feasible dataset can be alleviated with the use of SSA. As a non parametric method, the projection of time series into a smaller dimensional space ensures lagged signal components by itself. Most of the variance is retained, which explains the key signatures in the series (Carniel et al. 2006; Elsner and Tsonis 1996; Hassani 2010; Hassani et al. 2011). A significant point of difference between conventional PCA and contemporary SSA is that Singular Value Decomposition (SVD) is carried out using PCA (Bhowmik et al. 2019a; Chao and Loh 2014; Groth and Ghil 2015; Kilundu et al. 2011; Lakshmi et al. 2017; Murotani and Sugihara 2005).

The reconstruction process in SSA ensures that the principal components are adopted from the signal subspace. This automatically removes any noise components associated with the signal and acts as a filter for cleaning datasets.

The primary function of SSA is to decompose a time series into simpler components and then reconstructing the acquired principal components onto a new time series. Predominantly, the simpler components arise either from slowly varying trend, harmonic component and/or noise. While the slow trend corresponds to slow varying additive constituents, the harmonic series makes up for the periodic nature of the time series. The effects of noise are removed through an automatic filter. Any aperiodic series can be treated as the noise that does not affect the functioning of the reconstruction process. SSA proceeds by evolving a Hankel covariance matrix from the single set of sensor responses obtained through monitoring. A sliding window is incorporated to resolve the principal components in their order of significance. This window is usually selected to have a lesser size (in terms of samples) as compared to the actual length of time series. This forms the first step in embedding. Subsequent decomposition of the matrix into elementary matrices according to decreasing eigen significance is commonly termed as the SVD step of the method. The automatic noise removal process is attributed to the truncation of matrices to approximate the original matrix—in a method called ‘grouping’—that includes principal components only from the signal subspace. Finally, the reconstruction phase involves a diagonal averaging approach which mimics the original time series in terms of signal constituents, eliminating all the noise components at this stage.

6.5.6 First-Order Eigen Perturbation (FOEP) Technique

With the advent of statistical signal processing techniques aimed at extracting the key features of damage, methods for structural damage prognosis function mostly in batch mode operations. In this context, a class of mathematically consistent

algorithms has been developed that solely address damage detection in *real time*. Succinctly, FOEP is stated as the way of expressing the eigenstructure of the $(k + 1)^{th}$ step in terms of the eigenstructure at the k^{th} step as the $(k + 1)^{th}$ data streams in. This can be accomplished by structuring the EVD of the response covariance matrix in terms of the rank one perturbation of the ensuing eigenspace updates (Bhowmik et al. 2020a, 2020b; Bhowmik et al. 2019a, 2019b). The initial covariance estimate from the streaming data is first obtained. A key advantage to using FOEP lies in its inherent formulation where the eigenspace is updated at each instant of time instead of the covariance matrix, which contributes to the reduction of computational complexities. Tracking these eigenspace updates in real time with the use of certain pre-defined DSFs indicates structural damage in real time.

To explore FOEP, consider the eigenvalues of a perturbed matrix $\mathbf{C} + \Delta\mathbf{C}$ to be of the form $\mathbf{\Lambda} + \alpha\alpha^T$, i.e., the *rank-one update* of the matrix $\mathbf{\Lambda}$. Using the following definitions:

$$\begin{aligned} \mathbf{C}\mathbf{V} &= \mathbf{\Lambda}\mathbf{V} \\ (\mathbf{C} + \Delta\mathbf{C})(\mathbf{V} + \Delta\mathbf{V}) &= (\mathbf{V} + \Delta\mathbf{B})(\mathbf{\Lambda} + \Delta\mathbf{\Lambda}) \end{aligned} \quad (6.35)$$

where, $\Delta\mathbf{V}$ and $\Delta\mathbf{\Lambda}$ are the perturbation matrices. The EVD of the diagonally dominant term can be expanded as follows:

$$\mathbf{C} + \Delta\mathbf{C} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T + \mathbf{V}\Delta\mathbf{\Lambda}\mathbf{V}^T + \mathbf{V}\Delta\mathbf{\Lambda}\mathbf{V}^T + \Delta\mathbf{V}^T\mathbf{\Lambda}\mathbf{V}^T + O(\epsilon^3) \quad (6.36)$$

Recognizing that $\mathbf{C} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T$ and realizing the fact that $\mathbf{V}^T = \mathbf{I}$, the EVD of the perturbed matrix $\Delta\mathbf{C}$ (ignoring second-order perturbation terms) can be written as:

$$\Delta\mathbf{C} = \alpha\alpha^T = \mathbf{V}\Delta\mathbf{\Lambda}\mathbf{V}^T + \mathbf{V}\Delta\mathbf{\Lambda}\mathbf{V}^T + \Delta\mathbf{V}^T\mathbf{\Lambda}\mathbf{V}^T + O(\epsilon^3) \quad (6.37)$$

The above expressions provide an insight into the data-driven nature of the FOEP strategy. Recent research has demonstrated the uniqueness of the FOEP technique to be applied for multivariate datasets—ranging from dynamic system monitoring to online surveillance and real-time compressive sensing.

6.6 Time-Frequency Methods

In the aerospace field, most dynamic structures show time-varying behaviour due to modifications in mass and/or geometry (Ni et al. 2016; Senba and Furuya 2008). And conventional identification methods that viably opt for stationarity of signals are sometimes not suitable for robust needs. Both parametric and non-parametric approaches have been considered for modelling and analysis of non-stationary signals in the narrowband domain. Under parameterized techniques, methods such as TARMA are noteworthy (Poulimenos and Fassois 2006; Spiridonakos and

Fassois 2009; Spiridonakos and Fassois 2012). Cases of non-parametric approaches require the time-frequency representations (TFRs) of the signal that not only accommodates information from the time domain but also combines the retrospect from the frequency domain. This provides more intuition of modal parameters such as natural frequencies, mode shapes and damping ratios which can be subsequently—and effectively—be used for damage detection purposes (Fan and Qiao 2011). Recent literature has amalgamated a mix of traditional linear algebra and derivate sub-band coding which essentially forms the opposite sides of the same coin—a complete damage identification framework—using a variety of transforms based on TFRs.

Due to the growing interconnections between the fields of linear algebra and sub-band coding, which are seen as two parts of a single framework (Vetterli and Kovačević 1995), a substantial number of transforms leading to different TFRs have been recently proposed in the literature.

6.6.1 Scalogram and Spectrogram

Predominant linear transforms involving STFT (Gabor 1946) and wavelet (Daubechies 1992) are most effective in identifying the state of structural damage from its inherent parameters. In a first of its kind, the STFT utilizes a complex function in fixed windows whereas, the WT effectively employs a family of more flexible interpretations and algebraic functions. In particular, the Short-Time Fourier Transform (STFT) of the signal $x(t)$ is defined as:

$$STFT_x(\omega, \tau) = \int_{-\infty}^{\infty} w^*(t - \tau)x(t)e^{-j\omega t}dt \quad (6.38)$$

where $w^*(t - \tau)$ indicates the complex conjugate of a window function $w(t - \tau)$, usually selected as a Gaussian window. Shifting this window in time allows the time-frequency description of the energy of a signal. The spectrogram is the energy distribution associated with the STFT, that is:

$$S(f, \tau) = |STFT(\omega, \tau)|^2 \quad (6.39)$$

Conversely, the continuous wavelet transform (CWT) of the signal $x(t)$ is defined as follows:

$$CWT_x(a, b) = \frac{1}{\sqrt{a}} \int_{\mathbb{R}} \psi^*\left(\frac{t-b}{a}\right)x(t)dt \quad (6.40)$$

where the factor $\frac{1}{\sqrt{a}}$ is to conserve the norm and $\psi_{a, b}(t)$ is a function that is scaled and translated to expand the signal in the time-scale domain. The energy distribution associated with the CWT is called a scalogram and its definition is similar to the one

of the spectrogram. Such techniques have been used by Nagarajaiah and Basu (2009) to successfully identify modal parameters in output-only conditions (i.e. using only the structural response). Moreover, Kijewski and Kareem (2003) have shown the ability of time-frequency representations to track accurately time-varying modal parameters.

The Wigner–Ville distribution (WVD) (Boashash 2003; Cohen 1995) has been increasingly adopted based on its inherent TFR fundamentals. Additionally, the approach does not employ any windowing function except for its autocorrelation, as follows:

$$WD_x(\omega, \tau) = \int_{-\infty}^{\infty} x\left(\tau + \frac{t}{2}\right)x^*\left(\tau - \frac{t}{2}\right)e^{-j\omega t} dt \quad (6.41)$$

However, multi-component practical applications are more involved due to their inherent bilinear structure that automatically creates cross-terms. This has the potential to undermine distribution readability and suffer from problems of closely spaced modes (in addition to vanishing and crossing modes of interest). In an attempt to improve these methods, reassignment and synchro squeezing have shown to be quite effective in identifying the underlying features of interest in the TF domain (Auger et al. 2014). Highly localized distributions can be derived using such techniques on spectrograms and scalograms. Of particular interest is SST—based on scalograms obtained through CWT—as an effective substitute to the traditional (yet effective) EMD (Daubechies et al. 2011).

Approach to elicit signal constituents corresponding to various instantaneous frequencies (which are aptly termed as intrinsic mode functions or IMFs). Without imposing a strong reliance on any basis function (Huang et al. 1998), the method is generally clubbed together with the Hilbert transform (HT)—resulting in the Hilbert–Huang Transform (HHT)—to identify patterns in the TF domain called the Hilbert spectrum. Although mode mixing has been identified as a typical cause of concern for EMD, the Ensemble Empirical Mode Decomposition (EEMD) has demonstrated an improvement using a noise-assisted procedure (Wu and Huang 2009).

Recently, multivariate techniques based on the concept of modulated multivariate oscillations were introduced (Lilly and Olhede 2012; Omidvarnia et al. 2012). Multivariate extensions of the WVD (Stanković et al. 2018), SST (Ahrabian et al. 2015) and EMD (Rehman and Mandic 2010) were recently presented.

In general, non-parametric methods require post-processing of some sort for modal parameter extraction in the TF domain. This is converse to the working of the EMD that extracts information from TFRs, which usually involves the decomposition into separate modal responses—mimicking a single degree of freedom system (Avenidaño et al. 2018; Iatsenko et al. 2016). Ridge extraction was typically performed to this aim, finding the local maxima of distributions over time (Delprat et al. 1992; Rankine et al. 2007). The basic structure of the TFR can be used to identify the modal parameters after extraction of the ridges. However, the frequency peaks associated with the information pattern might correspond to issues regarding complete reconstruction of decoupled modal responses.

Moreover, these procedures may suffer from issues related to non-stationary amplitudes, narrowband disturbances and noise (Staszewski 1998). Wang et al. (2013) proposed a dynamic optimization strategy that directs the use of penalty function for noisy signals in contrast to the use of the SVD approached by other authors in the same domain (Le and Paultre 2013; Özkurt and Savaci 2005). Quqa et al. (2020) proposed a decentralized algorithm for near-real-time extraction of modal responses suitable for wireless smart sensing nodes, based on a filter bank that must be updated at the occurrence of particular situations. In a further study, a fully-adaptive procedure called Decomposition Algorithm based on Modal Assurance (DAMA) was presented (Quqa et al. 2021), less sensitive to noise and narrowband disturbances. Also, image processing techniques can be applied to TFRs to extract useful information. De-noising of TFRs was first investigated by Liu et al. (2004) and progressed to attenuating the cross-terms of WVD (Gómez et al. 2011). Recent strategies has incorporated separation of modal components (Zhang et al. 2013) associated with the TF plane of the energy peaks of different vibratory modes. The watershed transform—a morphology inherent segmentation method (Vincent et al. 1991)—uses scalograms to separate narrowband seismic waves (Roueff et al. 2004). A persisting problem of these methods lie in the difficulty of effectively separating closely spaced modes of interest.

6.7 Drawbacks and Limitations

Some of the key limitations associated with the use of vibration-based methods are summarized as under:

1. In many cases of damage detection, the change in natural frequencies to determine the instant of damage is masked by the varying operational and environmental conditions. For this reason, successful damage detection facilities are carried out in a controlled environment and not implemented as field problems. To alleviate this drawback, a statistical damage detection model using a pattern recognition technique is needed to distinguish damage-induced changes from environment-induced changes. Damage markers should correctly describe the embedding of the environmental variables into the algorithm, which remains a field of development of the response-based methods.
2. Most of the vibration-based methods are currently offline in nature, i.e., they require batches of data to compare against a recorded set of data obtained from a pristine state of the structure. To track the changes in the structure as and when the vibration data streams in real time, the aforementioned methods need to be implemented in an adaptive fashion that could identify damage at a particular instant of time, without using pre-recorded data.
3. A limitation in response-based methods is the detection of damage for cases with *progressive damage* because of the time-varying nature of the vibrating system. Damage involving changes of stiffness over a period of time, progressively, is

captured only through a recursive implementation since the data from a particular instant is taken into consideration and compared against the data obtained from the previous timestamp.

4. Almost all the methods discussed in the previous section utilize the data gathered from a large number of sensors. In practical scenarios, the number of sensors instrumented is mostly less than the total number of active DOF due to cost considerations and accessibility issues. This requires an immediate development of algorithms that detect the exact instant of damage for underdetermined cases (where the number of sensors is less than the number of DOF) as well, in a single framework, utilizing the principles of recursion in a perturbation approach.

There are also opportunities. Response-based methods can be improved by benefiting from the more recent development of signal processing techniques, for example, Complete Ensemble Empirical Mode Decomposition (CEEMD) and its improvements (Colominas et al. 2014; Torres et al. 2011), empirical wavelet transform (Gilles 2013) and variational mode decomposition (Dragomiretskiy and Zosso 2014). Instead of extracting features using specific signal processing techniques, an alternative that is receiving more attention recently is through machine learning, in particular deep learning. Deep learning, e.g. using Deep Convolutional Neural Networks, provides a powerful tool to extract features in a data-driven manner, which often outperforms hand-crafted features (Lin et al. 2017). There are however still many open problems, including effective network architectures and interpretability of the results.

6.8 Case Studies

This section contains numerical and laboratory case studies relevant to aerospace applications. Following a brief description of the benchmark models, the main results obtained using some of the aforementioned identification methods are reported, together with the references to the related published works.

6.8.1 *Vibration-Based Damage Detection in a Composite Plate by Means of Acceleration Responses*

The case study presented in References (Limongelli 2017; Limongelli 2019) was reported herein, in which the SIM is applied to detect damage on a lab specimen. The investigated structure is a 500 cm \times 500 cm glass fibre/vinylester composite plate with 1 cm thickness. Free-free boundary conditions were simulated by suspending the plate with a bungee cord. The person holding the plate in the figure is for demonstration purposes only. Damage due to an impact is simulated through the removal of an area of material with dimensions 1.5 \times 1.5 cm and 0.6 cm depth. The

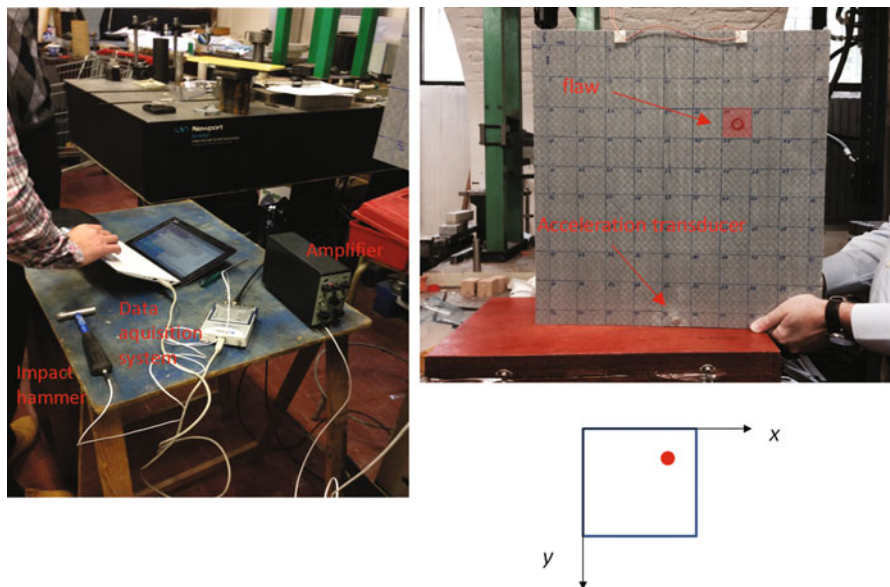


Fig. 6.3 Composite plate and flaw induced (Limongelli 2019)

plate was excited by an impulsive hammer at each point on the grid. The dynamic response of the plate in terms of acceleration is recorded in the reference and damaged configurations in all the nodes of a grid composed of 10×10 nodes. Figure 6.3 shows the location of damage and the layout of the grid. The Operational Deformed Shapes were determined as FRFs between each grid point and a reference one. The frequency range of the fundamental modes of the plate in the reference configuration is considered for the application of the SIM. Additional information on this experiment can be found in Limongelli (2017).

Figure 6.4 displays the results obtained by applying the SIM. The SIM can correctly localize damage since the damage index exhibits the highest values at the four nodes of the damaged elements. Some false alarm can be found at locations close to the flaw because of the interpolation process (Limongelli 2011). However, this phenomenon does not hamper the correct localization of damage since it influences locations close to the flaw only.

6.8.2 Numerical Comparison of Modal-Based Methods for Damage Detection

A numerical study has been carried out to compare the most popular modal-based methods. A laminated composite beam with length $l = 0.4$ m, thickness $h = 0.004$ m, width $b = 0.02$ m was considered. The following effective material properties were

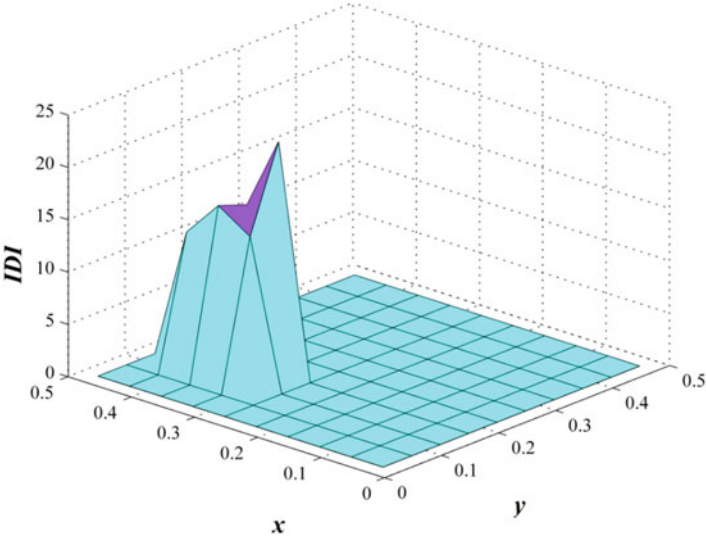


Fig. 6.4 Results for the damage induced in the composite plate (Limongelli 2019)

Table 6.1 Natural frequencies of the beam in the healthy and damaged conditions

Mode number	Frequencies Rad/s		Difference %
	Healthy	Damaged	
1	2.739284E+02	2.738425E+02	0.32
2	7.526039E+02	7.462084E+02	0.85
3	1.286927E+03	1.284473E+03	0.19
4	1.469070E+03	1.444934E+03	1.64
5	2.415473 +03	2.390013E+03	1.05

obtained and used in the calculations: Young modulus $E = 53.3 \cdot 10^9 \text{ N/m}^2$, density $\rho = 1866 \text{ kg/m}^3$, Poisson’s ratio $\nu = 0.4$. It was considered that due to delamination in a small part of the beam the elastic modulus is reduced to $E_d = 37.33 \cdot 10^9 \text{ N/m}^2$. The length of the damage is 3.75% of the total length of the beam.

Table 6.1 shows the computed first five natural frequencies of the beam. The computations were done by MSC NASTRAN software, discretizing the intact and damaged beams with 80 linear beam elements. The beam was modelled by the Timoshenko beam theory. It can be observed that the differences in the natural frequencies of healthy and damaged beams are very small—the first three ones are <1%. Such small damages cannot be identified experimentally on the base of the measured natural frequencies.

Figure 6.5 displays the most popular modal-based damage indices (based on modal displacement, modal slope, modal curvature, and strain energy) constructed for this beam. The location of the damage is highlighted in red (elements 17, 18 and 19). All considered damage indices have a maximum at the damage location. For modal shapes, modal slopes squares and even for the modal curvatures, some

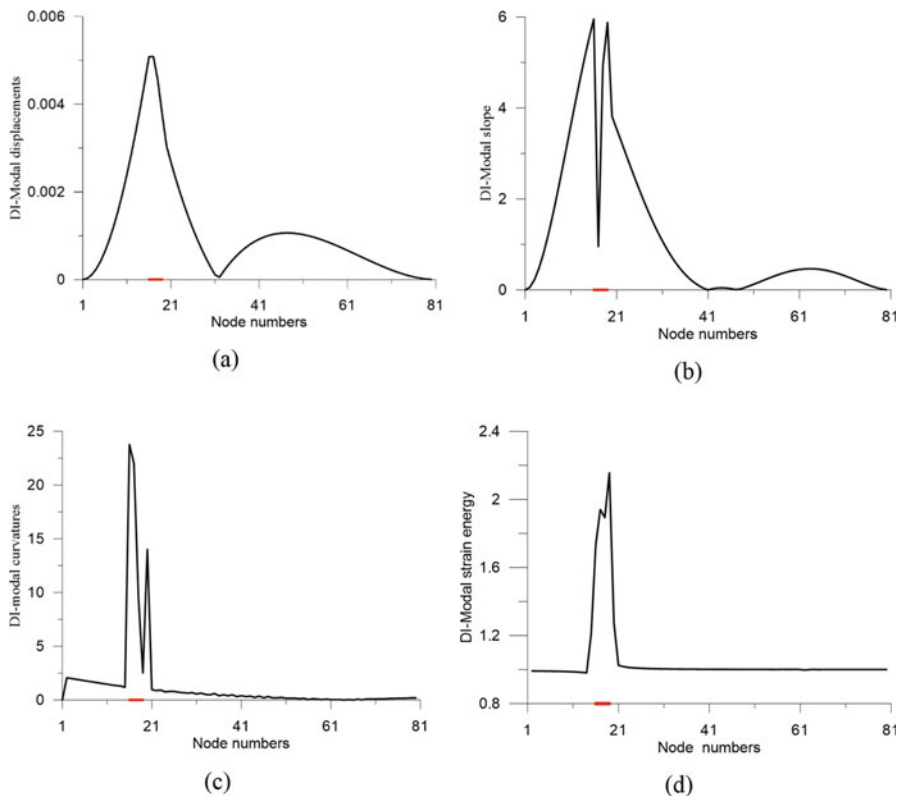


Fig. 6.5 Modal-based damage indexes for the damaged beam. (a) modal displacements DI; (b) modal slopes squares DI; (c) modal curvatures DI; (d) strain energy DI

additional peaks are observed in the curves of the damage indexes. This could confuse the potential user of these indexes, especially in the case of multiple damages. Moreover, the shapes of the peaks in the first two indexes are long-winded and could not be indicative of the extent of the damage. The best possibility to predict the damage are its location and extend of the damage index based on the modal strain energy. The considered indices are constructed on the base of the first normal mode.

6.8.3 Vibration-Based Monitoring of a Scaled Wind Turbine Blade by Means of Acceleration and Strain Responses

This case study reports the analyses presented in Jaksic et al. (2016a) where a 1.4 m-long polypropylene wind turbine blade (1.7 kg) is monitored with strain gauges, accelerometer (Microstrain G-Link) and a laser Doppler vibrometer (Polytec

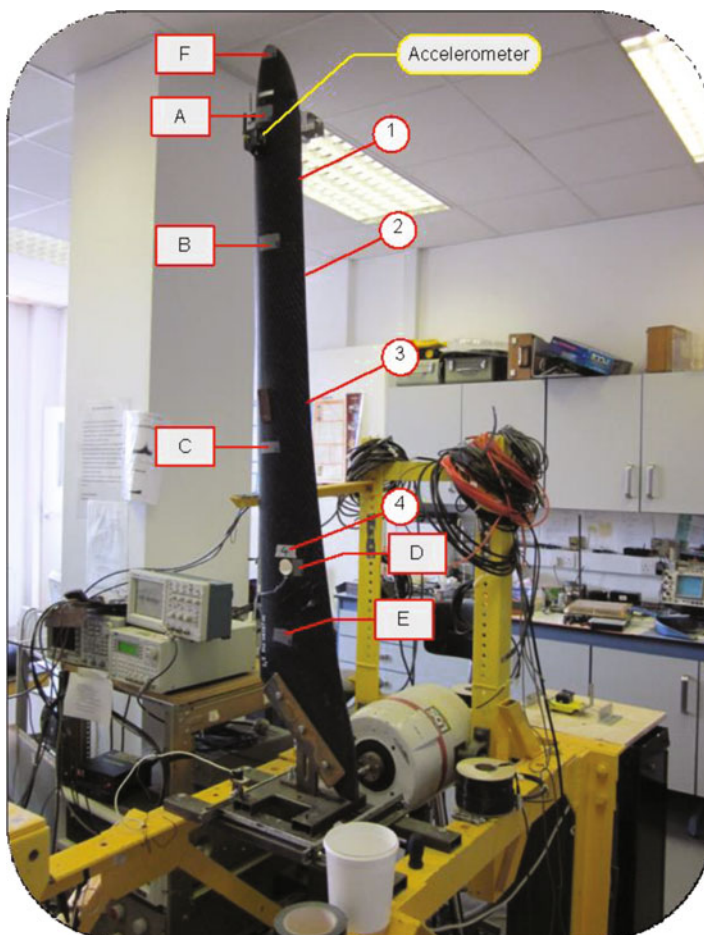


Fig. 6.6 Monitoring of a wind turbine blade in a laboratory setup using strain gauges, accelerometer and laser Doppler vibrometer (adapted from Jaksic et al. 2016b)

RSV-150), as shown in Fig. 6.6. The blade was excited through an electrodynamic shaker, clamped at base, through harmonic resonance (2 Hz–7 Hz), sine sweep (3 Hz–5 Hz) and white noise in keeping with the natural frequency of the blade. Examples of time-domain responses from these sensors are provided in Fig. 6.7a–c.

A delay vector variance analysis (Jaksic et al. 2016a) is employed as a marker for monitoring this blade under various excitations and compare signal nonlinearities of the responses with respect to a Gaussian benchmark with a time lag equal to unity and the embedding parameter set to 3, for computational efficiency, convenience and comparability to other results. Figure 6.8 presents an example of how such

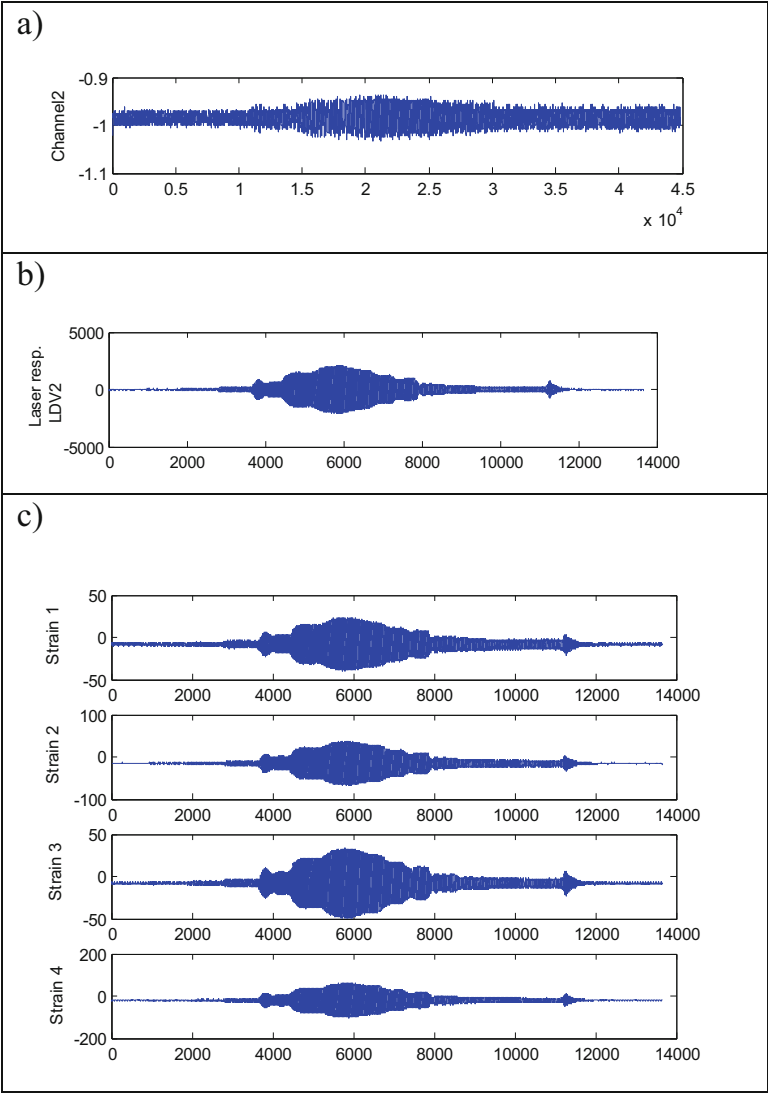


Fig. 6.7 Dynamic responses of a scaled wind turbine blade using from (a) accelerometer, (b) laser Doppler vibrometer and (c) strain gauges.

monitoring can effectively consider several sensors together for various types of excitations. Similar monitoring is also possible for floating wind turbine platforms (Jaksic et al. 2015a), even with control (Jaksic et al. 2015b) and possibly with damage (O'Donnell et al. 2020).

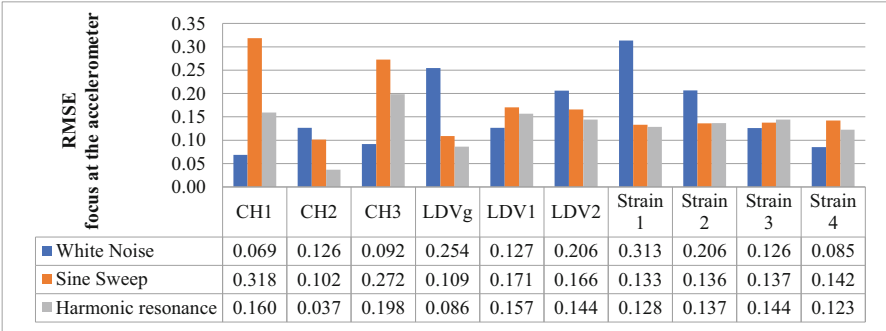


Fig. 6.8 A delay vector variance–based marker driven comparative analysis of a scaled wind turbine blade responses using various sensors and excitation techniques.

6.9 Conclusions

In this chapter, several methods for identification of losses of stiffness using the response to vibrations are presented. The merits, advantages and the drawbacks and limitations of the presented approaches were discussed. These are methods that enable to detect and localize damage using global parameters of the structure and without requiring any prior knowledge about its location. The limitation that descends from the global nature of the damage features is the lower sensitivity to small damages of these approaches with respect to local non destructive damage detection methods. Response-based methods are more feasible for online real-time damage identification purposes with respect to model-based approaches thanks to the lower computational efforts they require. However, the results provided by such methods may prove to be more affected by environmental and operational factors that have to be carefully accounted for in the analysis of the damage features.

When this is done, response-based methods can be reliably applied to detect and localize damage whereas the assessment of type and severity of damage usually requires the development of a calibrated numerical model.

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