Modeling Symmetric Minimal Surfaces by Mesh Subdivision

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Abstract. Thanks to the great diffusion of additive manufacturing technologies, the interest in lattice structures is growing. Among them, minimal surfaces are characterized by zero mean curvature, allowing enhanced properties such as mechanical response and fluidynamic behavior. Recent works showed a method for geometric modeling triply periodic minimal surfaces (TPMS) based on subdivision surface. In this paper, the deviation between the subdivided TPMS and the implicit defined ones is investigated together with mechanical properties computed by numerical methods. As a result, a model of mechanical properties as a function of the TPMS thickness and relative density is proposed.

Keywords: Lattice structures · Additive manufacturing · Triply periodic minimal surfaces · Design for additive manufacturing

1 Introduction

The spreading of additive manufacturing (AM) technologies makes it possible to produce parts with unprecedented complexity, such as biomimicry products, organic shapes and lightweight components. Nevertheless, anisotropic materials, surface finish, costs and security are just some of the open issues [1]. Among the lightweight structures, cellular solids or lattice structures are characterized by superior properties: they have high specific stiffness and strength, and they are good heat exchangers, energy absorbers and acoustic insulators [2, 3, 4]. Lattice structures, indeed, can find application from microscale to macroscale, from tissue engineering [5], to construction [6]. Minimal surfaces are a kind of lattice structures defined as surfaces with zero mean curvature or surfaces that minimize the surface area for given boundary conditions [7]. Minimal surfaces repeating themselves in three dimensions are called triply periodic minimal surfaces (TPMS) [8]. These surfaces have received huge attention in the research community due to their intrinsic properties [9], especially in the biomedical field, where it is mandatory to use porous scaffolds designed to allow fluid exchange and tissue regrowth [10, 11]. Furthermore, due to their curvature continuity, the stress concentration at nodal points is eliminated and fatigue life is improved compared to beam-like lattice structures [12].

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TPMS can be modeled adopting different methods: boundary representation (BRep), volume representation (VRep) and constructive solid geometry (CSG) [2, 13]. As emerged in recent studies, current mesh or parametric models are not ideal for modeling lattice structures, due to the significant computational resources required, processing times, robustness, rendering and visualization issues [14, 15]. To overcome these limitations, a novel method for geometric modeling variable thickness triply periodic surfaces based on a subdivision surface algorithm has been recently proposed [13]. Nevertheless, the geometrical accuracy of the modeling method and the mechanical characterization of the thickened structure still need to be investigated.

As an extension of the aforementioned work, in this study, the geometrical deviation of the subdivided TPMS P-Surface from the implicit defined one is investigated, then the mechanical properties of the unit cell are computed by finite element analyses. As a result, a model of mechanical properties as a function of the cell relative density is proposed and compared to the ones available in literature. The findings show the accuracy of the proposed modeling method; more, the numerical model allows to relate the thickness of the part to the mechanical properties by simulating a single cell inside the lattice structure, saving computational time, and giving directions for tailored applications with lattices that present variable properties in the design volume. Furthermore, knowing the properties of a single cell is a key feature when integrating topology optimization in the design workflow and also for applying the homogenization method [16] during the mechanical characterization of lattice structures.

2 Methods

A 1 mm unit cell of P-Surface type was modeled as described in [13]. A coarse mesh was used to model the unit cell of a P-surface; the Catmull-Clark subdivision surface algorithm was then adopted to achieve a smooth geometric model; finally, a thickness is assigned adopting a differential offset algorithm.

The deviation of the subdivided mesh at the third iteration of the Catmull-Clark subdivision scheme from the reference minimal surface was then computed in Rhinoceros 6 by "Mesh-Mesh Deviation" tool from "Rhino Open Projects" [17, 18]. The reference minimal surface was built in Netfabb introducing the minimal surface implicit equation in the mathematical part library (accuracy at 0.05 mm).

The finite element analysis software ANSYS R18.1 was then used to obtain the mechanical properties of the TPMS cells: Young's modulus (E), Poisson's ratio (ν), and Shear modulus (G). The used material was a titanium alloy, Ti6Al4V ELI (Grade 23), with the following bulk properties: $E_0 = 113800$ MPa, $\nu_0 = 0.342$, $G_0 = 42400$ MPa [19]. The stl file of the cell after three iterations of the subdivision algorithm was imported in the software and a second order tetrahedral elements mesh was used. Then, a set of constraints was imposed to the single cell. Even if a single cell is studied, the boundary conditions have been imposed in order to simulate an entire lattice structure, so the

cell must deform accordingly. As Fig. 1 shows, for E and ν determination, a 0.05 mm displacement along Y axis is imposed to the upper face, while X and Z directions are free; the bottom face is fixed along Y axis (i.e., a 0 mm displacement is imposed), and X and Z direction displacements are free. Remote points are used to control the degrees of freedom of the cell faces; this technique allows to relate a point (the remote point) to a node, a face, or to the entire body, and to control the behavior of the connected part by directly imposing loads and/or constraints to the remote point. A remote point with a deformable behavior, i.e. the geometry is free to deform, is assigned to the bottom face; remote points are also connected to the lateral faces of the unit cell, along the normal direction of each face, with a coupled behavior. To take into account the presence of the adjacent cells, coupling equations are assigned to the nodes of opposite faces [16], so each node of a face moves with the same displacement of the opposite node. To obtain E and ν, the reaction force of the bottom face and the contraction of lateral faces are evaluated. Similarly, another set is imposed to the P-Surface cell to determine G. A displacement along the X direction is imposed to the upper face, and reaction force of the bottom face are used to calculate G; the remote displacement and coupling equations of opposite faces are also assigned to ensure a displacement compatible with adjacent cells. The simulations were run four times, targeting four different thickness of the P-Surface cell, 0.1 mm, 0.2 mm, 0.3 mm and 0.4 mm, corresponding to a relative density of 0.23, 0.45, 0.64, 0.80, respectively.

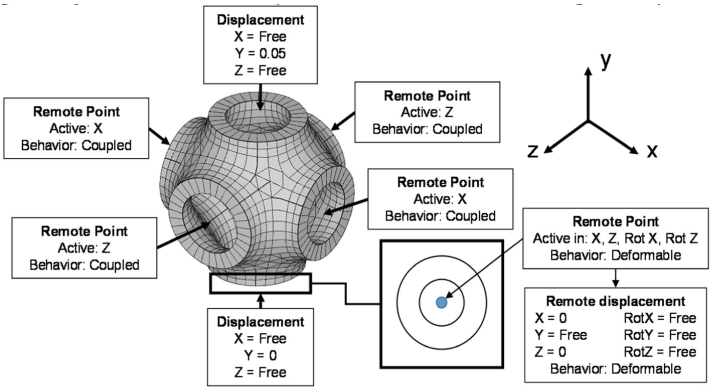

Fig. 1. Set of constraints for E and ν determination.

3 Results

Figure 2 shows the deviation map between the subdivided and the implicit minimal surface on a 1 mm unit cell. Subdivided minimal surfaces slightly differ from the ones defined by implicit equations, less than 1.3% of the cell dimension.

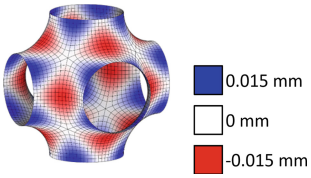

Fig. 2. Deviation map between subdivided and implicit minimal surfaces on a 1 mm P-surface unit cella. Max range: ±0.0127 mm.

Figure 3 shows the results for the elastic modulus E. The ratio between the obtained modulus E and the modulus of the bulk material E_0 is plotted against relative density (ρ). A power law well fits the results (Standard Deviation = 0.0146). The experimental data are also described by a quadratic equation so, as a first approximation, the model can be simplified (Standard Deviation = 0.0301).

In the same graph, a comparison with the results by Bobbert et al. [9] and Lee et al. [20] is presented. Lee's results are close to the ones obtained in this study; the trend found by Bobbert et al., instead, slightly differs but their results were experimentally obtained from compression tests, while the ones of the present study refer to numerical analyses of tensile tests. The proposed numerical model also extends in a wider relative density range, 0.2–0.8, if compared to Bobbert's, 0.3–0.5, and Lee's, 0.02–0.3. Graphs in Fig. 4 show the results for the determination of ν and G.

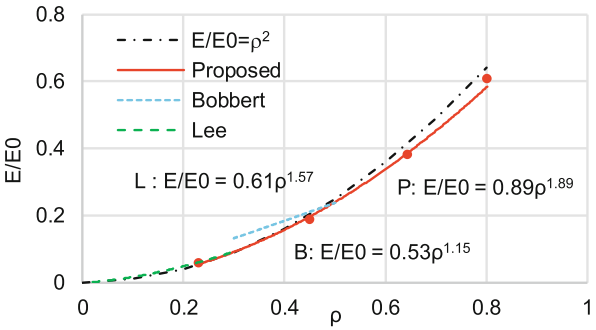

Fig. 3. Elastic modulus simulation results for P-Surface cell, and comparison with the literature.

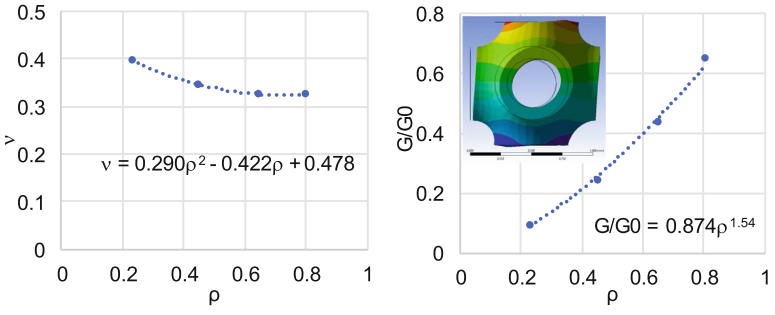

Fig. 4. Poisson's ratio (left) and shear modulus (right) simulation results for P-Surface cell. The figure in the shear plot highlights the effects of the deformable and coupled conditions.

4 Conclusions

In this work, a TPMS P-Surface modeled with a subdivision surface method was geometrically and mechanically characterized, and a model of the mechanical properties as a function of the relative density was obtained. The numerical method that was used only requires a single unit cell resulting in accurate solutions, and reducing computational time, since computational time and cost increase cubically as the number of cells increases.

The results show that Young's and shear modulus increase with relative density. The results for Young's modulus, if compared with data available in the literature, present a good agreement and extend for a wider relative density range.

Mechanical characterization of subdivided TPMS opens new possibilities for the implementation of the topology optimization in the modeling workflow of lattices with variable properties and allows to simulate this type of structures using the homogenization method, reducing time and computational costs.

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