# Chapter 7
# Our Enduring Legacy from Otto Stern

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**Abstract** Otto Stern's scientific legacy continues to animate discoveries on a rapidly advancing research frontier.

## 1 Introduction

Otto Stern's scientific legacy was commemorated and celebrated in 1988 at the centenary of his birth by a Festschrift [1]. In the three decades since then, scholarship has enriched our understanding of Stern's achievements. (See Chap. 5 of this volume). The goal of this essay is to show *how* Stern's legacy has grown through his links with new generations of scientists in Atomic, Molecular, and Optical (AMO) Physics. To keep the discussion tractable, it focuses on AMO's Nobel Laureates.

## 2 Preface: A View of Otto Stern's Legacy in 1988

For the centenary Festschrift, Norman F. Ramsey provided an overview of Stern's legacy. Ramsey was well-positioned to appreciate that legacy because he had worked with I. I. Rabi, since joining his group as a new graduate student in 1936. Rabi's career at Columbia had been launched by Stern in 1930. Stern was still active in Pittsburgh and Rabi spoke of him often. Ramsey became a leading figure in the world of physics, particularly molecular beam physics, and he was able to recognize Stern's achievements from first-hand knowledge.

In the decades since the Stern centenary, the field of Atomic, Molecular, and Optical Physics was transformed by advances that nobody could have predicted. Nevertheless, Ramsey's overview at the time of the Centennial provides a panoramic summary of Stern's impact on science at the dawn of this revolution in AMO physics. Here it is:

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*MOLECULAR BEAMS, OUR LEGACY FROM OTTO STERN* [2]

*by Norman F. Ramsey*

1. *Velocity distribution of molecules—Stern and others.*
2. *Space quantization—Stern and Gerlach.*
3. *Spin of electron = 1/2—Stem and Gerlach.*
4. *Anomalous magnetic moment of the proton—Frisch and Stern.*
5. *Nuclear spin measurements—Rabi and others.*
6. *Nuclear magnetic moments (stable and radioactive)—Rabi, Nierenberg and others.*
7. *Deuteron quadrupole moment and nucleon tensor force—Kellogg, Ramsey, Rabi and Zacharias.*
8. *Molecular beam electric and magnetic resonance methods—Rabi and Ramsey.*
9. *Anomalous magnetic moment of the neutron—Bloch and Alvarez.*
10. *Lamb shift in hydrogen hyperfine structure and quantum electrodynamics—Lamb and Retherford.*
11. *Anomalous H hyperfine structure and relativistic quantum electrodynamics—Nafe and Nelson.*
12. *Anomalous magnetic moment of the electron—Kusch.*
13. *Nuclear octupole moments—Zacharias and others.*
14. *First masers—Gordon, Zeiger and Townes.*
15. *Cs atomic clocks—Rabi, Zacharias, Essen, Ramsey and others.*
16. *Atomic hydrogen maser—Ramsey and Kleppner.*
17. *Accurate H, D and T hyperfine structure—Ramsey, Kleppner, Crampton and others.*
18. *Accurate atomic magnetic moments of H and D and reduced mass corrections—Ramsey, Valberg and Larson.*
19. *Rotational Magnetic moments of molecules—Stern, Ramsey and others.*
20. *Nuclear data—Magnetic moments, quadrupole moments and octupole moments.*
21. *Molecular data—Rotational moments, spin- rotational interaction, spin-spin interactions, quadrupole moment of molecules, orientation dependence of susceptibilities,* etc.
22. *Atomic scattering cross sections.*
23. *Reaction cross sections.*
24. *Van der Waals molecules.*
25. *Highly excited and Rydberg atoms.*
26. *Multiphoton atomic beam spectroscopy.*
27. *Jet sources and cluster beams.*
28. *Laser spectroscopy, excitation and detection of molecular beams.*
29. *Chemistry in detail—State to state reaction studies.*
30. *Measurement of parity non-conservation.*
31. *Laser cooling and trapping.*
32. *Tests of time reversal symmetry.*

## 3 Portraying Our Enduring Legacy Today

The “Our” in “Our Enduring Legacy” are members of the scientific community that is rooted in the work of Otto Stern. In the United States and abroad the community is known as Atomic, Molecular, and Optical Physics (AMO Physics). Stern’s seminal research launched AMO Physics and his legacy continues to nourish it.

Portraying Stern’s influence is a formidable challenge. In 1967 Rabi’s influence was summarized by the Rabi Tree, an illustration of all the researchers influenced by Rabi [3]. At the roots of the Rabi Tree, prominently displayed, is the name *Otto Stern.* Considering the explosive growth of atomic physics in recent decades, a Stern Tree, with its additional trunks in chemistry and nuclear physics, would be intractable.

We shall summarize Stern’s influence by showing connections from his work to the Nobel Prize laureates in AMO physics. The Nobel Prize is generally agreed to honor important advances, although focusing on it neglects important achievements that did not happen to have been awarded the Prize. Nevertheless, many of the laureates are linked to Stern by student-teacher or colleague-colleague experiences. It is difficult to think of stronger evidence for the value of Stern’s legacy to science.

The achievements of the Nobel Laureates are well-documented elsewhere and will not be stressed here. The comments below focus on the laureates’ personal links to Stern, or to Stern/Rabi. In large part, it is by such links that Stern’s legacy thrives. A few cases in which no link is evident will be noted.

*The origins of the legacy.*

As described in Chap. 5 of this volume, Stern’s scientific productivity was extraordinary. At the time that his work was interrupted by political interference in 1933, Stern’s achievements included:

- The first measurements of molecular speeds
- The Stern-Gerlach experiment demonstrating spatial quantization
- The demonstration of recoil from absorption of a photon
- The first demonstration of atom diffraction
- Discovery of the anomalous magnetic moment of the proton
- Discovery of the magnetic moment of the neutron

This sequence of discoveries reveals Stern’s uncanny ability to identify important problems. In addition, he obviously had great stamina and powers of concentration, what in German is called *Sitzfleisch.* According to his student and colleague Otto Frisch [4], Stern also had great talent for enjoying life. (See also the personal testimonials by his niece and great-nephew in Chaps. 3 and 4 of this volume.)

Stern’s first position was as assistant to Einstein in Prague and Zurich from 1912 to 1914, see also Chap. 6. In his biographical memoir of Otto Stern, Emilio Segrè wrote [5]: “*It was from Einstein that he learned what were the really important problems of contemporary physics: the nature of the quantum of light with its double aspect of particle and wave, the nature of atoms, and relativity*.” Stern and Einstein remained scientific friends long after their careers parted.

Rabi was ever mindful of Stern's legacy and he preserved it for the coming generations. From time to time Rabi's friends—including this author—would hear him talk of it. In paying tribute to Wolfgang Pauli and Otto Stern, Rabi wrote [6]: "*From Stern and Pauli I learned what physics should be. For me it was not a matter of more knowledge. I learned a lot of physics as a graduate student. Rather it was the development of taste and insight; it was the development of standards to guide research, a feeling for what was good and what is not so good. Stern had this quality of taste in physics and he had it to the highest degree. As far as I knew, Stern never devoted himself to a minor question.*"

In 1933 Stern's research was terminated by the Nazi regime. He was forced to abandon his University and his culture and flee his country. He was appointed as a professor at Carnegie Institute of Technology where he worked with Immanuel Estermann, one of his first Ph.D. students and a life-long collaborator. Stern's highly productive years had come to an end but his legacy continued to grow, nourished in part by his post-doctoral associate at Hamburg, I. I. Rabi.

Rabi met Stern when he visited the University of Hamburg for over a year in 1927–1929. He owed his research career to that time with Stern. He had planned to work with Wolfgang Pauli but happened to meet Stern and became interested in Stern's work. Rabi suggested an approach to magnetic deflection that avoided the magnetic gradients which bedeviled the Stern-Gerlach experiment. Stern invited the would-be theorist to try out his idea in the laboratory. It worked! Rabi was elated and his career abruptly headed in a new direction. He was appointed to a junior position at Columbia University and in 1930 he started a research program there.

From today's perspective, Stern and Rabi seem to be almost a single force. Rabi started building his research program at Columbia less than two years after his visit with Stern. Three years later Stern had to flee to his country. Stern's mainline research never recovered its momentum but as Stern's program was slowing, Rabi's was gathering speed. Rabi's research centered on the spins and magnetic moments of nuclei, atoms and molecules and on fundamental issues in the quantum properties of atoms—as had Stern's. Rabi's creative style, his experimental designs and his sense of scientific fitness were all evocative of Stern's.

Rabi made his major discovery—molecular beam magnetic resonance—in 1937 [7]. A second major discovery, the quadrupole moment of the deuteron, followed within two years [8]. Then, in 1940, war in Europe disrupted the research. In November, Rabi left Columbia to become the scientific director of the program to develop radar at the MIT Radiation Laboratory (the Rad Lab), bringing to a close a decade of innovative physics.

Stern and Rabi are viewed collectively here because their research created a continuous narrative. Because of Stern's influence on Rabi, and Rabi's deep appreciation for Stern's teachings and innovations, their legacies in AMO physics often meld.

## 4 The Nobel Prizes of Stern and Rabi

A natural place to launch this narrative is with the Nobel Prize awards to Stern and Rabi themselves.

**1943 Nobel Prize**: **Otto Stern** "*for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton.*"

Otto Stern received the 1943 Nobel Prize and the following year the Nobel Prize was awarded to Rabi. The Prizes were awarded in New York City in 1944 at the same ceremony. As described in Chap. 5, Stern received an overwhelming number of Nobel Prize nominations—more than any previous winner. Considering his many significant scientific achievements, this is hardly surprising. Curiously, the Stern-Gerlach experiment is not mentioned in the citation. During Stern's career a revolution in physics was underway and one senses that the Nobel Committee was overwhelmed by the barrage of epochal discoveries and the confusions of the onslaught of perplexing new knowledge.

**1944 Nobel Prize: Isidor Isaac Rabi** "*for his resonance method for recording the magnetic properties of atomic nuclei.*"

Isidor I. Rabi's Nobel Prize was awarded quickly following his discovery of molecular beam magnetic resonance in 1937. At that time one could point to two significant achievements: magnetic resonance and discovery of the deuteron quadrupole moment. The Prize Committee was prescient to realize the vast potential of magnetic resonance. Rabi's discovery of molecular beam magnetic resonance led to the creation of powerful new tools for atomic, molecular, and nuclear physics and his ideas diffused into adjacent fields. Rabi's enormous impact on science has been well documented [9]. He emerged from the war years as a statesman of science and his career as a statesman was extraordinary.

In the decade following the war Rabi conceived and led the creation of Brookhaven National Laboratory (with some help from Norman Ramsey [10]) and sparked the creation of CERN [11]. He also sparked the creation of the President's Science Advisory Committee (PSAC) which was influential for several presidencies, and he steered the U.S. national policy to keep nuclear technology under civilian control and he spent considerable effort trying to achieve international control.

Rabi's career as a statesman left him little time for basic research. After the war his single paper on fundamental physics was on the hyperfine structure of hydrogen (discussed below). Nevertheless, Rabi's impact on the Columbia Physics Department was enormous. The judgments about people and physics that he exercised while leading the Physics Department led to the discoveries of the Lamb Shift, the anomalous magnetic moment of the electron, the creation of the theory of relativistic quantum electrodynamics (QED) and the creation of the laser, and garnered seven Nobel prizes for work carried out at Columbia [12].

## 5 Links Connecting the AMO Nobel Laureates to Otto Stern

**1952 Nobel Prize: Felix Bloch and Edward Mills Purcell** "*for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith.*"

The invention of nuclear magnetic resonance was an early spin-off from Stern's measurement of the magnetic moment of the proton and Rabi's invention of the magnetic resonance technique.

Felix Bloch had a distinguished early career, having studied with Peter Debye at ETH in Zurich and Heisenberg in Leipzig. His career was interrupted when he had to flee from Germany in 1933. He was appointed to the faculty at Stanford University and became interested in Stern's discovery of the neutron magnetic moment. He designed a neutron beam magnetic resonance experiment which employed spin polarizers and analyzers that used magnetic scattering rather than Stern-Gerlach magnets. Rabi was visiting Stanford and the two worked together on preliminary version of the experiment. Its results were published in 1940 [13]. In the course of this he became interested in using magnetic resonance to measure magnetic fields. The proton magnetic moment was then known and Bloch realized that a sample of protons—for instance in water—would have magnetic susceptibility and that in a magnetic field there would be a significant difference in the spin-up and spin-down populations. A radio frequency field would cause transitions, creating a rotating polarization that would induce a current in a conducting loop. The frequency that induced the current would reveal the value of the field.

Edward Mills Purcell, then an assistant professor at Harvard, spent the war years at the MIT Radiation Laboratory (the Rad Lab) working on radar. Rabi was the scientific director of the Rad Lab and they interacted frequently. At the end of the war, Rabi asked Purcell and other experts to stay on to document their work. It was during this period that Purcell, working in the evenings with borrowed equipment, demonstrated Nuclear Magnetic Resonance. The magnetic moment of the proton was well known and Purcell realized that a mass of the protons would exhibit bulk magnetization that would absorb power at the resonance frequency. He detected absorption almost simultaneously with Bloch's discovery and they announced their discoveries in side-by-side abstracts at the 1946 spring meeting of the American Physical Society (APS), the first post-war meeting [14, 15].

At that meeting they realized that Bloch's Nuclear Induction and Purcell's Nuclear Magnetic Resonance were essentially identical. They had amiable personalities and in discussing their findings they agreed that it would be best to have a single name. They agreed on *Nuclear Magnetic Resonance* and the acronym NMR entered the lexicon of science.

**1955 Nobel Prize: Polycarp Kusch** "*for his precision determination of the magnetic moment of the electron*", and **Willis Lamb** "*for his discoveries concerning the fine structure of the hydrogen spectrum.*"

Suspicions that there were flaws in the Dirac theory of the electron inspired a series of AMO experiments at Columbia University in the late 1940s, where Rabi chaired the Physics Department. The results led to the creation of the theory of relativistic quantum electrodynamics (QED) shortly after the war—widely regarded as a triumph for Physics.

Three experiments were pursued in this quest.

1. The anomalous moment of the electron: Kusch and Foley [16]
   The magnetic moment of the electron was believed to be exactly one Bohr magneton. Detection of an anomaly—a departure from unity—would pose a fundamental problem in the theory. The Kusch-Foley experiment discovered such an anomaly. Polycarp Kusch had joined Rabi's group in 1937 and Rabi later appointed him to the Columbia faculty. With Henry M. Foley, Kusch carried out atomic beam magnetic resonance on three different atoms that had the same total angular momentum but different combinations of spin and orbital angular momentum. By studying radiofrequency resonances in a fixed magnetic field, they discovered a small anomaly and measured it to a precision of about 4%.

2. The Lamb Shift: Willis Lamb [17]
   According to the Dirac theory the energy levels in hydrogen with the same total angular momentum have the same energy. Willis E. Lamb showed that the energies of $2S_{1/2}$ and $2P_{1/2}$ states were not identical. The energy difference is known as the Lamb shift.
   In 1947 Rabi attracted Willis E. Lamb to the Columbia Physics Department from Berkeley where he had been working with Robert Oppenheimer. Somewhat to Rabi's surprise, Lamb capitalized on his experience developing microwave technology during the war and designed and executed an experiment. Using an atomic beam of metastable hydrogen atoms in the $2S_{1/2}$, and working with graduate student James Retherford, he observed the transition: $2S_{1/2} \rightarrow 2P_{1/2}$.

3. The Hyperfine Energy of Hydrogen: Rabi et al. [18]. The hyperfine energy of hydrogen depends on the product of the magnetic moments of the electron, proton, as well as other accurately known factors. A precision measurement of the hyperfine transition frequency would provide an independent value for the magnetic moment of the electron.
   Although the measurement did not add significant new knowledge, it had an important impact—it convinced Julian Schwinger to become engaged with the problem. Schwinger had been an undergraduate prodigy when Rabi brought him to Columbia before the war and mentored him. Schwinger received his undergraduate degree and also completed his Ph.D. thesis at the age of 19. He worked with Oppenheimer at Berkeley before the war and at the Rad Lab during the war. Schwinger then joined the faculty at Harvard where he returned to the problem of the anomalous magnetic moment of the electron. In 1965 he shared

the Nobel Prize in Physics with Richard P. Feynman and Sin-Itoro Tomanaga for creating Relativistic Quantum Electrodynamics to account for the values of the electron moment anomaly and the Lamb shift.

**The 1964 Nobel Prize: Charles Hard Townes, Nikolay Basov and Alexander Prokhorov** "*for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle.*"

The maser preceded the laser and provided the foundation for its invention. The invention of the laser advanced essentially every branch of science and it transformed society. The maser was a new type of molecular beam resonance device.

In 1947 Rabi persuaded Charles Townes to join Columbia's physics faculty. Townes received his Ph.D. from California Institute of Technology in 1939 and joined the staff at Bell Laboratories. Townes appointed Arthur Schawlow to his postdoctoral staff and they co-authored the magisterial monograph *Microwave Spectroscopy* [28]. Townes had a particular interest in detecting the ammonia molecule by microwave spectroscopy of its inversion line, about 23 GHz. He employed a molecular beam with an electrostatic state separator, essentially the first half of a molecular beam resonance apparatus. He conceived the idea of observing a resonance transition by detecting the energy the molecules radiated as they passed through a resonator tuned to the molecular resonance. The operation of the maser was reported in 1955 [19].

N. G. Basov and A. M. Prokhorov also published a proposal for a similar device although few details are available [20].

As an amplifier, the maser found applications in radio astronomy and it inspired Ramsey's creation of the hydrogen maser, a device employed in frequency control laboratories and in GPS systems. The biggest impact of the maser is that it inspired Townes and Schawlow to propose a maser that could operate at optical frequency—the laser [21].

**Nobel Prize 1966: Alfred *Kastler*** "*for the discovery and development of optical methods for studying Hertzian resonances in atoms.*"

The invention of optical pumping created a major new stream of AMO physics. The work of Stern and Rabi helped Kastler to develop optical pumping and its first application: optical double resonance. One of Kastler's early papers is entitled (in French) *Some suggestions concerning the production and detection by optical means of inequalities in the populations of levels of spatial quantization in atoms. Application to the Stern and Gerlach and magnetic resonance experiments* [22].

Alfred Kastler was born in Alsace in 1902 and studied at École Normale Superieur (ENS) from 1931 to 1936. His career started as a teacher in lycées in Alsace and Bordeaux. He became engaged in optics and spectroscopy and the transfer of angular momentum with circularly polarized light. This led him to conceive the idea of polarizing atomic nuclei by successive absorption of polarized photons [23]. Kastler became a professor at Bordeaux in 1938 and in 1941 he was invited to ENS to help establish the physics teaching program.

In 1945 Kastler was approached by a young ENS graduate, Jean Brossel, who asked to pursue research with him. Brossel had entered ENS in 1938 and spent two years in the Army before returning to finish his studies.

Kastler had had a correspondence with Francis Bitter, a professor at the Massachusetts Institute of Technology best known for his creation of high magnetic fields, and Kastler asked him if he could take Brossel into his laboratory for thesis research. Bitter agreed and Brossel started with him in 1948. During his time abroad, Brossel and Kastler kept in touch by frequent correspondence. (The correspondence is preserved in the MIT Bitter archives). Brossel demonstrated nuclear polarization by the successive absorption of circularly polarized photons, soon to be named *optical pumping*. It was first observed using a simple atomic beam of mercury. Later, at ENS, they discovered that the nuclear polarization is stable against gaseous and surface collisions, allowing the effect to be observed in a glass cell rather than a molecular beam. This enormously simplified its usage.

A technique for polarizing and analyzing atoms provides a natural platform for magnetic resonance. The technique is called *double resonance* and the possibility was recognized early in the Kastler-Brossel collaboration.

During his time at MIT Brossel developed the complete theory of double resonance [24]. He received his Ph.D. for this work shortly after returning to Paris in 1951.

The invention of optical pumping and double resonance opened a new branch of atomic physics. The Stern/Rabi methods center on interactions of atoms and molecules with magnetic fields while optical pumping centers on their interactions with light. This encompasses a much broader range of phenomena including light-induced energy level shifts, multiphoton processes and quantum optics.

There are no direct links between Kastler and Rabi although Bitter knew both of them. He corresponded extensively with Kastler, and he remained a close friend of Rabi after their graduate student days at Columbia. Bitter invited Kastler to visit MIT and arranged an invited talk at an APS meeting, which Kastler accepted, but the state Department denied him a visa. The U.S. was suffering a "red scare" and Kastler had been in a left-leaning organization. His visit to the U.S. never took place.

In viewing the scientific heritage of AMO physics, one sees Kastler standing alongside of Stern and Rabi.

**Nobel Prize 1965: Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman** "*for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles.*"

None of these theorists would be identified as members of the AMO community although the overwhelming experimental evidence that led them to create their theories of relativistic quantum electrodynamics (QED) all came from Columbia under Rabi's reign. Furthermore, Schwinger's engagement with QED was directly due to a Rabi experiment.

Julian Schwinger had been an undergraduate prodigy when Rabi brought him to Columbia, see above. After the war Schwinger joined the faculty at Harvard.

The Rabi, Nafe, and Nelson experiment started just as the war ended and the first results, though not definitive, were strong enough to cause Schwinger to start work on his theory of QED [25].

**Nobel Prize 1981: Nicolaas Bloembergen and Arthur Leonard Schawlow**, "*for their contribution to the development of laser spectroscopy.*"

Nicolaas Bloembergen studied at the University of Utrecht for two years before emigrating to the United States to work with Purcell at Harvard University. He arrived shortly after NMR had been discovered by Purcell, Torrey and Pound [14] and became interested in nuclear relaxation. The results of his work with Purcell and Pound led to the publication "Nuclear Relaxation," [26] which became a citation classic. When Townes reported operation of the ammonia beam maser [19], Bloembergen recognized that the essential element of maser operation was an inverted population and that many other systems should be capable of displaying this. He chose an ionic crystal system to illustrate his ideas, using microwave pumping to invert the populations. The solid-state maser he proposed was realized and became a useful tool for radio-astronomy, including the discovery of the cosmic background radiation by Penzias and Wilson.

When Townes and Schawlow published their analysis of an optical maser—the laser [21]—Bloembergen realized that operations must always involve the nonlinear response of a medium to the incident radiation. Nonlinear optics became the central theme of his research career and it revealed a cornucopia of new effects: optical doubling, three-and four-wave mixing, parametric generation, high-harmonic generation, line narrowing methods. His entire career was at Harvard in the Division of Engineering and Applied Physics, close to Ramsey and Purcell.

Arthur L. Schawlow received his graduate degree in molecular spectroscopy from the University of Toronto and joined Townes at Columbia in 1949. They worked together on microwave spectroscopy of molecules, work which was summarized in what became the classic monograph on the subject, *Microwave Spectroscopy* by Townes and Schawlow [27]. When Townes invented the maser [19], he was interested in extending its operation to shorter wavelengths and he and Townes together wrote a paper proposing how to do this [21]. The short wavelength maser was soon renamed the laser: this paper launched its creation.

In 1961 Schawlow joined Stanford University and started a program in laser spectroscopy with a young colleague, Theodor W. Hänsch. Previously, spectroscopy was carried out with incoherent light sources—thermal sources of gaseous discharges. Laser light is coherent and tunable, providing vastly improved resolution and a tool for investigating previously inaccessible states and, eventually manipulating the atoms themselves. They rapidly made the laser a practical research tool, inspiring new research, launching Hänsch in a lifetime career of ever-increasing precision and innovations in optics.

**1989 Nobel Prize: Norman Ramsey** "*for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks.*"

**1989 Nobel Prize: Hans G. Dehmelt and Wolfgang Paul** "*for the development of the ion trap technique.*"

In September 1936, Norman Ramsey joined Rabi's group as a graduate student. (Rabi famously tried to discourage him on the grounds that the interesting things with molecular beams had essentially all been done. A few months later, Rabi discovered molecular beam magnetic resonance [28].) Among the group's most important discoveries was that the deuteron has a quadrupole moment, in which Ramsey played a major role [29]. After the war he helped Rabi found Brookhaven National Laboratory and he served as its first Head of the Physics Department. He started a group in molecular beam research whose summer schools eventually morphed into the International Conference on Atomic Physics (ICAP). This meeting continues today, providing an ongoing monument to the vitality of Otto Stern's heritage.

In 1947 Ramsey joined the faculty at Harvard, where he remained for the rest of his career. His class in molecular beams educated generations of graduate students and his monograph *Molecular Beams* [30] became the standard text on that topic. The book is noteworthy for its attention to Stern's work.

*The separated oscillatory field method*: [31] In 1950 Ramsey invented the separated oscillatory fields method, a technical advance that improved the accuracy of molecular beam magnetic resonance for his studies of magnetic interactions in molecules. This topic remained at the core of his research throughout his long career. The method also extended the Rabi method to high frequency, enabling the creation of the first atomic clock—the cesium beam clock—which remains in use until today [32] and has numerous metrological applications. Figure 1 shows Ramsey together with Rabi in 1959.

In recent years, a different aspect of the separated oscillatory field method has been recognized: In the region between the oscillating fields the atom can exist in an entangled state, thus providing a tool for research in quantum optics and quantum information theory.

*The Hydrogen Maser*: [33] Increasing the precision of a quantum measurement of energy or frequency, such as in an atomic clock, requires increasing the measurement time. Ramsey hit upon the idea of *storing* the atoms during the measurement process by confining them in some sort of container. The goal for creating the maser was to confirm the effect of gravity on the rate of a clock, which was eventually achieved.

Hans G. Dehmelt was a student of Hans Kopfermann at the University of Göttingen. Dehmelt initially studied NMR problems based on Bloch and Purcell's work as well as the magnetic resonance techniques of Rabi and Kastler. He moved to the University of Washington at Seattle and innovated techniques for trapping charged particles based on the radiofrequency trapping techniques developed by Paul as well as static magnetic-electric confinement. He refined his methods to the point where he could observe a single ion and trap a single electron, a "mono electron oscillator." [34] This initiated single particle spectroscopy and opened the way to a measurement of the electron magnetic moment to an accuracy of 0.28 parts per trillion [35], which remains the most precise measurement achieved in physics.

**Fig. 1** Norman Ramsey (left) and Isidor Rabi at the Brookhaven Conference on Molecular Beams held in Heidelberg, Germany, in June 1959

Wolfgang Paul was also a graduate student of Kopfermann and moved with him to Göttingen where there was an active molecular beams group. Detecting atoms and molecules was a perpetual problem for molecular beam physics. Paul invented a mass spectrometer based on static and oscillating electric fields which provided high mass resolution and high efficiency. He went on to develop methods for trapping ions in oscillating fields—the “Paul Trap.” The trap was useful for the spectroscopy of ions and was employed in the first observations of a single particle. He also developed the Penning trap which was used by Dehmelt and Gerald Gabrielse to probe the limits of QED through measurements of the magnetic moment of the electron [35].

**1997 Nobel Prize: William D. Phillips, Claude Cohen-Tannoudji and Steven Chu** "*for development of methods to cool and trap atoms with laser light.*"

Observation of Bose-Einstein condensation (BEC) in an atomic gas was announced in the summer of 1995 [36, 37]. The achievement was immediately recognized as a major discovery and was awarded the Nobel Prize in 2001. In anticipation of that award, the 1997 Prize was awarded for the breakthrough that made the discovery possible—laser cooling, an optical technique for cooling atoms to unbelievably low temperature.

The history of laser-cooling constitutes a saga of experimental physics that is narrated in the Nobel Prize lectures of the laureates: Phillips [38], Cohen-Tannoudji [39], and Chu [40]. Principal events include:

- The demonstration of atom-slowing by laser light by William D. Phillips and Harold Metcalf and the discovery of excess cooling [41].
- The demonstration of three-dimensional cooling by Steven Chu [42].
- The theory for the unexpected cooling mechanism, "Sisyphus cooling", by Claude Cohen-Tannoudji [43].

Claude Cohen-Tannoudji was a student of Alfred Kastler and was deeply immersed in theory and experiment in optical pumping and optical double-resonance from the start of his research career. Previously, in magnetic resonance phenomena the oscillating field was treated classically, following Rabi's approach. Early in his career Cohen-Tannoudji developed, with assistance from Serge Haroche, a quantum theory for the atom and field, the "dressed atom" theory [44]. This provided a new language for describing magnetic resonance and the interactions of atoms with electromagnetic fields. The dressed atom theory ultimately explained and guided the development of laser cooling, including the surprising "Sisyphus effect."

Steven Chu was a graduate student of Eugene D. Commins at Berkeley: Commins did his Ph.D. research in Rabi's group at Columbia. Chu joined the staff at Bell Laboratories and became interested in Arthur Ashkin's research on manipulating small particles with light. (Ashkin received the Nobel Prize for this work in 2018, see below.) Chu extended the research to manipulating atoms with light. He joined the faculty at Stanford University and, with Schawlow, devised a method for reducing the speed of atoms by using laser light tuned slightly below the resonance frequency. The Doppler shift would retard the motion of atoms approaching the laser. In a standing wave, the motion would be opposed in either direction. In three perpendicular standing waves, *all* motion would be retarded [41].

Such a gas was called "optical molasses" because atoms behaved as if they were in a viscous medium. This technique was key to the cooling schemes that ultimately achieved BEC.

William D. Phillips did his graduate research at MIT with me: I was a student of Ramsey. Phillips, disregarding his advisor's advice, took a position at the National Institute of Science and Technology (NIST) rather than a university. At NIST he developed a research group that studied light forces and atom slowing. He carried out the first demonstration of atom slowing by laser light in an experiment with Harold Metcalf: An atomic beam of sodium was retarded by a laser beam tuned to the principal transition. As the atoms slowed, their resonance wavelength shifted due to the Doppler effect, but by applying a tailored longitudinal magnetic field the Zeeman energy shift effect compensated the Doppler shift for the length of the apparatus [45]. The atoms were slowed—their motion could even be reversed—though in one dimension only. Nevertheless, this set the stage for laser-cooling.

After optical molasses had been discovered, Phillips developed a method for measuring the temperature of the atom cloud. He turned off the confining radiation causing the cloud to drop and imaged the expansion. The temperature found was significantly *lower* than theory predicted. This discrepancy led Cohen-Tannoudji to develop the theory of Sisyphus cooling.

*A postscript on Otto Stern and laser cooling*: The scientific legacy of Otto Stern animates the history of laser cooling, even though the direct connection was not appreciated until after the discovery. The roots of atom cooling lie in Einstein's 1917 paper on radiation. The first part of the paper introduces the concepts of absorption, stimulated emission and spontaneous emission and the Einstein A and B coefficient. The second part, not as well known, is responsible for the discovery that photons ("light quanta") carry momentum. Einstein showed how a gas of atoms comes into equilibrium with a thermal radiation field by absorbing and emitting radiation, taking into account Doppler-shifts. He proved that equilibrium is possible only if the radiation field is described by the well-known black body thermal distribution, and only if photon carries momentum = energy/c.

Light momentum was exactly the type of phenomenon that attracted Stern because of its underlying fundamental nature, although its detection would be extremely difficult. Nevertheless, he searched for the deflection of an atomic beam of sodium that was transversely irradiated by light from a sodium discharge. The deflection was minute but was observed by Otto Robert Frisch and Stern in the final moments of Stern's Hamburg laboratory (see Chap. 5). Stern omitted his name from the publication likely to assist Frisch in his search for a new position.

**2001 Nobel Prize in Physics: Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman** "*for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates.*"

The histories of the prize winners are of particular interest. Eric Cornell and Carl Wieman worked as a team at the Joint Institute for Laboratory Astrophysics (JILA) of the University of Colorado and the National Institute of Standards and Technology (NIST) in Boulder Colorado. The history of their discovery is described in a joint paper based on their Nobel Prize lectures [46]. Wolfgang Ketterle worked at the Massachusetts Institute of Technology Cambridge, Massachusetts. His Nobel lecture is also published [47].

With respect to the Stern/Rabi heritage, the laureates personal histories reveal some commonalities. Eric Cornell did his graduate research at MIT with David

E. Pritchard, working on high precision mass spectroscopy. At the time Cornell received his Ph.D., Pritchard had become interested in atom cooling and had made some valuable contributions. Cornell was intrigued and went to JILA as a postdoc, where he started collaborating with Carl Wieman. Wieman had worked with me as an undergraduate at MIT and then went to Stanford for graduate training where he worked with Ted Hänsch. Wolfgang Ketterle was a student of Herbert Walther in Garching but he had no experience in atom cooling when Pritchard appointed him to a postdoctoral position in his group. Ketterle quickly revealed talents that called for a faculty position. In a discipline such as AMO physics, the MIT Physics Department does not appoint a junior person to collaborate with a senior faculty member. To resolve the dilemma, Pritchard stepped aside, turning over the laboratory for atom-cooling to Ketterle. To complete the connections: David Pritchard was my graduate student when I was an Assistant Professor at Harvard. (We worked on molecular beam differential spin-exchange scattering.) When I moved to MIT, Pritchard came along to finish his research. We had all been members of Ramsey's group at Harvard, and Ramsey took over formal responsibility for Pritchard. There were few places where Stern's heritage burned as brightly as it did in Ramsey's group.

The search for Bose-Einstein condensation in an atomic gas (BEC) is one of the great scientific adventure stories of twentieth century physics. Laser-cooling was an essential development but that was only one part of the final success. New concepts needed to be created and new technologies needed to be developed. Many groups were involved and many postdocs launched their careers working in the search. Histories of the discoveries of BEC and developments since then have been presented in the Nobel lecturers that are referenced by Proukakis et al. [48].

**2005 Nobel Prize in Physics: Roy J. Glauber** "*for his contribution to the quantum theory of optical coherence.*" (Stern/Rabi links not identified)

**2005 Nobel Prize in Physics: John H. Hall and Theodor W. Hänsch** "*for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique.*"

John L. Hall did his undergraduate, graduate and postdoctoral research at the Carnegie Institute of Technology. In 1962 he went to the Joint Institute for Laboratory Astrophysics (JILA) and dedicated his career to the pursuit of high precision [49]. His influence on the AMO community is widespread through JILA's programs for students and visiting scientists which over the years brought many of today's AMO leaders to Boulder.

Theodor Hänsch graduated from Heidelberg University and pursued graduate research there in laser physics—then in its infancy—with Peter Toschek, a former student of Wolfgang Paul. In 1970 he joined Arthur Schawlow at Stanford University. The collaboration sparked a revolution in spectroscopy and metrology, culminating thirty years later in the creation of the optical frequency comb [50]. In 1986 Hänsch returned to Germany to become a professor at the Ludwig-Maximilians-Universität of Munich and to lead the Division of Laser Spectroscopy at Max-Planck-Institut für Quantenoptik in Garching.

The optical frequency comb: a revolutionary advance in metrology and control that extends radiofrequency and microwave techniques into the optical regime, is cited in both of these awards. It was developed independently and essentially simultaneously by the two laureates.

**2012 Nobel Prize in Physics: Serge Haroche and David W. Wineland** "*for ground-breaking methods that enable measuring and manipulation of individual quantum systems.*"

Serge Haroche did his graduate research at Ecole Normale Supérieure when the laboratory was under the direction of Alfred Kastler and Jean Brossel. Haroche collaborated with Claude Cohen-Tannoudji in developing the dressed atom theory and then exploring its applications experimentally. In 1970 he went to Stanford University and worked with Arthur Schawlow. His scientific history—which could be summarized as the evolution from using photons to study and control atoms to using atoms to study and control photons—is described in his Nobel Lecture [51].

David J. Wineland was well linked to the Stern/Rabi tradition through his graduate research with Norman Ramsey. He did postdoctoral research with Hans Dehmelt at the University of Washington and went to the National Bureau of Standards (now NIST) in Boulder, Colorado. There he directed a program of research on trapped ions, precision measurements, quantum logic and other quantum phenomena including ion cooling, as recounted in his Nobel lecture [52].

**2017 Nobel Prize in Physics: Rainer Weiss, Barry C. Barish and Kip S. Thorne** "*for decisive contributions to the LIGO detector and the observation of gravitational waves*"

The concept of LIGO and the experimental search for gravitational waves originated when Rainer Weiss—then an MIT dropout—wandered into the laboratory of Jerrold Zacharias and volunteered to help out on electronics. He became fascinated with research on an atomic clock whose goal was to observe the effect of gravity on time. Zacharias mentored Weiss through graduate school and in his early career on the MIT faculty. The history of the birth of gravitational astronomy is narrated in Weiss's Nobel lecture. Zacharias was a postdoctoral fellow in Rabi's laboratory in the 1930s and worked on the first demonstration of molecular beam magnetic resonance. Following the war Zacharias started a molecular beams laboratory at MIT.

**2018 Nobel Prize in Physics: Arthur Ashkin** "*for the optical tweezers and their application to biological systems.*"

Arthur Ashkin was an undergraduate in physics at Columbia University. He graduated in 1947 and went to Cornell University to study nuclear physics where he received the Ph.D. in 1952. He went to Bell Laboratories for the rest of his career. He initiated the use of laser light to control the motion of small particles and later collaborated with Steven Chu in the development of "optical tweezers" for manipulating molecules and atoms.

**2018 Nobel Prize in Physics: Gérard Mourou and Donna Strickland** "*for their method of generating high-intensity, ultra-short optical pulses.*" Links to the scientists in the Stern/Rabi chain have not been identified.

## 6 Otto Stern's Heritage in Chemistry

This study has focused on Otto Stern's heritage in AMO Physics but his influence reaches well beyond that. His molecular beam method was a direct influence on those in pursuit of chemistry "under single-collision conditions."

The following is a summary of Nobel Prize winners who have benefited from the heritage of Otto Stern and passed it on.

**1986 Nobel Prize in Chemistry: Dudley R. Herschbach, Yuan T. Lee and John C. Polanyi** "*for their contributions concerning the dynamics of chemical elementary processes.*"

Otto Stern's molecular beam method was a direct influence on those in pursuit of chemistry "under single-collision conditions." In Chap. 1 of this volume, Dudley Herschbach details his path to "doing chemistry" in crossed molecular beams. Herschbach set out on that path after taking Norman Ramsey's course in molecular beams at Harvard in 1955 (the author was a classmate). Ramsey was enthusiastic about Dudley's ideas and encouraged him to pursue them.

Dudley Herschbach had a lion's share in raising awareness about the legacy of Otto Stern—through the centennial Festschrift and numerous publications since as well as his many talks, including his Nobel Lecture. Herschbach also served as the honorary chair—together with Jan Peter Toennies—of the Otto Stern Fest in 2019 in Frankfurt.

Apart from Nobel laureates who were under direct influence—or spell—of Otto Stern and Isidor Rabi, there are a number of awardees whose connection to the founders of AMO Physics was more tangential or remote. Their work was nevertheless nourished by the AMO and Chemical Physics communities that produced the directly related laurates. Prominent among them are:

**1991 Nobel Prize in Chemistry: Richard Ernst** "*for his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy.*"

**1996 Nobel Prize in Chemistry: Robert Curl, Harold Kroto, and Richard Smalley** "*for their discovery of fullerenes,*" using molecular beams and mass spectrometry. The carbon polyhedron $C_{60}$ was named for a geodetic dome designed by the architect Richard Buckminster Fuller; also the $C_{60}$ pattern exacts a soccer ball!

**1999 Nobel Prize in Chemistry: Ahmed Zewail** "*for his studies of the transition states of chemical reactions using femtosecond spectroscopy.*"

**2002 Nobel Prize in Chemistry: John Fenn and Koichi Tanaka** "*for their development of soft desorption ionization methods for mass spectrometric analyses of biological macromolecules*" (a.k.a. electrospray).

**2007 Nobel Prize in Chemistry: Gerhard Ertl** "*for his studies of chemical processes on solid surfaces.*"

## 7 Epigraph

The advances in AMO physics from the time of Otto Stern to the present follow a persistent theme; ever increasing control. The Stern-Gerlach experiment permitted control of the electronic spin state of a beam of atoms; Rabi discovered how to transfer atoms from a hyperfine state to one of the many hyperfine levels and Ramsey discovered how to transfer atoms into a coherent superposition state, which we would now describe as an entangled state. In inventing the laser, Townes made it possible to generate radiation in a single mode of the radiation field and create lasers that can transfer atoms to any desired electronic state. Kastler discovered how to transfer the nuclei in a gas of atoms into a single nuclear spin state. Dehmelt discovered how to capture and study a single electron; Paul discovered how to catch and hold ions in an ion trap. Stern's first beam measurements—of the speeds of atoms—initiated a history of increasingly precise control of atomic motion, culminating in laser cooling that gives total control of *all* the quantum states of atoms, external and internal. Beyond that lies the world of ultra-cold chemistry where molecules can be assembled one atom at time and the world of optical lattices where the spatial structure of a many-atom array can be controlled; atoms can be transferred to known vibrational states and their interactions with neighboring atoms can be controlled. The frontiers of atomic physics have been pushed into many-body physics where the many bodies are controlled with the full precision that quantum mechanics permits, and their dynamics can be observed as the systems are manipulated. The discovery of gravitational waves by the LIGO interferometer is the most recent advance in this ongoing process of ever-increasing control. By controlling space and time at the level of 1 part in $10^{21}$, LIGO revealed a world of cosmic black-hole events never before seen. LIGO grew from the dream of Rainer Weiss when he was a postdoc in Jerrold Zacharias' molecular beams laboratory. Zacharias was the first postdoc in the laboratory of I. I. Rabi, a protégé of Otto Stern.

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## Appendix: A Summary of Links between the AMO Nobel Laureates and Stern/Rabi

Key → was a student or post-doc of
… indicates some other association
Nobel laureates are in bold face

**Bloembergen**→**Purcell**→**Stern/Rabi**
**Chu** → Commins → **Stern/Rabi**
**Cohen-Tannoudji** → **Kastler…Stern/Rabi**
**Cornell** → Pritchard → (Kleppner/**Ramsey**) → **Stern/Rabi**
**Haroche** → **Cohen-Tannoudji** → **Kastler**... **Stern/Rabi**
**Kastler…**(indirect links)**…Stern/Rabi**
**Ketterle** → (Pritchard) → (Kleppner) → **Ramsey** → **Stern/Rabi**
**Kusch** → **Stern/Rabi**
**Lamb…Stern/Rabi**
**Phillips** → Kleppner → **Ramsey** → **Stern/Rabi**
**Purcell**... **Stern/Rabi**
**Ramsey** → **Stern/Rabi**
**Schawlow** → **Townes…Stern/Rabi**
**Townes…Stern/Rabi**
**Weiss** →Zacharias→ **Stern/Rabi**
**Wieman** → **Hänsch** → **Schawlow** → **Townes…Stern/Rabi**
**Wineland** → **Ramsey** → **Stern/Rabi**

The autobiographies of the Nobel Laureates are available at the NobelPrize.org website.

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