Chapter 1
Theoretical Background

Abstract This chapter starts by briefly presenting the theoretical background of welfare economics and introducing key aspects such as the indirect utility function, the expenditure function, or the concepts of compensating surplus or equivalent surplus. Next, it draws attention to willingness to pay and willingness to accept, essential measures in environmental valuation. Finally, the chapter summarises the basic mathematical notation of the random utility maximisation models used throughout the book.

1.1 Welfare Economics

Environmental valuation departs from the assumption that the goods and services provided by nature can be treated as arguments of the utility function of each individual. The main purpose of environmental valuation is to obtain a monetary measure of the change in the level of utility of each individual as a consequence of a change in the provision of these goods and services (Hanemann 1984). These individual measures can subsequently be aggregated across society and compared against the costs of implementing the change and thereby inform policymakers whether the proposed change is value for money, or more formally constitutes a potential Pareto improvement to society (Nyborg 2014).

For this purpose, it is imperative to establish a link between utility and income. In microeconomic theory, this is achieved by assuming that an individual derives utility from consuming goods and services provided by nature (e.g. clean water or recreation). Individuals maximise utility subject to a budget constraint. Hence, income and prices together define the feasible set of consumption patterns. The outcome of this optimisation process is a set of (Marshallian) demand functions, where demand depends on income, prices and environmental quality. An important distinction that needs to be made is between direct and indirect utility. Direct utility is the utility obtained from consuming goods and is unconnected to prices and income. For a connection with income and prices, we thus need to look at changes in optimal behaviour. This is where indirect utility comes into play. That is, we know through the demand functions how individuals respond to price, income and quality changes. Hence, the term indirect utility represents the utility derived at the optimal demand

P. Mariel et al., *Environmental Valuation with Discrete Choice Experiments*,
SpringerBriefs in Economics, https://doi.org/10.1007/978-3-030-62669-3_1

levels. In the DCE literature, most authors refer to indirect utility functions when they mention utility functions.

Benefit estimation departs from inferring the net change in income that is equivalent to or compensates for changes in the quantity or quality in the provision of environmental goods and services (Haab and McConnell 2002). More formally, we start by defining an individual's direct utility function in terms of z, a vector of market commodities and q, a vector of environmental services:

$$u(z, q).$$

The individual may choose the quantity of z but q is exogenously determined. Further, the individual maximises utility subject to income, y, so that the problem can be reframed in terms of the indirect utility function, v:

$$v(p, q, y) = \max_z \{u(z, q) | p \cdot z \leq y\},$$

where p denotes the price of market goods. Similarly, the expenditure function associated with the utility change, which is the dual of the indirect utility function, can be defined:

$$e(p, q, u) = \min_z \{p \cdot z | u(z, q) \geq u\}.$$

The expenditure function defines the minimum amount of money an individual needs to spend to achieve a desired level of utility, given a utility function and the prices of the available market goods. The indirect utility function and the expenditure function provide the basic theoretical framework for quantifying welfare effects, having some useful properties: (1) the first derivate of the expenditure function with respect to price equals the Hicksian or utility constant demand function (also known as Shephard's lemma); (2) the negative of the ratio of derivatives of the indirect utility function with respect to price and income equals the Marshallian or ordinary demand curve (also known as Roy's identity); and (3) if the utility function is increasing and quasi-concave in q, the indirect utility function is also increasing and quasi-concave in q and the expenditure function is decreasing and convex in q. Finally, it is important to highlight that the above discussion relies on assuming that the indirect utility function is linear in prices and independent of income in order to arrive at a demand restricted to unity—i.e. what is commonly assumed in discrete choice models. For more in-depth discussion, interested readers may refer to Karlstrom and Morey (2003), Batley and Ibáñez Rivas (2013), Dekker (2014), Dekker and Chorus (2018) and Batley and Dekker (2019).

Welfare theory distinguishes two ways in which changes in environmental quality may affect an individual's utility: either by changes in the prices paid for marketed goods or by changes in the quantities or qualities of non-marketed goods. Although essentially similar, the measures of welfare impact differ, being compensating variation and equivalent variation in the former and compensating surplus (CS) and equivalent surplus (ES) in the latter.

Given that most environmental policy proposals involve changes in either the quantities or the qualities of non-market environmental goods and services where q is exogenously determined for the individual, we will describe welfare measures in terms of CS and ES here. For cases where individuals can freely adjust their consumption of both z and q, interested readers may refer to Freeman et al. (2014) for similar deliberations of the compensating and equivalent variation measures.

If q changes, the individual's utility may increase, decrease or remain constant. The value of a welfare gain associated with a change in the environmental good from the initial state q^0 (usually known as *status quo*) to an improved state q^1 is defined in monetary terms by the CS

$$v\left(p, q^1, y - CS\right) = v\left(p, q^0, y\right) = v^0, \tag{1.1}$$

and the ES

$$v\left(p, q^1, y\right) = v\left(p, q^0, y + ES\right) = v^1 \tag{1.2}$$

It is important to note that even though CS and ES are both welfare measures of the same improvement in q, the two measures differ in their implied "rights" when income effects are present. The CS implies that the individual has the right to the status quo (i.e. the individual does not have the right to the improvement in q). Hence, the welfare gain is measured keeping utility fixed at v^0. On the other hand, the ES implies that the individual has the right to the change, and, hence, measures the welfare gain keeping utility fixed at v^1. This difference in definition leads to differences in how the CS and ES are measured in practice. CS for an improvement in q is measured by the monetary amount corresponding to the individual's maximum willingness to pay (WTP) to obtain the improvement. ES for an improvement in q is measured by the monetary amount corresponding to the individual's minimum willingness to accept (WTA) compensation for not obtaining the improvement. In other words, WTP and WTA are equivalent ways of measuring a welfare change: the change in income that makes a person indifferent to an exogenously determined change in the provision of an environmental good or service. The relationship between the Hicksian welfare measures and WTP/WTA is summarised in Table 1.1 for the welfare gain context described above, as well as for a welfare loss context, e.g. in terms of a deterioration of q.

The Hicksian welfare measures may be rewritten in terms of the expenditure function:

$$\begin{aligned} WTP &= e\left(p, q^0, u^0\right) - e\left(p, q^1, u^0\right) \text{ when } u^0 = v\left(p, q^0, y\right), \\ WTA &= e\left(p, q^0, u^1\right) - e\left(p, q^1, u^1\right) \text{ when } u^1 = v\left(p, q^1, y\right). \end{aligned}$$

Table 1.1 The relationship between Hicksian measures and WTP/WTA

	Compensating surplus	Equivalent surplus
Definition	Amount of income paid or received that leaves the individual at the initial level of well-being	Amount of income paid or received that leaves the individual at the final level of well-being
Welfare gain	WTP	WTA
Welfare loss	WTA	WTP

Source Adapted from Haab and McConnell (2002)

It is important to denote that while WTP is bound by the income level, WTA is not. Even though WTP and WTA are welfare measures of the same change, theoretically as well as empirically they may differ substantially. This disparity has been found both in real markets and hypothetical markets and both for private and public goods. It has been argued that it can be influenced by many factors, such as income effects, transaction costs and broad-based preferences (Horowitz and McConnell 2002).

In theory, which welfare measure to use depends entirely on what is the most appropriate assumption to make concerning the property rights in the specific empirical case (Carson and Hanemann 2005). However, the current state of practice of environmental valuation tends to favour WTP measures as they are more conservative (i.e. specially the case in valuation studies for litigation processes) and for incentive compatibility issues arising when using WTA measures (as will be discussed in Sect. 2.4). However, WTA has been found to be a better approach in practice when applying non-market valuation techniques in low-income countries. So the decision to focus on WTP or WTA remains an area for further research, ultimately dependent on the purpose of the study.

Discrete choice models work with indirect utility functions, although practitioners should realise that these functions derive from direct utility functions. Restrictions are therefore in place, particularly in the context of the inclusion of price and income variables, to work back to the original utility maximisation problem. Despite being underexplored, the use of indirect utility functions that are linear in costs and income may be recommended for now.

1.2 Random Utility Maximisation Model

The theoretical model commonly used for analysing discrete choices is the random utility maximisation (RUM) model, based on the assumption of the utility-maximising behaviour of individuals. Under the RUM, an individual n out of N individuals faces a choice among J alternatives in one or T repeated choice occasions. The individual n obtains from an alternative j in a choice occasion t a certain level of indirect utility U_{njt}. For simplification purposes, the rest of the text will refer to this indirect utility function as simply utility function, as commonly done in the RUM literature.

The alternative i is chosen by individual n in choice occasion t if and only if $U_{nit} > U_{njt}, \forall j \neq i$. The researcher does not observe the individual's utility but observes only some attributes related to each alternative and some characteristics of the individual. The utility U_{njt} is then decomposed as

$$U_{njt} = V_{njt} + \varepsilon_{njt}, \tag{1.3}$$

where ε_{njt} represents the random factors that affect U_{njt} but are not included in V_{njt}, often known as the deterministic (or representative) utility. The error ε_{njt} is assumed to be a random term with a joint density of the random vector denoted $f(\varepsilon_n) = f(\varepsilon_{n11}, \varepsilon_{n2}, \ldots \varepsilon_{nJT})$. The deterministic utility V_{njt} is usually assumed to be linear in parameters, that is $V_{njt} = x_{njt}^{'}\beta$, where x_{njt} is a vector of variables describing goods or attributes of goods (including their price) that relate to alternative j and β which are unknown coefficients.

If the utility of all alternatives is multiplied by a constant, the alternative with the highest utility does not change. Therefore, the model

$$U_{njt} = V_{njt} + \varepsilon_{njt} = x_{njt}^{'}\beta + \varepsilon_{njt} \tag{1.4}$$

is equivalent to

$$U_{njt}^{*} = \lambda V_{njt} + \lambda\varepsilon_{njt} = x_{njt}^{'}(\lambda\beta) + \lambda\varepsilon_{njt}. \tag{1.5}$$

The normalisation of the model is usually achieved through the normalisation of the variance of the error terms. For example, in a logit model, the errors are *i.i.d.* type I extreme value distributed with location parameter zero and scale one (also called the Gumbel distribution). As the variance of this distribution is $\pi^2/6$, we are implicitly normalising the scale of utility. In the case of independently and identically distributed normal errors with variance one, leading to the independent Probit model, the scale of utility is, therefore, implicitly normalised to a different value (Train 2009, Chap. 3).

References

Batley R, Dekker T (2019) The intuition behind income effects of price changes in discrete choice models, and a simple method for measuring the compensating variation. Environ Resource Econ. https://doi.org/10.1007/s10640-019-00321-2

Batley R, Ibáñez Rivas JN (2013) Applied welfare economics with discrete choice models: implications of theory for empirical specification. In: Hess S, Daly A (eds) Choice modelling. Edward Elgar Publishing, pp 144–171

Carson RT, Hanemann WM (2005) Contingent Valuation. In: Mäler KG, Vincent JR (eds) Handbook of environmental economics, vol 2. Elsevier, pp 821–936. https://doi.org/10.03248/S1574-0099(05)02017-6

Dekker T (2014) Indifference based value of time measures for Random Regret Minimisation models. Journal of Choice Modelling 12:10–20. https://doi.org/10.1016/j.jocm.2014.09.001

Dekker T, Chorus CG (2018) Consumer surplus for random regret minimisation models. J Environ Econ Policy 7:269–286. https://doi.org/10.1080/21606544.2018.1424039

Freeman AMI, Herriges JA, Kling CL (2014) The measurement of environmental and resource values : theory and methods. Routledge

Haab TC, McConnell KE (2002) Valuing environmental and natural resources. The econometrics of non-market valuation. Edward Elgar Publishing Limited, Cheltenham, UK

Hanemann WM (1984) Discrete/continuous models of consumer demand. Econometrica 52:541–561. https://doi.org/10.2307/1913464

Horowitz JK, McConnell KE (2002) A review of WTA/WTP studies. J Environ Econ Manage 44:426–447. https://doi.org/10.1006/jeem.2001.1215

Karlstrom A, Morey ER (2003) Calculating the exact compensating variation in Logit and Nested-Logit Models with income effects: theory, intuition, implementation, and application. Social Science Research Network, Rochester, NY

Nyborg K (2014) Project evaluation with democratic decision-making: What does cost–benefit analysis really measure? Ecol Econ 106:124–131. https://doi.org/10.1016/j.ecolecon.2014.07.009

Train K (2009) Discrete choice methods with simulation, 2nd edn. Cambridge University Press, New York