6 Transboundary Water Resource Management

6.1 Water Rivalry, Agreements, and International Water Rights

There are 276 international river basins worldwide which stretch over two or more countries (De Stefano et al. 2012). About 40% of the world population lives in international river basins (Water 2008). A major issue in transboundary rivers arises when claims for water exceed the available water quantity. Therefore, rules and legal paradigms are required to prevent tensions between competing consumers.

There exist two extreme legal paradigms:

- First is the principle of Absolute Territorial Sovereignty (ATS). Every state has the right to abstract and use the water in the basin on the basis of a souvereign decision of the state within its territory. This approach favors the upstream country which is able to fully cover its claims as long as enough water is available in the river.
- Second, the principle of Absolute Territorial Integrity (ATI) concerns the allocation of water between two states which are ordered sequentially along the course of the river. In that case, the downstream country must not be negatively affected by the upstream actor. For scarce water resources in the river, a diversion of water by the upstream state may increase the shortage and, therefore, shrinks the availability of water for the downstream country. Thus, a negative impact would occur for the downstream state and hence the diversion of water by the upstream state would not be allowed under the ATI principle. The ATI principle, therefore, favors, if any, the downstream state.

Currently, these two extreme approaches are diametral to each other and are commonly rejected in international water policy. Therefore, the "Territorial Integration of All Basin States" (TIBS) as well as the approach of Limited Territorial Sovereignty (LTS) are compromises between the conflicting ATS and ATI principle.

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- The TIBS principle states that the water in the river is a common resource and any riparian has the right to divert an appropriate share regardless of its river position and its river inflow contribution.
- The LTS principle enables each riparian to use the water while any other riparian is not harmed by the usage. Due to its flexibility and its room for interpretation, this LTS principle is widely accepted in international water policy (Moes 2013).

International laws for allocating water of transboundary sources mainly developed in the second half of the twentieth century. These range from a multitude of bilateral contracts to a number of UN conventions which are valid at a global scale. In this textbook, we focus on the development of the most important conventions.[1] These contain the

- Helsinki Rules on the Use of International Rivers agreed upon in 1966;
- UN Convention on the Protection and Use of Transboundary Watercourses and International Lakes (1997);
- Berlin Rules on Water Resources from the year 2004.

The Helsinki rules were agreed upon at the 52nd Conference of the International Law Association (ILA) in August 1966, and they regulate the usage of transboundary rivers and their connected groundwaters. The Helsinki Rules consist of a total of 37 articles which are split into six chapters (International Law Association 1966). Articles 4 and 5 are most relevant for transboundary river management. Article 4 entitles any riparian to a reasonable and equitable share in the use of water,[2] while Article 5 defines the criteria to estimate this reasonable and equitable share of water usage. These criteria are, for instance, the geography and hydrology of the basin, past utilization, economic and social needs, and comparative costs of an alternative.[3] The Helsinki Rules were a quite important inspiration for the UN Convention on the

[1]For a more detailed overview of the rules, we recommend, for instance, Van Puymbroeck (2003).
[2]Article 4 of Helsinki Rules: "Each basin State is entitled, within its territory, to a reasonable and equitable share in the beneficial uses of the waters of an international drainage basin."
[3]Article 5 of Helsinki Rules:

1. What is a reasonable and equitable share within the meaning of Article 4 is to be determined in the light of all the relevant factors in each particular case.
2. Relevant factors which are to be considered include, but are not limited to
   (a) the geography of the basin, including, in particular, the extent of the drainage area in the territory of each basin State; (b) the hydrology of the basin, including, in particular, the contribution of water by each basin State; (c) the climate affecting the basin; (d) the past utilization of the waters of the basin, including, in particular, existing utilization; (e) the economic and social needs of each basin State; (f) the population dependent on the waters of the basin in each basin State; (g) the comparative costs of alternative means of satisfying the economic and social needs of each basin State; (h) the availability of other resources; (i) the avoidance of unnecessary waste in the utilization of waters of the basin; (j) the practicability of compensation to one or more of the co-basin States as a means of adjusting conflicts among uses; and (k) the degree to which the needs of a basin State may be satisfied, without causing substantial injury to a co-basin State.

Protection and Use of Transboundary Watercourses and International Lakes in 1997, and were superseded by the Berlin Rules on Water Resources in the year 2004.

On May 21, 1997, the General Assembly of the UN passed the Law of the Non-Navigational Uses of International Watercourses, which is also known as the UN Watercourses Convention.[4] Until the present, it is the only treaty under international law with global validation which rules the non-navigational usage of international water sources including both surface and groundwater (Wehling 2018). It mainly aims to further the optimal and sustainable usage as well as to ensure the development and conservation of international water sources (Salman 2007). Because of their wide acceptance, the former explained principle of Limited Territorial Sovereignty (LTS) is incorporated in the convention. Based on Article 5, the water utilization has to be equitable and reasonable. The factors for such an equitable and reasonable usage are listed in Article 6 of the convention. These factors are, for instance, natural characteristics such as geographical and hydrological conditions, social and economic needs as well as the population dependent on the watercourse. Furthermore, the following articles oblige the riparian states to take appropriate measures to prevent significant harm to other watercourse states, to cooperate with each other on the basis of sovereign equality, territorial integrity, mutual benefit, and good faith, as well as to a regular exchange of available data and information on the condition of the watercourse (e.g., hydrological, meteorological, and water quality conditions).

The Berlin Rules on Water Resources—which replaced the Helsinki Rules—were passed at the 71st Conference of the International Law Association (ILA), August 21, 2004. While the Helsinki Rules and the UN Convention established the right for each riparian state to a reasonable and equitable share, the Berlin Rules emphasize the obligation to manage the shared watercourse in an equitable and reasonable manner (Salman 2007). The term manage is specified in Article 3 (14) of the Berlin Rules and contains the development, use, protection, allocation, regulation, and control of the waters. In contrast to the former principles, the Berlin rules are not only valid for international watercourses, but also relevant for national water sources. Furthermore, the Berlin Rules have downgraded the principle of equitable and reasonable utilization and have equated this principle with the obligation of not causing significant harm to other riparians (Salman 2007; Bourne 2004).

3. The weight to be given to each factor is to be determined by its importance in comparison with that of other relevant factors. In determining what is a reasonable and equitable share, all relevant factors are to be considered together and a conclusion is reached on the basis of the whole.

[4] This convention entered into force when the minimum quorum for ratification was reached on August 17, 2014, after Vietnam signed the ratification document as the 35th state.

6.2 Benefit Sharing Between Two Riparians

6.2.1 Principles of Benefit Sharing

In Sect. 3.7, we have analyzed the IWRM approach for water allocation along rivers in which the generated benefit in the entire basin was maximized. However, this analysis ignores the fact that generated benefits could be arbitrarily assigned between the riparians by realizing side payments between the riparians. This question about an efficient and incentive-compatible assignment of the basin's benefit is the main focus of the benefit sharing problem.[5]

In this section, we focus on the case with just two riparians at an international water body. This is quite common, as the majority of international rivers are shared by just two riparian states (De Stefano et al. 2012). There exist two possible cooperation scenarios in such a basin:

- Either the riparians act unilaterally in a noncooperative way where they maximize their individual benefit from water usage,
- or they form a joint arrangement where they act in a cooperative manner which means that both riparians consume the water in such a way that the common benefit in the entire basin is maximized.[6]

If both states act in a noncooperative manner, country 1 diverts the amount w_1^{NC} and generates the benefit $B_1(w_1^{NC})$, while country 2 receives the water amount w_2^{NC} and generates the benefit $B_2(w_2^{NC})$. Thus, the benefit generated in the entire basin is $B_1(w_1^{NC}) + B_2(w_2^{NC})$.

However, if both riparians make an agreement where they act and share the water in a cooperative manner, we assume that states 1 and 2 receive the water amount w_1^C and w_2^C, respectively. The resulting benefit in the entire basin is $B_1(w_1^C) + B_2(w_2^C)$. The cooperation gain Δ is the additionally generated benefit in the entire basin compared to the noncooperation scenario (see (6.1)):

$$\Delta = B_1(w_1^C) + B_2(w_2^C) - B_1(w_1^{NC}) - B_2(w_2^{NC}) \tag{6.1}$$

From an economic perspective, there is only an incentive for forming a joint arrangement if the cooperation gain is positive ($\Delta > 0$), which means

$$B_1(w_1^C) + B_2(w_2^C) > B_1(w_1^{NC}) + B_2(w_2^{NC}) \tag{6.2}$$

The generated benefit from consumption $B_1(w_1^C)$ and $B_2(w_2^C)$ results from the optimal water allocation in the joint arrangement. However, the assignment of benefit to

[5]In this context, incentive compatible means that each riparian has an incentive for realizing the social-optimal solution.

[6]A joint arrangement is only achievable if both riparians are willing to form a joint arrangement where they cooperate. However otherwise, if one or both riparians do not want to form a joint arrangement, both riparians act unilaterally in a noncooperative way.

the riparians is the focus of benefit sharing problems. The benefit of each riparian is not only affected by the benefit from consumption, but also by side payments paid or received.[7]

The benefit of the riparians 1 and 2 in a joint arrangement which are represented by the variables x_1 and x_2 results, therefore, from the benefit of consumption ($B_1(w_1^C)$ and $B_2(w_2^C)$) and the level of the side payments, with $sp_{1,2}$ representing the side payments made by riparian 1, while $sp_{2,1}$ stands for the side payments made by riparian 2:

$$
\begin{aligned}
x_1 &= B_1(w_1^C) + sp_{2,1} - sp_{1,2} \\
x_2 &= B_2(w_2^C) + sp_{1,2} - sp_{2,1}
\end{aligned}
\tag{6.3}
$$

Of course the assignment of benefits to the riparians has to be equal to the generated benefit in the joint arrangement which is represented as (6.4)

$$
x_1 + x_2 = B_1(w_1^C) + B_2(w_2^C) \tag{6.4}
$$

A solution in which the sum of the assigned benefits exceeds the total generated benefits in the joint arrangement ($x_1 + x_2 > B_1(w_1^C) + B_2(w_2^C)$) is not realizable and would therefore violate the feasibility condition. However, the contrary case in which the sum of assigned benefits falls below the total generated benefits ($x_1 + x_2 < B_1(w_1^C) + B_2(w_2^C)$) is Pareto-inefficient and would therefore violate the Pareto-efficiency condition. The determination of the assigned benefits to the riparians in a joint arrangement is the main focus of the benefit sharing problem.

If we assume that side payments are made by just one riparian, it is possible to derive from Eq. (6.3) that riparian 1 has to make side payments if its assigned benefit x_1 falls below its benefit from consumption $B_1(w_1^C)$, while similarly, riparian 2 has to make side payments if its assigned benefit x_2 falls below its benefit from consumption $B_2(w_2^C)$. The level of the side payments results from the difference between the assigned benefits (x_1 and x_2) and the benefit from consumption ($B_1(w_1^C)$ and $B_2(w_2^C)$) (see Eq. 6.5).

$$
\begin{aligned}
&\text{If: } x_1 < B_1(w_1^C) \Leftrightarrow x_2 > B_2(w_2^C) \text{ then: } sp_{1,2} = B_1(w_1^C) - x_1 = x_2 - B_2(w_2^C) \\
&\text{If: } x_1 > B_1(w_1^C) \Leftrightarrow x_2 < B_2(w_2^C) \text{ then: } sp_{2,1} = x_1 - B_1(w_1^C) = B_2(w_2^C) - x_2
\end{aligned}
\tag{6.5}
$$

In a cooperative arrangement, the riparians have to receive at least the benefit which they would have gained if they acted unilaterally. This requirement is also known as individual rationality, which has the following algebraic formulation:

$$
\begin{aligned}
x_1 &\geq B_1(w_1^{NC}) \\
x_2 &\geq B_2(w_2^{NC})
\end{aligned}
\tag{6.6}
$$

[7] Side payments are payment transactions between the riparians; the side payment is beneficial for the receiving riparian, while it is a financial burden for the paying one.

A riparian whose individual rationality condition is not met has an incentive to leave the joint arrangement and act in a noncooperative way.

Hence, due to the feasibility and Pareto-optimality conditions (see Eq. (6.4)) as well as the individual rationality condition (see Eq. (6.6)), sharing of the cooperation gain Δ is the main focus of the benefit sharing problem with two riparians.

6.2.2 UID, DID and the Shapley Solution

There are two extreme solutions for assigning the cooperation gain to the riparians, either the first or the second riparian receives the entire cooperation gain. Assuming riparian 1 is the upstream and riparian 2 is the downstream riparian, it is possible to distinguish between two extreme scenarios regarding the allocation of the cooperation gain:

- **Upstream incremental distribution (UID)**: When applying the UID approach, the cooperation gain is completely assigned to the upstream riparian 1, while the downstream riparian 2 just gets enough benefit to meet its individual rationality condition:

$$\begin{aligned} x_1^{UID} &= B_1(w_1^{NC}) + \Delta \\ x_2^{UID} &= B_2(w_2^{NC}) \end{aligned} \tag{6.7}$$

  If we assume that either riparian 1 makes side payments ($sp_{1,2} > 0 \wedge sp_{2,1} = 0$) which is the case if $x_1^{UID} < B_1(w_1^C)$ or riparian 2 makes side payments ($sp_{2,1} > 0 \wedge sp_{1,2} = 0$) which is the case if $x_2^{UID} < B_2(w_1^C)$, it is possible to determine the level of side payments:

$$\begin{aligned} sp_{1,2} &= \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) - \Delta = B_2(w_2^{NC}) - B_2(w_2^C) & \text{if: } x_1^{UID} < B_1(w_1^C) \Leftrightarrow x_2^{UID} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{UID} \geq B_1(w_1^C) \Leftrightarrow x_2^{UID} \leq B_2(w_2^C) \end{cases} \\ p_{2,1} &= \begin{cases} B_1(w_1^{NC}) + \Delta - B_1(w_1^C) = B_2(w_2^C) - B_2(w_2^{NC}) & \text{if: } x_1^{UID} > B_1(w_1^C) \Leftrightarrow x_2^{UID} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{UID} \leq B_1(w_1^C) \Leftrightarrow x_2^{UID} \geq B_2(w_2^C) \end{cases} \end{aligned} \tag{6.8}$$

- **Downstream incremental distribution (DID)**: In the DID approach, the cooperation gain Δ is completely assigned to the downstream riparian 2. The upstream riparian 1 receives benefit, such that its individual rationality condition is fulfilled.

$$\begin{aligned} x_1^{DID} &= B_1(w_1^{NC}) \\ x_2^{DID} &= B_2(w_2^{NC}) + \Delta \end{aligned} \tag{6.9}$$

Hence, the following side payments result:

$$p_{1,2} = \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) = B_2(w_2^{NC}) + \Delta - B_2(w_2^C) & \text{if: } x_1^{DID} < B_1(w_1^C) \Leftrightarrow x_2^{DID} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{DID} \geq B_1(w_1^C) \Leftrightarrow x_2^{DID} \leq B_2(w_2^C) \end{cases}$$
$$p_{2,1} = \begin{cases} B_1(w_1^{NC}) - B_1(w_1^C) = B_2(w_2^C) - B_2(w_2^{NC}) - \Delta & \text{if: } x_1^{DID} > B_1(w_1^C) \Leftrightarrow x_2^{DID} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{DID} \leq B_1(w_1^C) \Leftrightarrow x_2^{DID} \geq B_2(w_2^C) \end{cases} \quad (6.10)$$

Based on these two extreme cases, it is possible to find all possible realizations for x_1 and x_2 for the benefit sharing problem based on the linear combination of the extreme scenarios (see Eq. 6.11):

$$\begin{aligned} x_1 = \beta \cdot x_1^{UID} + (1-\beta) \cdot x_1^{DID} &\Leftrightarrow x_1 = B_1(w_1^{NC}) + \beta \cdot \Delta \\ x_2 = \beta \cdot x_2^{UID} + (1-\beta) \cdot x_2^{DID} &\Leftrightarrow x_2 = B_2(w_2^{NC}) + (1-\beta) \cdot \Delta \\ &\text{with: } 0 \leq \beta \leq 1 \end{aligned} \quad (6.11)$$

The value of parameter β is defined within the range [0, 1]. It becomes obvious from Eq. 6.11 that we get the UID or DID solution if β is set equal to 0 or 1, respectively. The higher the value of β, the more advantageous the benefit sharing solution for the upstream user 1, while the profit for the downstream user 2 raises with a decreasing value of β.

A further specific case is the determination of β with $\beta = 0.5$ which means that each riparian receives half of the cooperation gain. This solution results from applying the Shapley value approach for the case with two riparians (Shapley 1953). Therefore, this could be termed the Shapley solution. The Shapley solutions for x_1^{SH} and x_2^{SH} are

$$\begin{aligned} x_1^{SH} &= B_1(w_1^{NC}) + 0.5 \cdot \Delta \\ x_2^{SH} &= B_2(w_2^{NC}) + 0.5 \cdot \Delta \end{aligned} \quad (6.12)$$

For this Shapley solution, the following side payments result on the basis of Eqs. 6.5 and 6.12:

$$sp_{1,2} = \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) - 0.5 \cdot \Delta = B_2(w_2^{NC}) + 0.5 \cdot \Delta - B_2(w_2^C) & \text{if: } x_1^{SH} < B_1(w_1^C) \Leftrightarrow x_2^{SH} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{SH} \geq B_1(w_1^C) \Leftrightarrow x_2^{SH} \leq B_2(w_2^C) \end{cases}$$
$$sp_{2,1} = \begin{cases} B_1(w_1^{NC}) - B_1(w_1^C) + 0.5 \cdot \Delta = B_2(w_2^C) - B_2(w_2^{NC}) - 0.5 \cdot \Delta & \text{if: } x_1^{SH} > B_1(w_1^C) \Leftrightarrow x_2^{SH} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{SH} \leq B_1(w_1^C) \Leftrightarrow x_2^{SH} \geq B_2(w_2^C) \end{cases} \quad (6.13)$$

The benefit sharing problem in a basin with two riparians is also illustrated by Fig. 6.1. We draw riparian 1's assigned benefit (x_1) on the horizontal axis (abscissa), while the benefit of riparian 2 is illustrated on the vertical axis (ordinate). The benefits of the riparians 1 and 2 when acting unilaterally in a noncooperative way ($B_1(w_1^{NC})$ and $B_2(w_2^{NC})$) are pictured in this graph by the vertical and horizontal functions, respectively. In this diagram, these two functions have the algebraic expression: $x_1 = B_1(w_1^{NC})$ and $x_2 = B_2(w_2^{NC})$. The benefit generated in the basin when both riparians

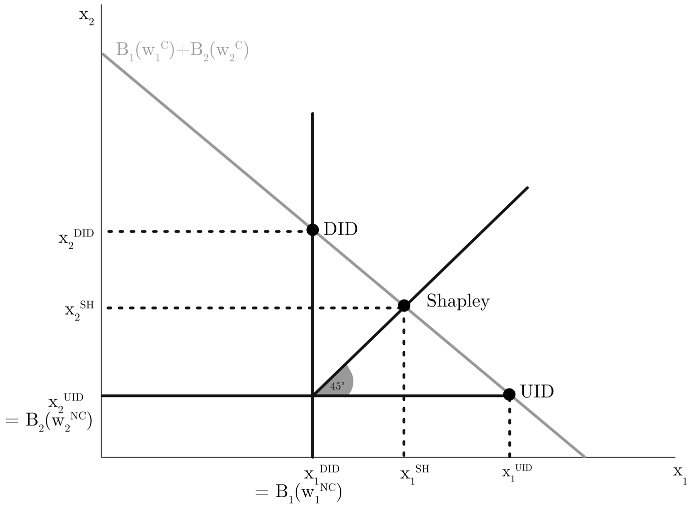


Fig. 6.1 Benefit sharing in a basin with two riparians. *Source* own illustration

form a joint arrangement is illustrated by the monotonous-decreasing diagonal line. In this diagram, the function has the algebraic expression: $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ due to Eq. 6.4. This means the higher the assignment of benefits to riparian 1, the lower the assignment to riparian 2 and vice versa. For meeting the feasibility and Pareto-efficiency conditions, the benefit sharing solution has to be located on the function.[8]

The function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ intersects with the horizontal and vertical axes at the level $B_1(w_1^C) + B_2(w_2^C)$. These points can be interpreted as the full assignment of the basin's benefit to just one riparian. Of course, these two benefit sharing solutions would meet the feasibility and Pareto-efficiency conditions, but would fail the individual rationality condition which states that the assignment of benefits to each riparian must be as high as the benefit the riparians would generate if they acted unilaterally in a noncooperative manner, which means $x_1 \geq B_1(w_1^{NC})$ and $x_2 \geq B_2(w_2^{NC})$ (see Eq. 6.6). Therefore, for meeting the individual rationality condition, the UID solution with $x_1 = x_1^{UID} = B_1(w_1^{NC}) + \Delta$ and $x_2 = x_2^{UID} = B_2(w_2^{NC})$ limits the assigned benefit to the upstream riparian 1 to the maximum level $B_1(w_1^{NC}) + \Delta$, while the DID solution with $x_1 = x_1^{DID} = B_1(w_1^{NC})$ and $x_2 = x_2^{DID} = B_2(w_2^{NC}) + \Delta$ determines the maximum possible assigned benefit for riparian 2 to the level $B_2(w_2^{NC}) + \Delta$. Therefore, the solutions which are on

[8]A solution which is located in between the area which is spanned by the axis and the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ would fail the Pareto-efficiency condition, because fewer benefits are allocated than generated, while a solution beyond the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ would fail the feasibility condition, because more benefits are allocated to the riparians than generated.

the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ and which are in between the UID and DID solution, which means $B_1(w_1^{NC}) \leq x_1 \leq B_1(w_1^{NC}) + \Delta$ and $B_2(w_2^{NC}) \leq x_2 \leq B_2(w_2^{NC}) + \Delta$, form the set of solutions for the benefit sharing problem with two riparians.

A specific focal point solution of this benefit sharing problem is the Shapley solution with $x_1 = x_1^{SH} = B_1(w_1^{NC}) + 0.5 \cdot \Delta$ and $x_2 = x_2^{SH} = B_2(w_2^{NC}) + 0.5 \cdot \Delta$. This solution could be found in Fig. 6.1 at the intersection point between the monotonous-decreasing diagonal function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ and the 45°-degree line whose origin is at the intersection of the vertical and horizontal lines that represent the benefit for noncooperative acting riparians $x_1 = B_1(w_1^{NC})$ and $x_2 = B_2(w_1^{NC})$.

6.3 Benefit Sharing Between More Than Two Riparians

In this section, we focus on methods to allocate the generated benefits in a basin with more than two riparians.[9]

6.3.1 Model of a River Basin

6.3.1.1 Superadditivity Condition

If the solution of the allocation problem is beneficial for all riparians, they have an incentive to form a cooperation scheme where they decide together about water management plans, water allocation strategies, and the allotment of benefits. An important condition for forming joint arrangements is the superadditivity of benefits which can be expressed by the following relation:

$$V(S) + V(T) \leq V(S \cup T) \text{ with: } S \cap T = \emptyset \tag{6.14}$$

The sets S and T stand for coalitions where participants, which are represented by elements of these sets, act in a cooperative manner. The sets could contain just one element or a multitude of elements. If the set S or T contains just one element, the corresponding riparian is not participating in a joint arrangement and, therefore, acts unilaterally in a noncooperative manner. However, if there are a multitude of elements in the set, the corresponding riparians act together in a sub-coalition. For the analysis of superadditivity, one riparian cannot be part of both sets S and T, hence $S \cap T = \emptyset$. In the case of a grand coalition, all riparians act in one joint arrangement, which means that the union set $S \cup T$ would contain all riparians.[10] The $V(\ldots)$ stands for the benefit which can be generated in the respective coalition, therefore, it is the

[9]The chapter-annex (Sect. 6.9) provides a full account to the mathematical derivations.

[10]The grand coalition cannot be represented by S or T, while a unilaterally acting player can not be represented by $S \cup T$. Sub-coalitions can be represented by S, T as well as by $S \cup T$.

value of the coalition. The superadditivity condition (6.14) states that the value of a joint arrangement between the sub-coalitions S and T must be at least as high as the sum of values of arrangements S and T.

If the superadditivity condition is fulfilled, the additional benefit due to the joining of S and T to a mutual arrangement can be expressed by the following equation:

$$V(S \cup T) - V(S) - V(T) \tag{6.15}$$

If the superadditivity condition holds for all cooperation arrangements, the highest benefits for the entire basin can be generated by forming the grand coalition.[11] Finally, for finding adequate solutions for sharing the benefit between the riparians of a coalition, methods from cooperative game theory can be applied.[12]

6.3.1.2 General Procedure for Solving a Benefit Sharing Problem

For solving the benefit sharing problem within a river basin, the first step is to set up a model of the river containing the riparians, their benefit and cost functions as well as the main hydrological conditions such as natural external inflows into the river and the flow direction. Afterwards, various options for cooperation in the river basin are addressed. Every riparian has the option either to cooperate and form an arrangement with other riparians, or not to cooperate and act unilaterally. Therefore, it is usually possible to form either one joint arrangement (grand coalition), or to arrange different forms of sub-coalitions between selected riparians. Further, it is also possible that no arrangement occurs in the basin, so that every riparian acts unilaterally. These different options of cooperation are quantified by calculating the joint benefit within the cooperation arrangement and by finding the individual benefit of each unilaterally acting riparian for any cooperation scenario. Finally, it is important to find a way to allocate the benefit generated in a joint arrangement between the participating riparians.

For this benefit sharing issue, various techniques from cooperative game theory can be applied. In this context, we focus on the concept of the core which gives a set of possible solutions to the question of how to share the benefit of a joint arrangement between its member riparians. Based on this, the bargaining power of each riparian can be found. While the core gives a set of possible solutions, there also

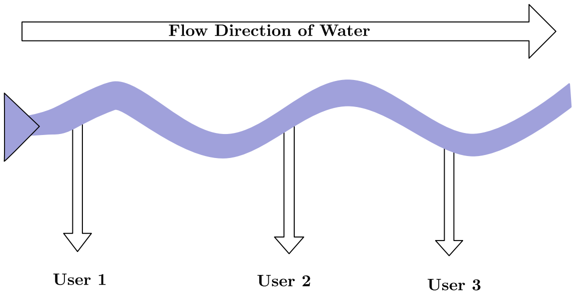
[11] In this textbook, we focus on sharing benefits. However, if there is a cost game (e.g., realizing a dam project under different coalition scenarios), the subadditivity condition has to be fulfilled which can be expressed by the following formulation:

$$C(S) + C(T) \geq C(S \cup T) \text{ with: } S \cap T = \emptyset$$

The $C(...)$ represents the cost for the related coalition. The subadditivity condition means that the cost under a cooperative arrangement between coalition S and T must not exceed the sum of costs if coalition S and T act separately. If this subadditivity condition is fulfilled for all coalition scenarios, the lowest cost is caused by forming the grand coalition.

[12] In contrast to the concepts of noncooperative game theory which are more popular in the economic literature, interaction and payments between the relevant actors are possible in cooperative game theory.

Fig. 6.2 Network of a hypothetical river basin. *Source* own illustration



exist focal point solution concepts for calculating concrete results for sharing the benefit: We focus on the concept of the Shapley value, the Nash-Harsanyi solution, and the nucleolus. The Shapley value allocates the benefits according to the strength of each player in the joint arrangement, while the Nash-Harsanyi solution maximizes the additional utility from cooperation in a joint arrangement compared to the non-cooperative case for each riparian. The nucleolus is a procedure for minimizing the maximum objection against the benefit sharing solution. Finally, parameters indicating the acceptability of a benefit sharing solution can be calculated. Nonacceptance of a solution results if one riparian views its payoff as unfair against the payoff of other players in the coalition. The higher the nonacceptance, the higher the risk that the unsatisfied player will leave the coalition.

The procedure which was explained for the general case above is now applied to a hypothetical river basin whose water is shared by three riparians. We assume a river basin (see Fig. 6.2) which consists of upstream, midstream, and downstream riparians, represented by index i with $i = \{1, 2, 3\}$. The river is fed by an inflow upstream of the first user; there are no other external inflows. By consuming the water, the riparians generate benefits. Assuming a constant marginal benefit, the benefit increases linearly with increasing consumption. If the available water amounts are consumed completely by the upstream or midstream or downstream user, the generated benefit is α or β or γ, respectively. Furthermore, we suppose the relation $\alpha < \beta < \gamma$, which means that the upstream riparian is the least productive one while the downstream user is the most productive one. Backflows to the river after withdrawing and consuming do not occur for this hypothetical case, hence the consumption of water is completely rivalrous. Furthermore, there are no limitations regarding the access to and extraction of water.

Options of Cooperation

For the hypothetical river network, there are different options of cooperation:

- All riparians act unilaterally which means that an arrangement in the river basin does not exist. The set which states the occurring coalition is, therefore, an empty set, $\emptyset$. The unilateral acting of riparians is stated with sets which just contain one element. This means if riparians 1, 2, and 3 act unilaterally, this is stated by the formulation $\{1\}$, $\{2\}$, and $\{3\}$, respectively.

Table 6.1 Generated benefits for different cooperation scenarios

Coalition	Benefit for ...							
	Non-cooperating			Coalition				Entire Basin
	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}	
∅	α	0	0	–	–	–	-	α
{1, 2}	–	–	0	β	–	–	–	β
{1, 3}	–	0	–	–	α	–	–	α
{2, 3}	α	–	–	–	–	0	–	α
{1, 2, 3}	–	–	–	–	–	–	γ	γ

- Two of three actors cooperate with each other. Hence, the following arrangements between the riparians are possible: {1, 2}, {1, 3}, and {2, 3}. The residual riparian that is not a member of the coalition acts unilaterally.
- All actors cooperate in one joint arrangement and thus form a grand coalition, which is symbolized by {1, 2, 3}.

Value of Coalitions

For the different options of cooperations, different levels of generated benefits can be found which are summarized in Table 6.1.

If no arrangement is formed between the riparians, the available water is completely consumed by the upstream user who is able to generate a benefit α. Further, downstream users do not receive any amount of water from the river and therefore cannot generate any benefit. The same water consumption pattern in the basin is also observable, if the mid- and downstream riparians form a coalition {2, 3} without the upstream user.[13] However, if the up- and downstream users arrange a cooperation without the midstream riparian, {1, 3}, there seems to be an incentive for the upstream one to leave water in the river, because the downstream riparian could generate the highest benefit for the coalition. However, the intermediate midstream riparian would seize the full amount of water, which is also known as leakage. (Ansink et al. 2012) This reaction of the midstream is anticipated by the upstream, hence the upstream would withdraw the total amount of water to maximize the benefit for the formed coalition. The generated benefit for the coalition would be α, while the midstream does not receive any amount of water.

If all the riparians form a joint cooperative arrangement, {1, 2, 3}, the upstream and midstream leave the total amount of water in the river, hence the downstream user is the only one who uses the water and is able to generate a benefit of γ for the grand coalition.

[13]This means that for the coalition {2, 3}, the same situation occurs as for the case that all riparians act unilaterally. The total amount of water is consumed by the upstream user who generates a benefit of α. The coalition of the mid- and downstream users {2, 3} does not receive any amount of water and gets a benefit of 0.

Superadditivity Conditions
For the presented case, the superadditivity condition holds for all cooperation scenarios:

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} \leq \underbrace{V(\{1,2\})}_{=\beta} \quad \rightarrow \ \alpha \leq \beta \ \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,3\})}_{=\alpha} \quad \rightarrow \ \alpha \leq \alpha \ \checkmark$$

$$\underbrace{V(\{2\})}_{=0} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{2,3\})}_{=0} \quad \rightarrow \ 0 \leq 0 \ \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \quad \rightarrow \ \alpha \leq \gamma \ \checkmark$$

$$\underbrace{V(\{1,2\})}_{=\beta} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \quad \rightarrow \ \beta \leq \gamma \ \checkmark$$

$$\underbrace{V(\{1,3\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \quad \rightarrow \ \alpha \leq \gamma \ \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2,3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \quad \rightarrow \ \alpha \leq \gamma \ \checkmark$$

Due to this fulfillment of the superadditivity condition, the grand coalition is the best option in order to maximize the total benefit in the entire basin. The result is also visible from the rightmost column of Table 6.1, which illustrates the aggregated benefit in the river basin for all possible cooperation scenarios.

Sharing the Benefits
After forming the grand coalition, the generated benefit with the level of γ has to be shared between all riparians. For this bargaining problem, different methods from cooperative game theory are available such as the core or various focal point solutions, for instance, the Shapley value, the Nash-Harsanyi solution as well as the nucleolus. For this analysis, the sets I, S, and G are defined. I contains as set elements those riparians which act unilaterally in a noncooperative way. Therefore, the value of the characteristic function for the set I, being $V(I)$, is based either on the benefits for the non-cooperating case in the entire river basin (see line $\emptyset$ of Tables 6.1 and 6.2) or on the minimum benefit which is gained under all coalition scenarios in which the relevant riparian is not a member (see Table 6.2).

Regardless of the applied approach, both procedures result in the same solution for this example. In the unilateral acting case, riparian 1 generates a benefit of α, while riparians 2 and 3 are not able to divert any water amounts and generate, therefore, a benefit of 0. The set S represents all possible sub-coalitions, which means that a cooperative scheme is formed between 2 or more riparians but does not contain all riparians of the basin. For the presented example, the three tuples $\{1, 2\}$, $\{1, 3\}$,

Table 6.2 Generated benefits of unilaterally acting riparians for different cooperation scenarios

Coalition	Benefit for ...		
	Non-cooperating		
	{1}	{2}	{3}
Ø	α	0	0
{1, 2}	–	–	0
{1, 3}	–	0	–
{2, 3}	α	–	–
MINIMUM	$\boldsymbol{\alpha}$	**0**	**0**

Table 6.3 Value of cooperations

$V(I)$			$V(S)$			$V(G)$
$V(\{1\})$	$V(\{2\})$	$V(\{3\})$	$V(\{1,2\})$	$V(\{1,3\})$	$V(\{2,3\})$	$V(\{1,2,3\})$
α	0	0	β	α	0	γ

and {2, 3} form the set S. However, the set G stands for the grand coalition, which means that all riparians of the basin form a coalition. For the presented example, the riparians 1, 2, and 3 would form a common cooperative scheme, hence the tuple $\{1, 2, 3\}$ is element of the set G. The characteristic functions of the cooperation scenarios, $V(I)$, $V(S)$, and $V(G)$, are based on the analysis concluded in Table 6.1 and are listed in Table 6.3.

The grand coalition is the optimal coalition due to the fulfillment of the super-additivity condition. Therefore, we assume that a grand coalition is formed and the basin's benefit of γ has to be shared between the riparians. The payoff (or imputation)[14] that each riparian i receives is symbolized by the variable x_i in the following. All the variables x_i develop the vector x, which contains for the presented example: $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. The determination of x is the main focus of the benefit sharing problem.

6.3.2 Benefit Sharing in the Grand Coalition: Four Approaches

6.3.2.1 The Core

The core is a set of payoffs which meets the four following conditions (Gillies 1959):

- Feasibility: Only benefits which are received can be allocated, which means $\sum_i [x_i] \leq V(G)$.

[14]The term imputation is used as a synonym of the term payoff.

- Pareto-efficiency: Nobody's payoff can be improved without worsening the payoff of another riparian. Therefore, there must not be an under-allocation of the gained benefits to the riparians, $\sum_i [x_i] \geq V(G)$.
- Combining the feasibility and Pareto-efficiency conditions leads to the requirement that the allocated benefits are equal to the generated benefits, $\sum_i [x_i] = V(G)$.
- Individual rationality: Each riparian rejects a payoff which is below its benefit when acting in a noncooperative way. Hence, $x_i \geq V(I)$. If this condition does not hold, the riparian would have an incentive to leave the grand coalition and to act unilaterally.
- Group rationality: Each sub-coalition of the grand coalition rejects a payoff which is below the benefit which is gained when the riparians act in a sub-coalition. Hence, $\sum_{i \in S} [x_i] \geq V(S)$. If this condition is not met, the relevant riparians $i \in S$ would have an incentive to leave the grand coalition and form the sub-coalition S.

Based on the presented example, it can be indicated whether the core is empty and has no solution ($Z < 0$), or there is only one solution ($Z = 0$) or there is a multitude of solutions ($Z > 0$) by solving the following optimization problem:

$$\begin{aligned}
&\max\ [Z = V(\{1,2,3\}) - [x_1 + x_2 + x_3]] \\
&x_1 \geq V(\{1\})\ ,\ x_2 \geq V(\{2\})\ ,\ x_3 \geq V(\{3\}) \\
&[x_1 + x_2] \geq V(\{1,2\})\ ,\ [x_1 + x_3] \geq V(\{1,3\})\ ,\ [x_2 + x_3] \geq V(\{2,3\})
\end{aligned}$$

which is equivalent to the following formulation:

$$\begin{aligned}
&\max\ \left[Z = \gamma - [x_1 + x_2 + x_3]\right] \\
&x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0 \\
&[x_1 + x_2] \geq \beta\ ,\ [x_1 + x_3] \geq \alpha\ ,\ [x_2 + x_3] \geq 0
\end{aligned}$$

Here, it is possible to calculate $Z = \gamma - \beta > 0$, hence, there are various payoff combinations which fulfill the conditions of the core.

Based on the core, the range of payoffs for each riparian can be indicated. There exists a lower bound and an upper bound for each riparian in the core:

- The riparian does not have an incentive to stay in the coalition until its payoff falls below the lower bound.
- However, if the payoff of an riparian exceeds its upper bound, another riparian certainly has an incentive to leave the coalition.

The lower and upper bounds of each player can be derived by solving the following optimization problems:

$$\begin{aligned}
&\textit{Lower Bound} \text{ or } \textit{Upper Bound} \text{ of riparian } i: \\
&\min [x_i] \text{ or } \max [x_i] \\
&s.t.\ [x_1 + x_2 + x_3] = V(\{1,2,3\}) \\
&\quad x_1 \geq V(\{1\})\ ,\ x_2 \geq V(\{2\})\ ,\ x_3 \geq V(\{3\}) \\
&\quad [x_1 + x_2] \geq V(\{1,2\})\ ,\ [x_1 + x_3] \geq V(\{1,3\})\ ,\ [x_2 + x_3] \geq V(\{2,3\})
\end{aligned}$$

Table 6.4 Lower and upper bounds of payments which are in the core

Riparian	Lower bound	Upper bound
Upstream riparian (User 1)	α	γ
Midstream riparian (User 2)	0	$\gamma - \alpha$
Downstream riparian (User 3)	0	$\gamma - \beta$

which is, for the presented river basin example, equivalent to the following formulation:

$$\begin{aligned}&\textit{Lower Bound} \text{ of riparian } i:\\ &\min\,[x_i]\\ &s.t.\ [x_1+x_2+x_3]=\gamma\\ &\quad x_1\ge\alpha\ ,\ x_2\ge 0\ ,\ x_3\ge 0\\ &\quad [x_1+x_2]\ge\beta\ ,\ [x_1+x_3]\ge\alpha\ ,\ [x_2+x_3]\ge 0\end{aligned}$$

$$\begin{aligned}&\textit{Upper Bound} \text{ of riparian } i:\\ &\max\,[x_i]\\ &s.t.\ [x_1+x_2+x_3]=\gamma\\ &\quad x_1\ge\alpha\ ,\ x_2\ge 0\ ,\ x_3\ge 0\\ &\quad [x_1+x_2]\ge\beta\ ,\ [x_1+x_3]\ge\alpha\ ,\ [x_2+x_3]\ge 0\end{aligned}$$

The objective contains the payment of the considered riparian i, which has to be minimized or maximized for finding the lower or upper bound of the core, respectively. The constraints of the optimization problem contain the conditions that have to be fullfilled for a payoff to be in the core:

- Feasibility and pareto-efficiency, $[x_1 + x_2 + x_3] = \gamma$;
- individual rationality, $x_1 \ge \alpha\ ,\ x_2 \ge 0\ ,\ x_3 \ge 0$;
- and group rationality, $[x_1 + x_2] \ge \beta\ ,\ [x_1 + x_3] \ge \alpha\ ,\ [x_2 + x_3] \ge 0$.

For the presented example, the lower and upper bounds of payments are listed in Table 6.4.

The lower bound of the riparians is determined by their individual rationalities. Therefore, the upstream user has to receive at least a payment of α, while the lower bounds of the mid- and downstream users are determined at the value 0.

Regarding the upper bound of payments, the upstream user could postulate a claim for the total generated benefits γ, because all constellations without the upstream user generate a benefit of zero,

$$\overline{x_1^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{2\})}_{0}\ ,\ \gamma - \underbrace{V(\{3\})}_{0}\ ,\ \gamma - \underbrace{V(\{2,3\})}_{0}] = \gamma\ .$$

However, due to the fact that the upstream user must receive at least a payment of α, the midstream user could get a maximal payment of $\gamma - \alpha$,

$$\overline{x_2^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{1\})}_{\alpha}\ ,\ \gamma - \underbrace{V(\{3\})}_{0}\ ,\ \gamma - \underbrace{V(\{1,3\})}_{\alpha}] = \gamma - \alpha\ .$$

The downstream riparian cannot claim more than $\gamma - \beta$, because of the threat that the up- and midstream riparians can form a sub-coalition $\{1, 2\}$ where they would generate a benefit of β,

$$\overline{x_3^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{1\})}_{\alpha}\,,\ \gamma - \underbrace{V(\{2\})}_{0}\,,\ \gamma - \underbrace{V(\{1,2\})}_{\beta}] = \gamma - \beta\,.$$

To sum up, the concept of the core gives a set of possible payments, which meets the feasibility, Pareto-efficiency, individual and group rationality conditions. By considering these conditions, a proposed payment which is in the core provides an incentive for each riparian to join the grand coalition. However, the core may be impractical in practice, because of the large amounts of possible solutions. Therefore, it might be advantagous to use focal point solution concepts for calculating a concrete payment vector x.

6.3.2.2 The Shapley Value

The Shapley Value which is based on Shapley (1953) shares the benefits in terms of the incremental value of the respective player for the coalition. The solution can be calculated using Eq. 6.16:

$$x_i = \sum_{\substack{I:i\in I\vee\\ S:i\in S\vee\\ G}} \left[\frac{(\#G - \#ISG)!\cdot(\#ISG-1)!}{\#G!}\cdot[V(...) - V(... - i)]\right] \tag{6.16}$$

The $\#ISG$ sign in Eq. (6.16) represents the number of riparians which form a coalition. These coalition scenarios can be

- unilaterally acting riparians, represented by set I with $I = \{\{1\}\ ;\ \{2\}\ ;\ \{3\}\ \}$;
- sub-coalitions, represented by set S with $S = \{\{1,2\}\ ;\ \{1,3\}\ ;\ \{2,3\}\ \}$;
- the grand coalition, which is represented by set G with $S = \{\{1,2,3\}\}$.

We already discussed that the set I stands for unilaterally acting riparians which act in a noncooperative way. Therefore, the set I contains the tuples: $\{1\}$, $\{2\}$, and $\{3\}$. There is just one element in these tuples, hence, $\#ISG = 1$ for these types of coalition scenario. The set S stands for the sub-coalitions which represent all coalitions between the riparians except the grand coalition. Therefore, the tuples $\{1,2\}$, $\{1,3\}$, and $\{2,3\}$ form the set S. It is obvious that there are two riparians which form a sub-coalition, hence, $\#ISG = 2$ for all sub-coalition-constellations. In case of a grand coalition G, all riparians of the basin form a common cooperation scheme, which means that the tuple $\{1,2,3\}$ is an element of set G. This coalition includes three riparians, hence $\#ISG = 3$. Furthermore, the parameter $\#G$ also represents the number of riparians in the grand coalition, hence $\#G = 3$.

Due to the formulation $\sum_{\substack{I:i\in I\vee\\ S:i\in S\vee\\ G}} [...]$ in Eq. (6.16), only those coalitions are addressed in which riparian i is a member. These involve the unilateral action situation for riparian i ($i \in I$), appropriated sub-coalitions ($i \in S$) as well as the grand coalition (G). Therefore, the following constellations are relevant for solving the Shapley solution of the riparians:

- Riparian 1: {1}, {1, 2}, {1, 3}, {1, 2, 3}
- Riparian 2: {2}, {1, 2}, {2, 3}, {1, 2, 3}
- Riparian 3: {3}, {1, 3}, {2, 3}, {1, 2, 3}

The term $\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!}$ in Eq. (6.16) is a weighting factor which comes from mathematical permutation, based on the normative assumption that every player in the coalitions is randomly ordered, with every ordering equally possible (Wu and Whittington 2006).

The term $V(...)$ in Eq. (6.16) represents the generated benefit for the addressed coalition, while $V(... - i)$ symbolizes the benefit of the coalition which is formed on the basis of the addressed coalition without the riparian i. Therefore, the difference $V(...) - V(... - i)$ could be interpreted as the incremental benefit of riparian i for the coalition.

The application of Eq. 6.16 for the presented simple example is concluded in Table 6.5. Because of the three users (upstream, midstream, and downstream), it becomes obvious that $\#G = 3$. Regarding riparian 1, there is the realization probability of $\frac{1}{3}$ each that this riparian acts unilaterally or works in a grand coalition. Furthermore, there exists the realization probability of $\frac{1}{6}$ each that riparian 1 forms a sub-coalition either just with riparian 2 or 3 (see column (IV) of Table 6.5). If we observe the cooperation scenarios {1}, {1, 2}, {1, 3}, and {1, 2, 3}, the benefits α, β, α, and γ could be generated in the coalition scenarios, respectively (see column (V) of Table 6.5). A coalition without the unilaterally acting riparian 1, does not exist and therefore generates a benefit of 0. The coalitions {1, 2}, {1, 3}, and {1, 2, 3} without riparian 1 result in the coalition constellations {2}, {3}, and {2, 3} (see column (VI) of Table 6.5) which are each characterized by the generated benefit of 0 (see column (VII) of Table 6.5). Hence, the incremental benefit of riparian 1 for the coalitions {1}, {1, 2}, {1, 3}, and {1, 2, 3} is α, β, α, and γ, respectively, which can also be found as the difference between the columns (V) and (VII) of Table 6.5 (see column (VIII) of Table 6.5). The incremental benefit of riparian 1 for each coalition scenario has to be multiplied with the realization probability of each coalition scenario, which can be found by the product of columns (IV) and (VIII) in Table 6.5 (see column (IX) in Table 6.5). By summing up all these weighted incremental benefits of riparian 1, we get the Shapley solution of riparian 1 which is $\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}$.

This procedure could be applied for riparians 2 and 3, analogously.

Therefore, it is possible to formulate the following payment vector as the focal point solution for the bargaining problem:

$$x^{SH} = \left(x_1^{SH}\ x_2^{SH}\ x_3^{SH}\right) = \left(\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}\ \frac{\beta+2\cdot\gamma-3\cdot\alpha}{6}\ \frac{\gamma-\beta}{3}\right)$$

Table 6.5 Calculation of Shapley value for simple river basin example

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
Riparian i	Coalition	$\#ISG$	$\frac{(\#G-\#ISG)!\cdot(\#ISG-1)!}{\#G!}$	$V(\ldots)$	Coalition without i	$V(\ldots - i)$	(V) - (VII)	(IV) · (VIII)	Shapley Value
User 1	{1}	1	$\frac{1}{3}$	α	Ø	0	α	$\frac{1}{3}\cdot\alpha$	$\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}$
	{1, 2}	2	$\frac{1}{6}$	β	{2}	0	β	$\frac{1}{6}\cdot\beta$	
	{1, 3}	2	$\frac{1}{6}$	α	{3}	0	α	$\frac{1}{6}\cdot\alpha$	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{2, 3}	0	γ	$\frac{1}{3}\cdot\gamma$	
User 2	{2}	1	$\frac{1}{3}$	0	Ø	0	0	0	$\frac{\beta+2\cdot\gamma-3\cdot\alpha}{6}$
	{1, 2}	2	$\frac{1}{6}$	β	{1}	α	$\beta-\alpha$	$\frac{\beta-\alpha}{6}$	
	{2, 3}	2	$\frac{1}{6}$	0	{3}	0	0	0	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{1, 3}	α	$\gamma-\alpha$	$\frac{\gamma-\alpha}{3}$	
User 3	{3}	1	$\frac{1}{3}$	0	Ø	0	0	0	$\frac{\gamma-\beta}{3}$
	{1, 3}	2	$\frac{1}{6}$	α	{1}	α	0	0	
	{2, 3}	2	$\frac{1}{6}$	0	{3}	0	0	0	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{1, 2}	β	$\gamma-\beta$	$\frac{\gamma-\beta}{3}$	

It becomes obvious that

$$x_1^{SH} > x_2^{SH} > x_3^{SH}$$

which means that the upstream riparian 1 receives the highest benefits, while the downstream riparian receives the lowest payoffs, which is reasoned by the hydrological power of the respective riparians. The more upstream the riparian is located in the basin, the earlier the riparian is able to abstract (or control the water amounts), which makes an upstream riparian more (hydrological) powerful than the downstream one. For instance, a coalition with just riparians 2 and 3 generates a benefit of 0. When riparian 1 joins this arrangement, which means that the grand coalition would be formed, it would generate the benefit of γ. Therefore, the incremental benefit of riparian 1 for this coalition is γ. The upstream riparian receives a higher proportion of the generated benefit than the downstream riparian.

6.3.2.3 The Nash-Harsanyi Solution

The Nash-Harsanyi solution maximizes the product of assigned benefits in excess to the benefits generated in the noncooperative case (Harsanyi 1958). Furthermore, the feasibility, Pareto-efficiency, individual rationality, and group rationality are also addressed, hence it is guaranteed that the solution is within the core. Therefore, the following general optimization problem can be formulated for finding the Nash-Harsanyi solution:

$$
\begin{aligned}
&\max \left[\prod_i (x_i - V(I))\right] && && \\
&s.t.\ \sum_i [x_i] = V(G) && && \text{(Feasibility and Pareto-Efficiency)} \\
&\quad x_i \geq V(I) && \forall I && \text{(Individual rationality)} \\
&\quad \sum_{i \in S} [x_i] \geq V(S) && \forall S && \text{(Group rationality)}
\end{aligned}
$$

which is for the presented river basin example:

$$
\begin{aligned}
&\max\,[(x_1 - \alpha) \cdot x_2 \cdot x_3] && \\
&s.t.\ [x_1 + x_2 + x_3] = \gamma && \text{(Feasibility and Pareto-efficiency)} \\
&\quad x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0 && \text{(Individual rationality)} \\
&\quad x_1 + x_2 \geq \beta\ ,\ x_1 + x_3 \geq \alpha\ ,\ x_2 + x_3 \geq 0 && \text{(Group rationality)}
\end{aligned}
$$

When applying this solution procedure for the presented river basin example, the following optimality conditions can be found:

$$
\begin{aligned}
&x_2 \cdot x_3 = (x_1 - \alpha) \cdot x_3 = (x_1 - \alpha) \cdot x_2 \\
&x_1 + x_2 + x_3 = \gamma
\end{aligned}
$$

Table 6.6 Additional benefits for Nash-Harsanyi solution in the Grand Coalition for the simple river basin example

	Upstream riparian (User 1)	Midstream riparian (User 2)	Downstream riparian (User 3)
Nash-Harsanyi solution	$\frac{1}{3} \cdot (2 \cdot \alpha + \gamma)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$
Benefit for noncooperative acting	α	0	0
Additional benefits in grand coalition	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$

The equation $x_2 \cdot x_3 = (x_1 - \alpha) \cdot x_3 = (x_1 - \alpha) \cdot x_2$ can be reformulated to the following expression:

$$x_1 - \alpha = x_2 = x_3$$

which is nothing else than

$$x_1 - V(\{1\}) = x_2 - V(\{2\}) = x_3 - V(\{3\})$$

Hence, the assigned benefit in excess of the respective noncooperative benefit is equal for each riparian, which is a typical characteristic of the Nash-Harsanyi solution. This characteristic results mainly from the objective function of the Nash-Harsanyi optimization problem.

The concrete Nash-Harsanyi solution of the presented river example is

$$x^{NH} = \left(x_1^{NH}\ x_2^{NH}\ x_3^{NH}\right) = \left(\tfrac{1}{3} \cdot (2 \cdot \alpha + \gamma)\ \tfrac{1}{3} \cdot (\gamma - \alpha)\ \tfrac{1}{3} \cdot (\gamma - \alpha)\right)$$

Therefore, the additional benefit in the grand coalition compared to the noncooperative case is the same for every player $\frac{1}{3} \cdot (\gamma - \alpha)$, which is also illustrated by Table 6.6.

For the unilateral acting case, the upstream riparian generates the highest benefit with the level α, while the mid- and downstream ones do not receive any water and therefore generate a benefit of 0. Hence, when applying the Nash-Harsanyi solution, the upstream riparian receives the highest payoffs while the mid- and downstream users receive equal payoffs:

$$x_1^{NH} > x_2^{NH} = x_3^{NH}$$

The total benefit in the basin when all riparians act unilaterally is α, because $V(\{1\}) + V(\{2\}) + V(\{3\}) = \alpha$, while the basin's benefit in case of a grand coalition is γ, because $V(\{1, 2, 3\}) = \gamma$. Therefore, the term $\gamma - \alpha$ can be interpreted as the cooperation gain in the basin. The Nash-Harsanyi solution shares this cooperation gain equally among the riparians, hence each riparian obtains the benefit $\dfrac{\gamma - \alpha}{3}$ in excess to those benefits the riparian would generate when acting unilaterally.

Table 6.7 Objection against the Shapley solution in the Grand Coalition

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff in {1, 2, 3} based on Shapley solution	$\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}$	$\frac{\beta+2\cdot\gamma-3\cdot\alpha}{6}$	$\frac{\gamma-\beta}{3}$	$\frac{\beta+2\cdot\gamma}{3}$	$\frac{3\cdot\alpha-\beta+4\cdot\gamma}{6}$	$\frac{4\cdot\gamma-3\cdot\alpha-\beta}{6}$
Objections of coalition against Shapley solution	$\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6}$	$\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6}$	$\frac{\beta-\gamma}{3}$	$\frac{2\cdot\beta-2\cdot\gamma}{3}$	$\frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}$	$\frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}$

6.3.2.4 The Nucleolus

The main goal of the nucleolus is to find a solution in which the maximum objection against a benefit sharing solution is minimized. This concept was first presented by Schmeidler (1969). The objection of a coalition against a benefit sharing solution results from the difference between the generated benefit of this coalition (if it was formed in the basin) and the payoff for this coalition due to the benefit sharing solution.[15]

For instance, the objections of the various cooperation constellations against the Shapley solution in the grand coalition are illustrated in Table 6.7.

The maximum objection against the Shapley solution is therefore

$$\max\left[\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6};\ \frac{\beta-\gamma}{3};\ \frac{2\cdot\beta-2\cdot\gamma}{3};\ \frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}\right]=\frac{\beta-\gamma}{3}$$

For minimizing the maximum objection under the consideration of the feasibility, Pareto-efficiency, individual rationality, and group rationality conditions, the following general optimization problems have to be solved to find the nucleolus solution (see Wang et al. (2003)):

[15]For the presented river basin example, the objections of the coalition constellations are

- Objection of riparian 1, $\{1\}:\ V(\{1\})-x_1$
- Objection of riparian 2, $\{2\}:\ V(\{2\})-x_2$
- Objection of riparian 3, $\{3\}:\ V(\{3\})-x_3$
- Objection of coalition $\{1,2\}:\ V(\{1,2\})-x_1-x_2$
- Objection of coalition $\{1,3\}:\ V(\{1,3\})-x_1-x_3$
- Objection of coalition $\{2,3\}:\ V(\{2,3\})-x_2-x_3$

$$\min\,[e]$$
$$s.t.\ \sum_i [x_i] = V(G) \qquad \text{(Feasibility and Pareto-efficiency)}$$
$$e + x_i \geq V(I) \qquad \forall I \qquad \text{(Individual rationality)}$$
$$e + \sum_{i \in S} [x_i] \geq V(S) \qquad \forall S \qquad \text{(Group rationality)}$$

which for the presented river basin example is equivalent to the following formulation:

$$\min\,[e]$$
$$s.t.\ [x_1 + x_2 + x_3] = \gamma \qquad \text{(Feasibility and Pareto-efficiency)}$$
$$e + x_1 \geq \alpha\ ,\ e + x_2 \geq 0\ ,\ e + x_3 \geq 0 \qquad \text{(Individual rationality)}$$
$$e + [x_1 + x_2] \geq \beta\ ,\ e + [x_1 + x_3] \geq \alpha\ ,\ e + [x_2 + x_3] \geq 0 \qquad \text{(Group rationality)}$$

The maximum objection against a benefit sharing solution is represented by the variable e. This variable e is a free variable, therefore, it is defined within the domain $[-\infty, \infty]$. If the variable e becomes zero or negative, the individual and group rationality conditions are certainly fulfilled. However, regardless of the value of variable e, the feasibility and Pareto-efficiency conditions are always met. The objective of the optimization problem sets the goal to minimize the value of e, which means that the maximum objection against the benefit sharing solution has to be minimized. As we already discussed, this is the main motivation of the nucleolus solution concept. The value of the variable e can not be set arbitrarily low, because the lowest possible value of e is restricted by the individual and group rationality conditions. If the superadditivity condition is met, the variable e takes a positive value. Therefore, the individual and group rationality conditions are fulfilled.

When applying the optimization problem to identify the nucleolus solution, we have to differentiate between two cases of parameter specifications α, β, and γ:

- Case 1: We specify β and γ such that $\beta \leq \frac{\gamma}{3}$. However, if in contrast $\beta > \frac{\gamma}{3}$, this case is also relevant for the specification $\frac{3 \cdot \beta - \gamma}{2} \leq \alpha$. Therefore, the compact description of this case is $(\beta \leq \frac{\gamma}{3}) \vee ((\frac{\gamma}{3} < \beta) \wedge (\frac{3 \cdot \beta - \gamma}{2} \leq \alpha))$.
- Case 2: This case becomes relevant when α, β, and γ are specified such that case 1 does not fit. Therefore, we know that $\frac{\gamma}{3} < \beta$. Furthermore, we also know that $\alpha < \frac{3 \cdot \beta - \gamma}{2}$. Hence, the compact description of this case is $(\frac{\gamma}{3} < \beta) \wedge (\alpha < \frac{3 \cdot \beta - \gamma}{2})$.

The optimality conditions which result from the application of the optimization problem are

$$\text{For case 1: } e = \alpha - x_1 = -x_2 = -x_3$$
$$\text{For case 2: } e = \beta - x_1 - x_2 = -x_3$$
$$\text{For cases 1 and 2: } x_1 + x_2 + x_3 = \gamma$$

Table 6.8 Objection against the nucleolus solution in the Grand Coalition (for case 1)

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff in coalitions based on nucleolus solution	$\frac{2\cdot\alpha+\gamma}{3}$	$\frac{\gamma-\alpha}{3}$	$\frac{\gamma-\alpha}{3}$	$\frac{\alpha+2\cdot\gamma}{3}$	$\frac{\alpha+2\cdot\gamma}{3}$	$\frac{2\cdot\gamma-2\cdot\alpha}{3}$
Objections against nucleolus solution	$\frac{\alpha-\gamma}{3}$	$\frac{\alpha-\gamma}{3}$	$\frac{\alpha-\gamma}{3}$	$\frac{3\cdot\beta-\alpha-2\cdot\gamma}{3}$	$\frac{2\cdot\alpha-2\cdot\gamma}{3}$	$\frac{2\cdot\alpha-2\cdot\gamma}{3}$

For *case 1*, it follows from the optimality conditions that the unilaterally acting riparians, denoted by {1}, {2} and {3}, state the maximum objections against the proposed nucleolus solution. Therefore, the level of the variable e which stands for the maximum objection is limited in its lowest value by the individual rationality conditions. The nucleolus solution for this case is

$$x^{nuc,1} = \left(x_1^{nuc,1} \; x_2^{nuc,1} \; x_3^{nuc,1}\right) = \left(\frac{2\cdot\alpha+\gamma}{3} \; \frac{\gamma-\alpha}{3} \; \frac{\gamma-\alpha}{3}\right)$$

This solution is equal to the Nash-Harsanyi solution. This means that the equal sharing of cooperation gains between the riparians (which is done by the Nash-Harsanyi solution) minimizes the maximum objections which are stated by the unilaterally acting riparians. The objections under this case 1 are listed in detail in Table 6.8.

The maximum objection under this case 1 is

$$e = \max\left[\frac{\alpha-\gamma}{3}\,;\; \frac{3\cdot\beta-\alpha-2\cdot\gamma}{3}\,;\; \frac{2\cdot\alpha-2\cdot\gamma}{3}\right] = \frac{\alpha-\gamma}{3}$$

which is stated by the unilaterally acting riparians 1, 2, and 3. Of course, this maximum objection determines the value of the variable e.

Under *case 2*, the sub-coalition between the riparians 1 and 2, denoted by {1, 2} and the unilaterally acting riparian 3, represented by {3}, state the maximum objection against the nucleolus solution which becomes apparent by the relevant optimality conditions. Therefore, the level of the variable e is limited in its lowest value by the group rationality of coalition {1, 2} as well as by the individual rationality of riparian 3.

Based on the assumption under this case 2 ($e = \beta - x_1 - x_2 = -x_3$) as well as the formerly presented optimality conditions, the following relations are valid:

$$e = \beta - x_1 - x_2 = -x_3$$
$$x_1 + x_2 + x_3 = \gamma$$
$$e + x_1 \geq \alpha,\; e + x_2 \geq 0,\; e + x_1 + x_3 \geq \alpha,\; e + x_2 + x_3 \geq 0$$

When solving this problem, we can find an explicit solution for x_3, which is

$$x_3^{nuc,2} = \frac{\gamma - \beta}{2}$$

Furthermore, we find that the sub-coalition containing riparians 1 and 2 has to receive a payoff in the level:

$$x_1^{nuc,2} + x_2^{nuc,2} = \frac{\beta + \gamma}{2}$$

However, there is no concrete solution regarding the assignment of benefits to riparians 1 and 2. However, for meeting the optimality conditions, we know that the solutions of x_1 and x_2 have to be in the following intervals:

$$x_1^{nuc,2} = \left[x_1^{nuc,2,min} \ , \ x_1^{nuc,2,max}\right] = \left[\frac{2 \cdot \alpha - \beta + \gamma}{2} \ , \ \beta\right]$$
$$x_2^{nuc,2} = \left[x_2^{nuc,2,min} \ , \ x_2^{nuc,2,max}\right] = \left[\frac{\gamma - \beta}{2} \ , \ \beta - \alpha\right]$$

Of course, we would like to have a focal point solution, which means that we want to find an explicit assignment of payoffs for riparians 1 and 2.

A possible opportunity for assigning the payoffs is just to apply the nucleolus procedure for the sub-coalition $\{1, 2\}$.[16]

In Exercise 6.2, we present the solution steps of the nucleolus procedure for a basin with 2 riparians in detail. Every riparian has to receive the benefit it would get when acting unilaterally (individual rationality condition). On top of that, the riparians get a share of the cooperation gain, which results from the difference between the payoffs the sub-coalition would receive in the nucleolus solution , which is $0.5 \cdot (\beta + \gamma)$, and the sum of the benefits the riparians would receive when acting unilaterally which is

[16]The optimization problem for finding the nucleolus solution of the sub-coalition $\{1, 2\}$ is

$$\min_{\{e,x_1,x_2\}} [e]$$
$$s.t. \ x_1 + x_2 = 0.5 \cdot (\beta + \gamma)$$
$$e + x_1 \geq \alpha$$
$$e + x_2 \geq 0$$

Table 6.9 Objection against the nucleolus solution (case 2)

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff based on nucleolus solution	$\frac{2\cdot\alpha+\beta+\gamma}{4}$	$\frac{\beta+\gamma-2\cdot\alpha}{4}$	$\frac{\gamma-\beta}{2}$	$\frac{\beta+\gamma}{2}$	$\frac{2\cdot\alpha-\beta+2\cdot\gamma}{4}$	$\frac{3\cdot\gamma-2\cdot\alpha-\beta}{4}$
Objections of coalition against nucleolus solution	$\frac{2\cdot\alpha-\beta-\gamma}{4}$	$\frac{2\cdot\alpha-\beta-\gamma}{4}$	$\frac{\beta-\gamma}{2}$	$\frac{\beta-\gamma}{2}$	$\frac{2\cdot\alpha+\beta-2\cdot\gamma}{4}$	$\frac{2\cdot\alpha+\beta-3\cdot\gamma}{4}$

α. When applying the nucleolus procedure in a coalition or basin with 2 riparians, we have to share the cooperation gain equally between the riparians.[17]

Therefore, we can set the following payoffs for the riparians 1 and 2:

$$x_1^{nuc,2} = \frac{2\cdot\alpha+\beta+\gamma}{4}$$
$$x_2^{nuc,2} = \frac{\beta+\gamma-2\cdot\alpha}{4}$$

Hence, the nucleolus solution for this case is

$$x^{nuc,2} = \left(x_1^{nuc,2}\ x_2^{nuc,2}\ x_3^{nuc,2}\right) = \left(\tfrac{2\cdot\alpha+\beta+\gamma}{4}\ \tfrac{\beta+\gamma-2\cdot\alpha}{4}\ \tfrac{\gamma-\beta}{2}\right)$$

The objections under case 2 are listed in detail in Table 6.9.

The maximum objection is:

$$e = \max\left[\frac{2\cdot\alpha-\beta-\gamma}{4};\ \frac{\beta-\gamma}{2};\ \frac{2\cdot\alpha+\beta-2\cdot\gamma}{4};\ \frac{2\cdot\alpha+\beta-3\cdot\gamma}{4}\right] = \frac{\beta-\gamma}{2}$$

[17]In case of unilateral acting, riparians 1 and 2 generate a benefit of α, i.e., $V(\{1\}) + V(\{2\}) = \alpha$. The sub-coalition between riparians 1 and 2 should receive a benefit of $0.5\cdot(\beta+\gamma)$ in the nucleolus solution, i.e., $x_1^{nuc,2} + x_2^{nuc,2} = 0.5\cdot(\beta+\gamma)$. Therefore, the cooperation gain is

$$\Delta = x_1^{nuc,2} + x_2^{nuc,2} - V(\{1\}) - V(\{2\}) = 0.5\cdot(\beta+\gamma) - \alpha$$

Half of the cooperation gain is $0.5\cdot\Delta = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha$. Therefore, the riparians receive the following benefits:

$$x_1^{nuc,2} = V(\{1\}) + 0.5\cdot\Delta = \alpha + 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha = 0.5\cdot\alpha + 0.25\cdot(\beta+\gamma)$$
$$x_2^{nuc,2} = V(\{2\}) + 0.5\cdot\Delta = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha$$

The maximum objection $e = \frac{\beta-\gamma}{2}$ is stated by the unilaterally acting riparian 3 and the sub-coalition between riparians 1 and 2.

Because of the hydrological power in the river basin, which results in the highest payoff for riparian 1 due to its position in the river basin, and the fact that the sub-coalition $\{1, 2\}$ states the maximum objection, the payoff for riparian 2 exceeds the one of riparian 3, hence

$$x_1^{nuc,2} > x_2^{nuc,2} > x_3^{nuc,2}$$

Box 6.1 Benefit sharing in the Nile river basin

With a total length of 6700 km, the Nile is the longest river in the world; its basin stretches over 11 countries: Egypt, Sudan, South Sudan, Ethiopia, Uganda, Kenya, Tanzania, Burundi, Rwanda, Democratic Republic Congo, and Eritrea. Similar to other international rivers (such as Euphrates and Tigris, Syr Darya, Amu Darya, and Ganges), there is a gap between the water quantity available in the basin and the water quantity claimed by the riparians. (Wu and Whittington 2006) Therefore, the riparians compete for the scarce water sources in the river. Wu and Whittington (2006) state a water defecit of about 50 billion cubic meter per year in the basin. Egypt which is the most downstream country in the basin contributes essentially nothing to the flow of the Nile, however, it currently consumes more than 80% of the Nile water due to its political and military power in the region (Wu and Whittington 2006). Ethiopia which is located in the upstream of the basin contributes over 85% of the water flow. It claims significantly more water resources than its current consumption for realizing its dam and irrigation project which became necessary to meet the needs of an increasing population.

There exist a number of Nile river models in the scientific literature which are explained in detail by, for instance, Nigatu and Dinar (2011). The Nile Economic Optimization Model (NEOM, see Wu (2000)) is a basin-wide economic optimization model which quantifies the economic benefits from water usage. Wu and Whittington (2006) use the NEOM to study conflict incentive-compatible resolution strategies based on various cooperation scenarios in the basin. Block and Strzepek (2010) set up the Investment Model for Planning Ethiopian Nile Development (IMPEND) which focuses on the impact of dams constructed for irrigation and hydropower purposes. The model which is applied by Nigatu and Dinar (2011) as well as by Dinar and Nigatu (2013) is based on the NEOM model and takes into account additional features such as the resource degradation, various climate change scenarios, and the possibility of introducing basin-wide water trade. It just focuses on the basin of the Blue Nile which involves the countries Ethiopia, Sudan, and Egypt.

Dinar and Nigatu (2013) distinguish different scenarios of allocation between the riparians in the Blue Nile. The scenario WRA-I, which allocates 12.2, 22.0, and 65.8%, respectively, to Ethiopia, Sudan, and Egypt, is based on the notion of Egypt's long-term use pattern. The scenario WRA-II, which allocates 38.4, 14.1, and 47.5% to Ethiopia, Sudan, and Egypt, respectively, is based on the notion of equitable access as reflected in the UN Water convention from 1997. The generated benefit for the different allocation scenarios are listed in the following table, with ETH, SDN, and EGY representing Ethiopia, Sudan, and Egypt, respectively. In cases of cooperation, the allocated water amounts are traded to another country of the cooperation scheme if increasing benefits result from this transfer. Therefore, the highest basin's benefit is generated for all allocation scenarios in the cooperation arrangement which involves all three riparians. The highest basin's benefit with 9.21 is generated under the allocation scenario WRA-II, followed by the allocation scenario WRA-I with 8.77.

Benefits under different allocation and cooperation scenarios. *Source* Dinar and Nigatu (2013)

Allocation Scenario	Unilateral Acting			Sub-Coalitions			Grand Coalition
	ETH	SDN	EGY	ETH+SDN	ETH+EGY	SDN+EGY	ETH+SDN+EGY
WRA-I	1.29	2.62	4.83	3.94	5.71	7.55	8.77
WRA-II	2.21	2.56	3.91	4.62	6.60	6.80	9.21

Assuming the three riparians form a joint arrangement, we focus on the question of how to allocate the common benefit to the individual riparians. This means that for the scenarios WRA-I and WRA-II, the basin's benefits of 8.77 and 9.21, respectively, have to be allocated to the riparians. The incremental benefit for any riparian results from the difference between its received benefit in the joint arrangement and the benefit which the riparian would generate when acting unilaterally. Applying the Nash-Harsanyi solution, the benefits are allocated in a way that the incremental benefits become equal for all riparians. For the scenario WRA-I, the benefits when acting unilaterally for Ethiopia, Sudan, and Egypt are 1.29, 2.62, and 4.83, respectively, which results in the basin's benefit of 8.74. The basin's benefit in the joint arrangement is 8.77, hence when applying the Nash-Harsanyi solution, the incremental benefit of every riparian becomes 0.01, because $\frac{8.77-8.74}{3} = 0.01$. This means that Ethiopia, Sudan, and Egypt are assigned a benefit of 1.30, 2.63, and 4.84, respectively, for the Nash-Harsanyi solution. For scenario WRA-II, acting unilaterally holds benefits of Ethiopia, Sudan and Egypt, respectively, which results in a basin's benefit of 8.68. The basin's benefit in the joint arrangement is 9.21. Hence, the incremental benefit of each riparian amounts to about 0.18 ($\frac{9.21-8.68}{3} \approx 0.18$) when applying the Nash-Harsanyi solution. Hence Ethopia, Sudan and Egypt receive a benefit of 2.39, 2.74 and 4.08, respectively. The Shapley and Nash-Harsanyi solution for the Benefit Sharing problem of the Blue Nile basin is illustrated in the following table.

Shapley and Nash-Harsanyi solution under different allocation scenarios

Allocation Scenario	Shapley			Nash-Harsanyi		
	ETH	SDN	EGY	ETH	SDN	EGY
WRA-I	1.20	2.79	4.78	1.30	2.63	4.84
WRA-II	2.33	2.61	4.27	2.39	2.74	4.08

Regardless of the allocation scenario, the upstream riparian Ethiopia prefers the Nash-Harsanyi to the Shapley solution, because 1.3 > 1.2 and 2.39 > 2.33. For the allocation scenario WRA-I, Sudan and Egypt prefer the Shapley (2.79 > 2.63) and Nash-Harsanyi solution (4.84 > 4.78), respectively, while for the other allocation scenario WRA-II, the contrary situation becomes obvious because Sudan and Egypt prefer the Nash-Harsanyi (2.74 > 2.61) and Shapley solution (4.27 > 4.08), respectively. The Shapley solution of the allocation scenario WRA-I is not in the core, because the assigned benefits to Ethiopia and Egypt with 1.20 and 4.78, respectively, are lower than the benefits Ethiopia and Egypt would generate under unilateral acting which are 1.29 and 4.83, respectively (see Table 6.10). Therefore, the Shapley solution for the allocation scenario WRA-I violates the individual rationality of Ethiopia and Egypt. However, the Shapley solution of the allocation scenario WRA-II is within the core. The Nash-Harsanyi solution is also in the core for both the scenarios WRA-I and WRA-II.

Source Dinar and Nigatu (2013)

6.3.3 Concluding Remarks on the Benefit Sharing Problem

The benefit sharing problem focuses on the question of how to assign benefits to riparians which are generated in a joint arrangement. The assigned benefit in this context is termed as payoff or imputation. The first important concept is the core which contains all payoffs which meet the feasibility, Pareto-efficiency, individual and group rationality conditions. Due to the feasibility and Pareto-efficiency conditions, the generated benefit in the joint arrangement has to be assigned to the riparians of this arrangement in total. The individual rationality condition means that any riparian of the joint arrangement has to receive at least as much payoffs as it would generate when acting unilaterally in a noncooperative way. However, the group rationality condition means that a coalition which could be formed by a subset of riparians acting cooperatively in the joint arrangement must receive at least as much benefits in the joint arrangement as it would generate if the respective coalition was formed in the basin. Usually, either the core is empty—which means that it is not incentive-compatible to form this joint arrangement—or there are a multitude of payoffs in the core. Therefore, we discussed the lower and upper bounds of the core for each riparian. The lower bound of each riparian is the minimum payment the respective riparian has to receive to have an economic incentive to join the cooperative arrangement, while the upper bound is the maximum payment

Table 6.10 Payoffs for riparians regarding the presented focal point solution concepts

	Rriparian 1	Riparian 2	Riparian 3
Shapley solution	$\frac{1}{6} \cdot (3 \cdot \alpha + \beta + 2 \cdot \gamma)$	$\frac{1}{6} \cdot (\beta + 2 \cdot \gamma - 3 \cdot \alpha)$	$\frac{1}{3} \cdot (\gamma - \beta)$
Nash-Harsanyi solution	$\frac{1}{3} \cdot (2 \cdot \alpha + \gamma)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$
Nucleolus (case 2)	$\frac{1}{4} \cdot (2 \cdot \alpha + \beta + \gamma)$	$\frac{1}{4} \cdot (\beta + \gamma - 2 \cdot \alpha)$	$\frac{1}{2} \cdot (\gamma - \beta)$

the respective riparian would get without setting an incentive for another riparian to leave the cooperative arrangement.

Furthermore, we also discussed some focal point solution concepts which state concrete solutions (a specific payment for each riparian): The Shapley value may be in the core, while the Nash-Harsanyi and nucleolus solutions are certainly in the core when the superadditivity condition is fulfilled. The Shapley value assigns the benefits depending on the (weighted) incremental benefit of the respective riparian for the coalitions. Particularly powerful riparians, e.g., those riparians that are located upstream in the river basin and, therefore, have hydrological power, benefit from this concept. The more the benefit in a coalition increases due to the joining of the respective riparian in this coalition—which is nothing else than the incremental benefit of the respective riparian for the coalition—the higher the proportion of benefits of the joint arrangement that goes to this respective riparian. However, the Nash-Harsanyi and nucleolus solutions do not focus as much on the power situation of a riparian, but more on aspects of justice. The Nash-Harsanyi solution allocates the benefits to the riparians in a way that the assigned benefit in excess to the respective noncooperative benefit is equal for each riparian. This means that the cooperation gain is shared equally between the riparians. The nucleolus is a solution concept in which the maximum objection against a payment solution is minimized. The objection of a coalition against a benefit sharing solution results from the difference between the generated benefit of this coalition (if it was formed in the basin) and the payoff for this coalition due to the benefit sharing solution.

For the presented river basin example, the realized benefit sharing solutions are illustrated in Table 6.10. Please note that we differentiate between two cases of parameter specification regarding α, β, and γ. The nucleolus and Nash-Harsanyi solutions just differ for the second case. For the first case, the nucleolus solution and Nash-Harsanyi solutions are the same.

The upstream riparian prefers the Shapley solution, because due to its upstream position (hydrological power) it generates high incremental benefit for the possible coalitions. For example, a coalition with just riparians 2 and 3 generates a benefit of 0. If riparian 1 joined this arrangement, it would generate the benefit of γ. Hence, the incremental benefit of riparian 1 for this coalition is γ. This relation illustrates why riparian 1 gets such a high proportion of benefit when applying the Shapley value.

Case 1: $x_1^{SH} > x1^{NH}$ Case 2: $x_1^{SH} > x_1^{nuc,2} > x_1^{NH}$

The midstream riparian also prefers the Shapley solution, because the Nash-Harsanyi solution and nucleolus solution are based on the low benefit level of 0, if this riparian acted unilaterally.

$$\text{Case 1: } x_2^{SH} > x_2^{NH} \qquad \text{Case 2: } x_2^{SH} > x_2^{nuc,2} > x_2^{NH}$$

The downstream riparian has the lowest hydrological power due to its position in the basin. Because of its limited hydrological power, the riparian has the highest aversion against the Shapley solution, while it prefers the Nash-Harsanyi solution:

$$\text{Case 1: } x_3^{NH} > x_3^{SH} \qquad \text{Case 2: } x_3^{NH} > x_3^{nuc,2} > x_3^{SH}$$

In the grand coalition, the benefit of the entire basin which is γ is generated by the water consumption of the downstream riparian. The less productive riparians 1 and 2 leave the water in the river, hence, they generate no benefit in the grand coalition. However, they have to receive payoffs for all presented benefit sharing solutions. Therefore, riparian 3 has to make side payments:

$$\text{Side payments made by riparian } 3 = \gamma - x_3$$

which means for the focal point solution concepts:

$$\text{Side payments made by riparian } 3 = \begin{cases} \frac{\beta+2\cdot\gamma}{3} & \text{Shapley solution} \\ \frac{\alpha+2\cdot\gamma}{3} & \text{Nash-Harsanyi and nucleolus solutions} \end{cases}$$

Riparians 1 and 2 receive the following side payments from riparian 3 in the level of their respective benefit sharing solution:

$$\text{Side payments received by riparian } 1 = x_1$$
$$\text{Side payments received by riparian } 2 = x_2$$

6.4 Bankruptcy Rules for Water Allocation

6.4.1 Principles of Bankruptcy Rules

The last two sections were based on the construction of monetary or utility-measured characteristic functions of cooperative game theory. Now, we turn to the so-called bankruptcy methods that distribute scarce water quantities *directly* to riparian states without calculating the economic value they create. Thus, the distributandum is not the monetary benefit, but the water itself. These methods have been developed in a completely different context: If a firm goes bankrupt, how should the residual liquidated wealth be distributed among its creditors? There is a plethora of rules and principles on how to allocate the insolvency assets to the creditors.[18] Should the residual assets be divided, for example, proportionally or equally? Suppose the first creditor lent 200 € to the company, a second 100 €. However, the remaining goodwill is only 200 €, so not all claims are covered. If we apply the proportional rule, then the first creditor receives two-thirds of the residual value, i.e., 133 €, while the second creditor receives only one third, i.e., 67 €. It is also possible to allocate equal shares to both creditors, in which case both would receive 100 €. Hence, it is necessary to develop bankruptcy rules according to which the residual value is distributed.

These methods have been applied to transboundary water issues as well, both theoretically and empirically (Box 6.2). The application of the bankruptcy rules is based on the Principle of the Territorial Integration of all Basin States (TIBS) as explained in Sect. 6.1. The whole catchment area is collectively owned by the riparian countries. If water becomes scarce, the allocation should not be based on the geographical position of these countries. Their claims are equally legitimate and are the sole information that will be taken into account within the allocation process.

On the one hand, it is advantageous that we do not need any complex models that reflect the link between water usage and economic welfare. It is all about water quantities that are measurable. On the other hand, by restricting water distribution alone, we give up the possibility of combined contracts in which other goods and services are specified in addition to water, which makes trade possible. As will be shown subsequently, one has to be careful when transferring these rules from the context of credit markets to water issues, because the very nature of claims in both sectors is rather different.

Suppose a set of $N = \{1, 2, 3, \ldots, n\}$ countries share a water resource R. Their claims can be summarized by a claim vector $c = \begin{bmatrix} c_1 \; c_2 \; \ldots \; c_n \end{bmatrix}$. Water is scarce, hence, we assume that

$$\sum_{i=1}^{n} c_i > R \tag{6.17}$$

Bankruptcy rules specify the allocation of R to the countries by a sharing rule function $x(R, c)$, where x is an n-dimensional vector. There is a variety of properties that are met by different distribution rules to varying degrees. These properties refer to consistency criteria that follow the principle of rationality and to normative criteria that take fairness considerations into account.

Basic properties are the following requirements that represent plausibility:

1. *Feasibility*: The implementation of bankruptcy rules must be feasible, i.e., the sum of water allotments required by the rules should not exceed the amount of

[18] A concise survey is Thomson (2002).

water available, i.e.,

$$\sum_{i=1}^{n} x_i(R, c) \leq R \tag{6.18}$$

2. *Efficiency*: Efficiency excludes waste. There is no water loss, i.e.,

$$\sum_{i=1}^{n} x_i(R, c) = R \tag{6.19}$$

3. *Claim boundedness*: Claim boundedness is not only a basic property that relates to plausibility but also to fairness considerations. The water allotment of a rule shall never exceed the claim stated, i.e.,

$$x(R, c) \leq c \tag{6.20}$$

A rationing scheme would be considered very unfair if it were to allocate more water than the claim of the respective riparian state.

There are additional, more specific properties to make a bankruptcy rule considerable for implementation. These rules refer to fairness considerations, and what the Helsinki Rules (Article 4) call "a reasonable and equitable share in the beneficial uses of the waters of an international drainage basin".

4. *Consistency*: Consistency refers not only to rationality but also to fairness. A water allocation which is considered as fair for all countries in a water treaty remains fair also if a subgroup shares the water allotted to them. Formally,

$$\text{for all } S \subset N \;\; x_i(R, c) = x_i\left(R - \sum_{i \in (N/S)} x_i, c_S\right) \tag{6.21}$$

where c_S is the vector of claims of the countries in S.
5. *Equal Treatment of Equals:* This condition is central to a fair water allocation. The same claims should lead to the same water allocation. Formally,

$$\text{for all } i, j \in N \text{ if } c_i = c_j \Rightarrow x_i(R, c) = x_j(R, c) \tag{6.22}$$

6. *Order Preservation*: Fairness also requires that those countries claiming more water receive more water under the sharing rule.

$$\text{for all } i, j \in N \text{ if } c_i \geq c_j \Rightarrow x_i(R, c) \geq x_j(R, c) \tag{6.23}$$

Of course, this requirement presumes that claims are legitimate and justifiable.

7. *Regressivity*: If a certain degree of inequity aversion prevails, regression may be required:

$$\text{for all } i, j \in N \text{ if } c_i \geq c_j \Rightarrow \frac{x_i(R, c)}{c_i} \leq \frac{x_j(R, c)}{c_j} \tag{6.24}$$

In other words, the relative fulfillment of the claims decreases with the amount of the claims. Whether this criterion makes sense depends heavily on the nature of the claims.

8. *Claim monotonicity*: Division rules should not be static. This means that they should not only refer to actual values of claims and water quantities but also be flexible with regard to changes in framework conditions. Fairness must also apply to changed input data. It is fair to say that the allocation of water resources increases for those riparian states whose justified claims increase. Formally,

$$\text{for all } i, j \in N \text{ if } c_i' \geq c_i \Rightarrow x_i(R, c_i', c_{-i}) \geq x_i(R, c) \tag{6.25}$$

where $c_{-i} = [c_1, c_2, c_{i-1}, c_{i+1}, c_n]$.

9. *Resource monotonicity*: The supply of water varies heavily depending on weather and climate conditions. The distribution rules must be fair for all possible water scarcity scenarios. Resource monotonicity is considered as fair. If there is less (more) water, every riparian state should get less (more):

$$\text{if } R' \geq R \Rightarrow x(R', c) \geq x(R, c) \tag{6.26}$$

6.4.2 Hydrologically Unconstrained Allocation Rules

The fulfillment of the individual properties defined above does not determine a unique allocation. In the literature, it is rather the case that different rules, seen as reasonable and fair, are proposed. The application of these rules leads to different allocations. In the following, we will present the most important ones, examine their properties and their practical applicability. We assume that the rules can be implemented hydrologically, i.e., the calculated allocations can also be physically transferred to the water users. This is the unconstrained case.[19]

Proportional Rule

Let us start with the most widely known and used rule. This rule is already mentioned in Aristotle's Nikomachian ethics.[20] The rule divides the available water in proportion to the claims, i.e.,

$$x_i^P = \frac{c_i}{\sum_{j=1}^{n} c_j} R, \quad i = \{1, 2, \ldots, n\} \tag{6.27}$$

[19]The constrained cases refer to river basins where the unidirectionality of the water flow may lead to the case that the calculated allocations cannot be physically realized. See the next subsection.
[20]Nikomachian ethics, book V. See the explanation in Young (1994), p. 64 ff.

This rule is self-evident and fulfills, it seems, the sense of justice at once. This is certainly due to the principle of accountability[21] that underlies this rule. Subsequently, we will compare this rule with other rules for different types of claims.

Adjusted Proportional Rule
This rule puts more weight to those countries with higher claims. It is derived in a two-step procedure. First, the so-called minimum rights have to be determined as

$$m_i = max\left[0,\, R - \sum_{j \neq i} c_j\right], \quad i = \{1, 2, \ldots, n\} \tag{6.28}$$

Minimum rights refer to the water allocation which is not contested. All $j \neq i$ concede this residual to i. It is simply the water that is left after serving the claims of all the other water users.[22] After the minimum rights have been distributed, the second step follows:

$$x_i^{AP} = m_i + \frac{c_i - m_i}{\sum_{j=1}^{n}(c_j - m_j)}\left(R - \sum_{j=1}^{n} m_j\right), \quad i = \{1, 2, \ldots, n\} \tag{6.29}$$

Here, the water division consists of the minimum rights plus the proportional portion of the residual water supply, which will be left after deduction of the minimum rights. The proportionality factor is formed with the help of the claims adjusted for the minimum rights. One should be careful with the concept of minimum rights. This is not a minimal provision of water in terms of human rights for water. It refers only to the amount of water the other competitors would leave without the request for negotiation.

Constrained Equal Award (CEA)
The CEA rule sets the water allocation in a very egalitarian way. Each country receives the same portion of available water regardless of its claims. Claims only play a role insofar as the equal shares apportioned may be higher than these. In this case, only the claims will be covered.

Formally,

$$x_i^{CEA} = min[E, c_i] \quad \text{where} \quad \sum_{i=1}^{n} min[E, c_i] = R \tag{6.30}$$

where E is the equal share provided E is less than both claims.

Constrained Equal Loss (CEL)
Instead of focussing on the distribution of the water, it is also possible to turn the observation around and look at the distribution of the water loss defined as the number

[21] See Sect. 3.3.

[22] Of course, if the residual $R - \sum_{j \neq i} c_j$ were negative, then the other claims are not met because negative minimum rights are excluded. That is reasonable and therefore acceptable.

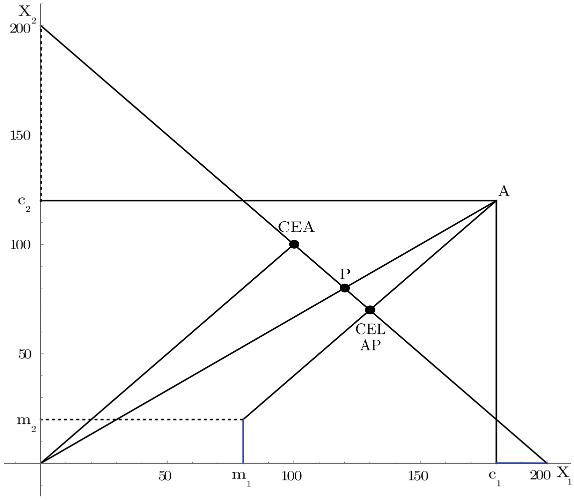


Fig. 6.3 Bankruptcy rules. *Source* own illustration

of claims not fulfilled, i.e., the allocation of the total loss $\sum c_i - R$. The CEL rule is defined as follows:

$$x_i^{CEA} = max[0, c_i - E] \quad \text{where} \quad \sum_{i=1}^{n} max[0, c_i - E] = R \qquad (6.31)$$

where E is the equal share of loss.

All the rules introduced do satisfy the properties with the exception of regressivity.[23]

Which rule is fair now? As we will see, this question cannot generally be answered, but depends on the circumstances[24]. By circumstances, we understand the object of allocation, in this case water, and the type and degree of legitimacy of claims. With the help of an example, we will discuss the appropriateness of sharing rules depending on the type of legitimacy of claims. At first, however, we want to look at a numerical example in order to examine the distributional effects of the four rules. Assume the following numerical values: $R = 200$, $c_1 = 180$, and $c_2 = 120$. Straightforward application of the rules allows to calculate the corresponding allocations and to insert the numerical values in Fig. 6.3.[25]

The budget line $x_2 = R - x_1$ represents all possible water distributions. The claims are plotted with a vertical and a horizontal line. Due to the scarcity of water, its intersection lies north-east of the budget line. The allocation according to the

[23]For the case of two countries, the proof is left to the reader; for the n-country case, the proof is more extensive, see, e.g., Thomson (2002).

[24]There are interesting studies which empirically determine the assessment of fairness of bankruptcy rules within the framework of experiments, see, e.g., Gaechter and Riedl (2006)

[25]Details can be found in Exercise 6.3.

proportional rule is the intersection of the budget line with the line connecting the origin with the claim point A, the slope of which is c_2/c_1 (see Fig. 6.3). Similarly, the CEA rule can be identified by the intersection of a line with a slope of 1 that starts from the origin. The available water is shared in half. This is due to this numerical example where the water allocation according to this rule falls short of both claims. The CEL rule is constructed by running a 45°-line from point A to the budget line. This line implements the requirement that the loss should be apportioned on a fifty-fifty base between the two countries. In the two country case, this allocation is identical to the adjusted proportionality rule. Here, we start from the minimal right point $\{m_1, m_2\}$ with a line of 45° degrees.

What we can see from this figure is that there is an ordering of rules with respect to the degree of equality. The CEA rule distributes the available water equally. Only if claims are fully covered, the rule deviates from the equal share principle (see point A). The proportional rule prefers the country with the higher claims somewhat whereas the CEL rule and the adjusted proportionality rule favor countries with higher claims.

But what is just, the complete equality of water allocations or the equality of the individual water losses suffered, measured by the degree of regressivity x_i/c_i? It turns out that only the proportional rule weakly satisfies this property in general, i.e., the ratio is constant with respect to different values of R. The CEA rule exhibits progressivity, i.e., the percentage of fulfilled claims rises with the number of claims and the CEL rule is undetermined. Progressivity and degressivity depend on the amount of water supplied. However, the question remains as to whether the allocation should be made more evenly distributed or whether the claims should be taken into account in the allocation. This question depends on the very nature of these claims or on the attributes claimants have. The following scenarios show that a sharing rule should only be decided upon once the legitimacy of claims has been clarified.

Scenario I

There are two countries with different population sizes. Country 1 (L1) is large compared to country 2 (L2), i.e., n_1 is larger than n_2, where n_i is the population size. The (culturally determined) subsistence level of water per person is υ. We assume that this subsistence level is the same in both countries. The claims are therefore

$$c_i = \upsilon n_i, \quad i = \{1, 2\} \tag{6.32}$$

We assume that the water available is less than the aggregated claims, i.e., $c_1 + c_2 > R$. Applying the proportionality rule immediately results in $x_i = (n_i/(n_1 + n_2))R$. Dividing the allocation by the respective population size yields the water allocation per capita, which is equal in both countries:

$$\frac{x_1}{n_1} = \frac{R}{(n_1 + n_2)} = \frac{x_2}{n_2} \tag{6.33}$$

Of course, due to water scarcity, this allocation is less than υ.

For water allocation under the CEA rule, we have to differentiate between two cases because of the claim boundedness: If $R/2 < c_2$, i.e., the water allocation is less than the lesser claim, both countries receive the same amount of water R/2. It follows that the water allocation per capita in L1 is lower than in L2. In other words, the more populous country gets less water per capita than the country with a smaller population. This result follows also for the case that $R/2 > c_2$. In this case, people from L2 receive $c_2/n_2 = (\upsilon n_2)/n_2$ (see Eq. (6.32)). Because water is scarce, i.e., $c_1 + c_2 > R$, per capita allocation in L1 is less than υ.

Both claims are well-founded, as they are derived from people's elementary needs. They should, therefore, not be called into question when drawing up a water contract. Fundamental needs should be met as good as possible. The more fundamental a need, the greater the role of equality that applies here to people, not countries. Therefore, the P rule is likely to be preferred to the CEA rule, because it is not acceptable that the more populous country has a lower per capita water supply due to the basic need property of water.

Scenario II
Let us resume the water allocation problem within the same mathematical structure, but with a different economic context. Again, there are two countries L1 and L2. Both countries have the same national product y and the same population size. However, the water consumption of the first country is higher than that of the second country, because L1 is more inefficient than L2, which leads to different water claims:

$$c_i = \epsilon_i y, \quad i = \{1, 2\} \tag{6.34}$$

where ϵ_i is the water intensity per unit of social product of the respective country. Since L1 is less efficient than L2, we have $\epsilon_1 > \epsilon_2$. Again, we assume water scarcity, which makes it necessary to apply a bankruptcy rule. Applying the P rule leads to

$$x_1 = (\epsilon_1/(\epsilon_1 + \epsilon_2))R \quad \text{and} \quad x_2 = (\epsilon_2/(\epsilon_1 + \epsilon_2))R \tag{6.35}$$

Hence, the inefficient country L1 gets more water than country 2. Dividing x_i by ϵ_i yields the national product under rationing, i.e.,

$$\frac{x_1}{\epsilon_1} = (1/(\epsilon_1 + \epsilon_2))R = \frac{x_2}{\epsilon_2} \tag{6.36}$$

Both countries end up with less affluence. However, the P rule allocates the water such that both countries bear the scarcity equally.

Again, if we apply the CEA rule, we have to distinguish between two cases: In the first case, i.e., $R/2 < c_2$, both countries are allocated the same amount of $R/2$. This implies that L2 can sustain a larger national product than L1:

$$\frac{x_2}{\epsilon_2} = \frac{R}{2\epsilon_2} > \frac{R}{2\epsilon_1} = \frac{x_1}{\epsilon_1} \tag{6.37}$$

The same result occurs if $R/2 > c_2$. In this case, the water allocation to L2 is $c_2 = y\epsilon_2$. Hence, L2 can sustain the social product of y. This implies that the affluence in L1 decreases due to water scarcity, i.e., the social product is less than y:

$$\frac{R - c_2}{\epsilon_1} < \frac{c_1}{\epsilon_1} = y \tag{6.38}$$

since $R - c_2 < c_1$.

Is it fair that the efficient country can maintain its standard of living, while L1, due to its inefficient water economy, has to accept a loss of welfare if the CEA rule is applied? Or should the proportional rule be applied, which will lead to an equal decrease in GDP in both countries? In contrast to scenario I, this might require more inquiries about the reasons for the different water efficiencies. Are countries accountable for that, or do these different efficiencies reflect geological properties countries cannot influence? In the latter case, they are not responsible in the sense of the principle of moral arbitrariness (as introduced in Sect. 3.3) and they should bear the scarcity equally. If the different water intensities are rooted in mismanagement, then the principle of accountability will apply with the consequence that the CEA rule is to be applied.

A comparison of both scenarios shows that bankruptcy rules should be weighted carefully before being adopted. Beyond the question of which rule has to be applied, the nature of the claims must also be examined. Are those derived from existential needs or are they economic wants? In analogy to Maslow's hierarchy of needs[26], we can put the claims in a prioritized order. This naturally means that the introduced water allocation rules have to be adapted to these hierarchies of needs. Let us go back to our numerical example and assume that the claims can be subdivided into basic needs and secondary needs. Let us assume that L1 is a developed country with a relatively small population ($n_1 = 4$) while L2 is a developing country with a large population ($n_2 = 10$). The subsistence minimum of water per capita is $w_s = 10$. Claims are the sum of basis needs and secondary wants. For L1, we have

$$c_1 = 180 = c_1^1 + c_1^2 = w_s n_1 + c_1^2 = 40 + 140 \tag{6.39}$$

and for L2:

$$c_2 = 120 = c_2^1 + c_2^2 = w_s n_2 + c_2^2 = 100 + 20 \tag{6.40}$$

We first look at the undifferentiated water allocation to the two countries (see Fig. 6.3). The numerical values are given in Table 6.11 for each country and for the three distribution rules considered.[27] The quantities of water per capita are calculated for each rule: These values differ considerably between the different rules. For the P rule and CEL rule, the water allocation per capita is below the subsistence minimum w_s.

[26] See Sect. 3.5 for Maslow's hierarchy.

[27] Notice, that in the two country case, the water allocation under the AP-rule is identical to the allocation under the CEL rule.

Table 6.11 Non-differentiated water allocation

	P Rule		CEA Rule		CEL Rule	
Countries	x_i^P	x_i^P/n_i	x_i^{CEA}	x_i^{CEA}/n_i	x_i^{CEL}	x_i^{CEL}/n_i
L1	120	30	100	25	130	32.5
L2	80	8	100	10	70	7

Table 6.12 Differentiated water allocation

	P Rule		CEA Rule		CEL Rule	
Countries	x_i^P	x_i^P/n_i	x_i^{CEA}	x_i^{CEA}/n_i	x_i^{CEL}	x_i^{CEL}/n_i
L1	92.5	23.125	80	20	100	25
L2	107.5	10.75	120	12	100	10

Needs of different priority make the direct application of bankruptcy rules to the aggregated claims questionable. Basic needs should definitely be met.[28] Thus, we should adopt a sequential approach. First, water should be distributed according to basic needs $\{c_1^1, c_2^1\}$. After that, the residual water $R - c_1^1 - c_2^1$ should be allocated taking into account the secondary claims $\{c_1^2, c_2^2\}$. The water allocation rule is then $x_i(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\})$. Total water assignments are

$$c_1^1 + x_1(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\}) \quad \text{and} \quad c_2^1 + x_2(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\}) \tag{6.41}$$

In Table 6.12, we have calculated these allocations for the three rules.

The sequential treatment of the requirements leads to the fact that the subsistence level is also fulfilled for the more populated country. The remaining water is then distributed to the two countries on the basis of the remaining entitlements, without taking into account the population figures. As shown in Table 6.11 and in Fig. 6.3, the CEA- and CEL rule take greater account of the asymmetry of claims than the P rule.

6.4.3 Sequential Allocation Rules

If we want to apply bankruptcy rules to a river system, its specific hydrological characteristics must be taken into account. The unmodified application of bankruptcy rules could lead to the fact that the resulting water allocations are not feasible. Imagine L1 is upstream, L2 downstream. The river system is characterized by a relatively low inflow in L1 and a big inflow in L2. A direct application of the CEA rule, for example, could lead to an equal distribution of the available water, i.e., the sum of all water

[28] Of course, if the sum of basic needs exceeds the water quantity available, we can apply the rules directly taking the basic needs as claims.

Table 6.13 Claims and inflows along a river

Countries	Inflows	Claims	Rule-based allocation	Downstream availability	Downstream excess claims
L1	$R_1 = 100$	$c_1 = 180$	x_1	$E_1 = R_1$	$cd_1 = (c_2 - R_2) = 20$
L2	$R_2 = 100$	$c_2 = 120$	x_2	$E_2 = R_2 + (R_1 - x_1)$	$cd_2 = 0$
Sum	200	300	200	–	–

inflows. This would not be possible considering the geographical position of the two countries and the low inflow in L1. For this constrained case, a sequential sharing rule has been proposed.[29] This bankruptcy rule takes the hierarchical order of countries and the unidirectionality of the river into account. As before, the sum of claims $c_1 + c_2$ is higher that the sum of inflows $R_1 + R_2$. However, due to the specific geography of the river, the feasibility of a water allocation must also be ensured. For this purpose, we define total available water in the two territories:

$$E_1 = R_1, \quad E_2 = R_2 + (R_1 - x_1) \tag{6.42}$$

where x_i is the water allocation (per country) according to a sharing rule to be specified. Equation (6.42) shows the typical water availability structure of a river depending on the position of the countries. Finally, to apply the bankruptcy rules to a river system, we have to define the downstream excess claims

$$cd_1 = (c_2 - R_2), \quad cd_2 = 0 \tag{6.43}$$

Equation (6.43) is of crucial importance for the sharing rules. Downstream country's claims are adjusted by its inflow R_2. The very reason for this approach is the insight that R_2 is always with country two due to the unidirectional flow of the river.

Starting with the water allocation for L1, we have to compare the available water for L1 with the excess claims of L2. In the following table, all the relevant variables are summarized. In addition to the numerical values in the example above, we have added numerical values for the inflows R_1 and R_2, so we can calculate the water allocations proposed by the sequential sharing rules. Table 6.13 lists all relevant parameters to apply bankruptcy rules. In the following, we will consider the sequential variants of the P rule, the CEA rule, and the CEL rule. Let us start with the P rule.

P Rule

In the first step, the water allocation for L1 is calculated, as shown in the first row of Table 6.14. x_1^{S-P} is a fraction λ_1 of the claim c_1. The amount of water remaining is the same fraction of the residual net claims of L2, the downstream excess claims. The proportionality factor λ_1 is simply the ratio between the availability of water

[29] See Ansink and Weikard (2012).

Table 6.14 The sequential proportionality rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Proportionality factor
L1	$x_1^{S-P} = \lambda_1 c_1$ $x_1^{S-P} = 0.5 \cdot 180 = 90$	$x_{cd1}^{S-P} = \lambda_1 cd_1$ $= 0.5 \cdot 20 = 10$	$\lambda_1 = E_1/(c_1 + cd_1)$ $= 100/(180 + 20) = 0.5$
L2	$x_2^{S-P} = E_2 =$ $R_2 + (R_1 - x_1^{S-P})$ $x_2^{S-P} =$ $100 + (100 - 90) = 110$	$x_{cd12}^{S-P} = 0$	——

Table 6.15 The sequential CEA Rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Award calculation
L1	$x_1^{S-CEA} = Min[c_1, \lambda_1]$ $x_1^{S-CEA} = 80$	$x_{cd1}^{S-CEA} = Min[cd_1, \lambda_1]$ $x_{cd1}^{S-CEA} = 20$	$x_1^{S-CEA} + x_{cd1}^{S-CEA} =$ E_1 $Min[180, \lambda_1] +$ $Min[20, \lambda_1] = 100$ $\rightarrow \lambda_1 = 80$
L2	$x_2^{S-CEA} = E_2$ $= 100 + (100 - 80) =$ 120	$x_{cd2}^{S-CEA} = 0$	—-

and the sum of claims $c_1 + cd_1$. After having determined the water allocation for L1, we go one step downstream and determine the water allocation for L2. In our simple two country case, we only have to allocate the remaining water supply E_2.

CEA Rule

The water shares that follow from the CEA rule are also calculated in a sequential way. Again, we begin with the upstream country L1. We start in the first line (L1) of Table 6.15 and split the available water R_1 to L1 and L2 downstream according to the CEA rule. The upper bounds (see the principle of claim boundedness Eq. (6.20)) are the claims c_1 and the excess claims of the downstream country cd_1. For the given numerical values, it follows that L1 receives 80 and the downstream country 20. We proceed to the second country L2 and calculate the residual water available, $E_2 = R_2 + (R_1 - x_1^{S-CEA})$ which yields 120. Finally, we have to calculate the water allocation sequentially for the CEL rule.

CEL Rule

Again, we begin with the first row for country L1 (Table 6.16), and calculate λ_1. First, we assume that there exists an equal share of loss. This is not feasible, because under this assumption the second term of the right-hand side would get negative. Hence, we assume that this term is nil which yields that $\lambda_1 = 80$. Having calculated x_1^{S-CEL}, we can compute $x_2^{S-CEL} = E_2$.

Discussion

The sequential sharing rules are somewhat exhaustive with regard to the calculation effort even if there are only two riparian countries. However, their logic is clear. The

Table 6.16 The sequential CEL Rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Loss calculation
L1	$x_1^{S-CEL} =$ $Max[0, c_1 - \lambda_1]$ $x_1^{S-CEL} = 100$	$x_{cd1}^{S-CEL} =$ $Max[0, cd_1 - \lambda_1]$ $x_{cd1}^{S-CEL} = 20$	$x_1^{S-CEL} + x_{cd1}^{S-CEL} = E_1$ $Max[0, 180 - \lambda_1]+$ $Max[0, 20 - \lambda_1] = 100$ $\rightarrow \lambda_1 = 80$
L2	$x_2^{S-CEL} = E_2$ $= 100 + (100 - 100) =$ 100	$x_{cd2}^{S-CEL} = 0$	—-

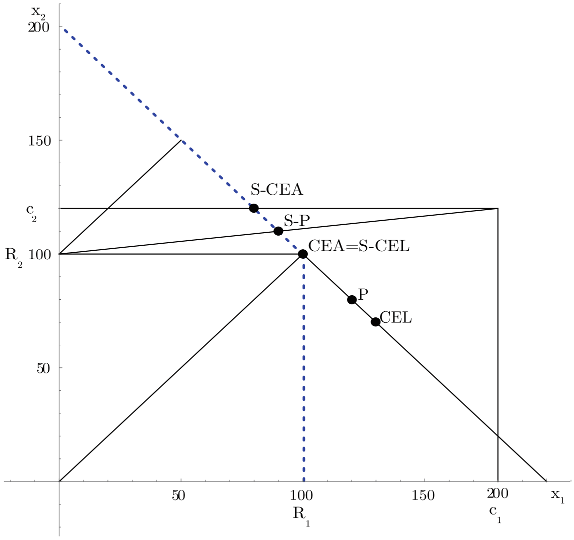
calculation begins upstream and first divides the available water on the basis of the existing claims between the first country and the remaining countries. If the water allocation for L1 is calculated, the calculation moves downstream and the remaining water is allocated to L2. The reason for the sequential approach is justified by the unidirectionality of the river system flow. However, one must be careful with the direct application of the sequential sharing rules, whenever water is to be distributed along a river.

Figure 6.4 shows the water allocation calculated in the last three tables graphically and compares it with the actual application of pure bankruptcy rules. The figure displays the water allocation from the bankruptcy rules $\{P, CEA, CEL\}$ depicted in Fig. 6.3. The P rule and the CEL rule are not feasible since these rules allocate more water to L1 than available by R_1. The river structure is depicted by the blue dotted water budget line with its kink at $\{R_1, R_2\}$. Feasible water allocations are only those along the blue dotted budget line. The sequential rules are by construction feasible. The P Rule (see Table 6.14) allocates the water on the basis of proportional shares. The proportional distribution is calculated with respect to inflow R1. In addition,[30] country 2 also receives inflow R2, which for hydrological reasons cannot be split between L1 and L2.

One has to be careful: The fact that the algorithm of the sequential sharing rule takes the hydrology of a river into account does not imply that these rules should always be applied for river basins. This is a normative decision. The consideration of the flow direction has the consequence that the claims from upstream are no longer considered in the calculation of the water allocation downstream. This follows from the construction of downstream excess claims (see, e.g., Table 6.14). Thus the calculated water allocations are compatible with hydrology, but at the expense of the lower weighting of upstream claims. The modification of the bankruptcy rules for unidirectional running waters is therefore not only of a technical nature, but also involves a normative adjustment. Thus, the decision as to which of the two sets of rules, the direct or the sequential, is to be applied remains not only a

[30]The line from $\{0, R_2\}$ to the point S-P and $\{c_1, c_2\}$ can be constructed from the L1-row in Table 6.14. Since, $x_1^{S-P} = \lambda_1 c_1 = R_1[c_1/(c_1 + (c_2 - R_2))]$ and $x_2^{S-P} - R_2 = R_1 - x_1^{S-P} = R_2 + R_1((c_2 - R_2)((c_1 + (c_2 - R_2))$ we can calculate $(x_2^{S-P} - R_2)/x_1^{S-P} = (c_2 - R_2)/c_1$.

Fig. 6.4 Bankruptcy rules and sequential sharing rules. *Source* own illustration



technical-hydrological one, but also a normative one. Table 6.14 clearly shows that the sequentiality favors the downstream country.

If the application of this sequential rule is perceived as unfair, one can return to the direct rule. Since it cannot be implemented in our example, a second best option is available, namely to come as close as possible to the desired allocation. This allocation is $\{x_1 = R_1, x_2 = R_2\}$. In Fig. 6.4, we see that this second best option is closest to the P rule and the CEL rule. At the same time, it is the water allocation according to the principle of absolute territorial sovereignty (ATS). Both riparian countries make full use of the water that originates in their territory. Notice, however, that this result is not based on the principle of sovereignty, but on fairness considerations.

The normative problem of the sequential rule is even clearer in Fig. 6.5. Here, we have chosen numerical values of the relevant variables such that both sets of sharing rules, the direct ones and the sequential ones, are hydrologically feasible.[31]

In this scenario, the choice of allocation only depends on normative criteria. The hydrology does not constrain the choice. The sequential sharing rules favor the downstream country with exception of the sequential CEL rule. In our two country example, both CEL rules, together with the direct CEA rule, lead to the same water allocation. This is due to the assumption that both countries have the same claims. The sequential versions of the P- and CEA rule do not satisfy the principle of Equal Treatment of Equals.[32] Thus, whenever the hydrology allows the direct application of bankruptcy rules, the application of the sequential rules cannot be justified. But even if the direct rules cannot be applied for hydrological reasons, the sequential rules cannot be applied automatically. It may turn out that second-best solutions are preferred for reasons of fairness, as shown above.

[31]The numerical values are $\{R = 180, c_1 = 160, c_2 = 160, R_1 = 120, R_2 = 60\}$.

[32]See the above-introduced properties. If $c_1 = c_2$ then $x_1 = x_2$. However, this might not be possible due to the hydrological conditions.

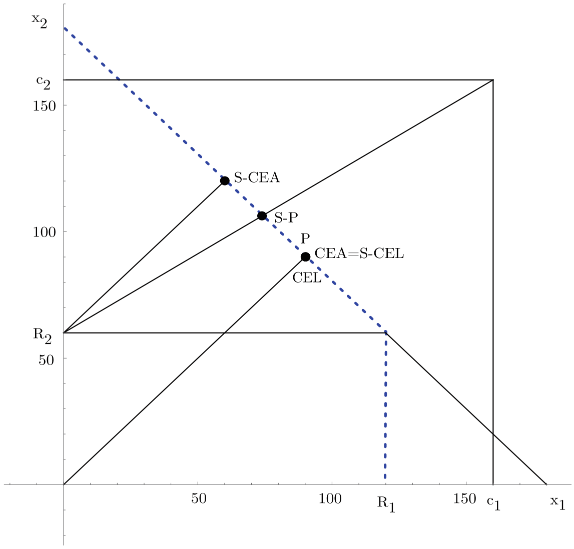


Fig. 6.5 Bankruptcy rules. *Source* own illustration

The discussion shows that we have to be careful when applying bankruptcy rules. The mere application of mathematical rules does not solve the sharing problem. These methodological tools are helpful but cannot substitute the intrinsic fairness problems of sharing scarce water resources. If the distribution rules are discussed in principle, the geographical position of the countries poses a principle problem. Geography could be regarded as morally arbitrary.[33] This implies that the actual order along the river must not result in any disadvantages for the individual countries. If this argument is valid, it is might be necessary to talk about the legitimacy of claims before applying a rule of division. Moreover, if moral arbitrariness is the basis of the TIBS principle, we might end up with the insight that water treaties have to be constructed in a more complex way. The pure allocation of water might not suffice to compensate for geographical disadvantages. This leads back to our discussion of welfare-based approaches which allow for side payments and other in-kind compensations. Of course, with this step we have to face all the problems discussed in Sect. 6.3.

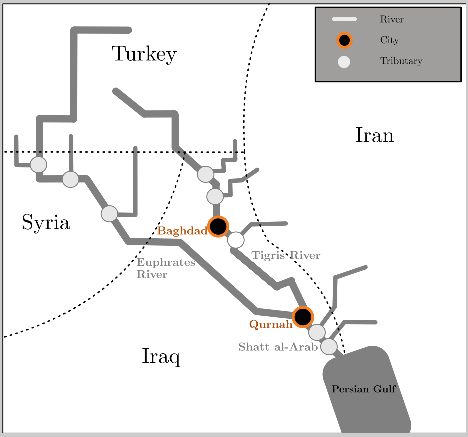
Box 6.2 Applying water bankruptcy rules to the Euphrates River

The Euphrates flows through three countries: Turkey, Syria, and Iraq. Together with the Tigris, it forms the water catchment area, which has been known as Mesopotamia ever since. From its springs in Turkey to the Persian Gulf, the Euphrates River stretches over a length of 2,786 km. The average annual water flow is 25 billion m^3 serving 23 million people in the transboundary catchment area. The water use in all three riparian countries relates mainly to irrigation

[33] See the fairness principles in Sect. 3.3.

(70%), hydropower, and drinking water supply. Data records of the past 70 years indicate a negative trend of water availability measured as a decrease in mean annual flows. The need for sustainable water treaties is, therefore, becoming increasingly important. Currently, there are two bilateral accords in force: an agreement between Syria and Turkey specifying the minimum average flow at the Syrian-Turkish border and another treaty between Iraq and Syria determining the water allocation of Euphrates water between these two countries.

The linear arrangement of neighboring states and the simple geography of tributaries make the Euphrates a good example of the sequential sharing rule. The following map shows the geographical structure. The main water inflow is provided from Turkey. In Syria, there are three tributaries contributing water to the Euphrates (the Sajur, the Balikh, and the Khabur). Iraq does not contribute to the watercourse.



Source: Jarkeh et al. (2016).

The following table summarizes all necessary information to apply the sharing allocation rules.

Claims and contributions				
Riparian Countries	Claim (MCM/year)	Claim %	Contribution (MCM/year)	Contribution %
Turkey	14,000	25.6	31,580	88.8
Syria	12,600	23	4000	11.2
Iraq	28,100	51.4	0	0

While water inflows are well measurable, the determination of the claims requires an estimate of the water demand components from the various economic sectors of the riparian countries. There are several studies in the literature, the results of which are gathered by Jarkeh et al. (2016) and then entered into the table as a best guess. The application of the sequential sharing rule yields the following water allocation for the three countries (as a percentage of the claims).

Sequential sharing rule. *Source* Jarkeh et al. (2016)

Riparian	Sequential sharing rule		
	P Rule	CEA Rule	CEL Rule
Turkey	62	100	32
Syria	66	86	62
Iraq	66	38	83

It is interesting to see that the percentage satisfaction is almost the same for all three countries in the case of the sequential P rule, despite Iraq's lack of inflow and its high claims. The application of the CEA rule leads to a complete coverage of Turkish water demand, while Iraq only receives about 40% of its claims. The CEL rule would yield exactly the opposite: Iraq achieves the highest fulfillment of claims while Turkey is allowed to use only extremely little water, 70% of its claims would not be covered. The remaining water is to cross the Turkish border for the benefit of downstream states. The question remains as to whether this water allocation has any chance of being implemented...

Sources UN-ESCWA and BGR (2013), Jarkeh et al. (2016)

6.5 Flexible Water Sharing

If one examines the emergence of water agreements between riparian states on an international water body, it becomes apparent that it often takes years to reach a successful conclusion. The Indus Waters Treaty, for instance, took over 6 years of bargaining until it was concluded with the assistance of the World Bank. Agreements are rather difficult to alter in response to unexpected changes of underlying hydro-climatological conditions. Specifically, if the volume and the pattern of the regional water inflow into an international catchment area changes, conflicts may occur. This instability is the result of the inflexibility of water agreements. New hydrological framework conditions are difficult to be taken into account in the treaties. Compliance with a treaty on the basis of outdated framework conditions can lead to a situation in which the conflict is more advantageous for some partner states than compliance with the treaty concluded. In the following, we will, therefore, investigate how different contract types can influence the behavior of the contracting parties in the event of unexpected changes in the hydrological conditions. We limit ourselves to investigating the case of decreasing water inflows into an international river.

Contract Types
Roughly, we can distinguish between three contract types:

- Complete contingent contracts: This complex type of contract would be the best answer to the variability of the water supply. For every conceivable hydrological and climatological scenario, the water quantities are allocated ex ante, possibly with corresponding non-water transfers. However, the amount of information required is very high. The concept of complex contingent contracts actually leaves out the problem of unexpected events. Thus, we subsequently only focus on the following two contract types.

- Fixed flow allocation: This type of water sharing rule is most common. A fixed amount of water for the downstream country is stipulated. In the following analysis, we assume that this fixed amount is accompanied by a non-water transfer from the downstream country to the upstream one.
- Proportional allocations: The water allocation follows a percentage rule. The downstream country is entitled to a certain percentage of the water supply available upstream. Again, we combine this type of agreement with a non-water transfer which is also proportional to the water received.

To analyze these two contract forms, we take up our example of a river with two riparian countries from Sect. 6.2. Upstream is labeled 1 and downstream is denoted by 2. In the following, we assume that the river is fed only by water upstream. There is no downstream tributary (i.e., $R_2 = 0$). The optimal allocation then can be derived from the following maximization program.

$$\max_{w_1, w_2} \quad B_1(w_1) + B_2(w_2) \quad w_1 + w_1 \leq R_1 \tag{6.44}$$

Let the water supply be scarce. Then, the optimal fixed water supply for downstream w_2^* satisfies the following condition

$$B_1^{'}(R_1 - w_2) = B_2^{'}(w_2) \tag{6.45}$$

Similarly, the proportional sharing rule can be fixed. The allotted amount of water downstream is expressed as percentage α^* of the total water available:

$$w_2^* = (1 - \alpha^*) R_1 \quad w_1^* = R_1 - w_2^* = \alpha^* R_1 \tag{6.46}$$

Thus if the water supply is constant, both allocations are identical. However, if the water supply R_1 decreases unexpectedly, the effects on both contract types are rather different. To show this, we first have to determine the non-water transfers in both contracts, the level of which depends on the bargaining power of both riparian countries. Of course, whatever the amount of money (or other non-water transfer vehicles) will be, the solution must lie in the core as defined in Sect. 6.2:

$$B_1(w_1) + T \geq B_1(R_1) \quad \text{and} \quad B_2(w_2) - T \geq 0 \tag{6.47}$$

where T is the non-water transfer. In the case of a fixed flow agreement, T is also a fixed amount. In the case of a proportional allocation of the water supply, T varies with the amount of water transferred from upstream to downstream whereby the price of water t is fixed.

$$T(r) = t(1 - \alpha)r \tag{6.48}$$

where $r \leq R_1$ is the actual water supply and t is the fixed water price. The proportional water agreement makes the non-water transfer contingent on the actual amount of water delivered to the downstream country. The fixed water price is calculated as

$$t = \frac{T}{R_1(1-\alpha^*)} \tag{6.49}$$

that is, t are the average payments per amount of water delivered to the downstream riparian at the time of the conclusion of the contract, i.e., when $r = R_1$. Inserting Eq. (6.49) into Eq. (6.48) yields

$$T(r) = \frac{r}{R_1} T \tag{6.50}$$

Robustness to Changing Hydrological Conditions
Now, the question needs to be answered on how the two types of contracts perform if, let's say as a result of climate change, the water inflow unexpectedly decreases. Three criteria are important here: efficiency, robustness, and fairness. Before the unexpected water reduction, both contracts are efficient and fair by construction. The sum of the benefits is maximized, both parties have agreed to the contract by appropriate choice of a transfer T in the core and the resulting distribution of the benefits is considered fair.

What happens now, if the water supply decreases, that is $r < R_1$? In both types of contracts, the quantities of water allocated differ from the quantities originally negotiated, with the result that the efficiency properties change. This also applies to the distribution of benefits. Further, it is unclear whether the two parties have an incentive to comply with the contract, i.e., how robust the contract is. In the following, we focus on this issue.

Let us assume that the water supply of the river r is falling, i.e., $r < R_1$. In the case of a fixed contract, the stipulated water allocation at the outset is w_2^* $(R_1 - w_2^*)$ for downstream (upstream). Let us assume that the fixed non-water transfer is calculated such that both countries are better off than in the case of no agreement, i.e.:

$$B_1(R_1 - w_2^*) + T^* > B_2(R_1) \quad \text{and} \quad B_2(w_2^*) - T^* > 0 \tag{6.51}$$

For example, we could calculate T^* such that total welfare of both riparian countries is distributed according to the Shapley values.[34] In this case,

$$T^* = \frac{1}{2}\left[B_2(w_2^*) - B_1(R_1 - w_2^*) + B_1(R_1)\right] \tag{6.52}$$

Now, let us analyze the robustness of this contract with the help of a numerical example: $B_i^{'} = a - bw_i$, $a = 100$, $b = 1$, and $R_1 = 100$. Let's start with the upstream country. The country will stick to the contract as long as it is better off than in the

[34] See Sect. 6.2 and Exercise 6.4.

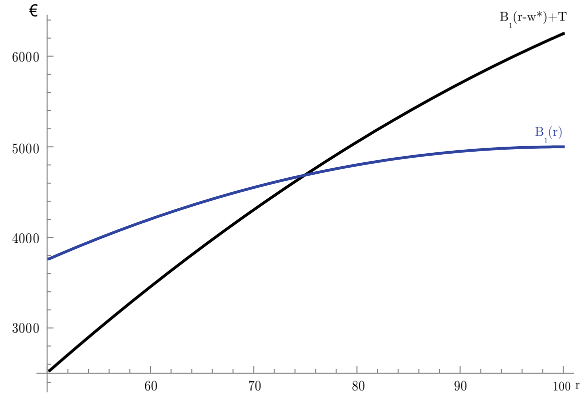


Fig. 6.6 Robustness of a fixed contract. *Source* own illustration

stand-alone case. This can be observed in Fig. 6.6. The blue line shows the welfare (utility) in the stand-alone case, i.e., in the case of conflict. In this case, country 2 does not receive water from country 1. The upstream country uses the whole water supply R_1. Of course, it does not receive any transfer because the downstream country has stopped to pay due to the breach of contract. The black line represents total utility for the case that the upstream country complies with the contract. It delivers the fixed amount of water w_2^* and receives in exchange T^*. As the water supply is decreasing, the water available for upstream decreases because the downstream country receives the fixed amount of water. There is a critical threshold $\hat{r}$, the intersection of both lines, where it does not pay for the upstream country to comply any more with the contract if r continues to drop. The length of the range $R_1 - \hat{r}$ indicates the robustness of this type of contract.

Let us turn to the proportional contract. At the outset, i.e., $r = R_1$, the agreement provides that downstream country receives w_2^* which can be expressed as a proportion of R_1 (see Eq. (6.46)). The amount of non-water transfer to the upstream country depends on the actual amount of water delivered downstream whereas the price is fixed according to Eq. (6.48). Thus, the countries' benefits including transfers are

$$B_1(\alpha^* r) + t(1-\alpha)r \quad \text{and} \quad B_2((1-\alpha^*)r) - t(1-\alpha)r \tag{6.53}$$

Figure 6.7 shows the stand-alone benefit of upstream as a function of the variable water supply r (blue line) again. The black line depicts benefits plus transfers under the fixed agreement and the gray line is net benefits of upstream under the proportional contract. The black and gray lines begin at $r = R_1$ at the same point "coop" where the difference to point "conflict" indicates the benefit increase for upstream to conclude a contract with downstream. However, this difference shrinks as r decreases. Then, comparing the intersections of both benefit lines of the two contract forms with the benefit lines in the conflict case (blue line) shows that the proportional contract is more robust than the contract with fixed quantities.

Durability of Agreements

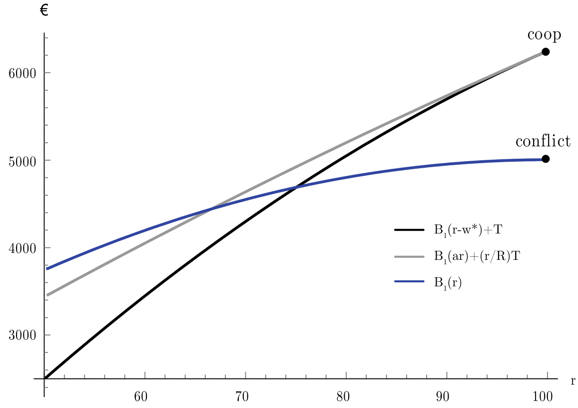


Fig. 6.7 Comparing fixed and proportional contracts. *Source* own illustration

The robustness of a water agreement depends on the flexibility of its construction. But the flexible design is not sufficient for the durability of an agreement:

- Water agreements are more viable if the participating countries share the risk associated with unexpected water shortage. A proportional contract provides a rule to share the risk. A fixed water agreement shifts the risk to the upstream country. Its only incentive to comply with the agreement is the anticipated threat of downstream to cancel the non-water transfer if no water is delivered. The stability of a water agreement depends on the flexibility of its items stipulated, specifically, the transfers agreed upon. If these non-water transfers are to be provided on a periodical basis, downstream can cancel its payment in reaction to a breach of contract by upstream. Here again, the fixed term agreement is less flexible than the proportional rule. If upstream does not deliver the amount of water agreed upon, downstream stops the payments. In contrast, the proportional contract allows (a bit) more flexibility. Less water delivered by upstream leads only to less payment from downstream according to the internal water price stipulated.
- The analysis so far assumes that the contract parties assess the advantage of the agreement by comparing the outcome under compliance with the stand-alone benefits. As long as the contract leads to more utility compared to the conflict situation (breach of contract), the terms and conditions agreed upon will be respected. However, whether to comply with the contract under a shrinking water supply might not be the only consideration of the parties. Even if the benefits under a contract are higher than in the conflict case (breach of contract), the distribution of benefits changes with less water supply. If the resulting benefit distribution is considered unfair, the contract might be broken even if the resulting conflict situation worsens the economic situation of one or both parties. We know from experimental economics that people do not only look at their benefits but also at the relative position, i.e., the distribution of benefits.[35]

[35]The ultimatum game has shown this with astonishing evidence, see, e.g., Thaler (1988).

- Even if a certain degree of flexibility is built into the treaty, there may be renegotiation because the countries are not satisfied with the scheduled outcome of the agreement when the supply of water has declined. Then it is important that the institutional framework of the treaty is operational. Regular contacts between representatives of the two countries create a basis of trust which makes successful renegotiation likely.

6.6 An Institutional Perspective on Transboundary Water Agreements

6.6.1 An Institutional Approach

In the previous sections, we have investigated designs to divide a transboundary water resource among riparians. It is evident that the solutions, such as the Shapley value, will not be translated into a real-world treaty as such. An institutional economic approach suggests that real-world transboundary management would not follow such a technocratic top-down approach. The study of the theoretical principles of water allocation, however, allows to clarify which division rules can be qualified as fair in principle and worthy of approval. These concepts are deeply connected with the fundamental principles of justice and ethics and have also shaped international water law. They certainly belong to what institutional economics calls the institutional environment, traditions, and informal institutional framework conditions shaped by cultural configurations (Ostrom 1990; North 1990). For example, the ancient Talmud's garment rule already contains a simple version of the constrained equal awards rule.

The two approaches presented here, benefit sharing and bankruptcy rules, differ with regard to the weighting of two traditions of thought in social philosophy and political theory. The concept of the core and Shapley value can be assigned to the concept of the social contract as the constitution of cooperation. Rational people come together and agree to divide the advantages of cooperation in a fair and acceptable way.[36] The underlying fairness concept of the Shapley value is certainly the accountability principle.[37] The Shapley value is calculated on the basis of the average marginal productive contributions of the individual partners to the overall result. Those who do not contribute to the cooperation receive nothing. The consideration of individual productivity also leads to the fact that the allocation is acceptable. In

[36] Remember that $\alpha < \beta < \gamma$ where the latter number is the outcome with cooperation.
[37] See Sect. 3.3.

this way, the Shapley value combines the concept of fairness (accountability) with the concept of rationality as exploiting mutual cooperation advantages.

The nucleolus is close to the concept of the "veil of ignorance" by John Rawls.[38] This approach implies that the allocation of resources should follow those who are most disadvantaged. The nucleolus first determines all "disappointments" that follow from a proposed allocation and selects the largest one. Then, in an iterative procedure, a new proposal is put forward with the aim of reducing the largest disappointment. This leads to disappointments of other sub-coalitions. Thus, the search process is continued until the maximum disappointment has been minimized. This approach is also in the tradition of the social contract: it is fair (in the sense of Rawls) and worthy of acceptance, i.e., it lies in the core and puts all partners in a better position.

However, these approaches have their limits in practice. They abstract from too many complex relationships that have to be considered if one wants to successfully conclude water contracts. To begin with, the contracting parties are not simply individuals, but state entities which themselves consist of a number of social groups with differing interests. We, therefore, consider the game theoretical allocation rules as an element of a comprehensive holistic approach to understand the development of international water treaties. Institutional economics[39] allows this broad perspective to be built up scientifically. Here, we distinguish between institutional environment (blue area in Fig. 6.8) and institutional arrangements (yellow bottom area).

The fundamental considerations of justice and its game theory specifications certainly belong to the first area together with culturally determined concepts of justice, religious belief systems, and grown principles of law, written and unwritten. In contrast, the institutional arrangements are the structure within which the members of a society act politically and carry out economic transactions (production, consumption). These structures have grown historically, a development process that is not solely the result of planning, but is often predetermined by the past. Historians speak here of path dependencies or lock-in effects. Socio-technical structures often exhibit an inertia that resembles a lock-in, such as an energy system based on fossil resources that does not change to a system of renewable energy production without deliberate energy policy measures. Similar retarding forces of grown institutional structures inhibit the further development of spatially bound infrastructures, such as waterways. Changed geographical settlement structures, for example, require new waterways instead of simply preserving the historic ones.[40] The interaction between environment (vegetation, landscape, terrestrial eco-system) and man-made infras-

[38]This is the concept underlying the social welfare function: If no one knew in advance how he would fare on earth because he had no prior information about it (veil of ignorance), he would argue in favor of improving the situation of the worst off in the world. In the context of the nucleolus, the worst off is the one who is most disappointed with respect to the difference of the utility apportioned to him in the grand coalition and the welfare level he can achieve by himself.

[39]Saleth and Dinar (2004) explain the importance of the institutional economics approach to understand the water sector.

[40]See Willems and Busscher (2019) for an analysis of the Dutch national waterways.

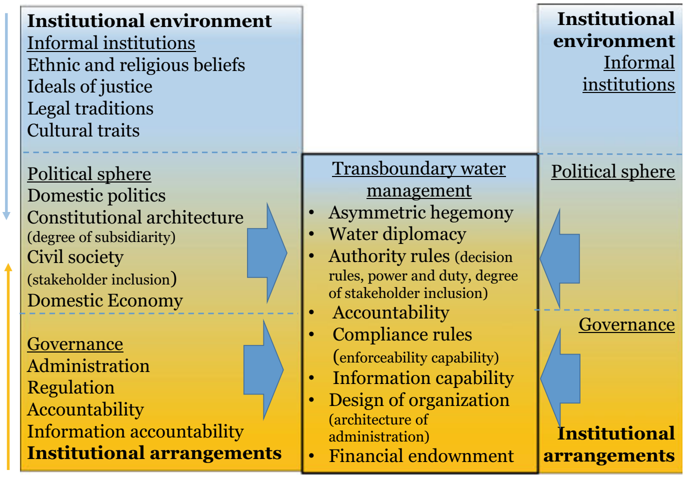


Fig. 6.8 The institutional embeddedness. *Source* own illustration

tructure determines the political level in which social and economic developments are embedded. Interventions in the water cycle can lead to ecological effects that extend beyond the boundaries of the local infrastructure. It is then not enough to assess infrastructure investments at the lower local political levels, but also higher levels must be involved in the decision-making process. This is entirely in the spirit of integrated water resource management, linked to the principle of subsidiarity. The integrative approach does not only refer to the geographical dimension, but also to the water users and indirect users of the water cycle. The latter are, for example, farmers who not only use water directly (irrigation), but also depend on a functioning ecosystem to ensure soil fertility. An inclusive approach to water management should be pursued at national level. All stakeholders should be taken into account in the sustainable shaping of the water cycle.

The problem of incomplete inclusion of all social groups in water management is not only due to an asymmetry of political power. Even if access to co-determination is guaranteed constitutionally and politically, it depends on the executive implementation of water policy plans. The level of governance is thus addressed. The effectiveness and functioning of water management institutions depend on an adequate design that takes into account the political environment, the inclusion of stakeholders and the incentives of employees at different levels of the institution.

6.6.2 Principles for Effective Institutional Development

In the following, some principles are presented that are important for the development of effective institutions, both for national authorities and for transboundary institutions. However, this should not give the perception that effective water management institutions can simply be assembled from a design toolbox. Institution building is always a laborious singular historical process, which refers to the respective individual case. Nevertheless, general principles should be considered in that context.

- **Purpose, objectives, and role**: It must be clear from the outset why management institutions have been set up. What are the actual objectives and purposes of the institution? This question must be asked at every administrative level. For example, is the purpose confined to the mere collecting and analyzing of relevant data? Is it about coordination of decentralized decision-makers (passive management), of advice, or does the institution established also have decision-making power (active management)?
- **Power and duties**: If water management is endowed with power, it is particularly important to precisely define its competences and, to communicate its limits. These boundaries may be of an economic or geographical nature. Is the authority able to take planning initiatives or does it only have a monitoring function to enforce the regulatory measures laid down by other institutions (regional parliaments, etc.)?
- **Decision rules** : The exercise of power requires legitimacy, otherwise, the implementation of water management measures will not be enforceable. It must be transparent how decisions have been taken, and this with reference to the constitutional legal basis.
- **Accountability and responsibility** The responsibilities of all participants must be clearly defined. This applies on the one hand to the managers or civil servants employed in the administrative units and on the other hand to the water users. In the course of the institutional implementation of water management, the assignment of duties and the takeover of responsibilities must be clearly communicated. This also includes the definition and description of sanction measures in the event that those involved do not comply with them.
- **Mediation**: Integrated water management is often about competing claims. Conflicts will inevitably arise. As a rule, these cannot be decided top-down. The institution must, therefore, build up the competence and capacity to resolve these conflicts in an orderly communication and negotiation process.
- **Competence and expertise**: Institutions do not function abstractly. Design alone does not ensure their effectiveness. It is important to build up a personnel development with regard to competence and expertise right from the start (capacity building). Administrative and decision-making units must have a critical mass of a well-trained and competent core staff. In the long term, an institution cannot rely on external consultancy (Biswas 1996).

6.6.3 Idealtypes of Governance

If the international water catchment area is regarded as a common complex ecosystem, the institutional structure should take into account the specific complex interre-

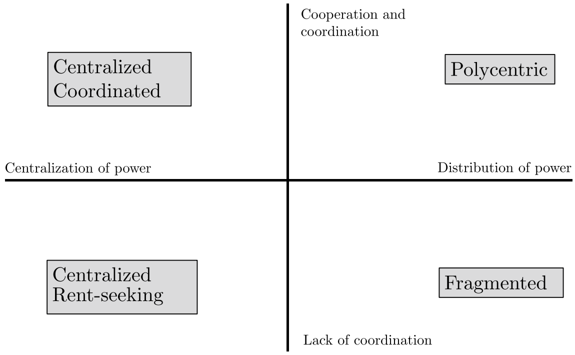


Fig. 6.9 Idealtypes of governance. *Source* Pahl-Wostl and Knieper (2014)

lations. The fitting of the management structure to the hydro-ecological conditions is called *adaptive management*.[41] For this approach, it is particularly important that the institutional structure must "mirror" the geographical, hydrological, and ecological complexity of a catchment area. An institutionalized top-down approach, for example, is usually not effective because decision-makers at the national level make water management decisions without necessarily taking into account regional impacts, leading to a so-called spatial scale mismatch. The spatial scale runs from the global level to the regional level, then to the level of regional rivers (lakes) and finally to subwater catchment areas. At all levels, effects can arise that must be perceived by suitable institutions (government agencies, NGOs, municipalities, etc.). The information must then be brought together promptly and effectively so that it can be processed at the respective institutional levels.

Basically, international waters should only be managed as a multilevel common pool within the framework of co-management of all open operational units. This can lead to problems of sovereignty, problems that must be solved in the underlying treaty. The increasing use of regional water systems and the increasing volatility of weather events (heavy rainfall, drought, etc.) require a very high degree of flexibility in the institutional structure, to be able to react effectively to these unforeseeable events and should therefore be polycentric in nature. This idealtypical structure has certain characteristics, which are illustrated by Fig. 6.9, based on Pahl-Wostl and Knieper (2014).

Along the horizontal axis, the degree of power decreases from the left with centralized power to the right pole, where power is equally distributed among all institutional units involved. These could be, for example, regional water authorities that are located at the same level without hierarchies. The vertical axis indicates the degree of cooperation or coordination between the sub-institutions. This can refer, for example, to the coordination of decisions or to the exchange of information. Coop-

[41] There is an extensive literature on this concept, see, e.g., Akamani and Wilson (2011).

eration/coordination is strongly pronounced at the upper end; it ends in a completely uncoordinated coexistence.

Four idealtypes result from this coordination system. At the top left is the coordinated, centralized water institution as found in top-down approaches. Its adaptability is low. The transparency of information is low, the degree of participation is just as low, and it derives its legitimacy only from the national level. At the bottom left, we find ourselves in a completely disintegrated situation. A few players, equipped with comparatively much power, pursue their own interests to the detriment of the international water catchment area. Economic literature refers to this constellation as rent seeking. This system is also very susceptible to corruption. The situation improves a little on the lower right because the un-cooperating institutions are endowed with little power. They cannot effectively implement their interests. However, this does not mean that the catchment area will be managed sustainably. Fragmentation does not allow the development of a targeted sustainability strategy. The polycentric structure is the only idealtype that has the prerequisites for adaptive management. There are no dominance structures, such that the various user interests can be balanced. The individual stakeholders are well networked and coordinated. It is therefore possible to react quickly to changing environmental conditions.

However, the polycentric configuration can only be understood as an ideal type. Whether the structure can be implemented at all in the respective political gravitational fields is a completely different question. It may be that, due to historical path dependency and cultural conditions, certain forms of adaptive management can only be implemented in the course of a long reform process. The development of typologies is nevertheless useful because it elaborates the necessary institutional prerequisites for successful transboundary water management. This makes it clear that there is a long way between fundamental considerations about the allocation of scarce water, as described in the previous sections, and practical implementation as recognized institutional structures.

6.6.4 Application to Transboundary Agreements

Institutional design of integrated water resource management is even more challenging once cooperation between sovereign states is required. Figure 6.8 highlights some institutional issues of transboundary water management. The political spheres of both countries play a role in the joint management. At the political level, negotiations are first held on the allocation of water, which is restricted by geographical patterns of watercourses (tributaries, lakes, direction of streams, etc.). Of particular importance is whether a multidimensional contract or only a contract for the quantities of water should be negotiated. The compromise space is much larger in the case of the multipurpose contracts because different economic sectors can be combined, for example, water use can be traded against energy supply.[42] This negotiation might be conducted in the presence of power asymmetries, whether due to the position of the riparian states (upstream, downstream) or due to economic and military dominance. These initial strategic positions become increasingly important as water scarcity increases. In some circumstances, riparians may not be prepared to negotiate the use of water because they feel strong enough to use water without taking into account the needs of other riparian states. This does not necessarily mean, however, that conflicts must arise. There may be something like a status quo under customary law in which the management of a transboundary water body takes place. However, these undefined floating conditions are likely to vanish as water scarcity increases.

When a contract becomes ready to be signed, implementation is an issue. This raises the question of the institutional nature of transboundary water management. Here, similar aspects to those described for the national or regional level must be considered. The organization to be formed is located in the gravitational area of sovereign states, which makes the institutionalization and administrative work considerably more difficult. The Damocles' sword of unilateral termination or simply noncompliance with the treaty by the contracting parties is hovering over the institution established.

The effectiveness of a transboundary organization depends not only on the principles introduced above, but also on the depth of cooperation granted to it by the contractual partners. A distinction can be made between different degrees of cooperation (see Vollmer et al. (2009)):

- *Shallow cooperation*: There is only a loose connection between the contracting parties. The cooperation is not "visible", i.e., there are no formalized structures, like joint committees, task forces, or established partnerships. There is only a loose direct contact with the respective national organizational entities of the riparian countries enclosed in a treaty. This minimal institutionalization is, of course, the result of a contract that does not explicitly regulate much, but rather represents a declaration of intent for cooperation.
- *Intermediate cooperation*: The operational level is visibly structured here. There are regular meetings between the responsible representatives of the state authorities, and a secretariat organizes this interaction, which also requires its own staff. However, there is no budget sovereignty.
- *Deep cooperation*: Within this framework, the established authority has a certain autonomy. It has an independent budget and its powers go far beyond preparatory work (information, organization). It has decision-making powers.

The varying degrees of cooperation reflect the level of allocative power conferred on the established institution. The wider the field of competence and organizational depth of the institution, the greater its clout. Within the framework of the Sustainable

[42] Benefit sharing takes account of this exploitation of exchange gains, while bankruptcy rules restrict themselves to water as a means of distribution.

Development Goal, UN Water has defined the effectiveness of the institutionalization of transboundary basin management (indicator 6.5.2).[43] A transboundary management institution is called "operational" if it meets the following criteria:

- *There is a joint body, joint mechanism, or commission (e.g., a river basin organization) for transboundary cooperation;*
- *There are regular (at least once per year) formal communications between riparian countries in [the] form of meetings (either at the political or technical level);*
- *There is a joint or coordinated water management plan(s), or joint objectives are set, and;*
- *There is a regular exchange (at least once per year) of data and information.*

UN Water collects data on the organizational implementation of formalized cooperation.[44] For surface water projects, 84 of the 155 international contractual cooperations responded to the survey 2018. 42 of these have a very high degree of organizational structure, mainly Europe and Northern America, and sub-Saharan Africa. However, some caution is called for when evaluating the empirical results. As UN Water notes, the degree of organization cannot be used automatically to draw conclusions about the results, such as better water quality or an improvement in the livelihood of people living in the international waters under the organizational structures implemented. There is a critical literature on this indicator.[45] The mere fact that an organizational structure has been established does not necessarily mean that the underlying contract is fair and inclusive in terms of sustainability, i.e., involves the various stakeholders in transboundary water management.

6.7 Exercises

Exercise 6.1 Benefit sharing in a river with two riparian states
Assume there are two riparians at one river, riparian 1 which is upstream and riparian 2 which is downstream. The riparians are indexed with the indices i, with $i = \{1, 2\}$. The natural inflow into the river upstream of riparian 1 is given with $R_1 = 100$, while the natural inflow downstream of riparian 1 but upstream of riparian 2 is given with $R_2 = 50$. Due to the diversion and consumption of water from the river with the level w_i, the riparians generate a benefit of B_i. The benefit functions are specified with

[43] McCracken and Meyer (2018) analyze the methodology of the SDG indicator 6.5.2 and report on empirical results.
[44] See Bertule et al. (2018).
[45] See Hussein et al. (2018).

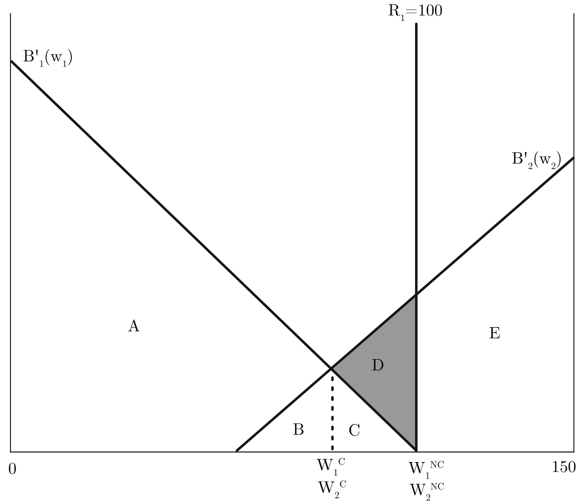


Fig. 6.10 Benefit sharing in a river basin with two riparians. *Source* own illustration

$$B_i(w_i) = a_i \cdot w_i - 0.5 \cdot b_i \cdot (w_i)^2$$

The parameters of the benefit functions are assumed with $a_1 = 100$, $a_2 = 60$, $b_1 = b_2 = 1$.

The situation in the river is shown by Fig. 6.10. The marginal benefit of riparian 1 is illustrated from the left vertical ordinate to the right direction, while we plot the marginal benefit of riparian 2 from the right vertical ordinate to the left direction. The length of the abscissa stands for the water amount $R_1 + R_2$, while the upstream external inflow R_1 is represented by the distance between the left origin of the diagram and the vertical line named with R_1. Therefore, the downstream external inflow R_2 is represented by the distance between the right origin of the diagram and the vertical line R_1. In the following explanation, we would like to find the UID, DID, and Shapley solution of the benefit sharing problem.

In the river basin, two cooperation scenarios are possible:

- The riparians act unilaterally in a noncooperative way. The water consumption amounts of the riparians are symbolized with w_1^{NC}. The consumption level of the upstream riparian (w_1^{NC}) is represented in Fig. 6.10 by the distance from the left origin of the diagram to the position of w_1^{NC}, while the consumption level of the downstream riparian (w_2^{NC}) is represented in Fig. 6.10 by the distance between the position of w_1^{NC} and the right origin of the diagram.
- The riparians form a joint arrangement and act in a cooperative manner. The water consumption amount for this scenario is represented by w_1^C. The consumption level of the upstream riparian (w_1^C) is represented in Fig. 6.10 by the distance from the left origin of the diagram to the position of w_1^C, while the consumption level of the downstream riparian (w_2^C) is represented in Fig. 6.10 by the distance between the position of w_1^C and the right origin of the diagram.

The first step of the benefit sharing problem is the calculation of the benefits under each cooperation scenario. Let's start with the unilateral acting. If the riparians act in a noncooperative way, any riparian would like to maximize its own specific benefit. Riparian 1 is upstream of riparian 2, hence riparian 1 will receive the natural inflow R_1 first. Therefore, we start with the benefit maximization problem of riparian 1. However, we have to note that the diverted amount w_1 is restricted by the water availability R_1. Therefore, we are able to formulate the following optimization problem:

$$\max_{\{w_1\}} [B_1(w_1)] \qquad s.t.\ w_1 \leq R_1 \tag{6.54}$$

Therefore, the following Lagrangian function can be formulated:

$$L_1 = B_1(w_1) + \lambda_1 \cdot (R_1 - w_1) \tag{6.55}$$

The resulting KKTs are

$$\begin{aligned} \frac{\partial L_1}{\partial w_1} &= B_1'(w_1) - \lambda_1 \leq 0 \perp w_1 \geq 0 \\ \frac{\partial L_1}{\partial \lambda_1} &= R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \end{aligned} \tag{6.56}$$

Both assumptions, on the one hand $w_1 \geq 0$ and $\lambda \geq 0$ and on the other hand $w_1 \geq 0$ and $\lambda = 0$, are leading to the optimal solution. For the assumption $w_1 \geq 0$ and $\lambda \geq 0$, it is possible to find the following solution:

$$\begin{aligned} (\boldsymbol{\lambda_1}):\ & R_1 - w_1 = 0 \\ \rightarrow\ & w_1^{NC} = R_1 = 100 \\ (\boldsymbol{w_1}):\ & B_1'(w_1) - \lambda_1 = 0 \\ \rightarrow\ \lambda_1 = B_1'(w_1^{NC}) = & a_1 - b_1 \cdot w_1^{NC} = 0 \end{aligned} \tag{6.57}$$

Based on the other assumption $w_1 \geq 0$ and $\lambda_1 = 0$, we find the same solution:

$$\begin{aligned} (\boldsymbol{w_1}):\ & B_1'(w_1) = 0 \\ \rightarrow\ & a_1 - b_1 \cdot w_1 = 0 \\ \rightarrow\ & w_1^{NC} = \frac{a_1}{b_1} = 100 \\ (\boldsymbol{\lambda_1}):\ & R_1 - w_1 \geq 0 \\ \rightarrow\ & R_1 = 100 \geq 100 = w_1 \end{aligned} \tag{6.58}$$

Therefore, the benefit of riparian 1 for unilateral acting is

$$B_1(w_1^{NC}) = a_1 \cdot w_1^{NC} - 0.5 \cdot b_1 \cdot (w_1^{NC})^2 = 5000 \tag{6.59}$$

which is represented by the illustration in Fig. 6.10 as the areas $A + B + C$.

After the consumption of riparian 1 (w_1) and the downstream headwater inflow R_2, the riparian 2 is able to divert and consume the water from the river. Due to the former water abstraction by riparian 1, just $R_1 + R_2 - w_1$ amounts of water are available for riparian 2. The optimization problem of the downstream riparian 2 is therefore

$$\max_{\{w_2\}} [B_2(w_2)] \qquad s.t.\ w_2 \leq R_1 + R_2 - w_1 \tag{6.60}$$

Hence, the following Lagrangian function can be formulated:

$$L_2 = B_2(w_2) + \lambda_2 \cdot (R_1 + R_2 - w_1 - w_2) \tag{6.61}$$

which leads to the following KKT:

$$\begin{aligned} \frac{\partial L_2}{\partial w_2} &= B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \\ \frac{\partial L_2}{\partial \lambda_2} &= R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \end{aligned} \tag{6.62}$$

Based on the assumption that riparian 2 will have a consumption ($w_2 \geq 0$) and that the available water for riparian 2 is fully used ($\lambda_2 \geq 0$), the solution of the optimization problem can be found as

$$\begin{aligned} (\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0 \\ \rightarrow\ w_2^{NC} &= R_1 + R_2 - w_1^{NC} = 50 \\ (\boldsymbol{w_2}):\ & B_2'(w_1) - \lambda_2 = 0 \\ \rightarrow\ \lambda_2 = B_2'(w_2^{NC}) &= a_2 - b_2 \cdot w_2^{NC} = 10 \geq 0\ \checkmark \end{aligned} \tag{6.63}$$

Therefore, the benefit of riparian 2 for the noncooperative acting in the basin is

$$B_2(w_2^{NC}) = a_2 \cdot w_2^{NC} - 0.5 \cdot b_2 \cdot (w_2^{NC})^2 = 1750 \tag{6.64}$$

which is represeted by area E in Fig. 6.10.

If the riparians form a joint arrangement, in which they allocate the water in a way that the benefit in the entire basin is maximized, the following optimization problem can be formulated:

$$\max_{\{w_1, w_2\}} [B_1(w_1) + B_2(w_2)] \qquad s.t.\ w_1 \leq R_1,\ w_2 \leq R_1 + R_2 - w_1 \tag{6.65}$$

Similar to the problems (6.54) and (6.60), the water consumption of any riparian is restricted by the available water at the respective abstraction point (see constraints of problem (6.65)). The available water for riparians 1 and 2 is R_1 and $R_1 + R_2 - w_1$, respectively. Based on problem 6.65, the following Lagrangian function can be set up:

$$L = B_1(w_1) + B_2(w_2) + \lambda_1 \cdot (R_1 - w_1) + \lambda_2 \cdot (R_1 + R_2 - w_1 - w_2) \tag{6.66}$$

Therefore, it is possible to formulate the following KKT:

$$\begin{aligned} \frac{\partial L}{\partial w_1} &= B_1'(w_1) - \lambda_1 - \lambda_2 \leq 0 \perp w_1 \geq 0 \\ \frac{\partial L}{\partial w_2} &= B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \\ \frac{\partial L}{\partial \lambda_1} &= R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \\ \frac{\partial L}{\partial \lambda_2} &= R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \end{aligned} \tag{6.67}$$

The assumptions $w_1 \geq 0$, $w_2 \geq 0$, $\lambda_1 = 0$, and $\lambda_2 \geq 0$ lead to the following optimality condition[46]:

$$\begin{aligned} (\boldsymbol{w_1}):\ & B_1'(w_1) - \lambda_2 = 0 \\ (\boldsymbol{w_2}):\ & B_2'(w_2) - \lambda_2 = 0 \\ (\boldsymbol{\lambda_1}):\ & R_1 - w_1 \geq 0 \\ (\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0 \end{aligned} \tag{6.68}$$

It is therefore possible to find the optimal level of consumption based on the following system of equations:

$$\begin{aligned} (\boldsymbol{w_1}) \wedge (\boldsymbol{w_2}) :\ & B_1'(w_1) = B_2'(w_2) \\ \rightarrow\ & a_1 - b_1 \cdot w_1 = a_2 - b_2 \cdot w_2 \\ (\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0 \end{aligned} \tag{6.69}$$

The solution of the system of equations is

$$w_1^C = 95\ ,\ w_2^C = 55$$

This solution is optimal, because there are no contradictions within the optimality conditions or assumptions:

$$\begin{aligned} (\boldsymbol{w_1}) \wedge (\boldsymbol{w_2}) :\ & \lambda_2 = B_1'(w_1^C) = B_2'(w_2^C) \\ \rightarrow\ & \lambda_2 = a_1 - b_1 \cdot w_1^C = a_2 - b_2 \cdot w_2^C = 5 \geq 0 \\ (\boldsymbol{\lambda_1}):\ & R_1 \geq w_1\ \rightarrow\ 100 \geq 95 \end{aligned} \tag{6.70}$$

The benefits which result from consumption are therefore

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[46]We assume that both riparians consume water and therefore, $w_1 \geq 0$ and $w_2 \geq 0$. Furthermore, we assume that the upstream riparian 1 does not consume the entire available water at its abstraction point and leaves water in the river, hence, $\lambda_1 = 0$, while the downstream riparian 2 abstracts the total amount which is available, hence, it can be assumed that $\lambda_2 \geq 0$.

$$B_1(w_1^C) = a_1 \cdot w_1^C - 0.5 \cdot b_1 \cdot (w_1)^2 = 4987.5\,,$$
$$B_2(w_2^C) = a_2 \cdot w_2^C - 0.5 \cdot b_2 \cdot (w_2)^2 = 1787.5$$

The benefit from consumption for riparians 1 and 2 are represented in Fig. 6.10 by the areas $A + B$ and $C + D + E$, respectively. Based on these benefits from consumption, it is possible to calculate the cooperation gain, which is

$$\Delta = B_1(w_1^C) + B_2(w_2^C) - B_1(w_1^{NC}) - B_2(w_2^{NC}) = 25 \tag{6.71}$$

which is represented in Fig. 6.10 by area D.

In the joint arrangement, any riparian has to receive at least as much benefits as it would generate for the unilateral acting case, which means $z_1 \geq B_1(w_1^{NC})$ and $z_2 \geq B_2(w_2^{NC})$. Therefore, the question of how to share the cooperation gain is the main focus of the benefit sharing problem for a basin with 2 riparians.

For the *UID approach*, the total cooperation gain is assigned to the upstream riparian, hence,

$$x_1^{UID} = B_1(w_1^{NC}) + \Delta = 5000 + 25 = 5025,\, x_2^{UID} = B_2(w_2^{NC}) = 1750$$

The benefit of the riparians 1 and 2 are represented by the area $A + B + C + D$ and E in Fig. 6.10, respectively. For realizing the UID approach, the upstream riparian has to receive side payments from the downstream riparian:

$$sp_{2,1}^{UID} = B_2(w_2^C) - z_2^{UID} = 1787.5 - 1750 = 37.5$$

which is represented by the area $C + D$ in Fig. 6.10.

However, the total cooperation gain is assigned to the downstream riparian 2 for the *DID approach*, therefore,

$$x_1^{DID} = B_1^{w_1^{NC}} = 5000\,,\; x_2^{DID} = B_2^{w_2^{NC}} + \Delta = 1750 + 25 = 1775$$

These benefits for riparians 1 and 2 are represented in Fig. 6.10 by the areas $A + B + C$ and $E + D$, respectively. For realizing this solution, the downstream has to make side payments to the upstream riparian of the level:

$$sp_{2,1}^{DID} = B_2(w_2^C) - x_2^{DID} = 1787.5 - 1775 = 12.5$$

which is represented by the area C in Fig. 6.10.

The UID and DID solution sets the minimum and maximum bound for the assigned benefits to the riparians in the joint arrangement (see Eqs. 6.72 and 6.73). Furthermore, the generated benefit has to be assigned to the riparians in total to meet feasibility and pareto-efficiency conditions (see Eq. 6.74).

$$x_1^{DID} \leq x_1 \leq x_1^{UID} \rightarrow 5000 \leq x_1 \leq 5025 \quad (6.72)$$

$$x_2^{UID} \leq x_2 \leq x_2^{DID} \rightarrow 1750 \leq x_2 \leq 1775 \quad (6.73)$$

$$x_1 + x_2 = B_1^{w_1^C} + B_2^{w_2^C} \rightarrow x_1 + x_2 = 4987.5 + 1787.5 = 6775 \quad (6.74)$$

The Shapley solution is a specific focal point solution of the benefit sharing problem, in which both riparians receive half of the cooperation gain:

$$x_1^{SH} = B_1(w_1^{NC}) + 0.5 \cdot \Delta = 5000 + 12.5 = 5012.5\,,\; x_2^{SH} = B_2(w_2^{NC}) + 0.5 \cdot \Delta = 1750 + 12.5 = 1762.5$$

The assigned benefit for riparian 1 is represented by the areas $A + B + C + 0.5 \cdot D$ in Fig. 6.10, while the benefit of riparian 2 is represented by areas $E + 0.5 \cdot D$. This Shapley solution could be realized by side payments made by riparian 2:

$$sp_{2,1}^{SH} = B_2(w_2^C) - x_2^{SH} = 1787.5 - 1762.5 = 25$$

which is represented by areas $C + 0.5 \cdot D$ in Fig. 6.10.

Exercise 6.2 Applying the focal point solution concepts of benefit sharing to a water body with two riparians

Assume a water body with two riparians (1 and 2). Both riparians can either act unilaterally (noncooperation scenario) or they can form a joint arrangement where they act in a cooperative way:

- If both act unilaterally, we assume that the water consumption of riparians 1 and 2 is w_1^{NC} and w_2^{NC}, respectively. Based on the consumption levels, the riparians 1 and 2 generate a benefit of $B_1^{NC}(w_1^{NC})$ and $B_2^{NC}(w_2^{NC})$. For simplification reasons, we will term the benefit in the case of noncooperation of riparian 1 by B_1^{NC} and the one of riparian 2 by B_2^{NC} in the following. The benefit generated in the entire basin is $B_1^{NC} + B_2^{NC}$.
- If both form a joint arrangement, the riparians allocate the water in a way that the benefit in the entire basin is maximized. Therefore, the riparians 1 and 2 receive the water w_1^C and w_2^C, respectively. Based on the consumption, they generate a benefit of $B_1^C(w_1^C)$ and $B_2^C(w_2^C)$, which can be simplified as B_1^C and B_2^C. The generated benefit in the basin is $B_1^C + B_2^C$. The cooperation gain Δ results from the difference of the benefit in the entire basin between cooperation and noncooperation:

$$\Delta = B_1^C + B_2^C - B_1^{NC} - B_2^{NC}$$

Therefore, the benefit in the entire basin under the case of cooperation $B_1^C + B_2^C$ can be also formulated as

$$B_1^{NC} + B_2^{NC} + \Delta$$

Assume the joint arrangement is formed and the generated benefit in the basin $(B_1^{NC} + B_2^{NC} + \Delta)$ should be assigned to the riparians 1 and 2. For solving this benefit sharing problem, we want to apply the three formerly explained focal point solution concepts which are presented in Sect. 6.3, for finding the Shapley, Nash-Harsanyi, and nucleolus solutions.

The Shapley Solution

We apply Eq. (6.16) for finding the Shapley solution, which is explained in detail in Sect. 6.3.[47]

The Shapley value solution of one riparian is affected by the weighting factor and the incremental benefit of this user for various cooperation scenarios.

Regarding the weighting factor, there are just two cooperation scenarios which can be realized, either the unilateral acting or the joint arrangement. We assume in the Shapley approach that both cooperation scenarios have the same realization probability, which is a purely normative assumption from the Shapley approach (see (Wu and Whittington 2006)). Hence both cooperation scenarios have a realization probability of 0.5:

- Unilateral acting: this cooperation scenario is represented by the sets {1} and {2}. There is of course per definition just one riparian in these sets, hence $\#ISG = 1$. We have two riparians in the basin, hence the grand coalition $\{1, 2\}$ consists of these two riparians and therefore $\#G = 2$. Inserting these parameters in the weighting factor, we get

$$\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} = \frac{(2-1)! \cdot (1-1)!}{2!} = \frac{1! \cdot 0!}{2!} = 0.5$$

- Joint arrangement: this situation is represented by the set $\{1, 2\}$ which consists of two riparians, hence $\#ISG = 2$. We already discussed the level of $\#G$, which is $\#G = 2$. Inserting these parameters in the weighting factor, we get

$$\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} = \frac{(2-2)! \cdot (2-1)!}{2!} = \frac{0! \cdot 1!}{2!} = 0.5$$

[47] The formula for finding the Shapley solution is

$$x_i = \sum_{\substack{I : i \in I \vee \\ S : i \in S \vee \\ G}} \left[\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} \cdot [V(\ldots) - V(\ldots - i)] \right] \tag{6.16}$$

The incremental benefit of riparian i, which is the second main element of the Shapley approach, is represented in Eq. (6.16) by the term $[V(...) - V(... - i)]$. This incremental benefit is

- in case of unilateral acting for riparians $\{1\}$ and $\{2\}$: the level of the respective benefit the unilaterally acting riparian generates.
- in case of a joint arrangement $\{1, 2\}$: the difference between the generated benefit in the joint arrangement and the level of benefit the other riparian generates under the situation of unilateral acting. Hence, the incremental benefit of riparian 1 is the difference between the generated benefit in the joint arrangement and the generated benefit of the unilaterally acting riparian 2, $V(\{1, 2\}) - V(\{2\})$, while the incremental benefit of riparian 2 is the difference between the generated benefit in the joint arrangement and the generated benefit of the unilaterally acting riparian 1, $V(\{1, 2\}) - V(\{1\})$.

We know that the benefit in the grand coalition is $V(\{1, 2\}) = B_1^{NC} + B_2^{NC} + \Delta$, while the benefit of the unilaterally acting riparian 1 is $V(\{1\}) = B_1^{NC}$ and the benefit of the unilaterally acting riparian 2 is $V(\{2\}) = B_2^{NC}$.

The riparian 1 is just part of the coalition scenarios $\{1\}$ and $\{1, 2\}$, hence the Shapley solution is

$$\begin{aligned} x_1^{SH} &= 0.5 \cdot V(\{1\}) + 0.5 \cdot [V(\{1, 2\}) - V(\{2\})] \\ &= 0.5 \cdot B_1^{NC} + 0.5 \cdot (B_1^{NC} + B_2^{NC} + \Delta - B_2^{NC}) \\ x_1^{SH} &= B_1^{NC} + 0.5 \cdot \Delta \end{aligned}$$

while the riparian 2 is just part of the coalition scenarios $\{2\}$ and $\{1, 2\}$, hence, its Shapley solution is

$$\begin{aligned} x_2^{SH} &= 0.5 \cdot V(\{2\}) + 0.5 \cdot [V(\{1, 2\}) - V(\{1\})] \\ &= 0.5 \cdot B_2^{NC} + 0.5 \cdot (B_1^{NC} + B_2^{NC} + \Delta - B_1^{NC}) \\ x_2^{SH} &= B_2^{NC} + 0.5 \cdot \Delta \end{aligned}$$

The following table at the next page can be also used as an auxiliary tool for finding the Shapley solution:
The Nash-Harsanyi solution

The optimization problem of the Nash-Harsanyi solution concept is (see Sect. 6.3)

$$\begin{aligned} &\max_{\{x_1, x_2\}} \left[(x_1 - B_1^{NC}) \cdot (x_2 - B_2^{NC}) \right] \\ &s.t.\ x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta \\ &\quad B_1^{NC} \le x_1 \\ &\quad B_2^{NC} \le x_2 \end{aligned}$$

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
Riparian i	Coalition	$\#ISG$	$\frac{(\#G-\#ISG)!\cdot(\#ISG-1)!}{\#G!}$	$V(\ldots)$	Coalition without i	$V(\ldots - i)$	(V) - (VII)	(IV) · (VIII)	Shapley Value
User 1	$\{1\}$	1	0.5	B_1^{NC}	$\emptyset$	0	B_1^{NC}	$0.5 \cdot B_1^{NC}$	$B_1^{NC} + 0.5 \cdot \Delta$
	$\{1, 2\}$	2	0.5	$B_1^{NC} + B_2^{NC} + \Delta$	$\{2\}$	B_2^{NC}	$B_1^{NC} + \Delta$	$0.5 \cdot (B_1^{NC} + \Delta)$	
User 2	$\{2\}$	1	0.5	B_2^{NC}	$\emptyset$	0	B_2^{NC}	0	$B_2^{NC} + 0.5 \cdot \Delta$
	$\{1, 2\}$	2	0.5	$B_1^{NC} + B_2^{NC} + \Delta$	$\{1\}$	B_1^{NC}	$B_2^{NC} + \Delta$	$0.5 \cdot (B_2^{NC} + \Delta)$	

Therefore, the following Lagrangian function results:

$$L = (x_1 - B_1^{NC}) \cdot (x_2 - B_2^{NC}) + \mu \cdot (B_1^{NC} + B_2^{NC} + \Delta - x_1 - x_2) + \lambda_1 \cdot (x_1 - B_1^{NC}) + \lambda_2 \cdot (x_2 - B_2^{NC})$$

And hence, we are able to set up the following KKT conditions:

$$(x_2 - B_2^{NC}) - \mu + \lambda_1 \leq 0 \perp x_1 \geq 0 \tag{6.75}$$

$$(x_1 - B_1^{NC}) - \mu + \lambda_2 \leq 0 \perp x_2 \geq 0 \tag{6.76}$$

$$B_1^{NC} + B_2^{NC} + \Delta - x_1 - x_2 = 0\ , \ \mu \text{ is free} \tag{6.77}$$

$$x_1 - B_1^{NC} \geq 0 \perp \lambda_1 \geq 0 \tag{6.78}$$

$$x_2 - B_2^{NC} \geq 0 \perp \lambda_2 \geq 0 \tag{6.79}$$

Suppose that both riparians receive benefits which exceed their respective individual rationality conditions. Hence, we have to assume that $x_1 \geq 0$, $x_2 \geq 0$, $\lambda_1 = 0$, and $\lambda_2 = 0$.[48] Based on Eqs. (6.75)–(6.77), we can set up the following system of equations:

$$x_1 - B_1^{NC} = x_2 - B_2^{NC}$$
$$x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$

The solution is

$$x_1^{NH} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{NH} = B_2^{NC} + 0.5 \cdot \Delta$$

This solution meets the conditions (6.78) and (6.79).[49]

<u>The nucleolus solution</u>

The nucleolus solution can be calculated on the basis of the following optimization problem (see Sect. 6.3):

$$\min_{\{x_1, x_2, e\}} [e]$$
$$s.t.\ x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$
$$e + x_1 \geq B_1^{NC}$$
$$e + x_2 \geq B_2^{NC}$$

[48] The variable μ is a free variable, because it is related to an equality constraint.

[49] Based on condition 6.78, $x_1 \geq B_1^{NC}$. Due to $x_1 = B_1^{NC} + 0.5 \cdot \Delta$, Eq. 6.78 is met. Based on condition 6.79, $x_2 \geq B_2^{NC}$. Due to $x_2 = B_2^{NC} + 0.5 \cdot \Delta$, Eq. 6.79 is met.

Therefore, we can formulate the following Lagrangian function:

$$L = e + \mu \cdot (x_1 + x_2 - B_1^{NC} - B_2^{NC} - \Delta) + \lambda_1 \cdot (B_1^{NC} - e - x_1) + \lambda_2 \cdot (B_2^{NC} - e - x_2)$$

and hence, we are able to set up the following KKT conditions:

$$\mu - \lambda_1 \geq 0 \perp x_1 \geq 0 \tag{6.80}$$

$$\mu - \lambda_2 \geq 0 \perp x_2 \geq 0 \tag{6.81}$$

$$1 - \lambda_1 - \lambda_2 = 0\,, \ e \text{ is free} \tag{6.82}$$

$$x_1 + x_2 - B_1^{NC} - B_2^{NC} - \Delta = 0\,, \ \mu \text{ is free} \tag{6.83}$$

$$B_1^{NC} - e - x_1 \leq 0 \perp \lambda_1 \geq 0 \tag{6.84}$$

$$B_2^{NC} - e - x_2 \leq 0 \perp \lambda_2 \geq 0 \tag{6.85}$$

Please note that e and μ are free variables, which means that they can have a positive or negative value. Under the assumption that $x_1 \geq 0$, $x_2 \geq 0$, $\lambda_1 \geq 0$, and $\lambda_2 \geq 0$, we are able to formulate the following system of equations[50]:

$$e = B_1^{NC} - x_1 = B_2^{NC} - x_2$$
$$x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$

The solution is

$$x_1^{nuc} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{nuc} = B_2^{NC} + 0.5 \cdot \Delta$$

The maximum objection which is minimized by applying the nucleolus approach is $e = -0.5 \cdot \Delta$.

Comparison of Focal Point Solutions

It becomes obvious from this analysis, that in a basin with just two riparians the three presented focal point solution concepts lead to the same results:

$$x_1^{SH} = x_1^{NH} = x_1^{nuc} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{SH} = x_2^{NH} = x_2^{nuc} = B_2^{NC} + 0.5 \cdot \Delta$$

This means that each riparian receives the benefit it would generate when acting unilaterally in a noncooperative way and furthermore half of the cooperation gain. Therefore, the cooperation gain is shared equally between the two riparians.

[50] We suppose that both riparians receive benefits, hence we assume $x_1 \geq 0$ and $x_2 \geq 0$. If we furthermore assume that the maximum objection in the nucleolus solution, denoted by e, is based on the payoff of the unilaterally acting riparian 1 (which means $e = B_1^{NC} - x_1$) as well as on the payoff of the unilaterally acting riparian 2 (which means $e = B_2^{NC} - x_2$), it becomes obvious that the conditions 6.84 and 6.85 become binding, and hence, we have to assume that $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

Exercise 6.3 Water allocation under bankruptcy rules
The importance of rationing rules will probably increase in the next few years. In many international waters, inflows are decreasing due to climate change. In particular, justice issues will become even more important in the discussion. Thereby, it is difficult to determine which bankruptcy rule leads to a fair distribution of water. We have seen that this question is closely related to the legitimacy of claims. But even if an agreement has been reached on what a justified level of claims is, the question remains as to which of the bankruptcy rules is fair. We cannot answer this question a priori here. That remains to be decided on a case-by-case basis.

What we can do, however, is to investigate how water allocations develop as a function of the scarcity of water. Thereby, we are primarily interested in how the relative allocation of scarce water as a function of R develops. To do so, we first define the degree to which water claims are met[51]:

$$\gamma_i^{BR} = x_i^{BR}/c_i, \quad i = \{1,2\}, \;\; BR = \{P, CEA, AP\} \tag{6.86}$$

To compare this degree, we construct the relative fulfillment:

$$\Gamma^{BR} = \gamma_1^{BR}/\gamma_2^{BR}, \;\; BR = \{P, CEA, AP\} \tag{6.87}$$

From Eq. (6.27), it is easy to derive the claim satisfaction of both countries for the proportional rule:

$$\gamma_1^P = x_1^P/c_1 = \frac{R}{c_1 + c_2} = \gamma_2^P = x_2^P/c_2 \tag{6.88}$$

From Eq. (6.88), it is clear that the relative fulfillment of claims does not change with respect to R. This bankruptcy rule is obviously fairness-stable, i.e., the relative claim fulfillment does not vary with R.

The same applies to the CEA rule, as can easily be shown. In our example in Sect. 6.4, the CEA allocation led[52] to $x_1^{CEA} = R/2 = x_2^{CEA}$. Thus,

$$\Gamma^{CEA} = \gamma_1^{CEA}/\gamma_2^{CEA} = \frac{(R/2)/c_1}{(R/2)/c_2} = \frac{c_2}{c_1} \tag{6.89}$$

which shows that the CEA rule is also fairness-stable with regard to a decrease of the water supply.

It remains to analyze the Adjusted Proportional Rule. From Eq. (6.28), we have

$$x_1^{AP} = (R - c_2) + \frac{c_1 - R + c_2}{2(c_1 + c_2 - R)}\{R - (R - c_2) - (R - c_1)\} \tag{6.90}$$

[51] The superscript BR refers to bankruptcy rules.

[52] We have assumed the following numerical values: $R = 200$, $c_1 = 180$, and $c_2 = 120$. This has led to water allocation as depicted in Fig. 6.3.

which can be reduced to

$$x_1^{AP} = \frac{R + c_1 - c_2}{2} \quad \Rightarrow \quad \gamma_1^{AP} = x_1^{AP}/c_1 = \frac{R + c_1 - c_2}{2c_1} \tag{6.91}$$

Finally, utilizing the claim fulfillment of country 2, we get

$$\Gamma^{AP} = \gamma_1^{AP}/\gamma_2^{AP} = \left(\frac{c_2}{c_1}\right)\left[\frac{R + c_1 - c_2}{R + c_2 - c_1}\right] \tag{6.92}$$

It is left as an exercise to the reader to prove that

$$\frac{\partial \Gamma^{AP}}{\partial R} = \left(\frac{c_2}{c_1}\right)\frac{2(c_2 - c_1)}{(R + c_2 - c_1)^2} \tag{6.93}$$

Our numerical example $c_1 = 180 > c_2 = 120$ implies that with increasing R, the relative claim fulfillment for the upstream country gets worse. Thus, with a lower water supply the relative claim fulfillment decreases for the downstream country.

What is the lesson of this task? It shows that the riparian countries choose bankruptcy rules not only depending on the outcome for a certain water supply currently provided by the regional hydrological cycle but also on the characteristics of these rules when the water supply changes.

Exercise 6.4 The robustness of water agreements

In many international waters, water inflow has declined in recent years. This is partly due to climate change. This unexpected change in the water cycle is often not taken into account in international water treaties. We have addressed this problem in the section on the robustness of water contracts. In the following, we will examine the stability of fixed and proportional contracts with the help of a very simple numerical example.

Assume two identical countries, country 1 (upstream) and country 2 (downstream). The benefit function of each is $B_i(w_i) = aw_i - (b/2)w_i^2$, $i = \{1, 2\}$ Let $a = 100$ and $b = 1$. As in Sect. 6.5, we assume that water inflow takes place only upstream and is $R = 100$. Since both countries are identical, the optimal allocation would be simply $w_1^* = w_2^* = R/2 = 50$, i.e., the upstream country allows half of the water supply to flow through to country 2. It remains to analyze the non-water transfer of the downstream country to the upstream country. Let us assume, that this transfer is constructed such that the joint benefit of cooperation is distributed according to the Shapley value. We know that the Shapely value lies for the case with just two riparians in the core. The formula is

$$s_1 = B_1(R) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] \tag{6.94}$$

$$s_2 = B_2(0) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] \tag{6.95}$$

This benefit division can be accomplished with the help of a transfer of country 2 to country 1. We have

$$s_1 = B_1(R) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] = B_1(R/2) + T \tag{6.96}$$

where T is the non-water transfer from downstream to upstream. From this equation it is easy to calculate T:

$$T = \frac{1}{2}[B_1(R) + B_2(R/2) - B_1(R/2)] \tag{6.97}$$

Notice thereby that $B_2(0) = 0$. Since we have assumed identical countries $B_2(R/2) = B_1(R/2)$ and, hence, the transfer is simply

$$T = \frac{1}{2}B_1(R) \tag{6.98}$$

the downstream country ends up with

$$s_2 = B_2(R/2) - T \tag{6.99}$$

It is now easy to determine the degree of robustness of both types of contracts analyzed in Sect. 6.5 for the Shapley value. We begin with the fixed contract.

The upstream country delivers the fixed amount of water $R/2$ for a fixed payment of T. As the water supply drops, the net benefit of the upstream country decreases. Note that the benefit of country 2 is not affected by the water decrease. The contract is robust as long as

$$B_1(r - (R/2)) + T = B_1(r - (R/2)) + (1/2)B_1(R) \geq B_1(r) \tag{6.100}$$

If we insert the quadratic benefit function, we can find the critical value of r for which Eq. (6.100) is an equality:

$$r = (3/4)R \tag{6.101}$$

This result can be found in Fig. 6.6. If the water decrease is less than 25% of R, the water contract is stable. However, a larger water decrease would lead to a dissolution of the agreement if the parties feel that the decrease will be long term.

To derive the r-range of the proportional contract, we start with the benefit of the upstream county. The contract specifies that half of the available water flows downstream. Utilizing Eq. (6.50), total benefit of country 1 is

$$B_1(r/2) + T(r) = B_1(r/2) + (r/R)T \tag{6.102}$$

where T is defined in Eq. (6.98).

To determine the range where total benefit defined in Eq. (6.102) within the contract is higher than or equal to the conflict option, we have to set it equal to $B_1(r)$. If we substitute the quadratic utility function into Eq. (6.102), it is an easy task to calculate the critical r-value[53]:

$$r = (2/3)R \tag{6.103}$$

It remains to check whether the downstream country sticks to the contract as the water supply drops. The respective constraint is

$$B_2(r/2) - (r/R)T = B_2(r/2) - \frac{r}{2R}B_1(R) \geq B_2(0) = 0 \tag{6.104}$$

Substituting the quadratic benefit function into this constraint yields after some algebraic manipulation $r \leq 2R$ which is always satisfied, since $r \leq R$. The downstream country never has an incentive to break the contract. It is always worse off without water from upstream.

This analysis assumes that both countries only compare their benefits with respect to the conflict case, i.e., the situation without a cooperative solution. However, we know from the history of the bargaining process preceding the conclusion of a water agreement that the distribution, i.e., the relative position of the contract partner, is of high importance. Let us assume that in our example both countries were satisfied with the distribution of the benefits at the outset. The allocation of water and the transfer determined by the Shapley approach is deemed fair. As the water supply drops, both net benefits decrease and the question remains whether the resulting distribution of benefits continues to be regarded as just if the contract concluded for $r = R$ still applies. If not, it may happen that the contract will be broken even if the conflict position makes the parties worse off. Such behavior due to an injured sense of justice may well occur, as we know from experimental economics and also from everyday life.

6.8 Further Reading

International environmental agreements as well as international water agreements are often analyzed in economics as a sequence of strategic negotiation steps. Noncooperative game theory is particularly suitable for this purpose. Each negotiating participant tries to maximize his advantages, taking into account the behavior of the other participants. In cooperative game theory, the main purpose is to determine joint action and the distribution of cooperative gains. The theory assumes that binding contracts can be concluded, i.e., the parties to the contract adhere to the agreed arrangements. A very useful introduction to cooperative game theory with a number of relevant examples from international water agreements is Dinar et al. (2007). Wu

[53] See in Fig. 6.7, the intersection between the blue and the gray line.

and Whittington (2006) apply the concepts of the Shapley value and the nucleolus to a water-sharing game of the Nile. In addition to the calculation of the benefits of cooperation, the authors have also included hydrological constraints.

Contrary to cooperative game theory, bankruptcy rules deal with zero-sum games in a noncooperative setting. What one gets, the other does not have, and vice versa. It is obvious that with zero-sum games, considerations of justice are of particular importance. Thomson (2002) gives a very instructive overview. Which rules of division fulfill which axioms or criteria of justice? The relationship with cooperative game theory is made. This concept can also be applied in the context of zero-sum games (e.g., the Shapley value). Dagan and Volij (1993) compare different bankruptcy rules with the bargaining approach of game theory. This is an interesting approach because the criteria and properties of bankruptcy rules are interpreted as the result of cooperative negotiations. For example, the authors show that the cooperative Nash solution leads to CEA. One must be careful when applying bankruptcy rules to transboundary water agreements. This applies in particular to river basins where there is a hierarchical structure of claims due to the unidirectionality of watercourses. It is possible that a water allocation resulting from the direct application of a bankruptcy rule cannot be implemented at all for hydrological reasons. Ansink and Weikard (2012) have therefore developed modified bankruptcy rules (sequential sharing rules) that take hydrological restrictions into account.

The effects of climate change will also affect international waters. For some time now, scientists have been working on the question of how the increasing water scarcity and variability of the water supply should be taken into account in international water agreements. Cooley and Gleick (2011) analyze how existing contracts can accommodate these changes. The allocation rules must be made dependent on the amount of water available. This requires a functioning monitoring system embedded in a well institutionalized transboundary management system. Ansink and Ruijs (2008) analyze the exact effects of sharing rules in a formal model when the average available water quantity decreases and the variability increases at the same time. They conclude that the increasing scarcity of water reduces the stability of international treaties, but that increasing variability can even lead to a strengthening of contractual cooperation. This analysis is deepened in Ambec et al. (2013). Different contract formats (fixed and variable) are examined with regards to their vulnerability to increasing water scarcity. An important finding of the authors is that contracts are stable when their contractual components (water and compensatory transfers) are contingent on a variable water supply. In this case, the contract becomes self-enforcing, i.e., there is no incentive for the contracting parties to violate the contract.

The literature on the institutionalization of international treaties on water use is very extensive. It is interdisciplinary and transdisciplinary. One of the latter is Biswas (1996), who has worked on transboundary water management both as a scientist and as an expert and practitioner. This also includes Draper (2007), who is active as a researcher and political planner of water infrastructures. He developed criteria as a necessary prerequisite for effective water-sharing agreements.

In addition to more descriptive studies, such as Vollmer et al. (2009), which bring the institutional diversity of transboundary management into a taxonomic order, there are analyses based on institutional economics, such as the comprehensive work of Saleth and Dinar (2004). Here, the network of institutional structures at different administrative and political levels is examined with regard to their effectiveness. This depends primarily on the goals of the institutional units and the existing incentive mechanisms.

The interdependence of institutional units, e.g., between water authorities at the regional level and transboundary institutions, which are made up of representatives of different riparian states, is also the focus of research dealing with adaptive management. However, the question is broader: How can the complexity of the ecological system of an international water catchment area be combined with a correspondingly adapted design of water management to form a sustainable and resilient ecological-social integrated system? Akamani and Wilson (2011) and Pahl-Wostl et al. (2008) give an overview. The basic philosophy is presented in Folke et al. (2005), and Karkkainen (2004) provides two very instructive examples of how joint water management goes far beyond the rigid implementation of a treaty at a national level (Chesapeake Bay Program, US-Canadian Great Lakes Program).

6.9 Chapter-Annex: Step-by-Step Solution of Optimization Problems of Sect. 6.3

The Core: Identify the Number of Solutions in the Core

Optimization problem:

$$\max\left[\gamma - x_1 - x_2 - x_3\right] \qquad s.t.\ x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta\ ,\ x_1 + x_3 \geq \alpha\ ,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = \gamma - x_1 - x_2 - x_3 + \lambda_{\{1\}} \cdot (x_1 - \alpha) + \lambda_{\{2\}} \cdot x_2 + \lambda_{\{3\}} \cdot x_3$$
$$+ \lambda_{\{1,2\}} \cdot (x_1 + x_2 - \beta) + \lambda_{\{1,3\}} \cdot (x_1 + x_3 - \alpha) + \lambda_{\{2,3\}} \cdot (x_2 + x_3)$$

KKT Conditions:

$$-1 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} \leq 0 \perp x_1 \geq 0$$
$$-1 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} \leq 0 \perp x_2 \geq 0$$
$$-1 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} \leq 0 \perp x_3 \geq 0$$
$$x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0$$
$$x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0$$
$$x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0$$
$$x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0$$
$$x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0$$

Assumption:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 = 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{1,2\}} \geq 0$$

Solution: (Please note, the objective $\gamma - x_1 - x_2 - x_3$ is denoted as Z)

$$x_1 = \alpha + \delta,\ x_2 = \beta - \delta,\ \lambda_{\{1,2\}} = 1$$
$$Z = \gamma - x_1 - x_2 - x_3 = \gamma - \beta \geq 0$$

The core: The Lower Bound of Each Player in the Core

Optimization problem:

$$\min \left[x_j\right] s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = x_j + \lambda_{\{1,2,3\}} \cdot (x_1 + x_2 + x_3 - \gamma) + \lambda_{\{1\}} \cdot (\alpha - x_1) - \lambda_{\{2\}} \cdot x_2 - \lambda_{\{3\}} \cdot x_3$$
$$+ \lambda_{\{1,2\}} \cdot (\beta - x_1 - x_2) + \lambda_{\{1,3\}} \cdot (\alpha - x_1 - x_3) + \lambda_{\{2,3\}} \cdot (-x_2 - x_3)$$

KKT Conditions of Dual Variables:

$$x_1 + x_2 + x_3 - \gamma = 0\ ,\ \lambda_{\{1,2,3\}} \text{ is free}$$
$$\alpha - x_1 \leq 0 \perp \lambda_{\{1\}} \geq 0$$
$$-x_2 \leq 0 \perp \lambda_{\{2\}} \geq 0$$
$$x_3 \leq 0 \perp \lambda_{\{3\}} \geq 0$$
$$\beta - x_1 - x_2 \leq 0 \perp \lambda_{\{1,2\}}$$
$$\alpha - x_1 - x_3 \leq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$-x_2 - x_3 \leq 0 \perp \lambda_{\{2,3\}} \geq 0$$

KKT Conditions of Primal Variables (primal variable is in the objective):

$$1 - \lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_1 \geq 0$$
$$1 - \lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_2 \geq 0$$
$$1 - \lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_3 \geq 0$$

KKT Conditions of Primal Variables (primal variable is not in the objective):

$$-\lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_1 \geq 0$$
$$-\lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_2 \geq 0$$
$$-\lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_3 \geq 0$$

Assumption for Problem of Riparian 1:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Solution for Problem of Riparian 1:

$$x_1 = \alpha$$
$$\lambda_{\{1\}} = 1,\ \lambda_{\{1,2,3\}} = 0$$
$$x_2 = \delta_2,\ x_3 = \delta_3$$
$$\text{with } 0 \leq \delta_2 \leq \gamma - \alpha, 0 \leq \delta_3 \leq \gamma - \alpha \text{ and } \delta_2 + \delta_3 = \gamma - \alpha$$

Assumption for Problem of Riparian 2:

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{2\}} \geq 0$$

Solution for Problem of Riparian 2:

$$x_2 = 0$$
$$\lambda_{\{2\}} = 1,\ \lambda_{\{1,2,3\}} = 0$$
$$x_1 = \alpha + \delta_1,\ x_3 = \delta_3$$
$$\text{with: } 0 \leq \delta_1 \leq \gamma - \alpha, 0 \leq \delta_3 \leq \gamma - \alpha \text{ and } \delta_1 + \delta_3 = \gamma - \alpha$$

Assumption for Problem of Riparian 3:

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{3\}} \geq 0$$

Solution for Problem of Riparian 3:

$$x_3 = 0$$

The core: The Upper Bound of Each Player in the Core

Optimization Problem:

$$\max \left[x_j\right] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = x_j + \lambda_{\{1,2,3\}} \cdot (\gamma - x_1 - x_2 - x_3) + \lambda_{\{1\}} \cdot (x_1 - \alpha) + \lambda_{\{2\}} \cdot x_2 + \lambda_{\{3\}} \cdot x_3$$
$$+ \lambda_{\{1,2\}} \cdot (x_1 + x_2 - \beta) + \lambda_{\{1,3\}} \cdot (x_1 + x_3 - \alpha) + \lambda_{\{2,3\}} \cdot (x_2 + x_3)$$

<u>KKT Conditions of the Dual Variables:</u>

$$\begin{aligned} \gamma - x_1 - x_2 - x_3 = 0,\ \lambda_{\{1,2,3\}} \text{ is free} \\ x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0 \\ x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0 \\ x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0 \\ x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0 \\ x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0 \\ x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0 \end{aligned}$$

<u>KKT Conditions of Primal Variables</u> (primal variable is in the objective):

$$\begin{aligned} 1 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1 \\ 1 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0 \\ 1 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0 \end{aligned}$$

<u>KKT Condition of Primal Variables</u> (primal variable is not in the objective):

$$\begin{aligned} \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1 \geq 0 \\ \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0 \\ \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0 \end{aligned}$$

<u>Assumption for Problem of Riparian 1:</u>

$$\begin{aligned} & x_1 \geq 0, x_2 = 0, x_3 = 0 \\ & \lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0 \end{aligned}$$

<u>Solution for Problem of Riparian 1:</u>

$$x_1 = \gamma,\ \lambda_{\{1,2,3\}} = 1$$

<u>Assumption for Problem of Riparian 2:</u>

$$\begin{aligned} & x_1 \geq 0,\ x_2 \geq 0,\ x_3 = 0 \\ & \lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0 \end{aligned}$$

<u>Solution for Problem of Riparian 2:</u>

$$\begin{aligned} & x_2 = \gamma - \alpha \\ & x_1 = \alpha,\ \lambda_{\{1\}} = 1,\ \lambda_{\{1,2,3\}} = 1 \end{aligned}$$

Assumption for Problem of riparian 3:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{1,2\}} \geq 0$$

Solution for Problem of Riparian 3:

$$x_3 = \gamma - \beta$$
$$\alpha \leq x_1 \leq \alpha + \delta,\ 0 \leq x_2 \leq \beta - \alpha - \delta$$
$$\text{with: } 0 \leq \delta \leq \beta - \alpha \text{ as well as } \lambda_{\{1,2\}} = 1 \text{ and } \lambda_{\{1,2,3\}} = 1$$

The Nash-Harsanyi Solution

Optimization Problem:

$$\max\,[(x_1 - \alpha)\cdot x_2 \cdot x_3] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = (x_1 - \alpha)\cdot x_2 \cdot x_3 + \lambda_{\{1,2,3\}}\cdot(\gamma - x_1 - x_2 - x_3) + \lambda_{\{1\}}\cdot(x_1 - \alpha) + \lambda_{\{2\}}\cdot x_2 + \lambda_{\{3\}}\cdot x_3$$
$$+ \lambda_{\{1,2\}}\cdot(x_1 + x_2 - \beta) + \lambda_{\{1,3\}}\cdot(x_1 + x_3 - \alpha) + \lambda_{\{2,3\}}\cdot(x_2 + x_3)$$

KKT Conditions:

$$x_2 \cdot x_3 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1 \geq 0$$
$$(x_1 - \alpha)\cdot x_3 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0$$
$$(x_1 - \alpha)\cdot x_2 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0$$
$$\gamma - x_1 - x_2 - x_3 = 0,\ \lambda_{\{1,2,3\}} \text{ is free}$$
$$x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0$$
$$x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0$$
$$x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0$$
$$x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0$$
$$x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0$$

Assumption:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Optimality Conditions:

$$\lambda_{\{1,2,3\}} = x_2 \cdot x_3 = (x_1 - \alpha)\cdot x_3 = (x_1 - \alpha)\cdot x_2$$
$$x_1 + x_2 + x_3 = \gamma$$

Solution:

$$x_1 = \frac{1}{3} \cdot (2 \cdot \alpha + \gamma),\ x_2 = x_3 = \frac{1}{3} \cdot (\gamma - \alpha)$$

The Nucleolus

Optimization Problem:

$$\min\,[e] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 + e \geq \alpha,\ x_2 + e \geq 0,\ x_3 + e \geq 0$$
$$x_1 + x_2 + e \geq \beta,\ x_1 + x_3 + e \geq \alpha,\ x_2 + x_3 + e \geq 0$$

Lagrangian Function:

$$\begin{aligned} L = e &+ \lambda_{\{1,2,3\}} \cdot (x_1 + x_2 + x_3 - \gamma) + \lambda_{\{1\}} \cdot (\alpha - x_1 - e) + \lambda_{\{2\}} \cdot (-x_2 - e) + \lambda_{\{3\}} \cdot (-x_3 - e) \\ &+ \lambda_{\{1,2\}} \cdot (\beta - x_1 - x_2 - e) + \lambda_{\{1,3\}} \cdot (\alpha - x_1 - x_3 - e) + \lambda_{\{2,3\}} \cdot (-x_2 - x_3 - e) \end{aligned}$$

KKT Conditions:

$$\begin{aligned} \lambda_{\{1,2,3\}} - \lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} \geq 0 \perp x_1 \geq 0 \\ \lambda_{\{1,2,3\}} - \lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} \geq 0 \perp x_2 \geq 0 \\ \lambda_{\{1,2,3\}} - \lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} \geq 0 \perp x_3 \geq 0 \\ 1 - \lambda_{\{1\}} - \lambda_{\{2\}} - \lambda_{\{3\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} = 0,\ e \text{ is free} \\ x_1 + x_2 + x_3 - \gamma = 0,\ \lambda_{\{1,2,3\}} \text{ is free} \\ \alpha - x_1 - e \leq 0 \perp \lambda_{\{1\}} \geq 0 \\ -x_2 - e \leq 0 \perp \lambda_{\{2\}} \geq 0 \\ -x_3 - e \leq 0 \perp \lambda_{\{3\}} \geq 0 \\ \beta - x_1 - x_2 - e \leq 0 \perp \lambda_{\{1,2\}} \geq 0 \\ \alpha - x_1 - x_3 - e \leq 0 \perp \lambda_{\{1,3\}} \geq 0 \\ -x_2 - x_3 - e \leq 0 \perp \lambda_{\{2,3\}} \geq 0 \end{aligned}$$

In accordance with the specification of α, β, and γ, we have to differentiate between two cases:

- Case 1 is valid if $(\beta \leq \frac{\gamma}{3}) \vee ((\frac{\gamma}{3} < \beta) \wedge (\frac{3\cdot\beta-\gamma}{2} \leq \alpha))$
- Case 2 is valid if $(\frac{\gamma}{3} < \beta) \wedge (\alpha < \frac{3\cdot\beta-\gamma}{2})$

Assumption under Case 1:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} \geq 0,\ \lambda_{\{3\}} \geq 0,\ \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Optimality Conditions Under Case 1:

$$\alpha - x_1 - e = -x_2 - e = -x_3 - e = 0$$
$$x_1 + x_2 + x_3 - \gamma = 0$$
$$\lambda_{\{1,2,3\}} = \lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}}$$

Solution Under Case 1:

$$x_1 = \frac{2 \cdot \alpha + \gamma}{3}, \; x_2 = x_3 = \frac{\gamma - \alpha}{3}$$
$$e = \frac{\alpha - \gamma}{3}$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2,3\}} = \frac{1}{3}$$

Assumption Under Case 2:

$$x_1 \geq 0, \; x_2 \geq 0, \; x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0, \; \lambda_{\{3\}} \geq 0, \; \lambda_{\{1,2\}} \geq 0$$

Optimality Conditions Under Case 2:

$$\beta - x_1 - x_2 - e = -x_3 - e = 0$$
$$x_1 + x_2 + x_3 - \gamma = 0$$
$$\lambda_{\{1,2,3\}} = \lambda_{\{1,2\}} = \lambda_{\{3\}}$$

Solution Under Case 2:

$$x_1 + x_2 = \frac{\beta + \gamma}{2}, \; x_3 = \frac{\gamma - \beta}{2}$$
$$e = \frac{\beta - \gamma}{2}$$
$$\lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,2,3\}} = 0.5$$

The nucleolus for the sub-coalition {1, 2} can be solved using the following optimization problem:

$$\min_{\{e, x_1, x_2\}} [e]$$
$$s.t. \; x_1 + x_2 = 0.5 \cdot (\beta + \gamma), \; e + x_1 \geq \alpha, \; e + x_2 \geq 0$$

The step-by-step nucleolus solution for a coalition with two riparians is explained in detail in Exercise 6.2.

Solution:

$$x_1 = \frac{2 \cdot \alpha + \beta + \gamma}{4}, \; x_2 = \frac{\beta + \gamma - 2 \cdot \alpha}{4}$$

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