Chapter 3
The Forest of Alternative Choices

Watch the path of your feet And all your ways will be established.

—Proverbs 4:26

In general, a path can be thought of as a sequence, timely ordered sequence of consecutive events or choices which can lead us far from the starting point.

Following a path means that we choose a specific sequence of steps from a pool of possibilities or alternative choices. So what does such a pool look like? Well, sometimes it is small, concrete and well-defined, while other times it is seemingly infinite and may be obscure and intricate. To illustrate, let's consider a very famous and classic event pool from European century.

In the eighteenth century, the city of Königsberg, Prussia was wealthy enough to have seven bridges across the river Pregel. The seven bridges connected four parts of lands separated by the branches of the river. The situation is shown in Fig. 3.1 where letters A, B, C, D denote the lands and the corresponding handwriting (ending with the B. and Br. abbreviations) mark the locations of the bridges. This scenario inspired the fantasy of the leisured inhabitants of Königsberg who made a virtual playground from the bridges and lands. Their favorite game was to think about a possible walk around the bridges and lands in which they cross over each bridge once and only once. Nobody could come up with such a fancy walk and nobody managed to prove that such a walk was impossible, until Leonhard Euler, the famous mathematician, took a look at the problem. Euler quickly noticed that from the perspective of the problem, most of the details of the map shown in Fig. 3.1 could be omitted and a much simpler figure could be drawn, focusing more on the problem (see Fig. 3.2).

This new representation contains only "nodes" marked with letters A, B, C, D in circles which represent the lands and "edges" drawn with curved lines between the nodes representing the bridges. A walk now can be described as a sequence of nodes and edges. For example the sequence A → E1 → C → E3 → D → E4 → A represents a walk starting from land A which proceeds to land C via bridge E1, then to land D via bridge E3, and finally back to land A via bridge E4. Using only the nodes and edges, all sorts of walks can be created. In fact, all the possible walks

A. Gulyás et al., *Paths*, https://doi.org/10.1007/978-3-030-47545-1_3

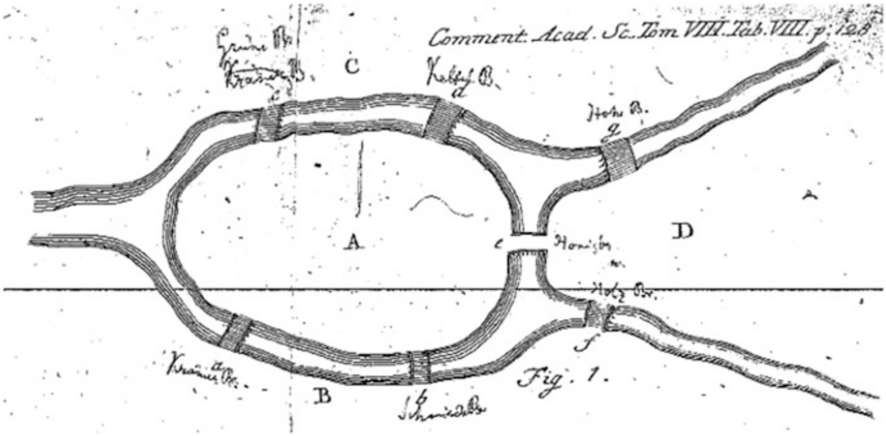

Fig. 3.1 Euler's Fig. 1 for the seven bridges of Königsberg problem from 'Solutio problematis ad geometriam situs pertinentis,' Eneström 53 [source: MAA Euler Archive; http://eulerarchive.maa.org/docs/originals/E053.pdf]

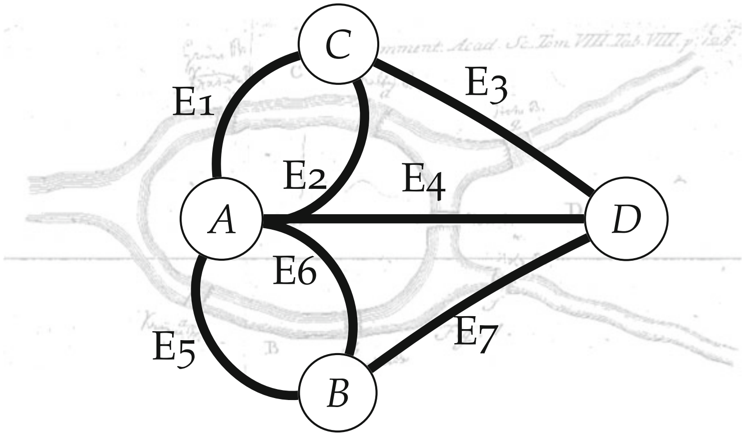

Fig. 3.2 Euler's idea of abstracting away the network underlying the Seven Bridges of Königsberg puzzle

that one can imagine throughout the bridges and lands is captured by this simple representation. The collection of nodes and edges called a network $N(n, e)$ turned out to be so powerful in modeling real-world problems that a whole new branch of mathematics, called graph theory,[1] was defined based on them.

[1]In the first ever graph theoretic argumentation Euler showed that to find a walk crossing each bridge once and only once requires that the underlying network can contain only two nodes with an odd number of edges. In Fig. 3.2 one can see that all nodes have an odd number of edges (A has five, while B, C and D has three), which makes the problem insolvable in this network.

We can observe that the walks around the lands and bridges are nothing more than the ordered sequences of consecutive events (bridge crossings) in Königsberg. These walks are very similar to our *paths* and the network $N(n, e)$ seems to effectively contain all the possible paths that can be taken, i.e., the paths[2] that can be differentiated by the sequence of bridge crossings in Königsberg. So, a network seems to be a good representation of the pools from which paths can take shape. The network in the case of the Königsberg bridges is very small and well-defined (contains four nodes and seven edges); however, the number of possible paths that people can take in this network is theoretically infinite as the length of the paths is not limited. In practice, the pool of all possible paths is much smaller as people become tired or bored after a few hours of walking. Even if we remove E2 and E3 and we are allowed to cross a maximum of ten bridges during the walk, there are still 2330 possible paths to choose from. Would people generally have a preference when choosing their afternoon walk? Will they choose randomly from all the possibilities? Or is there a hidden order affecting their choices? Those questions get even more complicated when, as in many real-life situations, a few more orders of magnitude of choices are at hand. For the sake of extending our scope for other connected systems, let us take a slightly more abstract network from the social sciences.

The small world experiments conducted by Stanley Milgram, a famous social psychologist, in the 1960s targeted to understand the network of human contacts in society. The main goal was to study the connectedness of people formed by their acquaintances. In the experiments, several random people were asked independently to send a letter (postcard) to a randomly chosen common target. Anyone, who did not personally know the target, was asked to send the letter to a friend who possibly would. Then the selected friends were subsequently asked to act the same way until the message arrived. In such a way, the persons were the nodes, and the friendships made up the edges of the network, while the traveler was the letter. Although many of the messages never arrived, those that did found a surprisingly short way through the chain of acquaintances. In many cases, even two or three middlemen were just enough for the letter to arrive (the average path length fell close to 6), in spite of the fact that the endpoints of the chain were carefully picked to be sufficiently far away from each other either geographically or socially. The so-called small world phenomenon is quite arresting in itself, the way the path is formed on the social graph is also fascinating. The arbiter behind the wanderer in this case is not a single entity but several independent ones, thus the established route is a collective phenomenon. Is there any similarity between the paths taken by individuals or the members of a community having a partially divergent perception on their environment? Do they achieve better at finding the shortest paths? Or do they require some superfluous sidesteps as well?

We will be able to answer such questions soon, but for now, let's be satisfied with finding networks as good representatives of all the imaginable paths belonging

[2]Note that the word path as used in this book corresponds to walks in the terminology of graph theory.

to a specific situation, because we will use them throughout this book. So, we have networks over which one can take paths by traversing nodes and edges in a particular sequence. But who or what will take the paths? Well, sometimes they are people as in the Bridges of Königsberg problem, sometimes a letter controlled by a small community as in the Milgram experiments. But in a broader scope, there can be many things that can take paths. Gossip, fashion styles, memes and all sorts of information seem to travel over social networks. If we look inside the human brain, we can identify the neurons as nodes and their axons as edges. What travels through this network? All kinds of information encoded into the specific firing patterns of neural cells. Similarly to networks (which can represent all kinds of paths), we need to find a name for the something which will travel through the network. From now on, we will call these travelers "packets". Networks and packets will be all we need to discuss paths in the broadest scope. Now let's consider a much more intricate network, over which the traveling "packets" will be indeed: packets.

The Internet is the greatest network man has ever built. Starting from a small research network funded by the US government, it became a huge interconnection network of thousands of computers all over the world. In its early phase, the Internet was similar in size to the network lying behind the bridges of Königsberg problem. It had so few nodes and edges that one could draw its map on a single piece of paper (see Fig. 3.3). After opening the network to the rest of the world, making it possible for almost anybody to connect, an interesting game began which still goes on today.

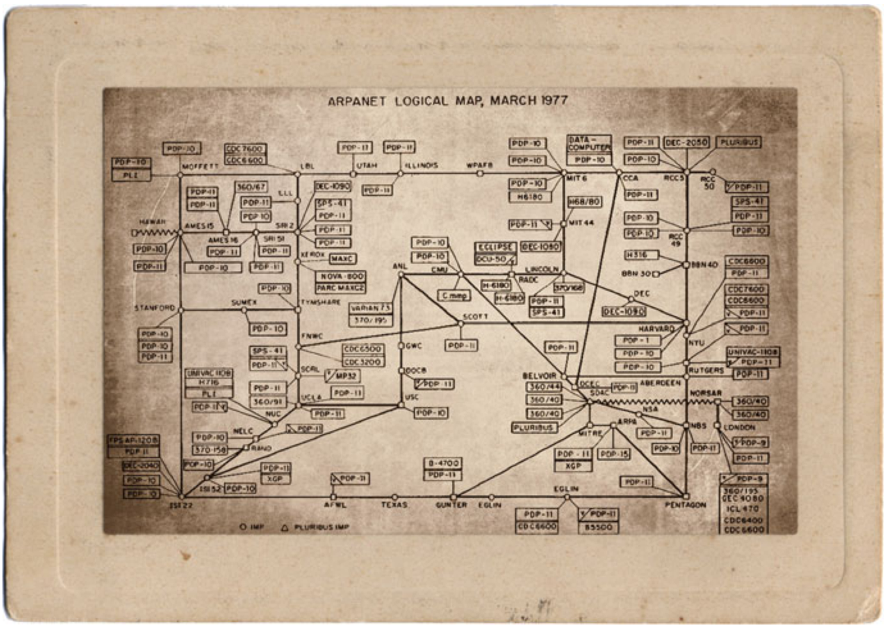

Fig. 3.3 Logical map of the ARPANET (the ancestor of the Internet) from 1977 [source: The Computer History Museum; https://computerhistory.org/]

Nodes started to join the network in an uncoordinated fashion, which meant that nodes and edges could have appeared almost anywhere in the network. As a result of this process, the Internet evolved into a large, complex network, the topology of which changes heavily day-by-day. Even drawing an approximate contemporary map was a great challenge for networking researchers and its visualization needed newly developed algorithms. Despite the researchers' best efforts, such maps were able to grasp only a limited subset of the edges present on the Internet. Above this large and evolving network, our emails, chat texts, web pages and videos travel day-by-day. All these data, converted to small information packets, are delivered through paths determined by the Internet's so-called "routing" system. This routing system has no central authority which could compute the paths for every single packet. Quite the contrary, the Internet's routing system is heavily decentralized, meaning paths are determined through the complex interactions of thousands of nodes. What kinds of paths come out of such a process? We know that latency is crucial if it comes to Internet services. Nothing is more irritating than a website that is slow to respond, a lagging video conference or a frozen video game. So it is natural that we expect the provisioning of low latency paths. But does it mean that we will have the shortest path between our computer and the desired service? Recalling the example of the Asian users of the open proxy system can make us suspicious. As usual, the truth will lie between the two extremes. But what exactly are these shortest paths? Now it is time to get acquainted with them.