Take-Home Message

<div align="right">**7**</div>

Let us try to gather our thoughts. What did we learn?

We have learnt that a mathematical model of a physical (or real) problem requires a good knowledge of the problem itself. Often – almost always – this knowledge must be shared with the experts on the problem, be these engineers, biologists, economists, physicists or physicians.

We have discovered that a mathematical model is fed by a set of data (generated by the real problem) and provides a solution that characterises the behaviour of the problem.

We have understood that to a given physical problem there may correspond several models. This should not surprise us that much. Modelling almost invariably requires that we simplify, by making assumptions meant to reduce the complexity of the initial problem without betraying its essence. Depending on the simplifying hypotheses made, we will find a corresponding mathematical model.

We have also discovered that this "multi-valued correspondence" is actually "invertible", because a specific mathematical problem may be used as a model for rather different physical problems. The meaning of the unknowns and of the data clearly changes from problem to problem, but the "structure" of the mathematical problem stays the same.

This feature is truly extraordinary and highlights the great encompassing power of mathematical models: they are able to bring into focus diverse physical processes using the same equations.

We have learnt that very often – practically always – a mathematical model cannot be solved by hand, since there is no magic formula to represent its solution. For this reason a mathematical model is paired with a numerical model. The latter can always be solved through an algorithm implementable on the computer by a programming language. We have further observed that to handle a numerical model several algorithms may be employed, each one characterised by a different computational cost (memory space, computing time).

This process, which starts from the mathematical model and ends with the computer-generated results, is the realm of Scientific Computing.