Chapter 8
Weather, War, and Chaos: Richardson's Encounter with Molecules and Nations

Jürgen Scheffran

Abstract Richardson's pioneering work on modeling conflict and arms races has demonstrated that mathematics can contribute to peace and conflict research, using system dynamics and stability conceptions to study both nature and society. Drawing from limitations and extensions of Richardson's model, including decision rules and chaos in arms races, an integrated modeling framework of social interaction among multiple agents is presented to study conflict phenomena in a complex world. Conditions for instability and chaos are discussed, potentially leading to arms races and violent conflicts, as well as transitions between conflict and cooperation. The model offers a basis for insights into the analysis of potential relationships of natural resources and climate change with social stability and conflict, building bridges between Richardson's research in atmospheric sciences and his work on peace and conflict.

8.1 On Molecules and Nations: Richardson's Scientific Conceptions

Lewis Fry Richardson (1881–1953), a British physicist, psychologist and pacifist, made important contributions to weather forecasting and conflict research and applied approaches and methodologies from physics, mathematics and atmospheric science to social phenomena (Ashford, 1985; Vulpiani, 2014). In particular, the concepts of equilibria and stability which are relevant for differential equations in meteorology and their solutions were transferred to the understanding of arms races (Richardson, 1956: 1247): 'stability is not the same as equilibrium; for on the contrary stable and unstable are adjectives qualifying equilibrium. Thus, an equilibrium is said to be stable, or to have stability, if a small disturbance tends to die away; whereas an equilibrium is said to be unstable, or to have instability, if a small

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disturbance tends to increase. … It is the instability which has the disastrous consequences.'

Regarding the notion of small differences versus large impacts, Richardson preempted concepts of chaos and complexity. In a letter to *Nature* he paid attention to the similarities between the behavior of nations and of gas molecules (Richardson, 1946a: 135). Starting from the observation that in a gas 'encounters of two molecules are much more frequent than encounters of three' he explained this by the product of three probability factors relevant in a theory of gas and the political world. He continued: 'Although three factors of the aforesaid sort are likely to appear in the theory of any chaos, yet their particular forms depend on circumstances; so that many varieties of chaos are conceivable. In the political world there were restrictions depending on geography and on sea-power. When they had been formulated, another effect became conspicuous, namely, the infectiousness of local fighting.'

This analysis has been expanded in two separate publications, concerning gases (Richardson, 1946b), and concerning the political world (Richardson, 1946c). In the latter, Richardson tested 13 theories of various degrees of complexity, and derived the interpretive idea of chaos, 'with its characteristic property that complicated events are rarer than simpler events. … the complicated events are regarded as built up from simpler elements … The more such elements, the less resultant probability.' (Richardson, 1946c: 138). He highlighted 'chaos, restricted by geography, and further modified by the infectiousness of fighting' (Richardson, 1946c: 130). By 'geography' he meant the 'opportunity of war for each country, depending on whether it was a worldwide sea power, a coastal state or a landlocked state'. By 'infectiousness' he meant the 'tendency to join the winning side.'[1]

Modern understandings of chaos and fractals were derived in the context of turbulence, which is different from a laminar flow where the volume of a fluid follows the same path as its predecessors. 'It is rather like the difference between the orderly progress of a well-disciplined company of soldiers and the wild rush of an unruly mob – the difference between order and chaos' (Ashford, 1985: 83). Chaos theory became prominent after Richardson's death, inspired by the Lorenz-Attractor in a simple weather model, and was extended to arms race models.

The complexity of Richardson's theory of atmospheric processes precluded manual weather forecasts, until the rise of computers after World War II. In 1946, John von Neumann proposed to the US Navy to apply high-speed, electronic, digital computing to dynamic meteorology. The ENIAC computer, derived for military purposes, was ready by March 1950 for the first test of simplified equations using meteorological observations, with promising potential to predict large-scale weather patterns (Ashford, 1985: 243). With the growing success of numerical weather prediction, Richardson's contributions became widely appreciated. To facilitate access to his annotated list of fatal quarrels, he presented a revised version

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[1] Both quotations from Ashford (1985: 209).

in machine-readable form (Ashford, 1985: 257). Long before Geographical Information Systems and cellular automata, Richardson suggested cell-based geographical approaches to conflict analysis, together with other conceptions that later became successful.[2] In the following, some are highlighted for his arms race model which served as a starting point for the conflict model derived by the author and applied to environmental conflict.

8.2 The Framework of the Richardson Arms Race Model

8.2.1 Stability and Balance of Power

Richardson's study of conflict modelling was inspired by his meteorological work (Hess, 1995). Similar to weather forecasting, he tried to predict war by finding general laws, common to all nations. Following his empirical analysis of World War I, he derived a set of differential equations to describe the arms buildup between major powers in Europe during the 1930s, possibly leading to major war (Richardson, 1960a, b). Richardson's model is based on the assumption that for two countries each increases its own armament level proportional to the armament of an opponent (weighted by the defense coefficients) and reduces it proportional to its own armament (weighted by the fatigue coefficients) plus a grievance term.

The equilibrium where the armament levels of both sides do not change, corresponds to the so-called 'balance of power'. Its stability is determined by the eigenvalues of the matrix of coefficients: For a positive eigenvalue a deviation from the equilibrium grows exponentially (corresponding to instability), for a negative eigenvalue it decays asymptotically (indicating stability). For two nations instability is given if the product of their defense coefficients exceeds the product of their fatigue coefficients, indicating that the drivers of arms-buildup exceed the dampening factors. Then the arms race becomes unstable and escalates, while for stability the armament levels approach the equilibrium, favoring disarmament. Richardson extended the equations to several nations for the arms races 1909–14 and 1933–39, using military expenditures as armament variables, which was modified to the difference between threat and cooperation, taking into account beneficial relationships between nations, in trade, travel and correspondence (Richardson, 1938). These calculations supported his view that 'foreign policy had then a rather machine-like quality' (Richardson, 1960a: 33) and lead him to conclude that increasing armaments could lead to war breaking out, while a constant level of armament corresponds to a steady state without war.

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[2] Cf. Gleditsch & Weidmann (2020) in this volume.

8.2.2 Critical Issues and Decision Rules

Richardson's model initiated a flood of publications on the armament dynamics and a debate about its applicability to real-world phenomena which raised several critical issues (see Smith, 2020, in this volume). Richardson himself was aware of the strengths and weaknesses of applying mathematics to social phenomena. Describing countries as structureless entities by one single variable was seen as questionable. Data on expenditure are not easily available and do not directly indicate security impacts. An arms race does not only have quantitative features, but also qualitative aspects, such as perceptions and doctrines. The Richardson model describes politics without personalities, where state authorities are black boxes and decisions are hidden in the budget. The fixed Richardson coefficients represent a linear and mechanistic interaction, where the initial conditions and coefficients determine the future, leaving no room for political decisions or control. Nations are assumed to have complete knowledge of the armament levels and react instantaneously. In reality, each side has limited information about other countries, and worst-case assumptions provoke reactions towards arms buildup, often with decision time lags.

The linearity and simplicity of the Richardson equations represents a few types of system behavior (oscillations, asymptotic decay, exponential increase). Reactions of real systems may be disproportionate and non-linear, showing qualitatively different modes of behavior. Decision-making may be better represented by time-discrete difference equations than by time-continuous differential equations. The arms buildup is not only an action-reaction process, driven by the opponents' armaments, but is also stimulated by a bureaucratic and budgetary dynamics with competing domestic interests. Although an arms race may provoke crisis-unstable situations, it does not necessarily lead to war if both sides want to avoid war or because one side reaches upper limits of armament which excludes an unlimited arms race.

Several extensions have been proposed to address the deficiencies of the Richardson model. Intriligator (1975) developed a framework for the strategic armament dynamics, based on decision rules that bridge the gap between desired and actual levels of missiles, taking into consideration the outcomes of a missile duel and the boundaries between deterrence and war initiation. This is represented by linear Richardson-type equations, whose coefficients can be derived from strategic considerations.

8.2.3 Chaos and Predictability in the Arms Race

While the Richardson model identifies basic system variables and relationships among countries, its rather simple structure does not represent the complexity of reality. Contrary to the well-ordered world of Newtonian mechanics, symbolized by the predictable swinging pendulum or the regular movement of the celestial bodies, complex systems, such as the turbulent weather patterns that Richardson studied, tend to be unpredictable. Since the 1970s, the natural sciences have begun to systematically explore critical phenomena, such as self-organization, tipping points, discontinuous phase transitions, and irreversibility. During the 1980s new mathematical concepts were developed, such as complexity, chaos, and non-linear dynamics. Chaos became not only a paradigm for the complex atmospheric dynamics but also for the turbulent transformation of the international system leading to the end of the Cold War in 1989 and the time after. Given the complexity of conflict, it seems appropriate to expand the Richardson model to non-linear phenomena.

The concept of chaos in arms race and war was introduced to show that simple non-linear deterministic arms race models may lead to the breakdown of predictability (Saperstein, 1984: 303). In chaos-like conflict situations, human actions and interactions are hardly foreseeable. Saperstein used a pair of non-linear difference equations with quadratic mappings for two variables, denoting the fractions of the available resources devoted to armaments which two countries pay annually. The problem of chaotic dynamics in arms race models was further investigated by Grossmann & Mayer-Kress (1989). The difference equations have a Richardson-like form with reaction parameters corresponding to the defense and fatigue coefficients and a grievance term, but with discrete time and a non-linear term that dampens armament expenditures at the upper cost limits. Factors provoking chaotic behavior are overshooting or underestimation, hectic responses, delay in information processing or discretization. The authors distinguish between chaos and instability: 'it is wrong to identify the general onset of bounded chaos with the outbreak of a war or another global crisis. The really dangerous case is instability' (Grossmann & Mayer-Kress, 1989: 702). Another non-linear time-discrete model, using decision rules for weapons procurement, was used by Saperstein & Mayer-Kress (1988) to simulate the impact of missile defense systems (Strategic Defense Initiative) on the East-West arms race. If production rates strongly increased, a chaotic transition from offense to defense occurred.

8.3 Multi-agent Interaction of Conflict and Cooperation

The Richardson model and other arms race models can be embedded into a broader framework of dynamic conflict modelling, bridging the gap between models for a few agents who optimize game strategies, and models for a large number of agents following dynamic decision rules. Inspired by Richardson's thinking about connections between the natural and the social world, in the following a dynamic agent-based modeling approach of social interaction is introduced that combines motivation and opportunity of multiple human agents to act upon and interact with their natural and social environment (for an overview see Scheffran & Hannon, 2007; BenDor & Scheffran, 2019).

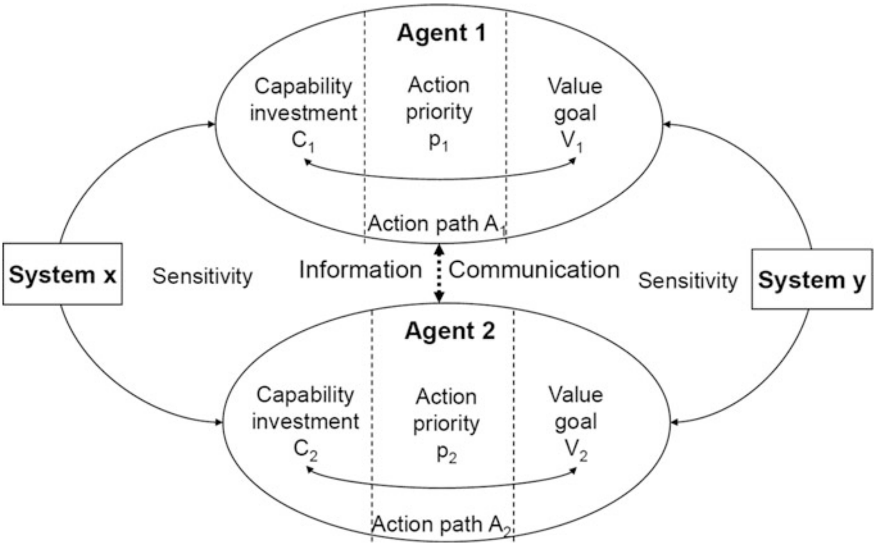

Fig. 8.1 Interaction between two agents and two environmental systems. Adapted from Scheffran et al. (2012)

8.3.1 *From Individual Action to Multi-agent Interaction*

The social interaction model follows the basic logic that individual agents act upon the environment, taking the opportunity to invest their capabilities to action pathways for achieving value-based goals which are a function of the benefits, costs, and risks of the actions taken. In repeated time steps and learning cycles agents mutually adapt their capabilities, action priorities and values, as a function of unit costs and values that represent the mutual sensitivities between agents and the environment (Figs. 8.1 and 8.2). Capabilities can change as a result of the dynamic interaction. Analytical conditions for conflict and cooperation as well as stability and chaos have been identified as a function of the value-cost efficiencies of each other's actions.[3]

Within the available capability limits, agents can adjust their investments and action pathways to meet their value goals according to decision rules, over time

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[3] In more formal terms, the VCX model describes the dynamic action and interaction of agents who use part of their available capabilities (K) as investments (C) with priorities (p) to given action pathways (A) that change their environment (X). The observed impacts of actions are evaluated in each time step based on the agent's values (V) and goals (V*). Important parameters are the sensitivity of human value to environmental change (v_x) and the inverse sensitivity (unit cost) of environmental change to human investment (c_x). The respective value-cost ratio $f = v_x/c_x$ indicates how sensitive human value is to human investment and thus how efficient an action is. Negative efficiencies f indicate a conflicting action path where agents hurt their values.

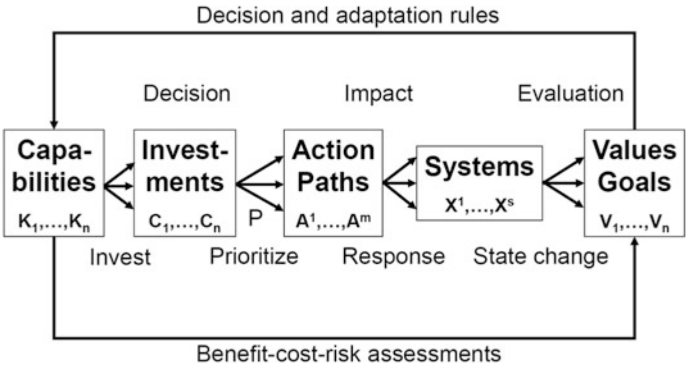

Fig. 8.2 VCX interaction model of multiple agents. Adapted from BenDor & Scheffran (2019)

switching to more efficient action pathways in a transition process. An interaction evolves for their responses to each other according to their respective decision rules which determine the fixed points where agents jointly achieve their goals. Decisive are the existence and location of the equilibria where all agents meet their goals within capability limits (corresponding to the 'balance of power' in Richardson's terminology) and the stability conditions of the multi-agent interaction matrix composed of the mutual action efficiencies.

Agents can control and stabilize or destabilize the dynamic interaction to some degree by using their capabilities and changing their action priorities to achieve the respective value goals. If the action priorities are directed towards hostile relations (damaging the values of other agents), the equilibria move towards higher investments, corresponding to an escalation. On the other hand, agents can switch to mutually beneficial cooperative actions lowering necessary investments. While this can be done independently, agents can also negotiate on their action priorities, leading to intertemporal dynamic games. In addition, they can form coalitions by pooling some of their invested capabilities and redistributing the gains (or losses) to the individual agents, or they may agree on the same values and goals, thus moving from individual to collective or institutionalized action and interaction (Scheffran, 2006).

8.3.2 *Conditions for Stability*

The type of social interaction is represented by the interaction matrix and its stability, mathematically determined by the eigenvalues around the social equilibrium. If agents are powerful in terms of their capabilities and efficient in pursuing their action goals, they can withstand, compensate, or counter-act a certain level of hostility by others, thus keeping eigenvalues in the negative range and avoiding major deviations from the equilibrium conditions. If the number and intensity of hostile actions exceed a critical threshold, then an unstable escalation may occur, leading to the breakup of the social system. As a general rule, stability of social interaction can be maintained if the positive (cooperative) effects of agents on each other (including on themselves) exceed their negative (conflicting) effects. This is a generalization of the stability condition found by Richardson. Thus, a social system can withstand a certain level of conflict and still satisfy the goals of its members.

For unstable eigenvalues, some agents are dissatisfied with existing action pathways and may select new ones, including those that damage others, e.g. though the use of violence, if they are not forced or incentivized to follow socially accepted action pathways. With a growing number of agents, the complexity of the interaction matrix as well as the number of eigenvalues increases, including those that are potentially unstable. This is known in systems theory as the ‘complexity-stability’ tradeoff, and raises the question whether complex systems are more unstable (Scheffran & Hannon, 2007; Gravel et al., 2016). Beyond tipping points complex systems may become destabilized and break apart into simpler ones, through cascading events and escalating conflict. Alternatively, mutual adaptations of actions or institutional control mechanisms can stabilize the interaction and contain conflict, e.g. by social security or other forms of support for the disadvantaged.

8.3.3 Connection to the Richardson Model

The presented multi-agent interaction model serves as a framework for the Richardson arms race model where countries are the main agents and their military expenditures are the investments. These are adapted according to decision rules driven by goal functions which are the differences between an adversary’s expenditures and one’s own (weighted by the defense and fatigue coefficients, serving as efficiencies) plus the grievance terms with strategic considerations. Both the Richardson model and the interaction model have a balance of power equilibrium and stability conditions for the interaction matrix when dampening coefficients exceed threatening ones. Richardson’s focus on two countries is compatible with his observation that multi-country encounters tend to be more unlikely, unstable and chaos-like, corresponding to the complexity-stability tradeoff. He was thinking about the effect of alliances and organizations in multi-country contexts (Richardson, 1946c). Thus, Richardson has presented a role model with key elements relevant in the general model of social interaction.

8.4 Model Applications

The described interaction model has been applied to different fields, including arms races and arms control, economic production and environmental sustainability, resource conflicts, energy security and climate change (see Scheffran & Hannon, 2007; BenDor & Scheffran, 2019). To demonstrate its relevance, a few cases are selected beyond Richardson's narrower focus, including issues where atmospheric processes, weather patterns, and anthropogenic climate change could affect emerging conflict landscapes.

8.4.1 Complex Conflict Landscapes and the Spiral of Violence

The VCX model was born in the final phase of the Cold War, as part of the author's Ph.D. thesis in physics, to understand the stability of the nuclear arms race between the two major rivals. The study analyzed potential transitions from nuclear deterrence to a world where the nuclear threat is contained through missile defense (as suggested by former US President Ronald Reagan) or abandoned through nuclear disarmament (proposed by then Soviet General Secretary Mikhail Gorbachev). Simulating a shift from worst-case to best-case perceptions, or from hostile to friendly attitudes, the model showed chaos-like events beyond a tipping point, leading to nuclear disarmament (Scheffran, 1989). Shortly after, the Cold War ended in a domino effect and a breakup of the Eastern Block (Scheffran, 2008).

The following globalized world was characterized by growing connectivity between multiple agents and security dimensions. Within the model, the fractal security landscape was represented by a bifurcation diagram, which for increasing response rates of agents moves from periodic oscillations via stable equilibria to a sequence of multiple fixed points (Scheffran, 2003), challenging the predictability beyond the edge of chaos. Adding to complexity is an unstable interaction of multiple agents, some of whom benefit from the interaction while others suffer, separating into groups of 'winners' and 'losers' (Scheffran, 2003).

Some conflicts are related to the 'security dilemma', where threats to the security of agents provoke reactions that threaten the security of others. Key lessons can be drawn from the study of World War I and the diffusion of threats leading to it, using social network analysis to understand the arms race among alliances before this war (Vasquez et al., 2011), as studied in Richardson (1938). Beyond critical thresholds of instability, violent acts provoke more violent acts, leading to a self-enforcing 'spiral of violence', which today can be found in fragile regions of Africa. A key question is how to induce a transition to a self-enforcing cycle of cooperation and peace-building, similar to Richardson's change from threat to cooperation as part of 'collective security' (Richardson, 1935).

8.4.2 Climate Change, Social Instability, and Violent Conflict

Richardson was aware that weather and climate are among the most complex systems. Although the climate system has been largely stable in the Holocene, human interference may become a destabilizing factor in the Anthropocene if critical tipping points are exceeded (Steffen et al., 2018). Weather extremes such as hurricanes, droughts, forest fires, floods, and heatwaves, often correspond to non-linear mechanisms such as phase transitions, critical thresholds, and chaos. Natural disasters are generally associated with extreme consequences that burden the stability of natural and social systems and overwhelm their adaptive capacities. The effects may be aggravated by compound events, i.e. the complex combination of multiple climate drivers and hazards. Together, they are more likely and risky than their independent occurrence, e.g. concurrent hot and dry summers (Zscheischler et al., 2018).

In this context, climate change has been called a potential risk multiplier that combines with other risks, including those to human security and societal instability (such as forced displacement, riots, insurgency, intervention, urban violence, and civil war). The implications of compounding risks have not been sufficiently addressed in climate-conflict research where some studies claim climate change to be a significant driver of violent conflict, while others find no clear causality. This deficit can be addressed by an 'agent-based approach to assess the interplay between capabilities and motivations for violence and the conditions for conflicting or cooperative interactions. … In the most affected regions, the erosion of social order and state failure as well as already ongoing violent conflicts could be aggravated, leading to a spiral of violence that further dissolves societal structures' (Scheffran et al., 2014: 369). One compound effect is the double vulnerability to violence and environmental hazard: environmental change can make societies more vulnerable to violence which in turn can make societies more vulnerable to environmental change, leading to a trap from which escape is difficult (Scheffran et al., 2014: 375).

These theoretical considerations increasingly attract empirical research on conflict sensitivity to climate change (von Uexkull et al., 2016: 12391): 'Results from naive models common in previous research suggest that drought generally has little impact. However, context-sensitive models accounting for the groups' level of vulnerability reveal that drought can contribute to sustaining conflict, especially for agriculturally dependent groups and politically excluded groups in very poor countries. These results suggest a reciprocal nature – society interaction in which violent conflict and environmental shock constitute a vicious circle, each phenomenon increasing the group's vulnerability to the other.'

Within the described model of social interaction, climate change may affect the allocation of investment to conflict, by undermining resource productivity (e.g. of agricultural output) and diminishing efficiency of human capabilities, or by providing incentives for violent resource capture, leading to stronger hostile actions.

In both cases, this can undermine the achievement of human goals und trigger more investments fueling conflict. Some effects could act over long distances, for instance large migration movements, interventions or humanitarian aid in remote regions affected by violent conflict. In this way, climate change may act as a global connector, adding to globalization, communication, transportation, and other linkages. To stabilize climate-induced interactions, agents could move towards mutually beneficial solutions (win-win), e.g. by innovation, resource sharing, risk management, and transition from high-emission to low-emission pathways within the temperature goals of the Paris Treaty (BenDor & Scheffran, 2018: Ch. 9).

Modeling climate-related conflict is still in an early stage and Richardson's work can provide some guidance, although he did not explicitly discuss the linkages between weather/climate and conflict, besides the observation that wars in the north temperate zone have ordinarily begun in spring or summer (Richardson, 1960b: 129). However, he pointed out that the probability of encounter in conflict is affected by geography (opportunity of war) and infectiousness (tendency to join the winning side) which are related to capability and motivation in conflict interaction. Richardson (1946c) also noted several factors that are important for conflict connectivity: 'Aviation is now tending to put every nation into contact with every other' (147) … 'The more persistent contact, the more opportunity for quarrels.' (152) … 'the trouble begins with the existence of a world-wide controversy' (155).

Apparently climate change is one such 'world-wide controversy'. Considering Richardson's general observations of multiple encounters, climate change could result in an increasing number of multi-actor and multi-factor encounters and related conflicts, which are not independent but result in compound risks. Further discussing these linkages could build bridges between Richardson's work in meteorology and peace.

8.5 Summary and Outlook

Starting from the Richardson arms race model and possible extensions, an integrative model framework of social interaction was presented in order to analyze conflict and cooperation, instability, tipping points, and cascading risks as well as transition and transformation processes. To cope with destabilizing consequences, affected systems need to adapt to the changing circumstances. Adaptive mechanisms influence critical decision points and adjust actions along multiple causal chains to protect human security and move from conflict to cooperation. The goal is to avoid risky pathways and facilitate a sustainable transformation, coping with conflict and climate change, developing social structures, political strategies, and institutional mechanisms that avoid or minimize social conflict and instability. Model expansions may contribute to improved understanding and forecasting of climate change and violent conflict in a turbulent world, encountering Richardson's research in atmospheric sciences and in peace and conflict studies.

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Jürgen Scheffran, b. 1957, Ph.D. in Physics (University of Marburg, 1989); Senior Researcher and Faculty Member, University of Illinois (2004–09); Professor of Geography, University of Hamburg (2009–), head of Research Group Climate Change and Security at CliSAP/CliCCS Clusters of Excellence; research fields: climate change, migration and security, rural-urban interaction, agent-based modelling, sustainable peace, juergen.scheffran@uni-hamburg.de