



Applying Multi-phase DES Approach for Modelling the Patient Journey Through Accident and Emergency Departments

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Abstract. Accident and Emergency departments (A&ED) are in charge of providing access to patients requiring urgent acute care. A&ED are difficult to model due to the presence of interactions, different pathways and the multiple outcomes that patients may undertake depending on their health status. In addition, public concern has focused on the presence of overcrowding, long waiting times, patient dissatisfaction and cost overruns associated with A&ED. There is then a need for tackling these problems through developing integrated and explicit models supporting healthcare planning. However, the studies directly concentrating on modelling the A&EDs are largely limited. Therefore, this paper presents the use of a multi-phase DES framework for modelling the A&ED and facilitating the assessment of potential improvement strategies. Initially, the main components, critical variables and different states of the A&ED are identified to correctly model the entire patient journey. In this step, it is also necessary to characterize the demand in order to categorize the patients into pipelines. After this, a discrete-event simulation (DES) model is developed. Then, validation is conducted through the 2-sample t test to demonstrate whether the model is statistically comparable with the real-world A&ED department. This is followed by the use of Markov phase-type models for calculating the total costs of the whole system. Finally, various scenarios are explored to assess their potential impact on multiple outcomes of interest. A case study of a mixed-patient environment in a private A&E department is provided to validate the effectiveness of the multi-phase DES approach.

Keywords: Discrete-event simulation (DES) · Healthcare modelling · Accident and emergency department (A&ED) · Phase-type models

1 Introduction

Accident and emergency department is a 24-h health area with prompt and appropriate service to care for those ill and injured patients in urgent need. A&E departments include resuscitation facilities and designated accommodation. Emergency services' objective is to avoid complications that could lead to even early death, considering that future quality of life and long-term mortality may be influenced by the early measures implemented in the A&E department. Competent services include access and availability to equipped facilities, specialized and trained professionals, and supporting services; all in an effective way [1].

But, A&E departments must face some difficulties as they are: overcrowding, long waiting times, patient dissatisfaction and high cost. That is why, in the last decade, some reforms and an increasing number of tools have been applied to A&E services to ensure that patients are seen, treated, admitted, and discharged appropriately [2].

A&E department crowding is an important and common international problem, so, researchers have an increasing interest in this field. Even in some countries, standards have been defined to monitor the maximum time in which a patient must be treated in these centers [3]. In the UK, the government set an operational standard that 95% of patients should wait <4 h in A&EDs [4]. In A&E departments, patient waiting time is another important factor in assessing the quality of health services and patient satisfaction. Several studies show that waiting time is a determining factor in patient satisfaction [5, 6].

One option to study, analyze and solve the problem is modeling and simulation. Modeling is usually used by healthcare professionals to represent all variables and scenarios involved in a real system and to understand its behavior. After building a model from observation or knowledge of a real system, it can be simulated [7]. Simulation is a methodology widely used to solve real-world problems, which mimics the operation of a real-world process or system, over time. It comprises the generation and observation of an artificial history of a system in order to produce inferences concerning the operating characteristics of the system under analysis. Simulation is used to describe and analyze the behavior of both existing and conceptual systems, perform diagnosis, and support the design of real systems [8].

Because of the complexity of providing quality services to health clinic users, Discrete Event Simulation (DES) has been used for at least two decades as an effective tool for allocating scarce resources and improving patient flow, while minimizing health service delivery costs and increasing customer satisfaction [9]. DES is a common stochastic analysis tool to perform experiments via computer modeling and test the likely effectiveness of different scenarios before their implementation.

An important advantage of DES is the possibility to represent the system-state description, which includes probability distribution of entity arrival, event duration, event status, and resources needed. Modeling and Simulation are key enablers to improve services in complex healthcare systems and propel major improvements in decision making, efficiency, and quality [7].

Our proposal is a conjunction of DES and the modeling of health processes in a multiphase approach. This approach consists of the representation of the real system

through the simulation of discrete events together with the modeling of A&E stage throughout the patient journey. We here use DES in order to (i) better analyze the resource utilization under system restrictions, (ii) identify non-value activities, (iii) effectively administer interactions (iv) evaluate potential interventions to improve the patient experience along the A&ED journey and (v) facilitate engagement with the health service managers through animation.

On a different tack, a complexity is modeling the processes and interactions among different areas. Areas can be, for example, radiology, laboratory, intensive care unit, hospitalization, pharmacology, morgue, etc. Given the nature of this A&E system, Markov chains, supporting the multi-phase framework, are used to reflect the probabilities of being transferred from one service to another. Thereby, the relationships between services can be modeled with high accuracy and robustness.

The proposed approach enables sector managers to simulate scenarios or improvement strategies on the virtual model and the results of a possible implementation can be assessed both operationally and economically. In addition, it facilitates the evaluation and applications of joint strategies in the wild including A&E and other healthcare departments.

The remainder of this paper is organized as follows: In Sect. 2, a review on the related studies is presented whereas the suggested framework is explained in Sect. 3. In Sect. 4, a case study of an A&E department from a Southamerican clinic is described. Finally, Sect. 5 presents conclusions and future work.

2 Related Studies

There is a lot of research using modeling and simulation to study different scenarios in A&E departments. We analyzed some papers published in the last ten years. For instance, some strategies were proposed to address the ambulance diversion and overcrowding. The authors aimed to develop a tool for evaluating the effectiveness of various ambulance diversion strategies [10].

An interesting review is presented in [11] about comparing statistical modeling approaches to describe and predict emergency department patient load and crowding. Regression models, mathematical equations, time-series analyses, queuing theory based models, and discrete event simulation models were contrasted.

In particular, DES has been used to study and provide solutions to problems in A&E departments. In [12], a DES model was developed to forecast near-future operating conditions and validate those predictions in several scenarios of crowding. In [13], the authors developed a DES model to determine the proper ICU bed capacity that balances between service level and cost-effectiveness. The objective was to increase the bed availability so that ambulance diversion and surgery cancellation can be avoided.

In [14], a quality improvement department used a DES to predict and test patient flow, staffing policy, and other process-level changes. Different methodologies and tools from the industrial area have also been used to redesign processes and improve the efficiency of systems. Such case is illustrated in [15] where Six Sigma was used to develop a classification and selection process for an emergency department. Some key

performance measures as length of stay and waiting time were considered. The results indicated a certain improvement in both parameters after DES implementation.

Discrete-event simulation models were implemented using SIMIO package. The study aimed at estimating the required number of doctors and examination rooms to achieve a service level of over 90%, at reasonable costs. The findings showed that modifying treatment processes and enabling more flexible staff hierarchy could enhance such service level [16].

On a different tack, phase-type models, used for describing flows with multiple outcomes and states, have gained increased prominence in healthcare applications [17]. For instance, in [18], a phase-type model was developed to describe the flow of each patient cluster based on the length of stay. The framework was also useful to provide predictions on bed occupancy. Another application can be evidenced in [19] where the authors used this approach to cost the stroke patient care from admission to discharge. In this case, the proposed methodology was used to underpin integrated planning, encompassing hospital and community services. However, in spite of these studies, the use of phase-type models is largely limited when addressing interactions with other healthcare services. Therefore, the novelty of our proposal is based on the combination of DES and Markov phase-type models to analyze interrelations and feedback between A&E departments and other healthcare settings (i.e. intensive care unit, hospitalization, and surgery) within hospitals so that holistic solutions can be provided to better assist health service managers in decision making (Fig. 1).

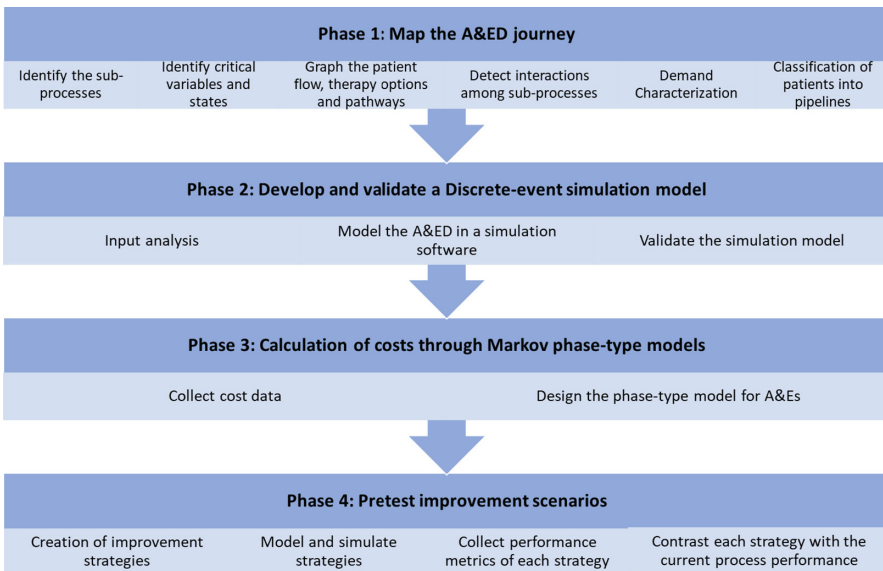


Fig. 1. The proposed framework for modelling and assessing operational changes in A&E departments and interactions.

3 The Proposed Framework

Phase 1 “Map the A&ED Journey”: In this phase, it is necessary to identify the patient pathways, states (i.e. diagnosis and treatment), sub-processes (i.e. lab testing, radiology), key parameters and associated process variables that characterize the patient journey within the A&E department under analysis. It is also required to characterize the demand of each triage level so that DES phase-type models can be more detailed and informative. Thereby, health service managers are able to predict the waiting time, profile patient cohorts, improve resource management and make better decisions. The process information is later summarized in a conceptual model where all the process details, assumptions and components can be fully identified. In order to ensure a high accuracy and robustness of the model, the modelers should work closely to the clinicians and healthcare managers since they can provide information on the procedures and protocols attached to the patient journey as well as the existing interrelations with other healthcare services (i.e. hospitalization, intensive care unit, and surgery).

Phase 2 “Develop and Validate a Discrete-Event Simulation Model”: A virtual model of the whole A&E department needs to be developed in order to obtain the current process behavior in terms of waiting time. To do this, modelers should collect appropriate data that represent the variables and parameters identified in Phase 1. In this regard, it is highly recommended to perform outlier analysis in order to detect special patterns and eliminate noise from datasets. This contributes to increasing the accuracy and reliability of the model as well as assessing the quality of measurement systems. Once the data are filtered, three statistical techniques are conducted to determine the correct manner of representing each process variable in the simulation model. Initially, an intra-variable dependence test is carried out to detect auto-correlations within a specific dataset. A homogeneity test is then carried out to categorize data into classes in accordance with similar profiles. The Kruskal-Wallis and log-rank tests can be employed for validating the heterogeneity assumption. If several classes are identified, a probability expression is then required for each class; otherwise, one distribution is enough to model the entire dataset. After this, the input analyzer feature of simulation packages is used to determine the probability expression (including parameter values) that better fits the data in case of randomness; alternatively, the modeler needs to find the formulae incorporating the dependency between the process variable and its (their) predictor (s). Once the statistical tests are complete, the resulting information is incorporated into the model. Such a model is suggested to be run 10 times in order to calculate the required number of replications that are necessary to represent the system uncertainty. After collecting the key variable value in each replication, a 1-sample t test is carried out to verify whether the virtual representation is realistic. If the resulting p-value is higher than the alpha level (0.05), then the model is concluded to be equivalent with the real A&E department; otherwise, it should be revised and improved until satisfying this condition. Finally, a capability analysis is performed to establish the current performance of the A&E department.

Phase 3 “Calculation of Costs Through Markov Phase-Type Models”: Phase-type models have been proposed in this study to better modelling and analysis of specific cost measures in a queuing setting. They also serve as a base for evaluating whether

certain scenarios are cost-effective for the A&E department. In this respect, the cost parameters to be employed in this intervention should be extracted from a reliable Financial Information System (FIS) so that real economic evaluation can be effectively performed. In this phase, homogeneity results from Phase 2 are considered to optimally stratify patients according to their triage level. After this, the length of stay (LOS) distribution of each group is modeled employing phase-type models. Such models also take into account the inter-arrival process of each patient class whose probability distribution can be also derived from Phase 2 through the Goodness-of-fit test. As a next step, transition rates for the phase-type models are calculated employing maximum probability estimation. Finally, the costs for the whole patient journey within the A&E department are estimated by implementing Markov classes.

Phase 4 “Pretesting Improvement Scenarios”: Different improvement strategies can be suggested by health service managers for reducing the waiting time in the A&E departments. The proposed approach presented in this paper enables policymakers to evaluate the potential impact of such strategies before implementation. The strategies are then run by the modeler and the resulting waiting times are collected. After this, the simulated performance is statistically compared to the real behavior using a 2-sample t (if the data are normally distributed) or Kruskal-Wallis (if the data follow a non-normal probability distribution). If the resulting p -value is significant (<0.05), the strategy is recommended to be executed in the A&E department since it may reduce the waiting time (C. L. = 0.95). If not, it should not be considered.

4 An Illustrative Example: Modeling an Accident and Emergency Department of a Clinic

4.1 Map the A&ED Journey

The patient journey of an A&ED from a private clinic was mapped (refer to Fig. 2) to provide a wide understanding of the interactions with other healthcare services and performance in terms of waiting time. Specifically, interrelations with six sub-processes (Hospitalization, Surgery, Intensive Care unit, Morgue, Laboratory, and Radiology) were identified. Our model was underpinned by a 1-year dataset extracted from the Patient Data Management System (PDMS) and consisting of all the admission registered between 1 January and 31 December ($n = 2506$ admissions). The journey starts with two types of arrivals: walk-in and ambulance. Here, the watchman registers the arrival time and type of arrival. Then, the patients enter the A&ED and some assistants proceed with collecting personal data, type of emergency, and the health promotion organization. However, some patients have to stay in the waiting room until the initial assessment due to bed and doctor availability. During emergency care, the doctor decides which treatment should be applied and which lab and radiology tests must be conducted to establish a correct diagnosis and intervention. Depending on the patient evolution, a transfer to other healthcare services may occur. In accordance with the data recorded by the PDMS, the discharges were the following: 46.4% (Home), 20% (Hospitalization), 18.1% (Surgery), 15.3% (Intensive care), and 0.2% (Morgue).

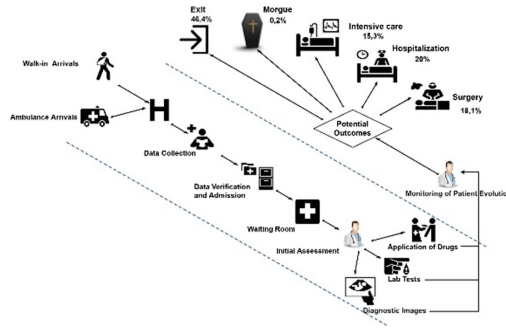


Fig. 2. Illustrative model for the A&E department

In the A&E department, there are 10 doctors who work in accordance with the schedule depicted in Table 1. Finally, four key variables were identified: (a) inter-arrival times for emergency patients, (b) initial assessment time (patients with chronic conditions), (c) initial assessment time (patients with minor complications) and (d) length of stay (LOS).

Table 1. Agenda of ED doctors

Doctor code	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
001	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X
002	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X
003	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X
004	8 am–10 am 3 pm–5 pm	8 am–10 am 3 pm–5 pm	8 am–10 am 3 pm–12 am	12 am–5 am	X	X
005	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X	7 am–7 pm	7 am–7 pm
006	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X	7 am–7 pm	7 am–7 pm
007	7 pm–12 am	12 am–7 am 7 pm–12 am	12 am–7 am	X	7 am–7 pm	7 am–7 pm
008	12 am–7 am	X	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am
009	12 am–7 am	X	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am
010	12 am–7 am	X	7 am–7 pm	7 am–7 pm	7 pm–12 am	12 am–7 am 7 pm–12 am

4.2 Develop and Validate a Discrete-Event Simulation Model

Once the data gathering regarding the three process variables is complete, an intra-variable dependence analysis was undertaken to verify the randomness assumptions.

The results are detailed in Table 2. The results demonstrated that all the variables were found to be random since the p-values were lower than 0.05. After this, a homogeneity test was carried through Analysis of Variance (ANOVA) in order to detect sub-groups of data in each variable (refer to Table 3). It was concluded that all the variables were homogeneous and can be therefore modeled without considering pipelines.

Table 2. The results of intra-variable dependence tests

Key variable	P-value
Inter-arrival time for emergency patients	0.569
Initial assessment time (patients with chronic conditions)	>0.15
Initial assessment time (patients with minor complications)	>0.15
Length of stay (LOS)	1

Table 3. The results of homogeneity tests

Key variable	P-value
Inter-arrival time for emergency patients	>0.15
Initial assessment time	<0.0005
Length of stay (LOS)	0.363

After completing the intra-variable dependence and homogeneity tests, the input analyzer feature of Arena 14.5 ® was employed to determine the probability expressions of each variable (refer to Table 4). For instance, the Kolmogorov-Smirnov test (p-value = 0.718) provided a good fit for the Weibull distribution of *length of stay (LOS)*.

Table 4. Probability expressions of each key variable

Key variable	Probability expression
Inter-arrival time for emergency patients	EXPO (18.8) min; Entities per arrival: DISC (0.9223, 1, 0.9917, 2, 0.9986, 3, 1, 4)
Initial assessment time (patients with chronic conditions)	UNIF (10, 45) min/patient
Initial assessment time (patients with minor complications)	UNIF (10, 15) min/patient
Length of stay (LOS)	WEIB (2.22, 202.58) min/patient

All the information derived from the input data analysis was included in the simulation model. Such a model was initially run 10 times in order to calculate the required number of replications that are required to represent the system variability. In this case, 91 runs were estimated as necessary for this particular aim. After collecting the resulting waiting times in each replication, a 1-sample t test was conducted for model

validation. Considering a p -value = 0.095 and T -value = -1.69 , the model is concluded to be comparable with the real A&E department and can be therefore used for capability analysis and exploration of improvement scenarios. After this, a capability analysis (refer to Fig. 3) was conducted with the aid of Minitab 17®. In this regard, A&E departments are called to meet the standard set by the National Health Institution (Maximum waiting time = 30 min/patients). The analysis revealed that the average waiting time is 34.69 min/patient with a shape of 2.36 min/patient and a scale of 39.13 min/patient. Furthermore, it was estimated that 624650 out of 1 million admissions will have a waiting time >30 min. Complementary to these measures, the P_{pu} (-0.07), P_{pk} (-0.07) and sigma level (1.28) evidence that the performance of the A&E department under study is low satisfactory and then requires deep interventions. These outcomes were presented to the general managers in order to explore various scenarios of improvement.

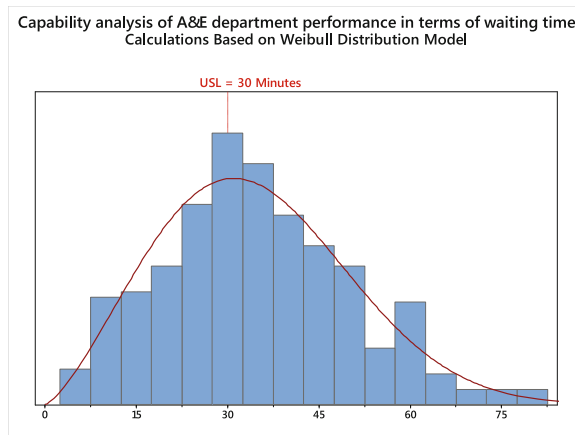


Fig. 3. Capability analysis to evaluate the performance of the A&E department under study

4.3 Calculation of Costs Through Markov Phase-Type Models

The healthcare costs used within the simulation were derived from the Financial Information System (FIS) of the A&E department under analysis. The parameters of the phase-type model were extracted from the Patient Data Management System (PDMS). In particular, the phase-type model has two absorbing states and four transitory stages (refer to Fig. 4). It will be also assumed that these probabilities do not vary over time in order to confirm the Markovian assumption.

The states will be labeled as presented below:

- 0:** A&E department (Emergency care)
- 1:** Intensive care
- 2:** Surgery
- 3:** Hospitalization

- 4: Discharge (Home)
- 5: Death

The phase-type model here described can be considered as an absorbing Markov chain where the long-term probability of both discharge and death states can be estimated. In addition, if the costs of each stage are fully known, it is then possible to calculate the average value-added cost per admission. The cost of each transitory state is enlisted in Table 5. With these considerations in mind, the matrix comprising of the long-term probabilities is finally derived (refer to Fig. 4).

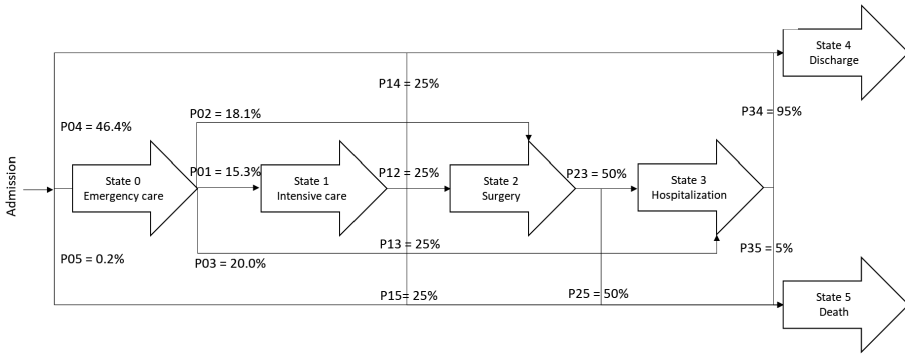


Fig. 4. Long-term absorption probabilities from state *i* to state *j* in the A&E department

Table 5. Daily costs for transitory states

Transitory state	Average daily cost (US\$)
Emergency care	309.85
Intensive care	870.46
Surgery	1410.43
Hospitalization	516.44

According to Fig. 4, it can be determined that a patient who is in the Hospitalization department (stage 3) has 95.0% of likelihood to survive (discharge-state 4) and die with 5%. As a next step, the fundamental matrix is achieved (refer to Table 6).

Table 6. Fundamental matrix for the A&E department

State <i>i</i>	State <i>j</i>			
	0	1	2	3
0	1.90	9.30	0.8	14.6
1	0	11.10	1.5	17.2
2	0	0	0.8	12.3
3	0	0	0	13.2

The fundamental matrix evidences the time (measured in days) that a patient spends in each transitory state of the A&E department. If a patient is in this area, the average stay will be 1.9 days and 9.3 days in intensive care. Furthermore, the admitted patient will spend 14.6 days (on average) in Hospitalization department. Considering the daily costs of each transitory state (refer to Table 5), it can be deduced that the average cost of a patient that is admitted in the A&E department and is later transferred to Hospitalization is US\$8218.73.

4.4 Pretesting Improvement Scenarios

Computer simulation enables health service managers to assess the impact of certain changes in the A&E department before their implementation. This is highly recommended to avoid errors, diminish cost overruns, and identify cost-effective scenarios. In spite of these advantages, only a few studies have been reported in the literature regarding the use of computer simulation for the assessment and comparison of various scenarios in healthcare delivery. Motivated by the need for bridging this gap and minimizing the waiting time, the modelers and emergency managers agreed to evaluate three possible scenarios: (a) Implement a doctor who filters the admissions by identifying which ones are real urgencies [20], (b) Add a doctor who works according to the shift established for 008, 009, 010 and (c) Classify patients through a triage system. These scenarios were later modeled using Arena 14.5 ® software. The performance of each scenario was statistically analyzed through 2-sample t tests (C.L. = 95%) [21–23].

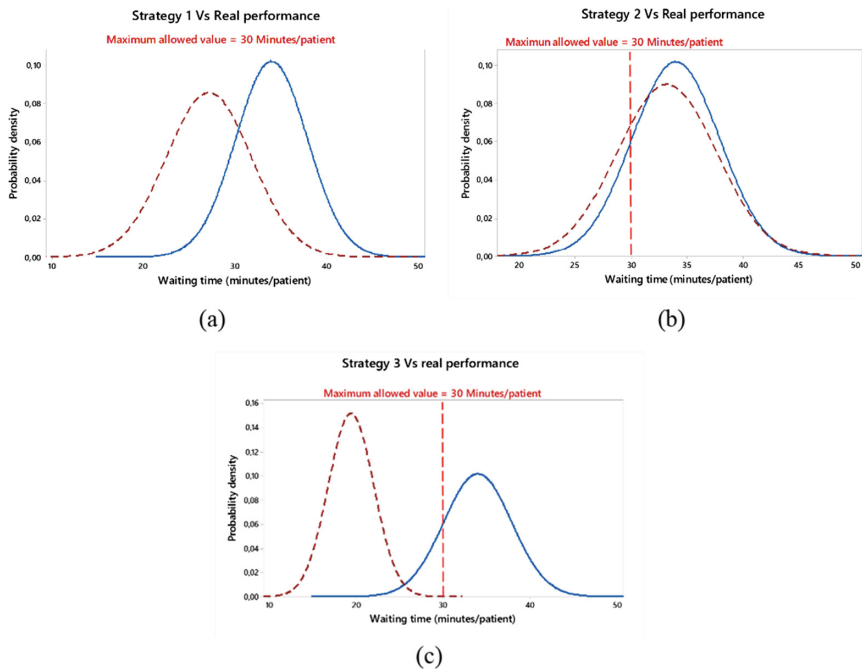


Fig. 5. The comparative analysis between the real behavior and (a) Strategy 1 (b) Strategy 2 (c) Strategy 3

Strategy #1 (refer to Fig. 5a) proposes to avoid the admission of non-urgent visits which can be instead redirected to priority outpatient care. In this case, the 2-sample t test revealed that the waiting time resulting from this scenario is lower than the current performance (p -value = 0; $T = 13.15$). On a different tack, Strategy #2 (refer to Fig. 5b) suggests augmenting the installed capacity by 48 h. However, no significant difference was detected between the actual performance and the proposed scenario (p -value = 0.095; $T = -1.69$). Finally, Strategy #3 (refer to Fig. 5c) proposes to implement the triage system which classifies patients into two groups: Triage 1–2 and Triage 3–5. In addition, it is necessary to aggregate 9 triage doctors who will be in charge of the classification process. In this scenario, the waiting time was concluded to be substantially minor in relation to the target and real A&E behavior (p -value = 0; $T = 26.25$).

5 Conclusions and Future Work

A&E departments are very difficult to model considering the multiple states, sub-processes, and interactions with other healthcare services. To correctly represent the real performance of these departments, it is necessary that modelers work alongside the health service managers so that the base structure, key variables, and parameters can be fully incorporated into the simulated version. The collection of appropriate data plays a relevant role in this aim. Thereby, more informative and detailed models can be provided to support decision making within A&E departments.

The proposed approach underpins the development of integrated models that serve as a basis for evaluating complex strategies involving other healthcare services. We believe that such a framework has the potential to be extended to other healthcare systems. Therefore, we plan in the future to intervene in outpatient care and emergency care networks so that the evidence base can be further developed and better planning can be granted in these scenarios.

The illustrative example presented in this paper provides a multi-phase DES model for A&E departments. In this particular case, it was proved that 624650 out of 1 million of admissions will have a waiting time > 30 min if this department operates under the current conditions. To tackle this problem, three alternatives were pretested. Based on the results, we recommend implementing a triage system that categorizes patients. Although, it is also suggested to combine this strategy with the addition of a doctor that filters admissions and redirects patients to priority outpatient care. On a different tack, the phase-type model enabled health service managers to elucidate costs and LOS along the patient journey within this department. For instance, it was detected that the average cost of a patient that is admitted in the A&E department and is later transferred to Hospitalization is US\$8218.73. Such information is highly valuable for supporting the economic evaluation of the improvement strategies and the deployment of cost-effective interventions addressing interactions with the associated healthcare services.

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