

# Chapter 7

## Teachers' Collective Work Inside and Outside School as an Essential Source of Mathematics Teachers' Documentation Work: Experiences from Japan and China



Takeshi Miyakawa and Binyan Xu

**Abstract** This chapter aims to report the results of a comparative study of teachers' documentation work in China and Japan, as well as to share some East Asian experiences that are less accessible to Western researchers. The Chinese case is gathered from teachers' collective work carried out inside school, and the Japanese case is taken from the group activities of a local mathematics teachers' association outside school. We analyze in each case teachers' documentation work as well as resources associated with such work. The comparison of the results of the analyses elucidates the commonalities between the two cases, such as the importance of textbook as a resource and the *practice-based* and *research-oriented* professional development, albeit the differences of contexts and institutional frameworks inside and outside school. We lastly discuss the perspectives for future research on the teachers' collective work with resources.

**Keywords** Mathematics teachers · Collective work · International perspective

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## 7.1 Teachers' Collective Work and Resources from an International Perspective

Today collective actions by teachers inside and outside school receive particular attention in mathematics education research (cf. Goos 2014; Jaworski 2014; Hart et al. 2011; Gueudet et al. 2013). A wide range of collective work is carried out in different parts of the world for different purposes: the collective development of resources for preparing day-to-day teaching, collective online writing of textbooks in France (Gueudet et al. 2016), Japanese lesson studies (Fernandez and Yoshida 2004; Isoda et al. 2007; Stigler and Hiebert 1999), Chinese Teaching Research Groups (Wang 2012; Gu and Wang 2003) for school-based professional development, and working groups in mathematics teachers' associations, to name but a few.

Resources such as teaching materials, lesson plans, and textbooks play a crucial role for any teacher collective work. Preparing lessons requires the teachers to investigate and develop multiple resources. Teachers' work cannot be dissociated from the use of resources. This is the rationale of the documentational approach to didactics (Gueudet and Trouche 2009), which investigates teachers' work and its evolution through the usage of resources.

In this chapter, we present a comparative study of teachers' documentation work in China and Japan from an international perspective. Our objective is to share selected illustrations of teachers' work that are less accessible to researchers outside China and Japan, and to provide some insights and questions to be investigated in future research. It will illustrate that China and Japan share a common culture which emphasizes integration and harmony and reflects the social orientation of its people.

The Chinese case is gathered from teachers' collective work carried out inside school, and the Japanese case is taken from the group activities of a local mathematics teachers' association outside school. These two cases are complementary to each other, in terms of the two kinds of teachers' collective work in East Asia, inside and outside school. Based on these two illustrative examples, we contend that teachers' collective work inside and outside school is an essential source of mathematics teachers' documentation work.

Teachers' work is complex and includes a variety of activities. In order to frame and organize our analysis of teachers' documentation work in the Chinese and Japanese cases, we share the view of the documentational approach to didactics. We rely especially on the idea that a *document* consists of (1) *resources* as artifacts and (2) a *scheme of utilization* on how to use these resources (Gueudet and Trouche 2009). "Scheme" in this case means "the invariant organization of activity for a certain class of situations" (Vergnaud 2009, p. 88), and it consists of four components: *goals*, *rules*, *operational invariants*, and possibilities of *inference*. Here we focus on the idea that the utilization scheme is specific to a certain class of situations. That is to say, if the situation was different, there would be another way to use the same resource. Through our examples of teachers' collective work in China and Japan, we will identify different classes of situations, inside and outside school,

requiring different utilization schemes, hence different teacher documentation work.

The guiding questions in this chapter are the following:

- What kinds of teachers' collective work are carried out, and in which ways?
- What kinds of resources are used and/or developed in the process, and in which ways?

The first question is intended to identify different classes of situations, while the second question aims to reveal the kinds of resources at stake and the related utilization schemes. In order to answer these questions, we carry out a comparative study between the Chinese case inside school (Sect. 7.2) and the Japanese case outside school (Sect. 7.3). In both cases, we present first institutional frameworks that create formally or informally opportunities for mathematics teachers to work together. It will show different structures of institutional framework in Japan and in China that should provide with various forms of teachers' collective work. In order to clarify such teachers' collective work, we will analyze one typical case from each, and elucidate teachers' documentation work as well as resources associated with such work. Finally, through the comparison of the results obtained in the case studies, we discuss the commonalities and differences between the two cases, as well as the perspectives for future research on teachers' collective work with resources (Sect. 7.4).

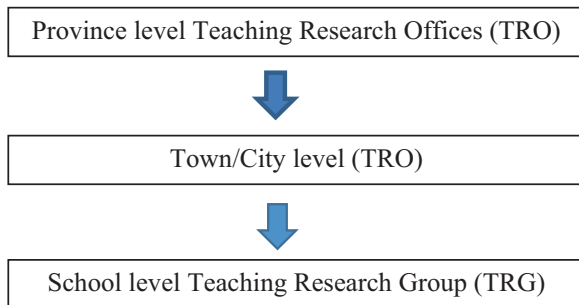
## 7.2 Inside School: The Chinese Case

Normally, Chinese teachers could and would work together inside schools because there are different kinds of working units that support teachers' collaborative works. Such working units support and contribute to teachers' professional development. In such professional development groups/units, teachers work collectively and with different resources. In the following, we explore mechanisms and development of teachers' work in such collectives in China.

### 7.2.1 *Context and Institutional Frameworks for Teachers' Work*

Entering the twenty-first century, the Chinese government has directed education to develop tasks to deepen education reform, to optimize the education structure, and to push forward the implementation of quality education (Pan 2005). Ministry of Education (MOE) published the Chinese mathematics curriculum standards (MOE 2001, 2003). Accordingly, standard-based textbooks were developed. At the same time, the curriculum reform suggested that teachers should not stick to such

**Fig. 7.1** Network of teachers' collective work in China



published textbooks; to the contrary, teachers would need to be able to reorganize textbooks and collect additional resources for mathematics teaching.

The curriculum reform and its implementation have generated much attention and discussion. How to work with and orchestrate abundant teaching resources, including textbooks, became teachers' essential work again. That means teachers are encouraged to be involved in the generation of teaching resources. This standard-based reform has been studied from international perspectives (e.g., Li 2007) as well as Chinese perspectives (e.g., Cao et al. 2006; Sun 2013). Sun (2013) mentioned that teaching materials are the basic foundation of teaching activities. He discussed the variety of teaching materials used by teachers collectively.

In China, teachers' collective work is aimed at enhancing student learning and teaching quality. One of the main tasks pays attention to school-based research. In order to help teachers to do research based on their teaching practical problems, a "three-level-institution" network was constructed by the government (see Fig. 7.1).

This is a top-down approach. From a macro point of view, it reflects an advantage of China's education system by playing an important role in managing and guiding school-based teaching research activities (Yang et al. 2013). The two teaching research officers (TROs) play the role of administration and professional guidance. From a micro point of view, in addition to researchers in higher learning institutions and school teachers, TROs<sup>1</sup> enlarge the team of professional researchers and play an important role in bridging the gap between teaching theories and instructional practice. TROs are powerful middlemen in the conducting of school-based teaching research activities. In this chapter, we focus on such research activities, which, in fact, are carried out in different typical teachers' collective work groups. In addition some new forms of collective work were developed in order to amend shortage of typical groups. One of the new forms would be analyzed.

<sup>1</sup>They work at TROs, called in Chinese 教研员. Normally they used to be experienced teachers and know development of certain educational theories very well. They provide school teachers with administration and professional guidance.

## 7.2.2 A Framework inside School: School-Based Groups of Teachers' Collective Work

In China, each school sets up different kinds of teachers' collective work groups. It includes *Lesson Preparation Group* (LPG), called in Chinese 备课组, *Teaching Research Group* (TRG) called in Chinese 教研组 and *Research Project Group* (RPG) called in Chinese 课题组. The three types of groups have different functions, at the same time they are integrated and implement a complex program.

### 7.2.2.1 Teachers' Documentation Work in LPG

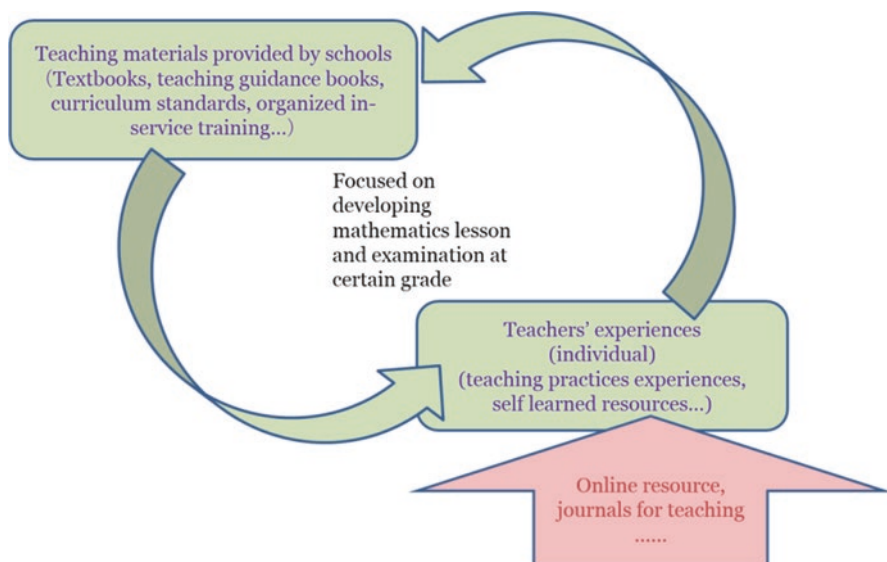
Teachers from the same grade and same subject gather to build a LPG which is led by an experienced teacher. Generally speaking, working in mathematics LPG can provide each mathematics teacher, especially novice teachers, with a clear structure of mathematics content for each unit and a concrete plan on how to implement each lesson. LPGs contribute to improving teaching practices and enhancing teaching effectiveness.

For Chinese teachers, textbooks are the most important resources in the mathematics classroom. One of teachers' essential work in China is to navigate and study textbooks in order to design instruction. Related to the LPG, there are two parts of resources that teachers may use: one is school related (e.g., the textbook, teaching guidance books, curriculum standards, organized in-service training); the other refers to teachers' individual experiences, including self-learned resources (e.g., own collection of books). In the digital age, most experiences derive from online resources, or from journals for mathematics teaching and learning. Often such individual resources play a role while teachers need to extend or deepen their understanding of school-related resources. In LPGs both kinds of resources are used while teachers prepare lessons (Fig. 7.2).

### 7.2.2.2 Teachers' Documentation Work in TRGs

The TRG is a popular group where teachers improve their teaching through collective study of practical problems. In the *Secondary School Teaching Research Group Rulebook (draft)* issued by Ministry of Education (MOE) in 1957, the function of the TRG was described.

The TRG is responsible for all mathematics teachers' professional development in the school (Fan et al. 2015). In TRGs teachers from the same subject in the school gather led by a subject head. It has three general foci: firstly, the TRG focuses on discussing teaching practices, including instructional design, mid-semester examination, and school-based open lesson. Secondly, the TRG orients on school-based research questions which reflect crucial teaching activities or misconceptions/problem areas, for example, how to improve geometrical under-



**Fig. 7.2** Resources structure related to LPG

standing with support of dynamic software or how to design mathematics lesson based on students' mathematical mistakes from their homework. Based on such research questions, the TRG tries to propose school-based research projects collectively. In order to encourage teachers or the TRG to implement projects, some schools also set up special project funding that can be applied for by teachers. The third role of the TRG is to be in charge of connecting city-level TROs. The TROs provide teachers with opportunities to participate in teaching evaluation, teaching competition, or participate in other tasks, which are assigned by city-level TRO. TRGs play an important role to help teachers to prepare collectively teaching competition.

Since the TRG undertakes different kinds of teachers' collective work, teachers within TRG utilize various resources as well as develop new resources. Teachers mainly use three types of resources, two of them are in accord with resources from the LPG. We can observe that the most important and valuable resources refer to resources generated at city-level TRO, including teaching evaluation, teaching competition, as well as research theories or methods from experiences with university researchers or researching institutes. Teachers' documentation work in TRGs happens in relative open environments, TRGs have good opportunities to interact with teaching practice groups from other schools, research groups from universities, and teaching administration groups. Working as a team, teachers in the TRG collect abundant information from those groups and may convert them into teaching resources. Figure 7.3 illustrates the resource system operated in TRGs.

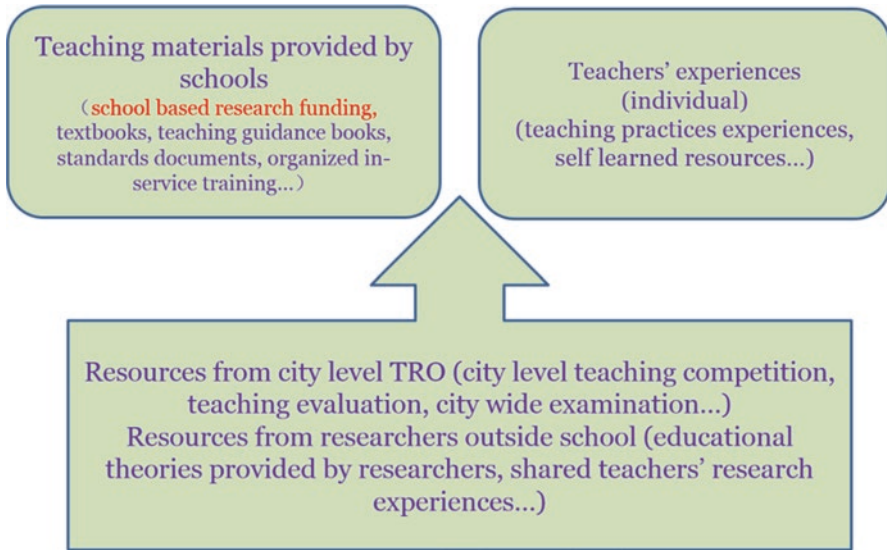


Fig. 7.3 Resource operated in TRG

### 7.2.2.3 Teachers' Documentation Work in RPG

Inside schools in China, there exists a working group called Research Project Group (RPG) that is led by a director of research in the school. The RPG aims at experiencing or implementing whole research projects, including literature review or analyzing practical teaching problems, design of research proposals according to particular research framework. RPGs also apply for research project funding at the city level, or province level, or national level.

In addition, the RPG has other opportunities to undertake a sub-project assigned by a research group outside school. For example, some schools got a sub-project of a national project, "construction of an innovative model to promote teacher professional development." One of the sub-projects may focus on a case study of mathematics teacher professional development. In order to finish such sub-project, the RPG would invite experts from outside school and discuss research methods, or research frameworks.

In RPGs teachers can have different resources that support teachers to do research related to teaching practices as shown in Fig. 7.4. Because practical problems are derived from all kinds of school-based activities, resources that RPG members use can be divided into three categories: developmental programs or previous research experiences at school level; teaching and learning practices at teacher individual level; and educational theories, relevant publications, research methods at theoretical level. The RPG teachers learn to manipulate different resources targeted.

Such structured mechanisms ensure that teachers experience collective working culture at schools. But often Chinese teachers express more "collective voice" than their individual voice. Such "collective work" cannot fully reflect requirements/needs of individual teachers for their professional development.

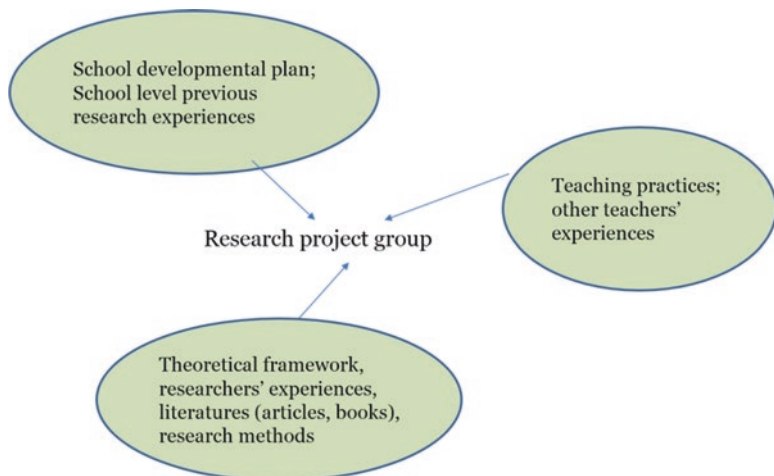


Fig. 7.4 Resources operated in RPG

### 7.2.3 A Case Study: A New Model of Teachers' Collective Work

Facing challenges of curriculum reform, school-based teaching research has been developed. School-based teaching research activities are not isolated; in other words, many schools open their doors and welcome or invite experts from outside. The boundaries of Teaching Research Group and Research Project Group become vague. Experts from city-level TRO or university researchers are actively involved in school-based research activities. Teachers have more opportunities to interact with academic colleagues. Teachers' collective work is full of thinking and enthusiasm.

#### 7.2.3.1 Collaboration Research Group Model

Hereon we introduce one of new models of teachers' collective work. This model is called Collaboration Research Group (CRG), in Chinese called 合作研究组, whose general nature is collaboration and sharing. The core ideas of the CRG target at researching for improving teaching, and thinking for enhancing development. This model has been established and implemented at schools for the past 10 years (Ye and Si 2017).

Members of CRG consist of school teachers, researchers from universities, and experts from city-level TRO. The three groups of members play different roles, respectively, while working and sharing collectively in CRG. School teachers will initiate research questions based on their teaching practices. Researchers provide with theoretical framework related to practical research questions and expound relevant theories to school teachers and TRO experts. School teachers will work



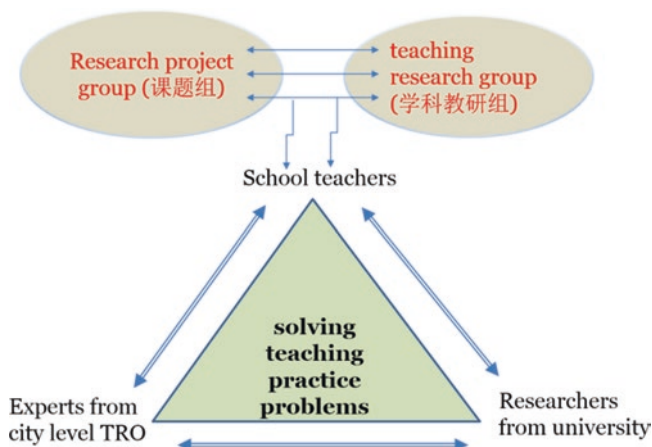


Fig. 7.5 Structure of Collaboration Research Group (CRG) in school

together with TRO experts and discuss how to apply theories into practices and design classroom teaching based on new theoretical perspectives. Researchers will observe and comment teaching practice designed by school teachers and TRO experts, and then will construct methods to evaluate function of CRG.

Only when school leaders attach importance to teacher professional development and encourage collaborative research working, they set up such CRG in schools. This is a research project-driven organization, instead of permanent administrative unit in school. The expectation of schools for CRG is that teaching practice problems should be explored. Figure 7.5 illustrates the structure of CRG.

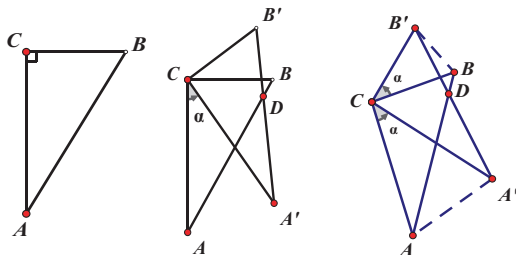
### 7.2.3.2 A Case of Implementing Collaboration Research Group

In one middle school of Zhejiang province, one of the key tasks of this school is to support teachers' professional development. One CRG has been set up in this school, with emphasis on exploring, analyzing, evaluating and improving mathematics teaching behaviors.

Firstly, one researcher from university and one expert from city-level TRO were invited by the school; they worked together with leaders from school-based TRG and RPG, in order to investigate mathematics classroom teaching and make decision on teaching problems driven research questions. The particular project was that how to understand and analyze mathematics classroom teaching based on video analysis. They believe that teachers will improve their teaching practice through engaging in such project.

Secondly, teachers who have interest in such topics were invited to work together with researchers; then the CRG is organized. At the beginning, all CRG members shared their own ideas focused on this topic, and then the researcher gave advises to participating teachers to design a practical research project.

**Fig. 7.6** Pictures in Lesson 1



Subsequently, researchers and teachers took on different roles. Researchers brought their research experiences and theoretical viewpoints into CRG, suggested one framework for analyzing classroom teaching, explained the theoretical background of the framework, and introduced particular analysis methods and tools. Teachers explained important or difficult mathematics content while teaching mathematics and designed lesson plan focused on concrete mathematics topics.

In the following, we show how teachers and researchers worked together. CRG focused on two mathematics lessons which had same mathematics topic, but different teaching strategies.<sup>2</sup> To design and implement the both lessons aimed at exploring characteristics of reviewing lesson of geometric for grade 9, and discussing how to embody mathematics core competencies in the classroom teaching.

### 7.2.3.3 Two Lessons of the Same Topic

Lesson 1 focused on *exploration of a rotational question, general repertory of geometric inquiry, made by S*; Lesson 2 was about *geometry exploration journey, made by Y*.

S designed three inquiring tasks for Lesson 1.

- Task 1: As pictured (Fig. 7.6), in  $\triangle ABC$ ,  $\angle C = 90^\circ$ ,  $\angle A = 30^\circ$ , if we let  $\triangle ABC$  rotate  $30^\circ$  counterclockwise around point C to get  $\triangle A'B'C'$ , what can you find from the image? (Please draw the image first.)
- Task 2: If we change the angle of rotation into  $\alpha$  ( $0^\circ < \alpha < 45^\circ$ ), do the conclusion from task 1 change?
- Task 3: When  $\triangle ABC$  is a general triangle, what have you found?

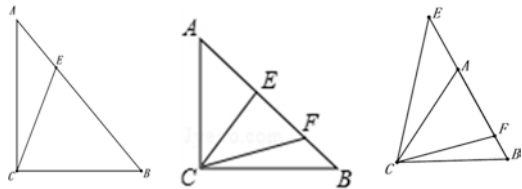
The teacher S (Fig. 7.7) gave conclusion of the lesson related to general repertory of geometric inquiry as follows:

<sup>2</sup>“Same mathematics topic, but different teaching strategies (同课异构)” is a particular teaching research activity in China. It means, focusing on one mathematics topic, teachers will design two different lessons using different methods, so that teachers can discuss or explore these lessons.



Fig. 7.7 Lesson 1 by S

Fig. 7.8 Pictures in Lesson 2



1. To determine the research objects (geometry elements—determine problems).
2. To explore the research contents (explore the invariant relationship in change, the relationship between geometric elements).
3. Inductive research methods (which were useful for solving tasks using special or general methods).

Y designed three tasks for students' inquiring for Lesson 2.

- Task 1. In Fig. 7.8, in the isosceles right triangle ABC, point E is a moving point on AB. Which relation existed between AE and BE?
- Task 2. In Fig. 7.8 in  $\triangle ABC$ ,  $\angle ACB = 90^\circ$ ,  $AC = BC$ , points E and F are two moving points on AB, and  $\angle ECF = \angle A$ . Which relation existed between AE and BE?
- Task 3. If  $\angle ACB = 90^\circ$  is changed to  $\angle ACB = \alpha$  ( $0^\circ < \alpha < 90^\circ$ ), other conditions remain unchanged. Do the above conclusions still exist?

Figure 7.9 showed that Y explained these three inquiring tasks.

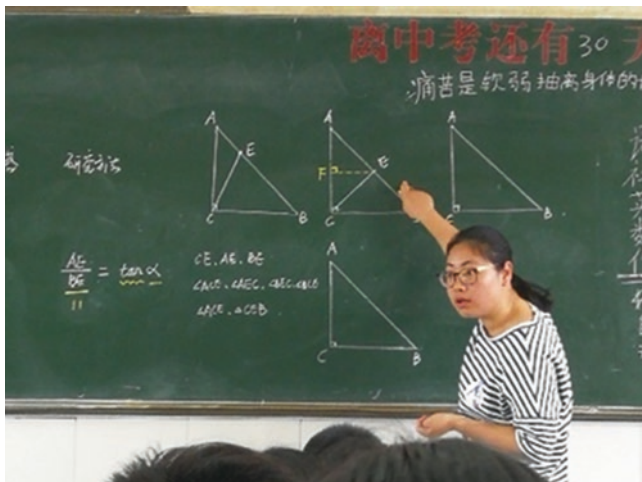


Fig. 7.9 Lesson 2 by Y

#### 7.2.3.4 Discussion After the Two Lessons

After implementing both the lessons, members of CRG shared their ideas and comments focused on the lessons. At first, according to the observation, the researcher Ye interpreted that these two lessons fully embodied the nature of the ninth grade review of geometry. He commented that these lessons grasped the object of geometry, used special-to-general thinking to explore the relationship between elements in the figures, so to find unique and concise conclusions.

He analyzed the differences of both lessons. Firstly, he observed the differences related to content. He said, for Lesson 1, the triangle ABC itself rotates. The two triangles (triangle ABC and triangle A'B'C') given here are two same triangles. The tasks evolve as follows: from determined angle rotation to any angle rotation; from isosceles right triangle to general triangle. For Lesson 2, we choose the moving point on one side of triangle ABC. The two given triangles (triangle ABC and triangle EFC) are probably not the same here. The tasks evolve as follows: from one moving point to two moving points; from isosceles right triangle to right triangle.

He also analyzed the difference related to instructional design. Teacher S uses this task to let students experience the process of how to explore the relations with moving points. The purpose of this task is to teach students the steps of exploring geometry relations: determine the objects (geometry elements), explore the contents, and generate the methods. So the task is used by teachers' guiding. For Lesson 2, she pointed out that this task is more open for students. The teacher Y hadn't demonstrated any conclusions before the class. So during the teaching process, the teacher decides or changes her way to teach based on the different responses given by the students. Which means teacher would use different teaching methods when working with different students. Students are more initiative to decide what to explore when working with the task.

The expert from TRO gave also comments, and teachers explained and reflect their teaching purpose. Such dialogues between members of CRG improve the teachers' professional development effectively.

There is a researcher (Sun) from the Zhejiang Provincial Teaching and Research Office who gave a high evaluation of these two lessons from the necessity, objective, structure, and effectiveness of a class. He encouraged school teachers to maintain such an atmosphere of teaching and research, trying and innovating, and to provide teachers meta motivation for quality education, teaching, and research.

In this case, school leaders were also concerned with CRG activities. The leader of mathematics TRG Lv represented other participating mathematics teachers. He said that he has learned a lot from the activities. After reviewing the original features of the lesson and geometry class, he must apply the ideas and methods learned from the activities to ordinary teaching practice, live up to the guidance and expectation of experts and predecessor.

### **7.2.4 Summary of Teachers' Collective Work Inside School**

Traditionally, in Chinese schools, there are different kinds of structured organizations (groups) where teachers can work collectively. They discuss and modify lesson plans, design examinations, share additional teaching resources, or study teaching practice problems and improve instructional quality. It is difficult to image that teachers can develop effective plans, tests or other resources, without collective work. Fortunately, school-based teacher collective work has expanded. Some research projects driven by collective work were carried out. Teachers, university researchers, and experts from other organization have had opportunities to gather at schools, and to undertake different tasks. Especially, researchers could share their research experiences and theoretical consideration with teachers, and in turn teachers' practical experiences were enriched with underpinning theory.

## **7.3 Outside School: The Japanese Case**

Japanese lesson study is well documented in the educational literature (written in English) as a format for teachers' collective work (Fernandez and Yoshida 2004; Isoda et al. 2007; Stigler and Hiebert 1999). Apart from it, one may find other kinds of formats for teachers' collective work outside school. One of them is organized by the local teachers' association, through monthly or biweekly meetings, where teachers get together, share, and discuss their teaching experiences and eventually carry out a lesson study project. This section presents the documentation work in teachers' meetings in Japan through two kinds of resources, *lesson plans* and *practice research reports*, in order to illustrate how teachers' collective work contributes to the sharing and development of their practice and knowledge.

### 7.3.1 *Context and Institutional Frameworks for Teachers' Work*

In Japan, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) determines the educational system, which is applied to all parts of the country and guarantees quality education. Until the end of lower secondary school education, the single-track system is adopted and all students are in a position to receive almost the same quality of education either in a city or in a rural area. The national curricula written by MEXT determine the number of hours dedicated to each subject and the teaching content (cf. MEXT 2008).

Concerning the teachers' work, there are different institutional frameworks, as in Chinese cases, that allow teachers to work collectively or individually inside and outside school. We classified them in Table 7.1 according to geographical levels, which also implies the different levels of educational management (see also Miyakawa and Winsløw 2017). In Japan, the board of education at the city level manages the public primary and lower secondary schools, and the board of education of the prefecture manages the administration related to the education of all cities, including upper secondary school.

Japanese teachers spend most of their working hours in school not only for preparing and teaching classes, but also for undergoing professional development such as lesson studies. At the city level, the board of education and teachers' associations provide teachers with opportunities for professional development. In this section, we take up the activities of teachers' associations at this local level.

**Table 7.1** Institutional frameworks of the different levels

Level	Institutional frameworks of teachers' work
Nation	Ministry of Education National Center for Education Associations of math teachers Commercial companies
Prefecture(s)	Board of Education Education center Teacher training university Schools attached to the university Associations of math teachers
City(s)	Board of Education Associations of math teachers
School	Schools

### 7.3.2 *A Framework Outside School: Local Teachers' Association*

In most Japanese cities, there exists a local voluntary association for mathematics teachers, which provides a place for them to work together and to improve their teaching practices. Such associations exist not only for mathematics but also for other subjects like science, history, and Japanese. In Jōetsu, a relatively small city in Japan, there is an association for primary and middle school mathematics teachers called *Jōetsu sūgaku kyōiku kenkyūkai* (Research Association for Mathematics Teaching in Jōetsu). Teachers take part in activities of this association without any obligation or reward. Different kinds of activities are proposed: monthly meetings, lectures by invitees (e.g., researchers, expert teachers), an annual congress, workshops for teachers, publication of a bulletin or book, and so forth. Our focus in this section is the teachers' documentation work in monthly meetings.

The monthly meeting of this association is held in the evening. It is open not only to the association members but also to anyone interested in discussing mathematics teaching. There are usually 10–15 different kinds of participants: teachers from primary and middle school, educational advisors, school principals (ex-math teachers), university professors, pre-service teachers (students), etc.

The meeting is devoted to two topics—with 45–60 minutes for each—brought up by two teachers. The presenting teacher always brings a material or a handout as a resource and distributes it to the participants. There are two kinds of resources. The first one is the *lesson plan* that describes the details of the designed lesson which will be taught in the classroom for different purposes, such as an *open research lesson*<sup>3</sup> in the context of school-based professional development. The second resource is the teaching *practice report* which describes the results of teaching practices that have taken place in the classroom. In general, these are the two principal resources that Japanese teachers develop and share in their ordinary activities inside and outside school. They are easily accessible on the websites managed by the educational center of the board of education.

### 7.3.3 *A Case Study: Monthly Meetings*

We go into the details of teachers' work promoted in the monthly meetings of local teachers' association. A case study is carried out through the analysis of the two types of resources discussed above, *lesson plan* and *practice report*. For each type of resources, we first present a lesson plan/practice report distributed at a monthly meeting with a brief analysis of its role, and then identify teachers' work associated

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<sup>3</sup>This is a lesson, called in Japanese *kōkai-jugyō* (*kōkai* means open or public), which is demonstrated to the colleagues of same school or the teachers of other school (cf. Miyakawa and Winsløw 2013). This is very often a part of lesson study or considered as a lesson study.

with it with a special focus on the resources used. We also present the discussion at a monthly meeting in order to clarify the nature of teachers' work and the resources involved.

### 7.3.3.1 Lesson Plan and Its Roles

The lesson plan is the most familiar resource since the preservice teacher training and throughout the profession of Japanese teachers. In a monthly meeting, the participants were discussing mathematics teaching with a lesson plan, which is given in Fig. 7.10. An individual primary school teacher was preparing this lesson plan for an open research lesson scheduled for the following month, and presented it as a material to be discussed at this meeting.

This plan consists of two A4 pages, and has a lot of text. At the top of the left page is the title, "View from inside a box" (first author's translation). The lesson plan is for a class on space geometry in grade 2 mathematics and proposes activities for the pupils (7–8 years old) that involve creating a big box or polyhedron, as shown in Fig. 7.11, so that they can feel and enjoy the breadth of the space from inside the box.

The lesson plan on the left-hand side consists of four sections: (1) Goal of activities; (2) Intentions of activities; (3) Actual state of the children; and (4) Characteristics of these activities. The first page is therefore describing the goal of teaching and its rationale with respect to the actual state of the pupils in the class. The second page (on the right-hand side of Fig. 7.10) is devoted to an explanation of one specific lesson, which would be demonstrated at the presenting teacher's school. One section devoted to a whole page is titled "5) About this lesson" and consists of three subsections: (1) Goal of this lesson, (2) Characteristics of this lesson, and (3) Progression of the class. Here, a goal is given again, since the goal of a single lesson is usually different from the overall goal of a unit. Further, the explanation of and justification for this lesson are given. Then, a table is used to describe the teaching process along with the timing, the expected pupil behaviors, and the teacher's instructional moves.

The lesson plan plays several roles. A principal role one may identify from the example above is to share with participants or readers the teaching practices of the designed lesson, as well as the teacher's ideas behind the series of lessons and the setting of these lessons. Further, a lesson plan is a tool for a teacher to justify and convince them of his/her choices. This is the reason why a lot of texts are necessary in the lesson plan, as seen in Fig. 7.11. Such a detailed lesson plan with justification would not be necessary if the designed lessons had already been shared with colleagues (such as in collaborative designing).

In contrast, the table showing the teaching process of the single lesson is relatively short. This implies that the lesson plan is a guide for participants or readers to help them understand the overall structure and activities of the lesson, rather than a guide for the presenting teacher on what to do during teaching. In general, the description of the lesson in the lesson plan does not provide precise teacher's





### 7.3.3.2 Teachers' Work Associated with Lesson Plans

What kinds of teachers' work are associated with this lesson plan in addition to the collective discussions at the monthly meeting? The lesson plan and the discussion in the meeting imply teacher's different kinds of work (in terms of the use of resources), which are generally carried out in Japan for developing a lesson plan. They are as follows:

- Understanding the goals.
- Designing tasks.
- Designing lessons.
- Writing a lesson plan.

Since mathematics classes are based on the national curriculum, the Japanese teacher is first of all required to understand the goals of a given unit and its lessons, through an analysis of the curricular resources, in particular the textbooks. This work before (or while) designing tasks is often called *kyōzai-kenkyū*, which literally means the study of teaching materials (*kyōzai*) (cf. Watanabe et al. 2008). It is noteworthy that the textbook has a special status as a resource for Japanese teachers. It should be approved by the ministry of education, and its use is compulsory by the Japanese regulation. The teacher covers almost all the materials given in the textbook (e.g., Becker et al., 1990). This is a reason why the textbook analysis is one of primary activities for preparing lessons. This was also the case for the teacher who wrote the lesson plan above. In fact, in the discussion of the monthly meeting, participants discussed what the textbook intends to teach.

The next work, *designing tasks*, involves coming up with tasks to be implemented in a series of lessons, through the exploration of different resources such as the internet, textbooks, and professional journals. This work is often called *kyōzai-kaihatsu*, which literally means the development (*kaihatsu*) of teaching materials. The teacher of the meeting invented the tasks that could not be found in the textbook.

*Designing lessons* refers to the development of the process or progression of teaching with the designed tasks. The resources necessary for this work tend to focus more on concrete teaching and learning actions or experiences in the classroom, such as learners' behaviors. This work is sometimes (not so often comparing to the above two kinds of work) called *jūgyō-kaihatsu* in Japan, which means the development of lessons.

Writing a lesson plan can be a different task from the three previous ones; it forces the teacher to explicitly formulate the ideas behind the lessons.

The distinction between these four kinds of work and the utilization of each term are not clear-cut in the Japanese educational community. They may overlap in some ways: *kyōzai-kenkyū* may at times include *kyōzai-kaihatsu*, and vice versa; *jūgyō-kaihatsu* may include the two former kinds of work.

It is also important to note that these four kinds of work are required especially in a lesson study or meetings with colleagues like the one documented in this chapter. In a day-to-day lesson, Japanese teachers often use the tasks given in the textbooks, and they will not write out a lesson plan like the one in Fig. 7.10, while an understanding of the teaching goals is always necessary.

### 7.3.3.3 Practice Report and Its Roles

Another monthly meeting was devoted to a discussion of the teaching practice report that would be presented at a regional congress for teachers several months later. What we call a *practice report* here is a report that a teacher writes after his/her teaching practices; it is called *jissen-hōkoku* in Japanese (*jissen* and *hōkoku* denote “practice” and “report,” respectively). It is written either in the context of a lesson study or as a part of individual or collective action research. As noted earlier, there are teachers' associations at different institutional levels. These associations organize the annual congress for the teachers to share their teaching practices. A teacher is sometimes asked to present their practice report at a congress or publish it in a professional journal. So, the activity of this monthly meeting was a preparation for the teachers who were to give oral presentations at the regional congress. A detailed analysis of how the activities of local associations relate to the activities of regional congresses is given by Miyakawa and Winsløw (2017). In this section, we will provide another example and an analysis from the perspective of teachers' documentation work.

A primary school teacher prepared a practice report for a monthly meeting (Fig. 7.12). This report consisted of six A4 pages and presented the results of his teaching practices carried out in grade 6 mathematics classes on the geometric unit on enlarged and reduced figures. According to him, this was the result of a lesson study carried out as school-based professional development. The structure of the report with our translations of section titles is given in Fig. 7.13. The title is given at the top of the first page. There is a section on the rationale for the selected theme (Section 1), and a “research hypothesis” is given (Section 2). The main part is Section 4. Four and a half pages are devoted to describing the teaching practices of the sequence of lessons. He selected some sessions and explained how the lessons had unfolded. He then concluded with the results and further issues (Section 5), followed by a reference list (only two sources are listed).

The report looks like a scientific paper. The term “research” (*kenkyū* in Japanese) is often used. This is because such kind of teacher's work is often regarded in Japan as research called “practice research” (*jissen-kenkyū*; see Miyakawa and Winsløw 2017). A lesson study is also considered to be a kind of practice research.

In this practice report, the teacher proposed a process of learning in the classroom that includes verbalization of the problem solving method, and posited the hypothesis that this activity would deepen students' understanding of geometric figures. One can see the main idea of his *research* in a diagram (Fig. 7.14); it shows the process and structure of pupils' learning, including the verbalization of the solving method during group work.

Fixing a hypothesis allows the teacher to focus on a specific aspect of the complex mathematics lesson. Further the scientific report format requires the teacher to deeply reflect on and investigate their teaching practices. In Section 4, for the description of the teaching practices, the presenting teacher identified the remarkable events, and described the learning process how the verbalization affected pupils' activities in the classroom.

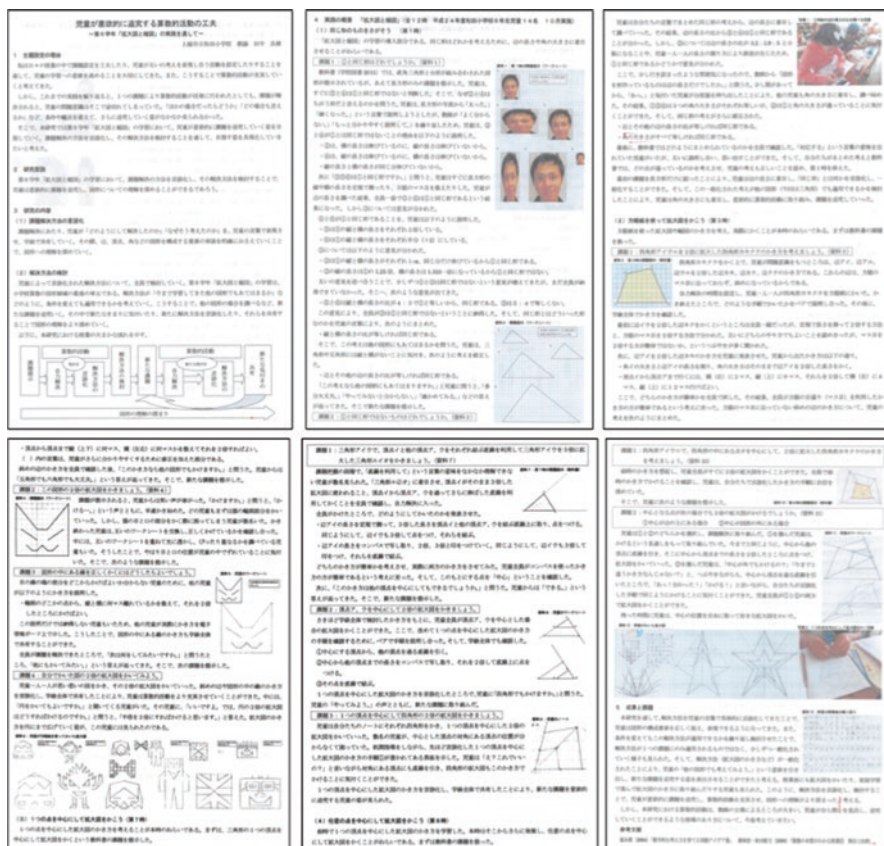


Fig. 7.12 Overall image of the distributed practice report. (Tanaka 2013)

The role of practice reports for the community of mathematics teachers is to share with colleagues or readers teaching practices and the ideas behind them. As in the case of the lesson plan, the practice report distributed at the monthly meeting targeted first the participants who did not know the presenting teacher's teaching practices, and second, the participants in the regional congress scheduled some months later. This resource provided participants with a specific subject to discuss and reflect on with concrete instances of mathematics teaching in a classroom.

The practice reports developed for and at the meetings are disseminated through oral presentations at the congress, and eventually through professional journals or books, becoming a resource to be explored in other practice research in different places. It should also be noted that the writing of a practice report is often a part of the professional development of an individual teacher. This role is discussed below in terms of the teacher's associated work.

Title: Mathematical activities in which children are motivated to participate: through the teaching practices of a grade 6 lesson: "Enlarged and reduced figures"

1. Reasons for the choice of this theme
2. Research hypothesis
3. Research content
  - (1) Verbalization of the problem solving method
  - (2) Discussion of the problem solving method
4. Overview of teaching practices: "Enlarged and reduced figures"
  - (1) Let's look for items of the same shape (Session 1)
  - (2) Let's draw an enlarged figure on the grid sheet (Session 3)
  - (3) Let's draw an enlarged figure using a specific center point (Session 7)
  - (4) Let's draw an enlarged figure using any center point (Session 8)
5. Results and further issues

References

Fig. 7.13 Structure of the practice report with translated section titles

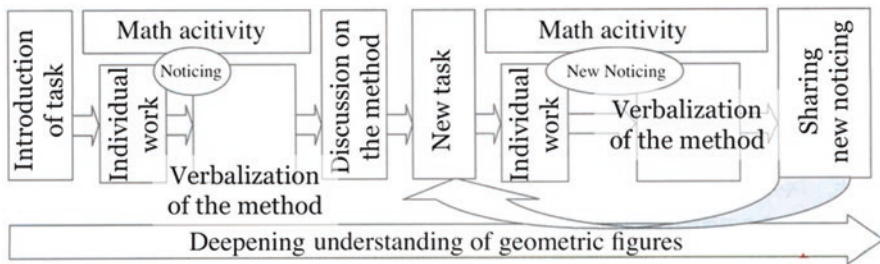


Fig. 7.14 Diagram of the lesson flow (translated by the first author)

### 7.3.3.4 Teacher's Work Associated with the Practice Report

As in the case of a lesson plan, one may identify, based on the practice report, different kinds of documentation work by the teacher, which include the tasks associated with the lesson plan we discussed in the previous section. In addition to those for the lesson plan, the teacher must carry out the teaching in the classroom and the collection of data, analyze or reflect on his own teaching, and write a practice report.

The analysis in the practice report is not as rigorous as in a scientific paper, but is the result of a "sincere reflection" by the teacher on what happened in the series of lessons. In fact, the analysis given in the practice report above was principally based on the pupil's worksheets and the teacher's memories (he regretted in the meeting that he had not audiotaped the discussions in the classroom). The main task

in writing a practice report is objectifying the teacher's own teaching, and summarizing and formalizing the main events in the lessons containing the pupils' work. Such work is rare in day-to-day teaching in Japan. The teacher usually relies on the textbooks and does not have enough time to reflect on his/her own teaching practices. Practice research creates situations that require the utilization of resources which are different from those used in the preparation of ordinary lessons. It is therefore an opportunity for teachers to better understand their own teaching, and to develop their professional skills and knowledge. Additionally, like in the case of a lesson plan, the teacher's collective work either as part of the discussion at the monthly meeting or as part of the oral presentation at the congress plays a crucial role in triggering this documentation work.

### 7.3.3.5 Discussion at a Monthly Meeting

We present here briefly what happens in the discussion of monthly meeting of the local teachers' association, and analyze teachers' work. In general, in any monthly meeting, there is a participant who moderates the session; the presenting teacher first explains his designed lesson in the case of a lesson plan, or his implemented lessons in the case of a practice report; then, the participants ask questions to understand the lesson and make comments to provide some helpful ideas.

The monthly meeting for the practice report above (Fig. 7.12) was held with a dozen or so participants of different kinds: primary and middle school teachers, university researchers, and pre-service teachers. The presenting teacher first took 15 minutes to explain his report, followed by 45 minutes of questions and comments from the participants. His explanation followed the structure of a distributed report, concluding with the issues that he identified. In the discussion, the moderator initially proceeded from the first page in order, soliciting questions and comments from the participants along the way, but partway through the process, the discussion gave way to an opinion-exchange session covering the entire document. The questions and comments offered can be roughly summarized as follows:

- The terminology of mathematics education (e.g., mathematical activities).
- The writing method of the teaching practice report.
- The content of the report: Consistency of the report (title and content), suggestion of some complementary contents ("you should add...").
- The participants' experience of teaching practices of the similar units.
- The pupils' behaviors and actions during the lesson; issues/problems faced by the teacher (aspects where the lesson did not go well).
- Its relationship with other teaching contents in primary and middle school.
- Suggestions for teacher's instruction, teaching materials, tasks, etc.
- Suggestions for new foci or themes of practice research.

As is apparent, a wide range of topics were discussed and considered. The principal (material) resource for the discussion was the practice report prepared by the presenting teacher. Further, the participants shared a lot of *cognitive resources*, the experiences of different kinds of participants. This is a specificity of this monthly meeting. The participants bring, according to their expertise, a variety of experiences to the discussion, and *re-source* each other. For example, an experienced teacher provided his experiences how to write a practice report and how to present it at a teachers' congress; a middle school teacher shared his experience of the similar topic (similar figures) and suggested to check the middle school textbooks; a primary school teacher brought up a perspective on the phrase "mathematical activities" in the title of the report, which is a concept emphasized in the Japanese national curriculum (MEXT 2008), and suggested that there had been insufficient consideration from that perspective; a university researcher shares the idea of additive reasoning and multiplicative reasoning as a related theme for the practice research.

It should also be noted that the teachers' collective work at such meetings is usually collaborative rather than cooperative, in the sense that the participants make different comments and do not necessarily look for a consensus. The presenting teacher receives these comments as resources for further reflections. This is also the case for the discussion on lesson plans. Principally in Japan, the teacher is charged with deciding what task will be used and how to teach in the open research lesson.

### **7.3.4 Summary of Teachers' Collective Work Outside School**

Teachers' collective work in associations outside school in Japan is very often intended for the development and sharing of teaching practices through *lesson plans* and *practice reports*. These resources are slightly different from the usual resources for classroom use, such as mathematical tasks and student worksheets. They serve to open up discussions with other teachers and to lead to sharing and acquiring ideas for use in future teaching practices. Further, developing such resources is a critical process that requires associated documentation work. Teachers' collective work is crucial here in the sense that it triggers teachers' individual documentation work for the sake of their professional development.

## **7.4 Discussion and Perspective**

In this chapter, we have shared teachers' collective work inside (China) and outside (Japan) school in China and Japan. We discuss here the commonalities and differences based on the comparison of our two cases, in terms of the teachers' collective work and the resources associated with such work. In addition, we provide perspectives for future research on the teachers' documentation work.

**Table 7.2** Comparison of the two cases

	CRG: a Chinese case inside school	Monthly meetings: a Japanese case outside school
Institutional frameworks	School Teaching research Office (TRO) of different levels University	Local mathematics teachers' association
Teachers' work	Development of lessons, based on individual/collective practice-based activities (analyzing curricular material, designing, implementing, and analyzing lessons) Research-oriented activities Discussion on teaching practices and their theoretical framework	Development of lesson plans or practice reports, based on individual/collective practice-based activities (analyzing curricular material, designing, implementing, and analyzing lessons) Research-oriented activities Discussion on teaching practices
Resources	Teaching materials (textbook, guideline, books, etc.) Research literature (theoretical framework, research method, etc.) Cognitive resources of different participants (experience teachers, university researchers, etc.) Publication opportunities in professional journals (associations or private publishers), proceedings	Lesson plans and practice reports Teaching materials (textbook, guideline, books, etc.) for the development of lessons Cognitive resources of different participants (experience teachers, university researchers, etc.) Publication opportunities in professional journals (associations or private publishers), proceedings

### 7.4.1 Comparison: Commonalities and Differences

We first summarized the results of the two case studies in Table 7.2. As these cases are in the very different educational contexts (inside and outside school), the table of comparison suggests the complementarities of teachers' documentation work in East-Asian countries, rather than the differences between China and Japan. The comparison does not mean therefore that the teachers' work and resources in China do not exist in Japan, or vice versa, but they are specific to each case regardless of country.

In both countries, different institutional frameworks at the different levels—ranging from school to national level—provide teachers with opportunities to work together. In the case of CRG, it is noteworthy that the university is involved as a principal sector, in addition to TRO which officially carries out TRG in a top-down approach. In contrast, the monthly meeting organized by the local teachers' association is a result of bottom-up action by voluntary teachers, which promotes in addition teachers' work in the associations of regional and national levels.

Regarding the teachers' work, China and Japan adopt a form of *practice-based* professional development, including the designing and implementation of teaching practices, instead of a form of lecture or workshop which is sometimes dissociated from the usual teaching practices with students. Remarkably, the *research-oriented* work is promoted, and the term "research" is very often used in both countries.



Teachers' work is considered to be a kind of research work focused on practical problems faced in teaching.

Further, there is a close relationship between the university researchers and the school teachers. The CRG Model in China and the monthly meeting in Japan both involve researchers from university, who bring cognitive resources into schools such as research experiences and theoretical viewpoints developed in mathematics education research. Moreover, the diversity of participants, which is a commonality in the two cases, is a critical aspect in order to enrich cognitive resources shared in the teachers' collective work.

In terms of resources, in both countries, the textbook occupies a prominent place among teachers' resources, and the study of textbooks is one of the essential works for teachers. In addition, it is noteworthy that the teacher communities are equipped with the publication infrastructure of professional journals or books that allow for the teachers to disseminate their work and hence to "re-source" other teachers.

### ***7.4.2 Perspectives for Future Research***

While teachers' collective work exists all over the world, the nature of collective work and the formats that promote such work differ based on the countries. We consider that the differences and commonalities between Eastern and Western countries in terms of collective work and associated resources are still to be further investigated. We discuss here the perspectives for future research in this respect through the Chinese and Japanese cases.

We consider, first, the need for a closer analysis of teachers' collective work in East Asian countries. This is, for instance, the case for the lesson study which is well known today and practiced in different places outside Japan (Hart et al. 2011; Lewis and Hurd 2011). The analysis of teachers' work in Japanese lesson study is still limited. In fact, while the collective work is very often emphasized in lesson study, there are a lot of associated individual work, and we do not yet understand how individual work is combined with the collective work, and how these different documentation work practices affect teachers' professional learning. The detailed lesson plan presented in this chapter would not make sense without such understanding. This need for closer analysis is also the case for Chinese teachers' collective work. We observed in China that teachers would sometimes follow the "collective voice," instead of expressing their own "voice". It will be important to go into the detail how teachers maintain their own identity while working collectively, in order to deeply understand how the collective work supports teacher learning.

Second, while we use similar resources with similar terms in Eastern and Western countries, their roles and functions and the associated teacher's work may be different, and it is necessary to carry out a closer analysis of such resources and teacher's documentation work. For example, in the case of textbooks, while we referred to the importance of textbook in East Asian countries, the textbook is usually a most important resource for teacher in any country (Pepin and Haggarty 2001; Pepin

et al. 2013; Mullis et al. 2012). However, the way this resource is used may vary from country to country, that is to say, there are different *utilization schemes* related to the textbook. As mentioned above, the deep and thoughtful analysis of textbooks is an ordinary and essential practice for Chinese and Japanese teachers. We also observed that teachers in China paid attention to students' cognitive development while using textbooks. This is, to some extent, due to the specific status of textbooks in East Asian countries: the textbooks should be approved by the government and their use made compulsory.

In such analyses on teachers' documentation work in East Asian countries, it is important to not only identify the differences between Western and Eastern countries, but to reveal the cultural elements that make such differences. Teachers' work is affected and shaped by several elements. A new system from a country cannot be transposed to another country without adaptation and appropriation. It is necessary therefore not only to learn from other countries, but also to understand, as researchers, the mechanism of teachers' documentation work in our own country, in order to improve teacher professional development and, consequently, improve teaching practices in the classroom.

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