Chapter 3
Searching for Alternatives for New Math in Belgian Primary Schools—Influence of the Dutch Model of Realistic Mathematics Education

Dirk De Bock, Wim Van Dooren and Lieven Verschaffel

Abstract We sketch the turbulent history of primary mathematics education in Belgium during the last (half) century. The outline starts with traditional mathematics in the period before and shortly after World War II, an approach that is often, but partly unjustly, labelled as 'mechanistic'. Then we focus on the rise of New Math or 'modern mathematics' in the 1970s. We briefly discuss its roots and describe how this structural approach, which basically followed the development at the secondary level, was implemented in Belgian primary schools. By the early 1980s, New Math was strongly criticised, which paved the way for its fall during the 1990s. This leads us to the current curricula that are strongly inspired by the Dutch model of Realistic Mathematics Education (RME), while maintaining valuable elements of the strong Belgian tradition in developing students' mental and written calculation skills and even some (minor) New Math accents. We describe in some detail the influence of RME on the different mathematical domains in these curricula, as well as some new challenges that arise on the horizon.

Keywords Mechanistic approach · New math · Primary level · Realistic mathematics education · Structural approach

3.1 Traditional Mathematics

The approach that dominated (primary) mathematics education before and in the first decades after World War II is often labelled as 'mechanistic' (Treffers, 1987).

D. De Bock (✉)
Faculty of Economics and Business, KU Leuven, Leuven, Belgium
e-mail: dirk.debock@kuleuven.be

W. Van Dooren · L. Verschaffel
Faculty of Psychology and Educational Sciences, KU Leuven, Leuven, Belgium
e-mail: wim.vandooren@kuleuven.be

L. Verschaffel
e-mail: lieven.verschaffel@kuleuven.be

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In a mechanistic approach, the focus of instruction is on factual and procedural knowledge (e.g., knowing how much 6×9 is, to know how to add or multiply multi-digit numbers, to know the formulas for computing the perimeter and the area of regular plane figures, etcetera). Learning is primarily seen as the acquisition of this type of factual and procedural knowledge through basic learning principles such as inculcation, memorising and repeated practice of technical computational skills, principles that were in the same period promoted and theorised by behavioural psychologists (e.g., Thorndike's law of exercise and law of effect). The instruction is heavily teacher directed, with the teacher being the dispenser or transmitter of the distinct specific pieces of knowledge and specific skills to be learned, as well as the taskmaster who decides what information and instruction the learners get, and when and how these are provided. In a mechanistic approach, there is little or no attention for conceptual understanding (the reasons behind the facts and procedures that are taught) and theory development, nor for 'realistic' applications (Freudenthal, 1991). Of course, in a pre-computer era, procedural knowledge and skills were considered more important than today, which partly explains the dominance of this approach in Belgium just like in many other places all over the world. Also in the Netherlands, the mechanistic approach to mathematics education was dominant at that time, but Van den Heuvel-Panhuizen and Drijvers (2014, pp. 521–522) also suggest a link with the science of mathematics (though this may rather apply to the secondary level):

> In the 1960s, mathematics education in the Netherlands was dominated by a mechanistic teaching approach; mathematics was taught directly at a formal level, in an atomized manner, and the mathematical content was derived from the structure of mathematics as a scientific discipline. Students learned procedures step by step with the teacher demonstrating how to solve problems.

To the best of our knowledge, mutual influences between the Belgian and the Dutch primary mathematics educational traditions during the 1960s and before, if there were any, have not yet been investigated.

It would be, however, a mistake to equalise all mathematics education approaches from the first half of the previous century as purely mechanistic. In Belgium, there was, from the 1930s on, a strong focus in primary education in general on child-centredness and on connecting school matter with children's concrete, daily-life experiences. From that time on, the official school curriculum was influenced by the so-called 'Reform Pedagogics', an international pedagogical movement, situated between 1890 and World War II, that strove for a harmonic and broad child development and of which the Belgian teacher and psychologist Dr. Ovide Decroly (1871–1932) was one of the main protagonists (Van Gorp, 2005). For mathematics, this reform-based curriculum involved an approach which showed similarities with what later would be called 'Realistic Mathematics Education' (RME). It was, for example, stated that arithmetic is not a goal in itself, but should always be connected to a concrete reality, that learning should start from observations and from students' own living environment, that long and tedious calculations should be avoided, and that word problems should be inspired by students' activities and interests. Likewise, in the domain of measurement it was recommended to only use measures that

the children would also use in everyday life. Unfortunately, the pedagogical principles that were central to the curriculum were not always faithfully implemented in practice: Commonly used textbooks still paid a lot of attention to long series of bare problems, without 'meaning' and apart from any applied context (De Bock, D'hoker, & Vandenberghe, 2011a).

Growing attention for the student and his learning process was also a main theme in the work of the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), created by Caleb Gattegno (1911–1988) in the early 1950s and bringing together leading mathematicians, mathematics educators and psychologists of that time. A common point of interest within CIEAEM at that time was related to the use of teaching aids, i.e., semi-concrete materials and models that could be used to stimulate students' thinking and conceptual understanding (De Bock & Vanpaemel, 2015; Gattegno et al., 1958; Vanpaemel, De Bock, & Verschaffel, 2012). These teaching aids included cardboard models, light projections, Meccano constructions, geoboards, films, electrical circuits and the famous Cuisenaire rods, a set of coloured sticks of different lengths that can be used as a didactical tool to discover and to explain various arithmetical concepts and their properties. They were invented by the Belgian primary school teacher Georges Cuisenaire (1891–1975) (Fig. 3.1) and promoted by no one less than Gattegno himself (Gattegno, 1954, 1988). They were widely used in Belgian primary schools from the end of the 1950s to support insightful teaching and learning of whole number arithmetic.

Fig. 3.1 Cuisenaire with his famous rods, ca. 1965

3.2   New Math

In the 1960s and 1970s, mathematics education in Belgium—as in many other countries—drastically changed: New Math or 'modern mathematics' broke through. This revolution first took place at the secondary level. From 1968 on, modern mathematics became compulsory in the first year of all secondary schools in Belgium (and from then on gradually in the subsequent years) and remained the prevailing paradigm in secondary mathematics education for about two decades. According to Georges Papy (1920–2011), professor of algebra at the Université Libre de Bruxelles and the figurehead of modern mathematics in Belgium, there were three main reasons to introduce modern mathematics: (1) the failure of traditional mathematics education, (2) the widening gap between mathematics 'as a living science' and mathematics as it was taught at various school levels, and (3) the growing importance of modern mathematics in a variety of other disciplines (Papy, 1976). New Math reacted against traditional, mechanistic approaches and instead emphasised insight in mathematical structure, often through the study of abstract concepts like sets, relations, graphs, algebraic structures, number base systems, etcetera. According to Papy (1976, pp. 20–21), excessive computational drill and practice ("dressage of children") lead to docility instead of free and creative thinking[1]:

> Regardless of the content, there are two main methods to teach mathematics. The most common one submits the students to the subject matter. They are trained and conditioned until they have sufficiently adapted and accept what is offered to them. This is accompanied by ritual and perpetual automatisms for calculating… This dogmatic method subjects the children to algorithms and thus makes frequent drill necessary. A recent document of the Institut de Recherche de l'Enseignement de la Mathématique of the Académie de Marseille recommended 'dressage of children' as an algorithm for subtraction. This method certainly contributes to educate children to become respectful citizens, disciplined soldiers, obedient employees […]. We suggest a diametrically opposite view on education, a method that allows the child to master a situation, to mathematise it, to learn to ask questions about it and to try to solve them, a method that is aimed at the development of personal creative freedom.

A main source of inspiration for the New Math or structural approach was found in the work of Nicolas Bourbaki, a collective pseudonym for a group of (mainly French) mathematicians who, from the late 1930s on, started the ambitious project to rebuild and restructure the mathematical knowledge of that time, a project quite similar to that of Euclid in the 3rd century BC (Bourbaki, 1939). Starting points were basic logical and mathematical structures and Bourbaki's method was a strictly deductive one. Modern mathematics is the application of the model that Bourbaki developed for the science of mathematics as a model for mathematics education (De Bock, Janssens, & Verschaffel, 2004). So, the reasoning by the reformers of that time was more or less as follows: If we start from basic, abstract and empty concepts, such as sets and relations, and we then gradually introduce the more concrete and rich concepts, using a clean deductive method, then students will be better able to understand and appreciate mathematics. To attain this latter goal, however, not only

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[1] In this chapter, all translations into English were made by the authors.

Fig. 3.2 Papy with a copy of *Mathématique Moderne*, Vol. 1, 1963
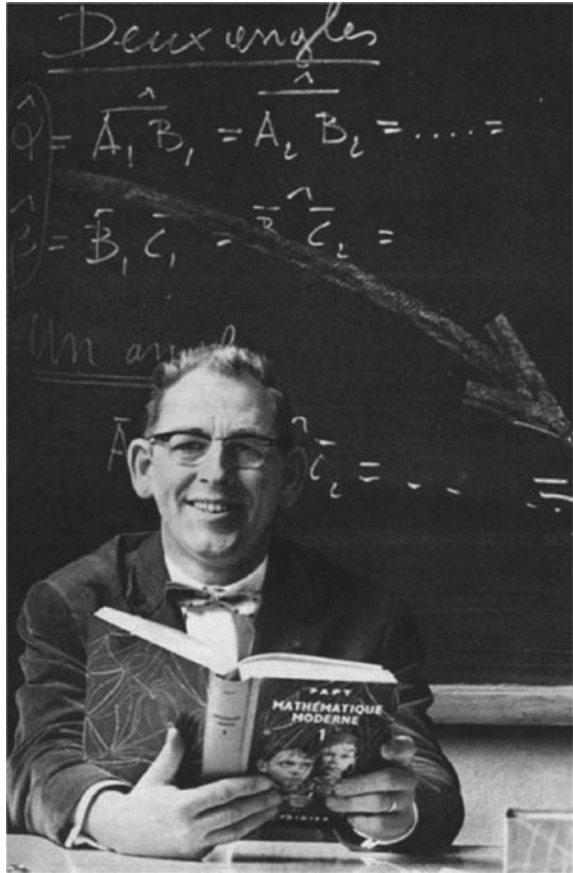
a modernisation of the mathematical content was needed, but also a modernisation of didactics and even of the language to communicate about these new mathematical ideas. Venn and arrow diagrams, Logiblocks and all kind of new terms, symbols, and conventions were introduced in the mathematics lessons. At that time Papy's multi-coloured textbooks *Mathématique Moderne* (Papy, 1963–1967) were translated in several languages and became quite influential in many European countries and even beyond (Fig. 3.2).

In contrast with the Netherlands, modern mathematics was also introduced at the primary school level in Belgium (and Belgium was even one of the nations that took a leading role and went furthest in this international reform movement at the primary level). In 1976 New Math was introduced in the Belgian primary schools of the Catholic network and two year later in the publicly run schools. Two main reasons were given to make modern mathematics compulsory at the primary level too. First, according to its defenders, the primary level had to prepare students for the modern mathematics of the secondary level. Second, Papy himself argued strongly for

starting the teaching of modern mathematics as soon as possible, at an age children are not yet conditioned by the bad habits of the old-fashioned mathematics education. Experiments at that time by Papy and his collaborators (Lenger & Lepropre, 1959; Papy, 1960), coordinated by the Centre Belge de Pédagogie de la Mathématique/Belgisch Centrum voor Methodiek van de Wiskunde, were quite promising in that respect and received ample attention in the international mathematics education community (see also De Bock & Vanpaemel, 2018). For example, Fielker (1961, p. 48) reported:

> Prof. Papy had taught sets to children from eight to twenty-five years old, and it was more difficult with the twenty-five year olds! Undergraduates were conditioned by the bad habits of traditional mathematics […]. Children of eight or ten were not so conditioned, and most success transpired with some fifteen-year-olds so poor in mathematics that they were uninfluenced by previous courses!

So, at the end of the 1970s, sets and relations became the most important ingredients of mathematics education at the primary level in Belgium, not only as educational goals and contents in themselves, but also as a vehicle to introduce all kinds of 'traditional' mathematical contents and to describe all kind of situations outside mathematics. Probably the most radical change took place in the teaching of geometry. The plane, represented by the symbol π, became an 'infinite set of points' and lines and geometrical figures became 'subsets of π'. In particular, the hierarchical order of the different plane figures was considered as essential (Fig. 3.3). Relations, such as 'all rectangles are parallelograms', were highlighted and visualised in the language of sets. Solving applied problems about geometrical figures was considered less important. Additionally, the correct use of an unequivocal terminology and symbol use was considered of the utmost importance. Therefore, inaccuracies from the pre-New Math programmes were eliminated. For example, a clear distinction was made between a 'circle' and a 'disk'. A circle only referred to the border of the plane figure, and thus its area was no longer πr^2 but 0. The geometry course also provided an introduction to transformation geometry. New topics, such as 'reflection through an axis' and 'axes of symmetry', had to prepare students to an extensive study of transformation geometry at the secondary level.

In the New Math period, primary school children were also introduced to what was called 'logical thinking'. In that part of the mathematics course, children learned to use correctly the connectives 'and' and 'or' and their negation by the logical operator 'not', typically by means of Logiblocks (a set of objects with restricted and well-defined features: rectangle, triangle or disk; yellow, blue or red; small or large, and thick or thin, see Fig. 3.4). They also were trained in correctly using expressions such as 'at least', 'at most', 'not all', 'only if', 'if and only if' and so on. There was even a frisky initiation to algebraic structures at the primary level. This should help students to understand more deeply the basic properties of operations.
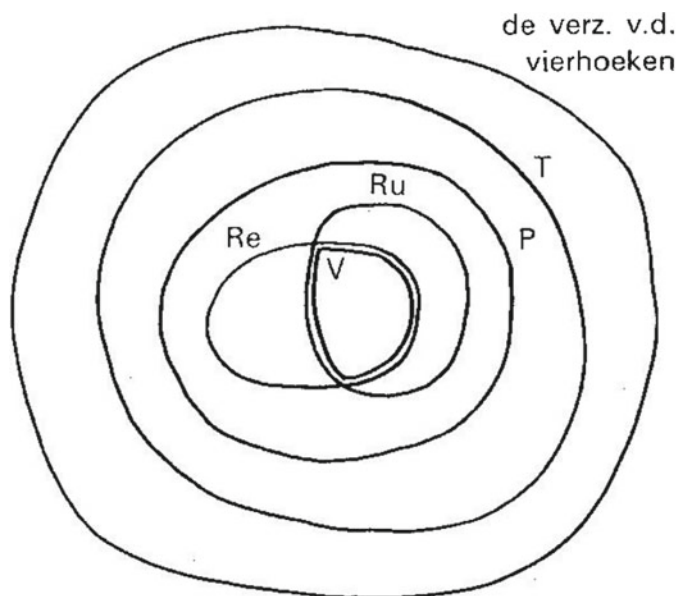
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                                              de verz. v.d.
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Fig. 3.3 Venn diagram to classify quadrilaterals ('de verz. v.d. vierhoeken' = the set of quadrilaterals; 'T' = set of trapezoids; 'P' = set of parallelograms; 'Re' = set of rectangles; 'Ru' = set of rhombuses; 'V' = set of squares); taken from *Vernieuwde wiskunde in de basisschool* (Renewed Mathematics in Primary School) (Ministerie van Nationale Opvoeding en Nederlandse Cultuur, 1976, p. 18)

Although the official Belgian primary mathematics curricula were seriously affected by New Math, it is unclear how drastically the daily mathematics lessons were actually affected by it. It is apparent that computation and measurement as well as word problem solving, parts of the 'old' curriculum, were not dropped by primary school teachers during the New Math period (especially not in Flanders and in the Catholic network where the influence of Papy and his collaborators tended to be less strong). These skills were still considered as important, although it was less evident to integrate them in the New Math philosophy.

Fig. 3.4 A set of Logiblocks, 1968 (© Les archives de la Fondation Vaudoise du Patrimoine Scolaire—CH 1400 Yverdon-les-Bains)

3.3　Critique on New Math

Although New Math was strongly criticised in international fora since the early 1970s (see, e.g., Kline, 1973), and in the Netherlands Hans Freudenthal (1905–1990) and his team had started the development of a 'realistic' alternative for the teaching and learning of mathematics at the primary level (the Wiskobas[2] project, see, e.g., Treffers, 1993), the Belgian mathematics education community remained remarkably silent. For about twenty years, official curricula in Belgium would follow faithfully the New Math or structural approach. Obviously, several mathematics educators and mathematics teachers were sceptical about this approach, but criticisms were rarely voiced in public (De Bock, D'hoker, & Vandenberghe, 2011b; Verschaffel, 2004). In 1982, this silence was suddenly broken by the Flemish pedagogue Raf Feys. In the *Onderwijskrant*,[3] an innovation-minded, independent and pluralistic journal on education, Feys wrote a virulent pamphlet in which he firmly criticised the starting points of New Math and the way it was introduced and dictated at the primary level (Feys, 1982) (Fig. 3.5). In his close contacts with schools, Feys did not see the appearance of a fascinating world, but "artificial results in a fake reality", and also little enthusiasm in children, but "more disgust, disorientation and desperation" (Feys, 1982, p. 3). He described New Math as "upper-level mathematics" which was in the

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[2] Wiskunde op de Basisschool (Mathematics in Primary School).

[3] Education newsletter.

Fig. 3.5 Cover of the *Onderwijskrant* in which Feys' (1982) critique on New Math appeared (moderne wiskunde: een vlag op een modderschuit = modern mathematics: a flag on a mud barge)

first place ballast, "i.e. an enormous extension of the programs, concepts that were misunderstood, mechanical learning and pedantry" (ibid., p. 6). Moreover, it created an obstacle for the acquisition of traditional mathematics, which he described as "mathematical-intuitive and practice-oriented lower-level mathematics". He further stated that "three-quarters of the reform involved the introduction of new terms and notations […], a formal language primary-school teachers are unable to cope with and which complicated the application of mathematics" (ibid., p. 8). The pamphlet ended with a call for a large-scale counter-action, addressed to "teachers, parents,

inspectors 'with free hands', parents' associations, labor movements, teacher training institutes, universities, centres for psychological, medical and social guidance" (ibid., p. 37).

In his pamphlet, Feys not only criticised New Math, he also suggested how mathematics education at the primary level should evolve, and the model he had in mind was clearly RME as developed by Freudenthal and his collaborators at the former IOWO.[4] According to the RME model, the starting point of mathematics education should not be the structure of mathematics, but children's intuitive, informal and real-world knowledge and skills, and these should be gradually developed. Feys (1982, p. 37) wrote:

> When evaluating the renewed mathematics education, we should not only compare with the old mathematics, but also with alternatives like the ones that are, e.g., developed in the Netherlands by Wiskobas. We need the courage to examine the alternatives thoroughly. […] We opt for an alternative reform along the lines of the Wiskobas approach of the IOWO, complemented, however, with a strong emphasis on the social-societal aspect of mathematical world orientation.

Although Feys' pamphlet enjoyed some resonance in the Flemish press and the author received some expressions of support by academics (e.g., by Leen Streefland, staff member of the IOWO, and by Lieven Verschaffel, whose letters were included in a subsequent issue of the *Onderwijskrant*), his point of view was not generally recognised and appreciated. Those responsible for primary mathematics education wrapped themselves in silence or disqualified Feys' analysis as inflammatory language of irresponsible 'doomsayers' (see, e.g., Verschaffel, 2002). They argued "that the innovation of mathematics education was a fact and that we, also as parents, could better express our belief in the revised approach" (quote from an interview of a member of the programme committee of the Catholic network as reported by Heyerick, 1982, p. 5). The discussion had clearly been launched, but the tide had not yet turned! An important follow-up event was the colloquium What Kind of Mathematics for 5–15 Year Olds? organised in 1983 by the Foundation Lodewijk de Raet, a Flemish socio-cultural organisation with a pluralist scope (Stichting-Lodewijk de Raet, 1983). At that occasion, proponents and opponents of New Math defended their positions. A strong delegation from the Netherlands (in casu from Utrecht) participated, including Aad Goddijn and Hans Freudenthal, who not only gave a lecture, but also firmly intervened in the discussion afterwards, with significant endorsement addressed to the opponents of New Math. Obviously, the colloquium elicited opposite points of views, but also strong dissatisfaction with the current situation ("no one wants to continue this way"; Stichting-Lodewijk de Raet, 1983, p. 29). It became at least very clear that something had to be changed and that Belgium (in this case, Flanders) could not neglect the evolutions that took place in other countries, especially in the Netherlands. At the end of the colloquium, again a call for action was launched, but the response to this call was minimal. In the subsequent years, no significant changes in the Flemish mathematics educational landscape occurred. Although interest in the

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[4] Instituut voor de Ontwikkeling van het Wiskunde Onderwijs (Institute for the Development of Mathematics Education).

Dutch alternative did not disappear and the RME approach received strong consideration in academic circles (Verschaffel, 1987) as well as in some so-called 'alternative schools' (based, for example, on the Freinet pedagogy), official curricula were not adjusted. Feys' (1982, p. 3) prediction that "the fateful choice for the New Math approach would also, for a very long time, impede valuable and necessary reforms" came true.

3.4 The 'Realistic' Alternative

At the end of the 1980s, the educational landscape in Belgium changed drastically. Belgium became a Federal State consisting of three Communities: the Flemish, the French and the (small) German-speaking Community. These Communities are based on a common language, or more broadly on 'culture'. Since January 1, 1989, the Communities are responsible for educational matters. We will focus further on the situation in the Flemish Community which has, due to the common language, most affinity with the Netherlands. One of the realisations of the Flemish Community with respect to mainstream education was the development of developmental objectives (for kindergarten) and attainment targets (for the primary and secondary level). Attainment targets are minimum objectives, determined by educational level, which the government considers necessary and attainable for the respective group of students. They are usually related to subjects and refer either to knowledge, to skills or to attitudes. The government determines the attainment targets, but the schools are responsible for reaching these targets with their students. Usually, for that purpose, schools follow the curricula developed by the educational network to which they belong. Hence, the approval of the attainment targets by the Flemish Government on July 15, 1997 was the occasion for the educational networks to develop new curricula for the different school subjects. For mathematics, it was the opportunity to renew this field in line with international developments and to officially break with New Math. The new curricula for mathematics for the three main school networks in Flanders (Gemeenschapsonderwijs, 1998; Onderwijssecretariaat van de Steden en Gemeenten van de Vlaamse Gemeenschap, 1998; Vlaams Verbond van het Katholiek Basisonderwijs, 1998) were implemented in 1998 and are still applicable. They differ slightly from each other, but not significantly. Flanders was (and still is) not ready for a cross-network curriculum for mathematics as advocated by Feys (1987). The curricular innovation of the 1990s was accompanied by new course materials for primary teacher education (Verschaffel & De Corte, 1995a, 1995b, 1995c, 1995d) and by new or renewed textbook series.

In all three networks, the curricula of 1998 differed strongly from the curricula from the New Math era. The typical topics from that period (sets and relations, logical thinking and the initiation to mathematical structures), as well as the abstract and formal spirit of the corresponding didactical approaches, had almost completely disappeared (although some attention for 'relationships between mathematical objects' and 'structuring' was maintained). On the one hand, there was a re-valuation of

traditional topics and skills. Curricular objectives again referred to classical math-
ematical domains such as numbers, operations and computation, measurement and
geometry. Traditional skills such as mental calculation and column arithmetic and
word problem solving were revised and renewed, and explicit attention was asked
for memorisation, automation and repeated practice, elements that characterised the
"rich Flemish tradition" (see, e.g., Vlaams Verbond van het Katholiek Basisonder-
wijs, 1998, p. 10). On the other hand, important new objectives were formulated,
objectives that were inspired by the Dutch RME model. For example, the curriculum
for the state schools (Gemeenschapsonderwijs, 1998, p. 2) stated that "Mathematics
in primary school should focus on mathematising reality. It is therefore necessary
to set mathematics education into a natural context". We further read that they want
to achieve that "children learn to describe situations derived from their own living
environment in the language of mathematics" (ibid., p. 3). In the curriculum for the
subsidised public schools (Onderwijssecretariaat van de Steden en Gemeenten van
de Vlaamse Gemeenschap, 1998, p. 11), we read that "mathematics starts from real
problems, problems that are experienced as 'real' by the students themselves". The
new orientation related to content was also accompanied by a plea for opening the
range of mathematical solution techniques to more flexible procedures, based on
students' insight in the structure of numbers or in the properties of operations, and to
informal strategies that students generate themselves. Although the inspiration from
the Dutch RME model was manifest—some of the general objectives are actually
almost copies of those formulated by Treffers, De Moor and Feijs (1989)—there
are also some differences. For example, the programme for the Catholic network
(Vlaams Verbond van het Katholiek Basisonderwijs, 1998) avoids using the term
'realistic' and instead speaks about 'meaningful situations'. Moreover, it first asks to
pay attention to standard arithmetical procedures and only afterwards to more flexi-
ble procedures. Such non-incidental details show that the Dutch realistic vision was
not copied blindly, but rather adapted to the Belgian or Flemish (historical) context.

Next to the objectives related to the traditional content domains of mathematics,
the curriculum developers introduced some objectives that exceeded these domains.
A first type of cross-domain objectives relates to the acquisition of problem-solving
skills and strategies and to their use in rich (and applied) problem situations, replac-
ing, in some sense, the traditional culture of word problem solving (Verschaffel et al.,
1998; Verschaffel, Greer, & De Corte, 2000). Problem-solving skills and strategies
refer to the process that leads to the solution of a problem. Main steps in that process
are the analysis of the situation, the selection or building of a mathematical model,
the application of mathematical techniques within that model, and the interpretation
and evaluation of the results. These steps clearly refer to a modelling perspective,
more specifically, to the so-called 'modelling cycle' (Verschaffel, Greer, & De Corte,
2000). Hence, word problems are no longer exclusively seen as a means to apply
mathematics that has just been learned, but also to introduce some basic ideas about
'mathematical modelling' at the primary level. Modelling brings a new question
to the forefront: Which mathematical model or operation is appropriate in a given
situation?

A second type of cross-domain objectives refers to attitudes. Examples are learning to value mathematics as a dimension of human activity, to appreciate smart search strategies in problem-solving activities, to develop a critical disposition towards facts and figures that are used, consciously or not, to inform, to convince, but also to mislead people. The explicit inclusion of cross-domain objectives in the curricula for the primary level implies that schools have to pursue these objectives, without necessarily having to (completely) reach them. These are considered as permanent objectives for mathematics education, even after primary school.

When we look at the actual RME inspired changes in the different mathematical domains, we notice that in numbers and operations, the attention shifted from obtaining insight in the structure of number systems to linking numbers to quantities. That way, numbers are no longer purely abstract entities, but objects that children learn to know and recognise in different forms (e.g., decimal numbers from reading monetary values). With respect to operations, the emphasis shifted from discovering and accurately formulating the commutative, associative and distributive laws to linking operations to concrete and meaningful situations. As well as mental calculation, estimation techniques and the competent use of calculators are also promoted, for example, for solving realistic problems or for checking the result of an operation. Hence, the importance that was previously attached to all kinds of tests for checking computational results (e.g., checking the result of an addition or subtraction by performing the 'inverse operation' or the method of casting out nines for multiplications and divisions) disappeared completely. As mentioned before, in addition to standard computational algorithms, solution methods based on heuristic strategies also acquired their place in the curriculum. A typical example is the use of ratio tables for calculations with proportional quantities (instead of the old 'rule of three').

Also, the domain of measurement changed drastically. While in previous periods, this topic was treated in a rather mechanistic way, with much emphasis on conversion between all kinds of units, often quite artificial ones, current curricula focus on understanding the attributes of length, weight, area, and so on, and on the process of measurement, namely choosing an adequate unit, comparing the unit to the object to be measured and reporting the number of units. Students are invited to visualise the results of their measurement activities in tables and graphs. In addition to standard units, natural units such as body parts are used to come to a better understanding of measurement. Lessons in measurement nowadays are really active ones in which students measure real objects with different tools or create objects of given sizes. Students are also encouraged to use estimations, and activities are provided to develop estimation strategies. To develop some measurement sense, students need some natural references, e.g., the volume of a dessert spoon is about one centilitre, the length of a football pitch is 100 m, or one metric ton is about the weight of a passenger car. There is still some emphasis on metric conversion, but only between natural units or units that are frequently used. Therefore, students can use conversion tables, which are the analogue of ratio tables. As well as gaining inspiration from the Dutch RME model, the new approach to measurement was also strongly influenced by the U.S. standards, published in *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, NCTM, 1989).

With respect to geometry education, Freudenthal (1973, p. 403) wrote:

> Geometry is grasping space … that space in which the child lives, breathes and moves. The
> space that the child must learn to know, explore, conquer, in order to live, breathe and move
> better in it.

Starting point in geometry is observation and experience. Students first learn to recognise geometrical shapes in planes and in space by seeing and doing. This experiential geometry that already starts in kindergarten, also matches the Belgian intuitive geometry of the pre-New Math era (Vanpaemel & De Bock, 2017), primarily conceived as a field in which you first see and only then formalise (Ministerie van Openbaar Onderwijs, 1957). A specific RME influence in the curricula of 1998 is particularly evident in various recommendations and clarifications asking to introduce preferably geometrical concepts and methods in realistic contexts. So, for example the concept of an angle is related to the angle made by an opened door, or straight lines first arise as vision lines used to determine an observer's position on a sketch or a picture. The concept of area is introduced intuitively. Starting point is the area of a rectangle that students can determine by counting squares. Area formulas for other regular quadrilaterals are found by cutting and pasting activities. Even the 'difficult' formula for the area of a circle is approximately determined in a similar way. Only later on, this experiential (intuitive, realistic) approach can lead to more abstract notions of geometry, such as parallel and perpendicular lines, equality of shape and size, or symmetry. However, the goal of geometry education is no longer the development of an abstract framework, but teaching students to apply geometry in solving realistic problems in the space in which they live.

In conclusion, we reiterate that the Flemish post-New Math curricula for the primary level were strongly inspired by the Dutch RME model, but did not simply copy that model. One may ask why the Flemish attainment targets (1997) and subsequent curricula (1998) did not choose a more radical implementation of the realistic alternative. Verschaffel (2002, 2004) reports two types of possible explanatory elements. First, since the late 1980s, the RME model was not only praised in Flanders, but critical questions and doubts about the value and feasibility of that model were also raised, and, strikingly enough, it was again Feys who played a pivotal role in these criticisms. Feys' critique focused, among other things, on the neglect of the mechanistic aspects of learning, on the lack of guided construction of knowledge, on the excessive freedom that is given to students to construct their own solution methods, on the limited attention for the process of de-contextualising, and on insufficient recognition of the value of mathematics as a cultural product (Feys, 1998). When comparing new RME methods with traditional Flemish (pre-New Math) methods, he esteemed the latter as superior to the first (Feys, 1989, 1993). Although not all mathematics educators in Flanders agreed with Feys's criticisms, it is likely that his judgments have contributed to the fact that particularly the more extreme elements and aspects of the RME vision were not implemented. Second and complementary to the first element, comparative international research of that period revealed the very high quality of Flemish mathematics education. Actually, Flanders outperformed the Netherlands, not only in large-scale international studies such as TIMSS (Mullis

et al., 2000), but also in some small-scale comparative studies only involving the Netherlands and Flanders (see, e.g., Luyten, 2000; Torbeyns et al., 2000). These results not only increased the self-confidence of Flemish mathematics educators, but also strengthened their hesitation to implement a more radical version of the Dutch RME model.

3.5 Math Wars

The negative reaction with respect to the value of the RME model, initiated in Flanders by Feys in the late 1980s, is akin to the position of one of the parties in the Math Wars that emerged around the same time in the United States. These Math Wars refer to a vehement debate held between reformers and traditionalists about mathematics education. This debate was triggered by the publication of the (reform-minded) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the widespread adoption of a new generation of mathematics curricula inspired by these Standards. The vision of the American Standards had much in common with the RME philosophy with, for example, much attention for self-discovery learning via rich interactions between teachers and students and between the students themselves, mathematical connections between the different mathematical domains, continuous vertical learning trajectories (from kindergarten to high school), multiple and flexible problem representations and solution strategies, a meaningful integration of new technologies and a plea to pay less attention to paper-and-pencil calculations and isolated skills. Especially from the side of the professional mathematicians, fierce criticism and even a real counter movement was initiated, blaming the Standards for dumping, without good reason, a number of traditional and tested values of the past, such as the memorisation of facts, the automation of skills and learning through direct classroom instruction. The opposite views between reform-based mathematics educators (of the NCTM) and traditionalists were the basis of the Math Wars in the United States (the further development of which is beyond the scope of this chapter, but which has been discussed by, e.g., Klein, 2007).

The Math War crossed the ocean, and in the Netherlands there also was a heated debate about the quality of mathematics education and its didactical approaches. The debate polarised between two groups that both partly relied on the (interpretation of) results of the Cito studies of the PPON[5] (see, e.g., Janssen, Van der Schoot, & Hemker, 2005) used to assess mathematics achievements of primary students in the Netherlands (Ros, 2009). The Dutch Math Wars were launched by Jan van de Craats, mathematician at the University of Amsterdam and co-founder of the action group Stichting Goed Rekenonderwijs (Foundation for good arithmetic education). Van de Craats (2007) stated that children in the Netherlands are no longer able to calculate, that the RME approach created chaos wherever good mathematics needs calm and abstraction, and that standard algorithms (such as long division) and automatisms—

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[5] Periodic assessment of education level.

which are, according to the traditionalists, especially helpful for children with medium and weak abilities—have totally disappeared from arithmetic education in the Netherlands. The scholars of the Freudenthal Institute, the successor of the IOWO, had to defend themselves and argue that there is no question of a general decline in the level of computational abilities and that Dutch children, as a result of the RME approach, are doing even better than 10–20 years ago on a number of aspects such as arithmetic in practical contexts, mental calculation, estimation techniques, working with percentages, and that they also have a better conceptual understanding of numbers and computational procedures (Van den Heuvel-Panhuizen, 2010). To clarify this situation, by the end of 2008 Dutch policy makers created a commission, under the auspices of the Royal Dutch Academy of Sciences (KNAW), which scrutinised all available research results. However, although that commission came to the conclusion that there is no demonstrable relationship between the level of computational ability of primary school children and the didactical method that is used ('realistic' or traditional) (KNAW, 2009), the debate in the Netherlands still goes on.

When it rains in Amsterdam, it drips in Brussels… In 2008, Feys and Van Biervliet published a special issue of the *Onderwijskrant*, titled "Mad Math en Math War", in which they informed their readership about the Math Wars in the United States and the Netherlands (Feys & Van Biervliet, 2008, p. 8). Not surprisingly, the authors unambiguously choose the camp of the traditionalists:

> The 'celestial' (too formal) New Math has been replaced by the 'terrestrial', contextual and constructivist approach, having too little attention for calculation skills and readily available knowledge, for generalisation and abstraction, and for mathematics as a cultural product.

The special issue, which also includes a contribution by Van de Craats, is certainly worth reading, but did not have the same strong impact as the 'A flag on a mud barge' issue from 1982. Lamentations about declining educational levels are of all times, but the feeding ground for a Flemish Math Wars seems to be missing. For that, several explanations can be given, but the most important is probably that the Flemish RME variant is less 'realistic' than the Dutch original, as argued above. This is also acknowledged by Feys and Van Biervliet (2008, p. 2) (and reported as their own achievement): "(We) succeeded to slow down the constructivist influence in primary education." Primary mathematics education in Flanders nowadays is eclectic, rather than (extremely) realistic. So, Verschaffel (2002) points out that in Flemish textbooks: (a) less time is spent on the informal, intuitive phase to switch more quickly to abstract, shorter and formal procedures; (b) there is more emphasis on practicing and automatisation; (c) fixed solution methods and schemes are more frequently used in mental arithmetic and word problems; (d) less use is made of new didactical tools and models, such as the reckoning rack and the empty number line, and older materials and models, such as square images, one hundred field and MAB materials,[6] are more frequently deployed; and (e) the principle of progressive schematisation is less consistently applied in the learning of (difficult) number algorithms than in Dutch methods. In conclusion, we can state that today's Flemish mathematics education

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[6] Multibase arithmetic blocks.

is—according to some people—a colourless mix; according to others a harmonious and workable balance between elements from the mechanistic and realistic traditions, with still some elements of the structural New Math vision.

3.6 Future Developments?

Of course, the fact that Flemish mathematics education seems to have found a good balance between elements from its different traditions does not mean that there is no room for improvement. One of the issues under discussion nowadays in Flanders is how to narrow the gap that exists between primary and secondary mathematics education. These two educational levels are still different worlds, with their own traditions, teachers and teacher education programmes. Students however just continue their school career and what they have learned at the primary level should help them instead of being an obstacle for what they have to learn at the secondary level. We mention three elements that could make the transition from the primary to the secondary level more fluent.

First, at the primary level, problems are typically approached in a purely arithmetical way, while at the secondary level, students switch to an algebraic approach (and an arithmetical approach is typically no longer accepted). Also, teachers at both educational levels make different use of and have different attitudes towards arithmetical and algebraic solutions methods (Van Dooren, Verschaffel, & Onghena, 2001, 2002, 2003). We think that the inclusion of some pre-algebra (methods) in the curricula and/or textbooks for the primary level is worth considering. Moreover, it would re-strengthen the 'structural' element in primary mathematics education, an element that is weakened since the elimination of New Math.

Second, we think that the learning of numbers and operations can be improved by better taking into account the results of recent research in this field. This research has, among other things, shown that prior knowledge about natural numbers often hinders students to understand rational numbers and operations with these numbers. This phenomenon is often referred to as the 'natural number bias'. For example, students may believe that 'multiplication always makes bigger', that 1/4 > 1/3 because 4 > 3, or that there are only two numbers between 0.2 and 0.5, namely 0.3 and 0.4 (see, e.g., Van Hoof, Verschaffel, & Van Dooren, 2015). Didactical approaches for both educational levels could better prepare students for this type of differences between natural and rational numbers and related operations, and for errors that may result therefrom.

Third, there is a need for a good, systematic and cross-level learning trajectory for (emergent) mathematical modelling and applied problem solving. This topic is included in the curricula for the (upper) primary level and is supported by a lot of research at that level (Verschaffel et al., 1998), but there is no clear continuation of that trajectory in the first years of secondary school. A revision of the topic of functions (for the first years of secondary school) in the direction of 'functions as

models' for various situations and phenomena, an approach that already exists at the upper secondary level in Flanders (Roels et al., 1990), could be considered.

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