

# Chapter 15

## Korean Mathematics Education Meets Dutch Didactics



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**Abstract** Dutch didactics—in Korean mathematics education society often referred to as Realistic Mathematics Education (RME)—has become one of the major perspectives on mathematics education which have been widely discussed and applied by Korean mathematics educators and mathematics teachers to reform Korean mathematics education over the past 35 years. This chapter briefly depicts how RME has been introduced in both theoretical and practical viewpoints through doctoral and master’s theses as well as through journal articles and curriculum documents in Korea. It turns out that RME has provided integral and meaningful issues to be constantly discussed among Korean mathematics educators since its introduction in the 1980s. In conclusion, RME has contributed largely to activating and reshaping Korean mathematics education in multiple ways although several barriers to overcome or perspectives to modify have emerged due to Korea’s different social and educational backgrounds. Parts of these barriers as well as recognised benefits come to the fore through feedback and reflections from the teachers and students who experienced RME in Korean contexts, as described at the end of this chapter.

**Keywords** Realistic Mathematics Education (RME) · Dutch didactics in Korean context · Mathematics curriculum · Mathematics textbooks · RME-based mathematics class

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## 15.1 Introduction

There is a long history, like in other countries, in which Korean mathematics educators have struggled with many serious issues in practice such as students' low understanding of mathematical concepts and blind memorisation of mathematical rules and procedures, poor connection between school mathematics and out-of-school mathematics, and teacher-centred mathematics teaching. Dutch didactics, in Korean mathematics education society often referred to as Realistic Mathematics Education (RME), has been considered one of the major perspectives by which Korean mathematics educators and mathematics teachers handle the aforementioned issues in Korean mathematics education practice. This article traces the efforts of the Korean mathematics educators who have tried to introduce RME to Korea's mathematics education society over the past 35 years. Section 15.2 describes how and by whom the RME perspective was introduced to Korea and how it has been interpreted, applied, and integrated into Korean mathematics education and its practice. Section 15.3 covers the influences of the RME perspective on the Korean mathematics curriculum, textbooks, and assessments. Section 15.4 discusses the recognitions and reflections of the teachers and students who experienced mathematics teaching-learning based on the RME perspective. Section 15.5 summarises the discussions and suggests conclusions.

## 15.2 The Research History of RME in Korea

In his paper published in 1980, Woo referenced RME for the first time in Korea. After that, Woo continually discussed RME in theoretical views (Woo, 1980, 1986, 1994, 1998), which has made significant contributions to the influence of RME on the research and practice of Korean mathematics education. In particular, Woo supervised his students to didactically analyse mathematical themes such as function, rational number, probability, variable, and calculus at elementary and secondary school levels for their doctoral theses (Han, 1997; Kim, 1997; Lee, 1996; Park, 1992; Yu, 1995), leading to broad evaluations on the RME perspective. These studies show how the RME perspective has influenced the research bases of Korean mathematics education, or how the mathematics educators have understood the RME perspective compared with other contemporary research perspectives affecting Korean mathematics education.

The RME perspective has been an important key to understand and reform the Korean mathematics curriculum, textbook, and education practices. The aforementioned doctoral theses focused on understanding RME in general and from a theoretical standpoint. At the same time, various papers published in the mathematics education research journals in Korea employed the RME perspective as a frame and reference of analysis to discuss Korean mathematics curriculum, textbook, and edu-

cation practices. The studies show the influence of the RME perspective on reflecting about and improving Korean mathematics education.

The next section will cover the understandings and discussions about RME in Korean mathematics education society, followed by its interpretations and applications in the Korean context.

### ***15.2.1 Understanding and Discussions About RME in the Korean Context***

The first discussion about RME in Korea was Woo's paper in 1980 titled "A Criticism about Anti-Piaget's Theories on the Mathematics Educational Point of View". In this paper, Woo refuted Freudenthal and Van Hiele who criticised Piaget and supported Piaget's viewpoint. However, in the paper published in 1986, titled "Some Remarks on the Van Hiele's Level Theory of Mathematical Learning", Woo (1986) recognised the significance of Freudenthal's approach. Then, he was talking about "a reasonable justification about honest confidence of a mathematics teacher in class based on a practical theory that is developed by an insight about the nature of mathematical thinking" (Woo, 1986, p. 91). Furthermore, at the end of this paper, Woo suggested mathematics teachers in Korea should focus more on mathematical thinking rather than on the content itself.

In 1992, the first doctoral thesis on mathematics education in Korea was published by Park under the supervision of Woo. In the thesis, Park criticised traditional teaching practice of function in Korea and suggested to introduce the perspective of Freudenthal's didactical phenomenology.

It [the traditional teaching practice in Korea] repeats a traditional deductive approach that exposes the essential of function to the students as it is, which should have been organized by the students themselves, and neglects most of the organization process of function. (Park, 1992, p. 159)

In the doctoral theses that followed, RME took an important role as a reference perspective to critically understand and to draw directions from for reform of Korean mathematics education. Chong's (1997) thesis, titled *A Study on Freudenthal's Mathematization Instruction Theory*, extracted didactical principles and its assessment principles based on mathematising, and suggested a direction for teaching function through progressive mathematisation. In Ko's (2005) thesis, titled *A Study on Active Construction of Number Concept at the Beginning of School Age* an active approach was suggested to instruct various aspects of the concepts of natural number in the early school ages, referencing Freudenthal's discussion about the organisation and instruction of the concept of natural number through activities. Also, Lee (2007) developed a learning model for mathematising activities in geometry and reported its results in his thesis, titled *A Study on the Development and the Effect of Realistic Mathematization Learning Model*, which was based on the RME theory of Freudenthal and the textbook series *Mathematics in Context* (MiC) (NCRMSE & Freudenthal

Institute, 1997–1998) developed at the University of Wisconsin in collaboration with the Freudenthal Institute at Utrecht University.

Among those mentioned above, the thesis of Chong compared the viewpoint of Freudenthal in mathematics education with those of Bruner and Dewey. These viewpoints were compared and discussed as follows.

Dewey regarded learning as a growth of experiences, while Freudenthal recognized it as an expansion process of mathematized reality. In fact, the meaning of ‘experience’ by Dewey and that of ‘reality’ by Freudenthal were used in a similar context. [...] ‘Experience’ used by Dewey, however, focused on its applications to more direct and concrete daily situations, while ‘reality’ by Freudenthal emphasized the world to be mathematized in addition. (Chong, 1997, p. 116)

In the same manner, Chong compared RME with Bruner’s theory and discussed how the RME perspective supplemented that of Bruner.

Comparative studies similar to Chong’s, which analyse the theories of other scholars with respect to the RME theory, are still one of the branches of Korean mathematics education research. This is attributed mainly to Woo, who introduced RME and has continued the supervision of doctoral theses about it. Also, the popularity of RME was due to its appropriateness to reveal the problems deeply rooted in the tradition of Korean mathematics education.

### ***15.2.2 The Interpretation and Applications of the RME in the Korean Context***

As mentioned above, Woo initially criticised the theory when he introduced RME. However, he did pay attention to its positive aspects and highlighted it as a potential perspective that would improve and compliment Korean mathematics education. This was mainly due to the fact that RME focused on the discussion about the education curriculum and didactics, as it provided specific directions and arguments that could be used to reorganise Korean mathematics textbooks and reform the mathematics curriculum. In particular, the two books by Freudenthal, *Mathematics as an Educational Task* published in 1973, and *Didactical Phenomenology of Mathematical Structures* published in 1983, had much influence on this trend.

For example, Kim and Na (2008) claimed that the context for the instruction of ratios and rates in the elementary school textbook was inappropriate and sequences of lessons of instruction should be changed. The basis of this claim can be found in the following discussion, which is related to Freudenthal (1983) and Streefland (1985).

From the perspective of ‘looking into our daily life,’ the context included in the current textbook for the introduction of ratios is very artificial. [...] A relation of ratio is not implied in the two given quantities, that is three boys and five girls. [...] In the current textbook, the definition of rates is given after the introduction of the value of ratios, and the instruction focuses on the value of ratios and not on rates. Although rates are a mathematical mean to compare the relative magnitude of various quantities, the textbook failed to provide the

experiences to students to learn the natural meaning of rates because of the intentional emphasis on the value of ratio rather than on rates in the meaning-rich context. (Kim & Na, 2008, pp. 314–316)

Kim and Na gave lessons using a version of the textbook that was reorganised according to the RME perspective, and they had very positive results. When adapting the textbook, they experienced that it was necessary to pay extra attention to contexts which contain cultural differences such as the currency unit.

Kang and Kang (2008) analysed the chapter on probability in the Korean textbook, and tried to reorganise it. The study pointed out that the instruction of probability in Korea focused on algorithms rather than on the concept. They suggested a four-step instruction reflecting the reinvention method of Freudenthal instead; that is a method containing (i) introducing a realistic context, (ii) devising informal solutions by the students, (iii) applying the devised solutions in various realistic contexts, and (iv) using progressive condensation and formalisation when the students are prepared for this. Kang and Kang suggested improvements in six directions in the Korean textbook. First, ample realistic contexts should be given to the students to learn the concept of probability in various contexts. Second, various concepts of probability should be taught that can be experienced in daily activities. Third, qualitative comparisons of probability as a measurement should precede quantitative comparisons. Fourth, various representations for probability should be taught. Fifth, the concept of probability should be followed by the concept of the number of cases. Sixth, formalisation should be placed in the last section of the course (Kang & Kang, 2008, pp. 85–86).

Lee and Lee (2006) introduced the RME perspective to improve the instruction method of irrational numbers. Based on the Korean mathematics textbook, irrational number is taught by defining the square root and rapidly formalising irrational number based on its definition. The students, therefore, have not had a chance to learn the concept of irrational number, which is considered to result in failure of understanding irrational number operation. In the study, Lee and Lee (2006, p. 299) hypothesised that “the instructional viewpoint of irrational number starts from rich contexts that stimulate reflective thinking about rational number, and acquires the essence of irrational number through progressive mathematization.” According to this viewpoint, the historic-genetic background of irrational number was investigated, from which the context to reveal the existence and necessity of irrational number was developed. After that, they suggested activities that promote relating students’ existing knowledge of rational numbers to irrational number. Unlike the then current Korean curriculum that introduced the Pythagorean theory after irrational number, the suggested new approach was teaching irrational number based on the intuitive level of understanding of the Pythagorean theory. Finally, the students understood the characteristics of irrational number by themselves through exploring the decimal expression of irrational number using a calculator (Lee & Lee, 2006, p. 310). The results from the instructional materials developed in the research were satisfactory and meaningful. The students recognised the necessity of the concept of irrational number and its existence, and found a suitable way to represent it.

To implement the suggestion of Freudenthal that ‘defining’ should be taught instead of ‘definition’, Cho and Park (2011) designed lesson plans that introduced the monster-barring approach as shown by Lakatos (1976) in *Proofs and Refutations*. The study showed examples and non-examples of prisms, and made the students experience ‘defining’ through the activities of defining and refining the definition of a prism. The instruction was designed in six steps. The first step was to find various properties from examples of prisms. The second step was to draft a definition of a prism from the properties found. The third step was to refine the definition to not include a non-example shown by the teacher. From this process, a second definition was made. From the fourth to sixth steps, an additional non-example was shown for each step to refine the definition. After the sixth step, the students refined their definitions to exclude the four non-examples. This study suggests that a definition can be acquired by an activity that transforms a naive definition to a sophisticated one by refining, rather than by a sudden creation. The results showed that the students felt difficulties in how to define a prism as they focused more on subsidiary properties than essential ones that were necessary for the definition. The initial definitions made by students were often too naive to be used as a definition, which necessitated the intervention of the teacher.

## 15.3 Mathematics Teaching-Learning in Korea and the RME

### 15.3.1 *Mathematics Curriculum*

The mathematics curriculum in Korea specifies the standards for students and the relevant teaching-learning methods. The contents of the mathematics textbooks are composed to comply with the curriculum guideline. Therefore, the mathematics curriculum in Korea has exercised great influence on the introduction of a certain mathematical content and its teaching-learning methods.

RME theory and the MiC textbook series have influenced the mathematics curriculum and the textbook development in implicit as well as explicit ways. Since the early 1980s, Korean mathematics educators have conducted research on the RME theory and on didactical phenomenology. In the studies which attempted a didactical phenomenological analysis on mathematical concepts such as function, negative number, and proportion, the researchers reflected the problems underlying the instruction methods of such concepts so far in Korea and proposed desirable instruction methods at a theoretical level. Those research results have exercised a concrete influence on the standards and the teaching-learning methods of mathematics since 2000.

Here the concept of function is chosen as a representative example of changes in the instruction method through a didactical phenomenological analysis. Research into didactical phenomenological analysis on the concept of function and its teaching-

learning method for mathematisation based on the RME theory has been conducted in Korea since the 1990s (Chong, 1997; Park, 1992). In particular, Chong (1997) analysed the research of Freudenthal (1983), Van Hiele (1986), Verstappen (1982), and Janvier (1980), and suggested an instruction method of function through activities aimed at progressive mathematisation, an approach that was introduced in RME by Treffers (1987). Regarding the instruction on function in Korea, Chong (1997) pointed out that the bottom level activities, which preceded formal instruction in the concept of function, such as the study of dependency relations as a mean for the organisation of phenomena, the study of patterns as a mean of the organisation of dependency relations, and the formation of a mental image about the concept of function, have been insufficient.

Those studies contributed to the 2007 *Mathematics Curriculum* (Ministry of Education and Human Resource Development, 2007), which has reset the achievement standard in order to teach the concept of function through progressive mathematisation. Before introducing the formal definition of function, the concept of function is introduced in an intuitive manner using “a correspondence relation of two quantities where one quantity is determined as the other changes” (Ministry of Education and Human Resource Development, 2007, p. 31). This helped the students to experience various phenomena that make the concept of function work as a mean for organisation and to form a mental object of the concept.

The teaching-learning method through progressive mathematisation is still emphasised in the 2015 *Mathematics Curriculum* (Ministry of Education, 2015b). This mathematics curriculum is changed so that the concept of function is introduced formally in the 8th grade, which is preceded by sufficient interpretations and explanations about the given graphs from various realistic situations in the 7th grade. This is to make the students experience various phenomena that show the properties of function and to express the changes through means such as linguistic expressions, qualitative graphs, and diagrams, which is followed by the formal introduction of the concept of function. The changes in the mathematics curriculum released in 2015 were intended to implement the progressive mathematisation of the concept of function in an active manner.

### 15.3.2 *Mathematics Textbook*

While the mathematical achievement of Korean students in the cognitive area is very high, their attitude towards mathematics is very negative (Kim et al., 2008, 2010). In this situation, Korean mathematics educators have tried various methods to increase students' interest in and positive attitudes to mathematics. One of the methods is to improve mathematics textbooks.

Korean mathematics educators have tried to develop a textbook which not only helps the students to have an interest in and positive attitudes to mathematics, but also sufficiently covers mathematical concepts, principles, and laws that need to be learned by the students. During the course of the textbook development, textbooks

from other countries have been investigated as benchmarks to reflect their advantages while keeping the strengths of the Korean textbook.

The MiC textbook is one of the textbooks from other countries which has inspired mathematics textbook developers in Korea since the 1990s. The mathematics textbook developers have studied the rich contexts that the MiC textbook contains, and tried to find suitable contexts that fit Korean students. Through the contexts, the students were expected to experience the fact that mathematics is a human activity existing near to them, to learn the principles and concepts of mathematics naturally through the activities, and to improve their interest in and positive attitude towards mathematics.

In the following it will be explained how Korean researchers have tried to improve students' learning of operating with integers by introducing various contexts and activities into the mathematics textbook, which shows direct or indirect influences by MiC textbook. As widely known, operating with integers is one of the subjects with which students have much difficulty. In particular, the operation that includes negative integers is very difficult for students to understand.

Figure 15.1 is extracted from the Korean mathematics textbook as an example in relation to the instruction of addition of positive and negative integers using multiple models that is one of the key features of the MiC textbook (Lee et al., 2008, p. 60). The figure shows that in the Korean textbook, a red and a blue ball are introduced to represent '+1' and '-1', respectively, to be used for the instruction of the operation method and its principle through the activity. Also, to go to the 'right' and the 'left' are introduced to represent '+' and '-', respectively, in order to help students to understand the addition operation of integers and the use of the number line.

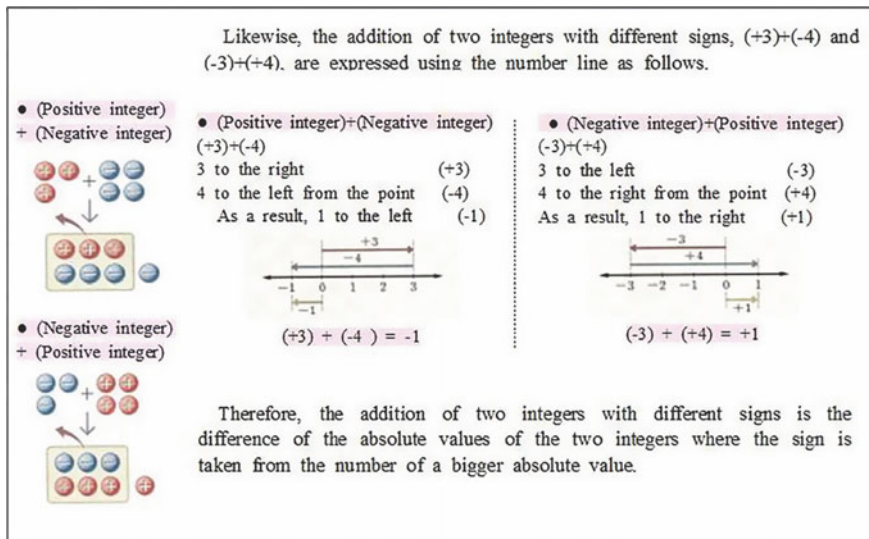
### ***15.3.3 Assessment of Mathematics Learning***

The direction of RME for the assessment of mathematical literacy has influenced the method of evaluating students' mathematical achievement in Korea. This section describes the contents of the unit on assessment included in the teacher guidebook for teaching mathematics in elementary school in Korea provided by the Ministry of Education (2015a).

The teacher guidebook gives the following five principles for the assessment of mathematics learning:

- To assess what the students know and think in mathematics
- To integrate the assessment into teaching
- To assess the overall viewpoint of mathematics and focus on broad mathematical tasks
- To design the problem situations that require applications of various concepts
- To employ various assessment tools including not only paper-and-pencil tests but also oral tests and performance tests etcetera. (Ministry of Education, 2015a, pp. 30–33).





**Fig. 15.1** Part of explanations on how to add two integers in a Korean mathematics textbook (Lee et al., 2008, p. 60)

A unit assessment is one of the summative evaluations, which is provided to the students after finishing a unit in the textbook. In the frame of the unit assessment as included in the teacher guidebook and shown in Fig. 15.2, number and operations, shape, measurement, pattern, and probability and statistics fall into the mathematical content domain suggested by the mathematics curriculum in Korea, while communication, problem solving, and reasoning fall into the mathematical process domain. This frame of the unit assessment has been organised based on the pyramid model of De Lange (2003).

The frame of the unit assessment is intended to provide the students with well-balanced assessment items and to realise the assessment principle of mathematics learning as mentioned above in a concrete manner by considering various assessment factors such as mathematical content, mathematical process, the level of understanding, and the problem context connected to daily life. In addition, the Korean researchers have tried to develop the tasks and problems in the unit assessments in reference with Van den Heuvel-Panhuizen’s (1996, pp. 140–153) suggestions to make paper-and-pencil tasks more informative.

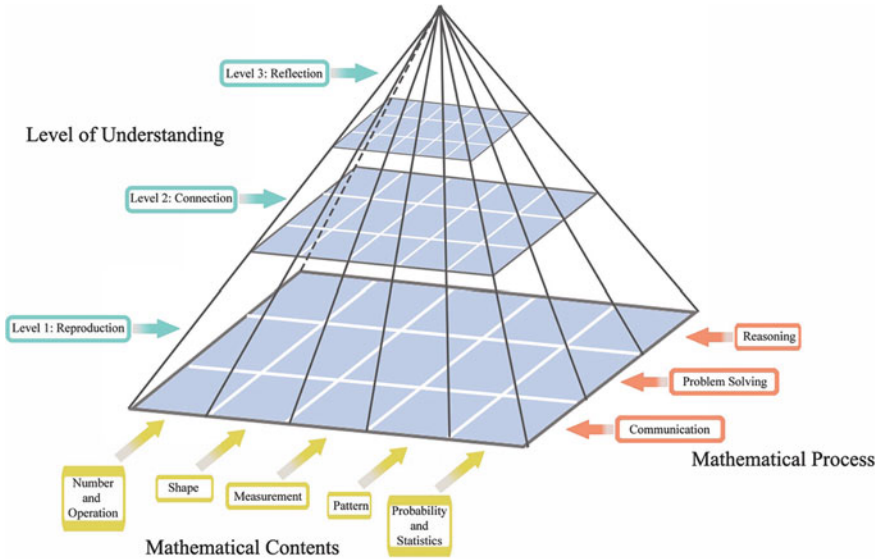


Fig. 15.2 The frame of the unit assessment

## 15.4 Voices from Korean Teachers and Students on RME

### 15.4.1 Voices from Teachers Regarding RME

Teachers in Korea who are interested in RME have tried to reflect the didactics of RME by practicing mathematics classes and mathematical activities based on RME through research meetings of either a school level or at an individual level. This section presents some teacher voices from a middle school and a research group.

#### 15.4.1.1 Voices from Teachers in Middle School IW

Although the application of the RME-based mathematics classes faces many difficulties as the traditional Korean mathematics teaching is very different from that based on RME, Middle School IW, an alternative school established in September 2003, has practised RME-based mathematics classes in the 7th and 8th grades using MiC textbooks translated into Korean since its opening.

The teachers in Middle School IW have studied the theories related with RME and shared their ideas and experiences so that the mathematics classes based on RME fit into the Korean education environments, and tried to extract and instruct the essence of the MiC textbook without distorting its overall composition (Park et al., 2010, pp. 71–73).

In such environments, students have drawn up various strategies of their own through investigation of the context, learned how to communicate in their own words that they fully understood instead of using only formal mathematical terms, and participated in the classes more actively. Teacher B testified the impressions he got from one of the classes based on RME as follows:

Who can be silent when he witnesses these various problem solving strategies? I was thrilled when the students presented their strategies. (Baek, 2004, p. 672)

Though they were often tempted to stop the mathematics classes based on RME due to the energy demanded for class preparations, such as continuous care and observations on the students and the arrangement of each class in the overall curriculum, and the pressure that the students should face in usual mathematics classes in the next grades, the teachers in Middle School IW have been encouraged by the changes to students and alumni. Teacher K testified about the impression on the mathematics class based on RME as follows:

The efforts and the changes that the students make and the fact that they create their own mathematics make me happy. (Park et al., 2010, p. 78)

Though it had been so difficult to go on, I have been encouraged by the alumni when they said: "Sir! We want you to keep going no matter what others say." (from a telephone interview in June 2015)

#### 15.4.1.2 Voices from the Teachers of Research Group G

A voluntary group of teachers, Research group G, has tried to apply the mathematics classes based on RME. It is a small group of middle school teachers in Gwangju, formed by Teacher J, who knew about RME when he joined the Korean Society of Teachers of Mathematics. The teachers in the group have become aware of RME through trainings, lectures at university, teacher's associations, and colleagues. Having reorganised the class contents through the process of instructional design, instruction implementation, and instruction analysis based on the MiC textbooks since 2010, they have been implementing the RME-based mathematics classes and providing the RME-based after-school activities to the students. They have continued discussions and reflections about their instruction designs on whether the designs were in line with RME as they had not studied the theory in detail.

A survey involving 21 teachers of Research group G in which these teachers were asked about the strengths and weaknesses of the RME-based mathematics classes (see Table 15.1) revealed that overall, the number of teachers responding positively was much larger than the number of teachers responding negatively.

The survey suggested that the RME-based mathematics instruction has a need for a course rearrangement that shortens the progressive mathematisation process to fit into the Korean education environment, and a need for explicit expressions of the principles and concepts, and the exercises for review. At the same time, the instruction should provide the atmosphere for the students to think by themselves and share their ideas, while keeping the strengths of the mathematics classes based

**Table 15.1** Teachers' opinions about strengths and weaknesses of RME-based mathematics classes

Strengths of RME-based mathematics classes	Weaknesses of RME-based mathematics classes
It is connected to daily life It helps to get to know mathematics through situations It helps to improve the ability to think mathematically It provides ample chances for mathematical inquiry and reasoning It provides natural motivations to learn mathematics The situation is natural and interesting Mathematical communication is natural and active Many activities are presented It exposes various thoughts and experiences of the students	The book is too verbose The flow is too slow to make a progress The structure is not organised well There are not enough exercises The expressions are too diverse and informal Though interesting, the context is not well aligned with the concept and the principle It does not look like a mathematics textbooks The objective of each lesson is not obvious

on RME, that is, its various contexts that are likely to happen in daily life and the emphasis on mathematical thinking, communication, and practical activities.

In addition to what the teachers themselves thought about the strengths and weaknesses of the RME-based mathematics classes they were also asked to describe positive and negative impacts on student learning (see Table 15.2).

## 15.4.2 *Voices from the Students Themselves Regarding RME*

### 15.4.2.1 **Voices from the Students in Elementary School J**

Under the supervision of Professor C, Teacher H of Elementary School J conducted the mathematics classes utilising the Korean version of the MiC textbook in a sixth-grade class during the Creative Extracurricular Courses for about eight months from April to December 2004. After the courses finished, the students were interviewed to understand how the characteristics of the course influenced them, which is shown in their reactions.

From the interview with Student Y:

The course presented many stories and examples. It was good to improve mathematical reasoning as well as writing skills. But, I have to understand and calculate at the end, and the Korean instruction method was also necessary because it emphasised calculation. The course content was similar to what we have really done in daily activities. It was tough because the course needed more thinking than the Korean instruction method and it focused on principles and understanding. But it was worth doing after all. It was useful to apply the contents to daily activities because the examples in the book were more relevant to daily life.

From the interview with Student G:

**Table 15.2** Teachers' opinions about positive and negative impacts on student learning in the RME-based mathematics classes

Positive impacts on student learning	Negative impacts on student learning
Students learned that mathematics was relevant to daily life	Students were uncomfortable because they had to think too much
Students liked that they could know various ways other students thought and found	Students felt difficulty to express what they thought
Students knew that mathematics was not only about formulas	Students who were poor at mathematics showed less satisfaction
Students found interests in mathematics	The instant effect seemed poor compared to the time and effort
Students liked to be actively involved in the class	More exercises were needed to understand
Students gained confidence during the course to solve challenging problems	Students were unfamiliar with multiple correct answers
Students liked that they could think more than just calculating only	Students were reluctant to expose their thought to other students
Students liked that they could learn a single subject in great detail	Students were not accustomed to understanding mathematics by context unlike the conventional instruction that teaches explicit principles and concepts
	Students were bothered by too much participation being demanded
	Students felt difficulty to find solutions by themselves
	Students were not familiar with the instruction style

The solution for a question in the Korean mathematics class is always determined. I mean about how to solve it. However, the Creative Extracurricular Courses did not make it determined, and allowed a second way. All the things were different from the Korean course, and the Creative Extracurricular Courses encouraged thinking and understanding while the Korean course taught the principle for calculation only. It was more comfortable and easy to understand and draw a result, because it dealt with common senses that we have already known without relying on tough calculations.

Student Y, whose academic achievement was mediocre, described that the RME-based instruction provided a chance to take pride in himself when he solved difficult problems, helped to improve the skills for mathematical thinking as well as for communication due to its diverse stories and examples presented, and was applicable to daily life because of its contexts linked with real situations. Student G, whose academic achievement was also mediocre, told that the instruction was easy and interesting because it focused on understanding by thinking and emphasised the diversity in thinking so that he could use his common senses.

After the course finished, students did show changes in their attitudes and view-points regarding mathematics, which is clearly shown in the following reactions.

From the interview with Student K:

Beforehand, I had thought calculation was all of mathematics. Now I have started to try various methods for a question and compared with others. It would be good if Korean textbooks present various methods.

From the interview with Student L:

Before I took the Creative Extracurricular Courses, I thought mathematics was to calculate according to formulas. But now I solve by my own methods that I create, and mathematics is interesting to me. Often, I think I find new aspects of mathematics when I try with a different method.

Student K, whose academic achievement was high, developed his viewpoint on mathematics that requires reasoning and provides chances to communicate with others, and hoped Korean mathematics textbooks would present various methods and reasons. Student L, whose academic achievement was low, said that he found new aspects of mathematics as he knew he could use his own methods, while he had thought mathematics was to apply given formulas to calculate before.

In summary, the students described the characteristics of the mathematics instruction as an instruction of mathematics that deals with situations and stories relevant to daily activities, encourages reasoning over calculation, suggests various methods over formulas, provides creative activities, is applicable to daily activities, and is fun.

#### **15.4.2.2 Voices from the Students in Elementary School G**

The teachers in Elementary School G conducted mathematics classes using Korean version of the MiC textbook for all Grade 4 students from September to December 2005, and studied the effects of RME-based mathematics instruction on the students' view on mathematics-related issues by comparing the results in this experimental group with Grade 4 students in a control group in which the students were taught in a regular way. Although the MiC textbook was developed for Grade 5–8, the teachers considered it appropriate for Grade 4 students because the students are relatively high achieving and they have learned necessary mathematical concepts such as angle (Shin, Park, Chong, & Chang, 2006).

The students were asked about professions that require mathematical competences and what they think about problem solving. Table 15.3 shows that the experimental group recognised more professions that need mathematical competence than the control group did and also showed more fluency in recognising these occupations. In addition, the experimental group was more positive about the process of reasoning for problem solving. The results support that RME-based mathematics instruction with the Korean MiC textbook is more effective to improve the recognition of mathematical competence required in professions and a positive attitude towards mathematical reasoning than the traditional mathematical instruction.

**Table 15.3** Students' view in the experimental and the control group on professions that need mathematical competence and on problem solving (Shin et al., 2006, p. 40)

Students' view	Experimental group			Control group			<i>t</i>	<i>p</i>
	<i>n</i>	<i>M</i>	<i>SD</i>	<i>n</i>	<i>M</i>	<i>SD</i>		
Fluency on recognition of professions that require mathematical competence	210	3.80	2.68	122	2.50	1.65	4.82	0.00
Diversity in recognition of professions that require mathematical competence	210	2.36	1.68	122	1.77	1.28	3.34	0.00
Originality in recognition of professions that require mathematical competence	210	0.55	0.91	122	0.33	0.69	2.36	0.02
Negative attitude to reasoning for problem solving	210	0.17	0.40	122	0.31	0.56	2.74	0.01

## 15.5 Concluding Remarks

Many research groups for mathematics education in Korea have paid attention to RME over 30 years. In Sect. 15.2, it was reported that RME has been actively discussed from both theoretical and practical viewpoints through doctoral and master theses as well as journals. As a result, RME has greatly influenced the research and practices of Korean mathematics education. In particular, it has been one of the useful perspectives and references that helped to identify and revise many issues in Korean mathematics education. However, the introduction of the RME perspectives to Korean mathematics education has been discussed very carefully because of its very different, and often contrary, aspects from what has been emphasised traditionally in Korean mathematics education.

In Sect. 15.3, the influences of RME and the MiC textbook on mathematics teaching-learning in Korea have been discussed, which focused on the curriculum, the textbook, and the assessments. In the adjustments on the standards of mathematics curriculum, the RME theory and its didactical phenomenological analysis influenced adopting progressive mathematisation. Regarding the textbook, the RME theory and the MiC textbook have inspired mathematics educators in Korea to find and develop appropriate contexts that improve the negative attitudes of Korean students to mathematics and help to experience mathematics as a human activity. The pyramid model of De Lange (2003) has played a critical role to reorganise the frame of unit assessments of the Korean mathematics textbook in elementary school. In addition, the suggestions of Van den Heuvel-Panhuizen (1996) to make paper-and-pencil tasks more informative have influenced the development of tasks and problems in the unit assessments of the elementary school mathematics textbook in Korea.

Section 15.4 reported the voices from teachers and students in Korea about RME. Both teachers and students told that mathematics instruction based on RME could change the recognition of mathematics to a positive stance, because it provided nat-

ural situations and activities that encouraged students to actively participate through diverse thoughts and communications. However, they added that it should be considered to shorten the process of mathematisation and to have repetitive exercises that fit the Korean education environment. Meanwhile, a study of the influence of RME-based mathematics instruction on the attitudes of students showed that the instruction improved recognition of the mathematical competence required in professions and attitude on mathematical reasoning. In order to extend the efforts done at the school level so far in relation with RME-based instruction, communication between the teachers and the researchers will be essential, which leads to sharing of perspectives on adapting the RME theory to Korean mathematics education.

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