Chapter 14
Methodological Issues in the Investigation of Values in Mathematics

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Abstract In this chapter, we first provide an overview on our previous investigations, then the methodologies employed are evaluated. A variety of research methodologies such as questionnaires, open-ended questions, interviews, classroom observation, episode writing, and mind maps have been employed at different stages of our intellectual journey. These methods are effective tools and helped us understand students' and teachers' beliefs and values about mathematics, mathematics learning and teaching. However, one's value is hidden and therefore not apparent. It is a deep-down notion in the sense that it is not easily unfolded. Thus, if we want to have a more holistic understanding on teachers' and students' values (in mathematics), we need a sharper methodology in which the participant is forced to make decision under a dilemmatic situation so that their values can be revealed. After providing examples of such situation that we used, we would argue that the use of hypothetical situations can enhance the investigations of values and related issues. What we used include (a) asking whether one is doing mathematics when provided with some hypothetical situations, (b) what a teacher would respond if his/her students asked whether one is doing mathematics in the given hypothetical situations, (c) how one respond to some famous mathematicians' quotes in which some controversial situations are involved, (d) hypothetical classroom situations and (e) hypothetical lesson planning. We believe that our discussions can enrich the study of values in mathematics education.

Keywords Research methodologies · Beliefs about mathematics · Values in mathematics · Hypothetical situations

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14.1 Theoretical Premises

Despite the fact that beliefs and values are important notions in mathematics education research, the methodologies for investigating these notions are still developing (Leder et al. 2002). By reviewing existing literatures related to this area, we found that beliefs and values are closely related but values are deeper than beliefs. According to Philipp (2007), value is "a belief one holds deeply, even to the point of cherishing, and acts upon" (p. 259). On the other hand, we think that one's value is hidden and therefore not apparent. Thus, it is not easy to be unfolded by simple methods such as observing one's behaviour or completing a questionnaire. In order to have a holistic understanding on students' and teachers' values in mathematics, a variety of research methodologies are needed. This is precisely the theme of this chapter. We will first review and reflect on the methodologies used in our own related studies. Then, we will argue that hypothetical situations can be a methodology which is complementary to other frequently used methodologies such as questionnaires and interviews. By hypothetical, we mean that the researcher confronts the participant (usually by means of interview) with a situation in which the participant may not actually have a chance to encounter before. More importantly, the situation is usually put in an extreme (dilemmatic) situation in which the participant needs to make a choice among different options. We believe that such choice can provide a 'window' for unfolding one's values.

14.1.1 Beliefs and Values as Affective Subdomains in Mathematics

Affect in general—as well as beliefs and values in particular—have drawn attention to the mathematics education community in recent decades. While Daskalogianni and Simpson (2000) exclaimed on the non-consensus definition of attitude that "… almost reducing it [the concept of attitude] to the pseudo-definition 'attitude is what attitude questionnaires measure'" (p. 217), the same might be true for affects, beliefs and values (see for instance, Hannula 2012). The interrelationships among these notions began to take shape in McLeod's seminal work (1992). Beliefs, attitudes and emotions are identified as three affective subdomains among which these three subdomains increased in the levels of affective involvement and intensity of response whereas they decreased in the levels of cognitive involvement and response stability. Beliefs play a significant role in students' development of attitudinal and emotional responses to mathematics. On the other hand, automatized repeated emotions can become an attitude.

Under the above framework, Goldin (2000) proposed the notion of affective pathways which sharpen the interrelationship among different affective subdomains.

It emphasizes the interaction of affective states and cognitive representations during mathematical problem solving. DeBellis and Goldin (2006) further developed a tetrahedral model which can be regarded as an extension of McLeod's framework. This model highlights the interacting relationship of **values** with the other three affective subdomains. According to DeBellis and Goldin (2006), "*values*, including *ethics* and *morals*, refer to the deep, 'personal truths' or commitments cherished by individuals" (p. 135, italic in its original). In around that period of time, Hannula (2012) re-conceptualized the notion of attitude, in which emotions, expectations and values are included as different aspects for analyzing one's attitude and its changes.

14.1.2 The Close Relationship Between Beliefs and Values

While DeBellis, Goldin and Hannula identified beliefs and values as subdomains of affect in mathematics, the close relationship between beliefs and values are emphasized by different scholars (see for instance, Barkatsas et al. 2018). Beliefs are usually related with one's knowledge or conceptions about something (Furinghetti 1998) whereas values are something that we consider to be important and worthwhile (Bardi and Schwartz 2003). This distinction and their relationships are stated by Philipp (2007). Beliefs are considered as "psychologically held understandings, premises, or propositions about the world that are thought to be true". In contrast, values are "the worth of something [and] a belief one holds deeply, even to the point of cherishing, and acts upon" (p. 259). It highlights the fact that values are **deeper** than beliefs because those beliefs held deeply by an individual are identified as one's values.

Even as early as 1990s, Bishop (1999) has pointed out that "values in mathematics education are the **deep** affective qualities which education fosters through the school subject of mathematics" (p. 2, emphasis added). Furthermore, values are something that are internalized as important and worthwhile, and thus they define how one sees the world (Seah and Andersson 2015).

Worldview is another notion commonly found in literature, which has close resemblance with value. It is "one's comprehensive set of beliefs about the nature of reality and how one should live in the light of those beliefs" (Heie 2002, p. 99). In other words, a person's worldview is an amalgamated product of multiple sources, including but not limited to one's beliefs and values. In the context of mathematics teaching and learning, one should not only focus on values in the subject discipline of mathematics but also need to address the values in general such as educational values, sociocultural values, or even religious values. This idea echoes to Bishop et al. (1999)'s identification of three kinds of values in mathematics teaching, that is: the general educational, the mathematical, and the specifically mathematics educational.

Although the definitions of value may vary among different scholars, all these definitions suggest that value is so deep down that it is not easily unfolded.

14.2 Our Journey on Investigating Beliefs and Values in Mathematics Education

Although we recognize that different scholars each has his/her intellectual journeys, we believe that our own journey is enough to reflect the methodological challenges one is facing when researching on beliefs and values. Thus, instead of doing a comprehensive literature review (which is not the goal of this chapter), we will review our journey on investigating mathematics-related affects (especially beliefs and values) in which the authors of this chapter have been involved. We will first introduce an overarching framework which guides our studies and then review the methodologies which have been employed.

14.2.1 The Lived Space of Mathematics Learning

A group of researchers in Hong Kong, led by the second author of this chapter, launched a project on mathematics beliefs/values in the mid-1990s. To begin with, they investigated students' conceptions of mathematics and their beliefs about mathematics/mathematics learning. How such beliefs affect their problem solving behaviour was also investigated. Teachers' beliefs were investigated at the same time, which later extended to teachers' teaching behaviours as well as their knowledge (Subject Knowledge and Pedagogical Content Knowledge included). How teachers' religion and their worldview might influence their teaching philosophy (including beliefs and values) as well as teaching per se was the most recent endeavour of the two authors of this chapter. All these investigations can be framed under the notion of *lived space* of mathematics learning (Wong et al. 2002).

As depicted in Fig. 14.1, the series of studies was nicely conceptualized into the framework of the *lived space*. First, how a phenomenon (learning phenomenon included) is perceived by either an individual or by a group generates a *space* of understanding or conception of that particular phenomenon. In terms of learning, that *space* would constitute an *outcome space* regarding that phenomenon (Marton and Booth 1997). The antecedent of this *outcome space* is students' *lived space* (which is largely shaped by the teacher) which is the result of their learning experiences. By reinterpreting earlier findings using the theoretical lens offered by phenomenography, we can conclude that a broader *lived space* leads to a richer *outcome space* of student learning (Marton and Booth 1997).

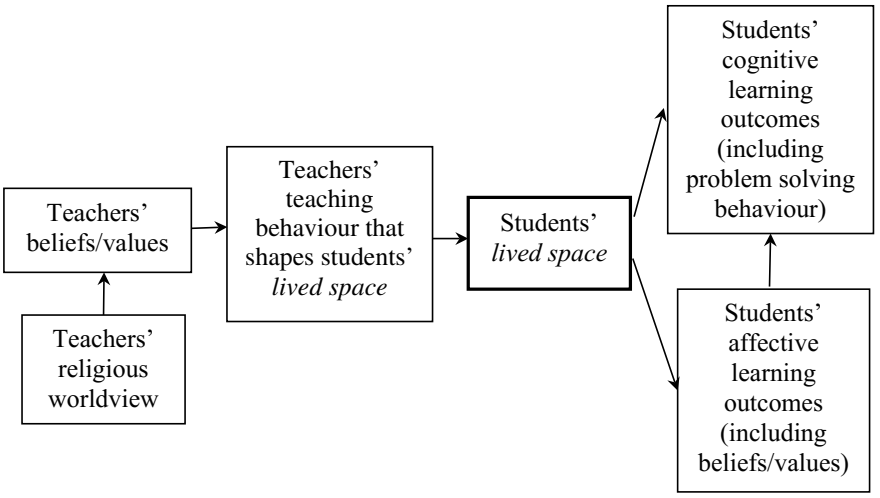

Fig. 14.1 The *lived space* of mathematics learning

14.2.2 Methodologies Used in Our Studies

Different methodologies have been used at different stages of our studies. Open-ended questions such as "Mathematics is …" was used in our study on students' beliefs about mathematics (Wong 1993). In our next study on students' beliefs about understanding mathematics, both open-ended questions (for instance, "When will you consider having understood a certain mathematics concept?") and episode writing (for instance, recall and write down any instance in which the participants understood a mathematics topic, formula, rule or problem) were employed (Wong and Watkins 2001). Besides arriving at other results, *the Mathematics Classroom Environment Scale* was developed. At that time, **we realized that it is not easy for students to tell us what mathematics is**. We adapted the ideas of Kouba and McDonald (1991) by utilizing hypothetical situations to investigate students' conception of mathematics. An example of such situations is "An elder sister lifted her younger brother. She said that he must weigh about 30 lb less than she. Did she do mathematics?" (Lam et al. 1999). Another inventory *The Conception of Mathematics Scale* was developed which was used in a number of studies, including an Ed.D. thesis on the use of history in mathematics teaching (Cheung 2014). In addition to the inventory, mind maps were used to tap students' conceptual change.

The above studies focused solely on students' beliefs about mathematics (and about mathematics learning). We moved one step forward to explore the linkage between beliefs and their mathematics learning outcomes, in particular, their performances in mathematics problems (Wong et al. 2002). In addition to computational and word problems, open mathematics problems were used (Cai 1995). Clinical interviews were conducted after the problem-solving process. Students' confined

conception of mathematics would lead them to approach mathematics problems in a mechanistically way, solving them by picking out routines. We hypothesized that this is the consequence of the narrow *lived space* and thus we proceeded to investigate the conception of mathematics/mathematics learning and teaching among the teachers.

We moved from studying **students** to studying **teachers**. Since teachers are adults and are trained in mathematics/mathematics education, **in spite the possibility that they just offer 'model answers' in answering what is mathematics, (projected) hypothetical situations were once again employed**, that is, asking teachers their expectation of their students' reactions when they were confronted with the above hypothetical situations and what would be their own responses. In addition, the teachers were asked to comment on quotes from famous mathematicians (which are often controversial). An example of such a quote is "The moving power of mathematical invention is not reasoning but imagination" (A. DeMorgan, cited in Graves 1889, p. 219). The study found that the teachers hold a slightly broader conception of mathematics than the students, yet there is much concord between the two (Wong et al. 2003). Semi-structured interviews (about mathematics, learning and teaching) were used again in Cai et al. (2009).

The close resemblance between the students' conception of mathematics and the teachers' conception of mathematics reinforced our premise that teachers' conception of mathematics contributes to the students' conception of mathematics. We need a further step to trace **how** the *lived space* of the students was shaped by the teachers. To achieve this, we entered the classroom. This is precisely the theme of Q. T. Wong's M.Phil. thesis (2003) (a summary of her work can be found in Wong et al. 2009). In her project, different methods were used to address different research agendas. In particular, the projected hypothetical situations of whether 'one is doing mathematics' (transformed to asking the teachers) were used once again. She also conducted classroom observation and follow-up interviews. In addition, she used hypothetical situations in classroom teaching to solicit participants' beliefs about mathematics teaching. First, she collected questions which often arouse debates among mathematics teachers. Then, she used these issues to stimulate her participants by asking them how they would react. The following is an example of the interview questions.

> When you are marking students' homework like "Each apple costs $3. What is the total price of 2 apples?", which of the following(s) – $3 × 2, $ (3 × 2) or $ (2 × 3) – do you regard as incorrect and would marks be deducted? And why? (Wong 2003, p.120)

Zhang's Ph.D. study (2010) moved one step further to include teachers' knowledge (a summary of his work can be found in Wong et al. 2009). He found that teachers' beliefs and their knowledge co-contribute to the shaping of students' *lived space*.

The Third Wave Project, which was initiated by Wee Tiong Seah, aimed at cross-regional comparisons of students' and teachers' values in mathematics learning and their co-valuing (Seah and Wong 2012). In this project, not only the notion of beliefs was extended to values, this is the first study (among those that we are involved with) that both students and teachers were investigated in a single study. In addition to widely used methods such as teacher journals, lesson observations

and semi-structured interviews (see for instance, Wiegerová 2013), a new method involving 'photo-voice' (Lim 2010) was utilized. Students were asked to take photo snapshots in 'aha' moments, that is moments that the students felt inspiring (including suddenly understood something) during the lesson. These photo snapshots formed the basis of both teacher and student post-lesson interviews. This kind of stimulus-recalled interviews enabled us to capture students' and teachers' espoused values in effective mathematics classrooms (Law et al. 2012).

This Project drew our attention to the relationships between values in mathematics and one's worldviews, teachers' religious beliefs in particular. Questionnaire and semi-structured interview were used in the first two studies (Chan and Wong 2014; Leu et al. 2015). Both studies led us to conduct our main study (Chan and Wong 2016) which investigated teachers' **enacted** values in mathematics that may be related to their religious values. In this study, hypothetical lesson planning was used. The respondents were requested to design a mathematics lesson which referred to their religious beliefs in mathematics teaching. It was hypothetical because the respondents could assume that they were permitted to do whatever things related to their religion in that lesson (which may or may not be true). This hypothetical lesson planning formed the basis of the follow-up semi-structured interviews which intended to capture the teachers' interplay between their religious values and their enacted values in mathematics.

While one can refer to our previous publications for details of examples of these hypothetical situations, let us provide another example of using hypothetical situations. First, participants were provided by a hypothetical mathematics lesson, in the form of a script (lesson plan), which is pre-analyzed and truncated into routines. At the junction of different routines, the participants were confronted with hypothetical questions like "If you were that teacher, what would you do next, why?"; "If you are not allowed to do what you proposed to do, what other teaching strategies can you think of?"; "If at this point, a student reflected that s/he does not understand, what other ways you can explain again?", "Please offer as many alternatives as possible and compare the strengths and weaknesses of these strategies". Since the entire situation is hypothetical, it leaves more room for both designing the lesson plan and the setting of the questions (Cai and Wong 2012).

14.3 Methodology Revisited

Though the description above is confined to the studies conducted by the authors of this chapter, it could project a picture of what other researchers are using because most of the studies were collaborative with other researchers overseas. In fact, if one explores standard texts on research methods like Punch (1998), the methodologies listed do not go beyond what we tried, thus ours may be much richer. We attempted many methodologies (sometimes used several methods simultaneously) because we realised that it is not easy to tap into one's values which are deep down and hidden in one's mind. The methods that we have attempted include conventional question-

naires, semi-structured interviews, teachers' journals, open-ended questions, episode writing, snapshots of critical moments, scenario-stimulated responses, clinical interview and hypothetical situations.

Apparently, no single means is more powerful than the others. It is common knowledge that each methodology has its limitations.

As the researcher is (part of) the instrument of qualitative methods, data-sensitivity of the researcher is crucial. Qualitative methods often face the challenge of being subjective. Results are usually not generalizable. Since the labour involved is intensive, the number of respondents is often limited. Social desirability (that is, the tendency of giving a socially acceptable response) is another issue if data collection involves face to face meetings (e.g., interviews). One is referred to standard texts like Creswell (2018), Denzin and Lincoln (2005), Kvale (1996), LeCompte and Preissle (1993), Rubin and Rubin (2012) and Silverman (2001) for easy reference. It may make the issue more intense when the topic is sensitive. Values could be one such issue.

Quantitative methods seem to be more 'objective'. However, quantitative means of data collection were often conducted in one-way paper-and-pencil fashion. Therefore, idiosyncratic issues (e.g. misinterpretation and having different respondents perceiving the same item differently) may arise. These issues could be caused by gender, cultural and other individual differences. We cannot rule out the possibility that some groups of respondents (Chinese in particular, who are typically modest) tend to pick middle response scores (Harzing 2006). Furthermore, quantitative methods such as Likert-type inventories appear to be objective, yet their responses are restricted by the questionnaire framework (construct dimensions) and guided by the questionnaire items. Quantitative methods are not as exploratory as qualitative methods. Thus, it is not easy to discover aspects beyond what is laid down by the questionnaire. The issue of social desirability, though not as prominent, is still there (Bernardi 2006; Fisher and Katz 2000; Lalwani et al. 2006).

One needs multiple sources of data to triangulate, piecing together different facets (from sources of data) into a relatively impartial picture. Therefore, mixed methods are becoming increasingly popular. Thus, we changed methodologies as the research progressed. We chose methodologies based on our specific needs (e.g., the number of participants, their age, as well as the sensitivity of the topic) because no single methodology serves all our needs.

On top of the above issues (including social desirability), what makes value research more challenging is that values may not be easily articulated. We need to go further to unfold such a notion which is deep down in one's mind. We are not saying that the above approaches and methods do not work. Nevertheless, some of these methods are just a setting to solicit informants' ideas. For instance, one can incorporate open-ended questions or episode writings in case studies or in focused group interviews. However, what is clear is that we need to overcome issues like having loaded questions, being too rely on self-reporting and social desirability. As such, we explored different possibilities in our academic journey.

Hypothetical situations are one that might fill the above gap. We are not advocating that the use of hypothetical situations is so powerful to replace all other means.

Rather, hypothetical situations can only be incorporated into other methodologies, e.g., interviews, or even paper-and-pencil 'questionnaires' (such as episode writing).

Let us re-iterate on the various kinds of hypothetical situations we used in our journey. Initially, we employed the method used by Kouba and McDonald (1991). With this method, students were confronted with situations and asked whether they considered them as doing mathematics. We adapted this by asking the teachers their expectations of students' responses when the students were asked whether these situations were considered as doing mathematics. Then we proceeded to use quotations of famous mathematicians in history. Hypothetical situations that concerns frequently debated issues in mathematics departments (within a school) were also used. Whether one regards these statements as mathematically correct caused much controversy in the wider mathematics teacher circle too. We confronted our informants about these frequently asked questions to unfold their beliefs and values about mathematics education. Other types of hypothetical situations were used in subsequent studies on how teachers' values about mathematics education are related to their religious values. Our respondents were asked to develop a hypothetical mathematics lesson that may express their religious values in the lesson.

The gist of hypothetical situations is that we provide participants with a situation that pushes them to an extreme so that they need to review their own values in order to make a response, thus revealing these values from the bottom of their hearts. In contrast, naturalistic situations such as observation of normal classrooms could be too routinized and hence unable to confront the participants through dilemmatic situations. In addition, the respondents' practical considerations such as administration or curriculum requirements would make it more difficult to extract the values behind the scene. We honour other data collection methods, but we do think that the use of hypothetical situations provides a means particularly relevant to value studies.

14.4 Conclusion

Values are deep down construct that no single method could possibly portray the whole picture. (Thus, some scholars even use the term 'value system'.) It is imperative in research involving values to triangulate the data with different methods. By triangulation we do not simply mean having different points (data collection methods) to cross check each other but to re-construct the 'real' picture through various points. We do not infer that the use of hypothetical situations is a panacea; we only see its high potential in value research. We would argue that values should only be explored by using such in-depth methodologies.

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