

# Chapter 12

## Modelling Tasks and Students with Mathematical Difficulties



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**Abstract** This study is a part of a bigger study of 23 fifth graders observed as they worked in heterogeneous groups on a sequence of 12 modelling tasks for eight months. This chapter focuses on nine students identified as having difficulties in mathematics. Our research goal was to identify the nature of the changes that occurred as they worked on these tasks and is exemplified by one case, that of Sami. The findings show how Sami's mathematical knowledge and modelling competencies developed and how, simultaneously, his group's attitude towards his contributions was affected. At the beginning of the process he did not understand the task situation, and even when he gave relevant realistic considerations his peers ignored him. Later he became more active not only in offering realistic considerations but also in suggesting mathematical ideas, and eventually Sami became dominant and effective in the group and was well aware of this change.

**Keywords** Mathematical difficulties · Modelling competencies · Modelling tasks · Students with learning difficulties

### 12.1 Theoretical Background

Researchers claim that teachers emphasize high thinking processes in good classrooms, while in classrooms of students with learning difficulties, they use methods of instruction that require only low order thinking (Shepard 1991; Raudenbush et al. 1993; Zohar et al. 2001). This tendency exists also when working in a heterogeneous class (Yair 1997). Page (1991) adds that teachers expect good students to deal

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with complex and challenging learning materials, while students with difficulties are expected to learn basic skills only. On the other hand, Haylock (1991) emphasizes the need to provide meaningful and relevant activities for students with difficulties, as well as the importance of working in small groups in coping with these tasks.

Characteristics of students with difficulties include a variety of problems such as: memory difficulties, difficulties in generalizing and transferring acquired knowledge to new and unfamiliar tasks (Kroesbergen and Van Luit 2003). Haylock (1991) also mentions difficulties in: language, reading, spatial perception and anxiety in mathematics. In addition, Peled (1997) and Bachor and Crealock (1986) emphasize the passive response to instruction as one of the main characteristics of these students.

In looking at the source of these difficulties, Abel (1983) claims that environmental factors are more significant than congenital factors. Ginsburg (1997) analyses many possible factors that might contribute to students' difficulties. He suggests that often instructional methods are to blame rather than some cognitive deficit. Thus, it can be concluded that low performance and achievement in mathematics can also result from inadequate instruction and lack of motivation (Barnes 2005; Ginsburg 1997; Karsenty and Arcavi 2003; Reusser 2000).

Peled (1997) claims that students who are passive and do not participate in class, often because of lack of confidence, experience less teacher reaction and miss the chance to improve their knowledge thus falling into a vicious cycle. She suggests that mathematics programs should encourage students with difficulties to participate more in class, receive more feedback and, thus, have a better chance of learning and progressing. In this study, our assumption is that, modelling activities can be appropriate and effective for students with difficulties in mathematics.

Mathematical modelling has been defined by Blum and Borromeo Ferri (2009) as a bi-directional translation process between the real world and mathematics. Peled (2007) defines modelling activities as a process of organization, analysis and observation of situations and phenomena through models and mathematical tools. Blum and Leiß (2007) describe the expected stages in a modelling solution as a cyclical process. The ideal modelling cycle starts in the "situation (usually realistic) world" with analysis, simplification, organization, and structuring of the given situation. This stage is followed by mathematisation of the structured situation by choosing and using mathematical models and representations, and by performance of some mathematical processes. The next stage involves interpretation of the results in terms of the situation, validation of the results and deliberation on whether new (realistic or mathematical) considerations are needed and another cycle is called for. The analysis of modelling competencies by Maaß (2006) and Maaß and Mischo (2011) is closely related to the modelling cycle because the problem solver should be able to perform each of the cycle's stages in order to follow this cycle. Niss et al. (2007) define competency as the ability to perform certain appropriate actions in a problem situation and mathematical modelling competency as the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution.

In this chapter, we have chosen to focus on the main competencies. This includes an awareness of the need to understand, analyse and simplify the situation and the

ability to do so; an ability to choose mathematical representations and use mathematical concepts in building mathematical models for describing the simplified situation; and a need to make sense of the results by reasoning and using argumentation. In addition, the whole process involves an organizational competency that includes the ability to take part in group work and keep track of it through clear documentation.

Maaß (2005) claims that, the close connection between reality and modelling activities makes mathematics more useful, interesting and understandable, especially for students with difficulties. Moreover, the work on modelling tasks is usually done in small groups (Zawojewski et al. 2003). This setting might result in having the better students dominate the discussion while the weaker students stay passive and are ignored when they try to contribute to the group discussion (Peled 1997). While this latter drawback exists, there is also some support for potential benefits of group work for these students. A report of an advisory committee in the USA (Steadly et al. 2008) suggests that low achievers and students with learning difficulties in mathematics might benefit from working with other students and listening to their peers' mathematical discussions.

In our own experience in several studies where we implemented a sequence of modelling tasks we noticed positive changes in the work of students identified by their teacher as having difficulties in mathematics (e.g. Filo and Peled 2012). Following these informal observations, we designed this study with the purpose of focusing specifically on these low-performing students and observing changes in their *modelling competencies*, their *mathematical knowledge* and how these changes are exhibited through the *nature of their participation*.

## 12.2 Method

The study was conducted in an Arabic primary school of a low-class population. The studied group was a fifth-grade class of 23 students, nine of whom were identified by their teacher as having difficulties in learning mathematics. Her definition matched that of the school and the Ministry of Education, which relies on the results of standard tests. The mathematics instruction method in this class was a whole class method, with the teacher being the source of knowledge and authority. The main mathematical topics in fifth grade are: fractions and decimals, geometry focusing on triangles, quadrilaterals, areas and circumference of polygons and deepening of the four arithmetic operations in natural numbers.

The study was conducted in school during regular school hours. It involved one lesson (of 45–60 min) each week for eight months during one of the lessons assigned as mathematics lessons. The task sequence consisted of 12 tasks, detailed in Table 12.1, including individual pre-test and post-test tasks. The study included all students in the class; they worked in heterogeneous groups of 4–5 students each group, two of them were students with difficulties. However, the observations focused on the students with mathematical difficulties in each of the groups. We followed their learning process in order to examine two questions:

**Table 12.1** Task sequence

Task	Context
1. Pre-test: <i>Fun day</i>	Planning a fun day schedule by choosing a sequence of activities
2. <i>Class party</i>	Ordering refreshments for a class party given items and prices
3. <i>Pizza</i>	Ordering pizza given two pizzerias, different items and prices in two places
4. <i>Cookie bakery</i>	Providing orders of cookies packaged in given package sizes
5. <i>Tangram</i>	Pricing different tangram parts given the total price of the game
6. <i>Ads-task A</i>	Preparing advertisement for a store sale of different products including toys, clothes, etc.
7. <i>Ads-task B</i>	A similar task as in part A with a more limited number of products
8. <i>School time</i>	Verifying the claim “most of the year is spent at school” (Maaß and Mischo 2011)
9. <i>Tents</i>	Deciding what tents in terms of size, number, and manufacturer the school should purchase
10. <i>Body relations</i>	Checking whether there is a constant relation between the head and body of a person
11. <i>Volleyball</i>	Choosing players based on a table of quantitative and qualitative data (Zawojewski et al. 2003)
12. Post-test:	Planning a schedule by choosing a sequence of activities
a. <i>Fun day</i>	Similar to the pre-test task 1 with different activities
b. <i>Farm visit</i>	Similar to the Fun day task with a different context and more complicated data

1. (a) Can modelling competences be developed among students with difficulties in mathematics? (b) Which modelling competences will develop through the implementation of a modelling task sequence?
2. Does this implementation of modelling tasks also have an impact on their mathematical knowledge?

The study was conducted using the Design Experiment approach developed by Cobb et al. (2003). This approach can be defined as a research method aimed at developing theories and materials, based on ‘how learning works’. It involves itera-

tive task design where tasks are evaluated and redesigned following observations of students' learning. Tasks were designed using contexts to motivate the students, and involved situations taken from their daily life. Further on in the study, some tasks were redesigned based on students' suggestions or needs.

The final task sequence (Table 12.1) consisted of twelve modelling tasks, some designed in advance and some constructed during the implementation of the sequence, as will be detailed further. All the tasks were designed using principles for constructing Model Eliciting Activities (Lesh et al. 2000), keeping in mind the goals of the study. The main features of these tasks involve the use of context in a way that will elicit and encourage an analysis of the given situation, organization of the situation, and making choices about representing and mathematising it. These tasks are expected to elicit the development of modelling competencies leading to a solution process that is depicted by the Blum and Leiß (2007) modelling cycle.

Table 12.1 details the twelve tasks with a short description of their context. The first task and the last task were similar; the difference between them was in the type of activities in order to suit the fun day at the end of the year, serving as individual pre-test and post-test for examining the change in modelling competencies in addition to the evidence collected from students' work through the whole sequence.

Our analysis of the progress of Sami and the rest of the students was conducted using a variety of data collection instruments: (a) individual pre-test and post-tests for examining modelling development competencies, (b) individual pre- and post-tests for examining mathematical knowledge (testing the concepts of multiplication and fractions), (c) observation notes and student work during implementation of a sequence of modelling tasks in which the students worked in heterogeneous groups and (d) individual interviews following the implementation of the modelling tasks and post testing. Pre-test and post-test data on the *Fun Day* and *Farm Visit* tasks were analysed for evidence of modelling competencies: awareness of the need to understand, analyse and simplify the situation and the ability to do so; an ability to choose mathematical representations and use mathematical concepts in building mathematical models for describing the simplified situation; and a need to make sense of the results by reasoning and using argumentation, and an organizational competency that includes the ability to take part in group work and keep track of it through clear documentation. In our observations through their implementation of the tasks sequence we focused on modelling competencies development through changes in the nature of their participation, chronicling any development from completely passive behaviour to becoming involved in the working and organisation of the group and constructing and documenting mathematical models.

The study was conducted as a part of the regular school day, one session per week for eight months with the exception of holidays. In addition to working on modelling tasks, the students continued with their mathematics classes according to the regular curriculum. As will be mentioned further on, the effect of the 'other' regular mathematics classes was controlled by data from another class, which did not participate in the study and served as a control group.

## 12.3 Findings

The information on the development of modelling competencies and mathematical knowledge was obtained by analysing data from the pre-tests and post-tests and from the observations during the task sequence. The comparison of the pre-test and the post-test for every student showed that seven of the nine students with mathematical difficulties had developed all competencies and showed progress in their mathematical knowledge.

We chose to describe in detail the follow-up on Sami's development in order to give sense and understanding to how the changes occurred. Sami was one of the seven students for whom we found a significant development in modelling competencies and mathematical knowledge. Sami's development was similar to the development of the other six students and hence serves as a representative example. The results of the pre-test for examining of mathematical knowledge were consistent with the assessment of Sami's performance in the standard school tests on these subjects.

In addition, the mathematics teacher noted at the beginning of the study that Sami did not show interest in mathematics lessons and did not participate in the class discussions despite his high verbal abilities expressed in social fields.

### 12.3.1 Sami's Pre-test in Modelling Competencies

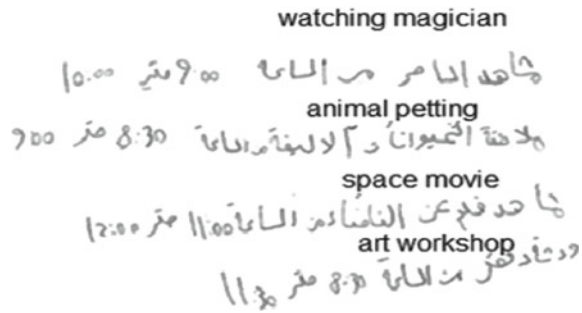
The pre-test *Fun day* (Appendix 1) examined several modelling competencies: simplifying and understanding the situation, setting up a mathematical model, reasoning and documentation. In this task, the students were requested to plan their own sequence of shows or activities from a given list with a starting time and duration of the shows.

Sami wrote down his favourite choices in a short and partial list without giving any explanation for his choices (Fig. 12.1). In addition, it would have been impossible to carry out his choice because of time overlaps between activities. For example, the art workshop overlapped all the other activities. Although he wrote the durations of the activities, he did not take them into account. He was thus demonstrating little to no modelling competence.

### 12.3.2 Sami's Performance and Role During the Task Sequence

As mentioned earlier, Sami worked in a heterogeneous group of five students. Two of the students, Sami and Noor were students with difficulties in mathematics. In the follow-up on Sami's work, we observed a gradual development of Sami's modelling competencies, which we describe below in four stages:

**Fig. 12.1** Sami's work in the modelling competencies pre-test



1. Passive.
2. Active in realistic considerations only.
3. Active in organization and partly active in setting up the mathematical model.
4. Dominant in all stages.

### 12.3.2.1 Passive

The first task (after the pre-test) in the group sequence, *Class party*, was a relatively simple decision-making task with limited data. It was supposed to create a bridge between traditional school word problems and modelling tasks. The students were requested to choose items from a given list of refreshments under the constraint that each student had to pay 6 New Israeli Shekels (NIS).

At the beginning of the group's work, Sami asked Noor, the other weaker student, not to interfere with the calculations: "you and I shouldn't do the calculation." This act expressed his concern and awareness of his own weakness in doing mathematics. Seeing the task as a kind of competition between the groups, he wanted his group to have the best mathematical solution, and therefore thought it would be better if the other group mates do the work.

As the group continued working, Sami did not stay completely passive. He *pointed out some realistic considerations* suggesting not to choose too many sweet drinks and snacks because they are not healthy. Unfortunately, his group mates totally ignored his ideas.

When the groups presented their choices, it was evident that Sami's group and all other groups provided similar solutions. They constructed some arithmetic expressions that would reach an amount equal to the number of students in class multiplied by the amount paid by each student, but their solutions did not take into account any realistic considerations.

The final stage in working on each of the modelling tasks involved a presentation of the groups' solutions to the whole class. This was supposed to give the different groups a chance to comment and discuss each other's ideas. However, as it turned out, the groups' presentations of this task did not trigger any discussion or argumentation of the presented solutions.

While the students did not make any comments or ask any questions, possibly not accustomed to doing so, the mathematics teacher made a move that could be considered an intervention. In, what seems to be, an effort to hint that the (new) rules (or didactical contract) allow discussing problem solutions, she raised a question about the validity of the solution: “Are the quantities of items you chose realistic?” As a result of the teacher’s question, Sami felt confident and said with excitement, “I was thinking that too many sweet drinks and snacks is unhealthy! I even told that to my group mates, but they did not listen!” At the end of this meeting all the groups asked to solve this task again.

### 12.3.2.2 Active in Realistic Considerations Only

Following their request, the groups were given the opportunity to work again on the *Class party* task. While they started to work, Sami suggested a criterion for choosing the products “Pita is most important, let’s order 24 pitas so that there will be an extra one for those who will still be hungry”. Then he added, “Two bags of snacks will be enough if you put them on plates so everyone can take, and we do not need a lot of juice, three bottles will be enough for the whole class”. This time his group mates listened to him and took his suggestions into account.

The next task, *Pizza*, was similar in text complexity to the *Class party* task but included more complex data and more choices. Again, Sami had an important role in *making decisions related to realistic considerations*. For example, he asked his group mates for their favourite extras and he looked in the table to find out which company supplied these. Although he *did not deal directly with calculations*, he showed interest in the written exercises and their solutions. In the presentation in front of the class, Sami listened to other groups, he criticized the solution of one of the groups that did not make (realistic) sense.

Up to this point, as just described, although Sami had become active in his group and in class discussion, his participation only involved realistic considerations. He was still avoiding taking any part in setting up the mathematical model.

### 12.3.2.3 Active in Organization and Partly Active in Setting up the Mathematical Model

The next task, *Cookie bakery* (Appendix 2), was complex and required quite an extensive situation analysis and data organization. The students’ task was to provide orders of different amounts of cookies. They were expected to role play cookie factory workers, make decisions, plan, and prepare efficient delivery of cookies that come packaged in given package sizes.

In the first meeting Sami took upon himself the role of *group work organizer*. At his initiative, he distributed empty pages to the group members. He asked each student to check 20 different numbers. Sami chose to check the small numbers, 1–20. He identified quantities that could be provided including all multiples of four and all



multiples of six (4, 6, 8, 12, 16, 18, 20). However, he did not notice that 10 and 14, combinations of the two package sizes, could also be provided.

Sami took his self-chosen role as an organizer seriously. At the end of this meeting he asked the researcher to keep the pages of all the members of his group in order to continue their work in the next meeting. As we will see, he continued taking upon himself this responsibility in the following lesson.

In the second meeting Sami was the first to ask the researcher for the documentation pages from the previous meeting. He took a blank page and began to collect the data from all the group mates. He suggested “I think we should draw three columns: one for the number of cookies, one to specify whether it is possible to provide or not and the third column to represent how to provide the cookies (number and type of packages)”. His group mates agreed with his suggestion.

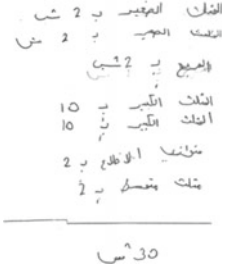
Sami presented the possible quantities that could be provided from the set of numbers he had examined. He said that it was possible to provide: 4, 8, 12, 16, 18 and 20 cookies. A mathematically strong student in the group told Sami that it was also possible to provide 10 cookies in one package of six and 1 package of four and that 14 is also possible using 1 package of six and 2 packages of 4. Sami acknowledged, “Oh, I did not think about that!”.

While checking the rest of the numbers, another mathematically strong student said that 26 was not possible to provide because it is not a multiple of four or six. Sami told her that he thought it was possible, “I found that the 20 is possible, so we will add another one package of 6. 26 could be provided with 5 packages of four and one package of six”. Sami listened carefully to the explanation of Noor, another mathematically weaker student, why 39 could not be provided: “38 cookies is possible: 5 packages of six and 2 packages of four. You cannot provide 39 because you can’t add one cookie. You know that! If it’s odd—you cannot provide it”. Sami wasn’t sure of Noor’s conclusion and began checking some odd numbers and said, “I think Noor is right”, and he suggested writing this conclusion in their group solution: “We cannot provide odd numbers of cookies”. He also said, “We need to check only the even numbers”.

It is interesting to note that Sami related to a solution that was better than his own as something exciting to be learned from. He did not see it as something that undermined his own discoveries, but rather as a group effort to progress together. Similarly, he listened carefully to his group mates and was able to identify, point out, and put on record good ideas.

The third meeting involved group solution presentations. When Sami’s group mates introduced the group’s solution, they asked the class to become active and participate in a simulation of ordering and providing cookies. The students were expected to make orders and the group would figure out “live” how to provide these orders, and if it was possible to provide a certain order at all. Sami actively participated in the role of the seller and was proud of himself for using his generalization about odd numbers and giving an immediate “not possible to deliver” answer when an odd number came up.

**Fig. 12.2** Sami’s pricing suggestion

	2 NIS small triangle 2 NIS small triangle 2 NIS square 10 NIS big triangle 10 NIS big triangle 2 NIS parallelogram 2 NIS medium triangle
<hr/> 30 ש"ח	<hr/> 30 NIS

During working on this task, Sami was very *active in organization and documentation* and he was able to learn and apply the knowledge acquired from his group and class mates.

The next task, *Tangram*, required pricing each part of the Tangram game-set under the constraint that the price of a complete set would be 30 New Israeli Shekels (NIS). Saying “I do not like shapes”, Sami resisted and withdrew when he saw the geometric shapes which were associated with the task.

Nevertheless, while the group started working Sami recommended pricing the parts by categorizing them into two sizes: large and small. The idea was to price each large part (the two large triangles) at 10 NIS ( $2 \times 10 = 20$ ) and each other part 2 NIS ( $5 \times 2 = 10$ ). The members of his group accepted his idea and priced them accordingly (see Fig. 12.2).

Following this first solution one of the mathematically stronger students in the group noticed that there were 3 different sizes and suggested another pricing solution: square 5, medium triangle 3, large triangle 7, parallelogram 3, small triangle 2.5. Sami liked this solution and said, “That is right, I noticed that two small triangles cover one square”.

In the class presentation, another group priced the shapes according to the exact ratio between the shapes areas. Their solution was: big triangle 8 NIS, parallelogram 4 NIS, medium triangle 4 NIS, square 4 NIS and small triangle 2 NIS. The total price of the various tangram parts, in this solution, amounted to 32 NIS. Sami noticed from their solution that the three shapes (parallelogram, medium triangle and square) are the same size. He said to his team members, “We did not do right”. A mathematically weaker student in this group, justified the fact that the total price of the seven pieces was higher than the price of the complete set (30 NIS), because the sale was in individual parts. Sami agreed with her and said, “It is always like that, when you buy in parts it is more expensive than buying the same parts as a set”.

Despite his initial dislike of the geometric shapes, Sami was an active partner and even dared to offer a pricing model for the group. His ability to learn from other students was evident during all the task stages. The task and the group work helped Sami overcome his dislike of geometry, as he noted at the end of this task, “At the beginning of the task I had a fear of the shapes but now I’m feeling that I even do like a little bit geometry”.

#### 12.3.2.4 Dominant in All Stages

All the tasks that were given after the *Tangram* task were more complex in terms of numbers and data types. In all these tasks, Sami was dominant in organizing the work of the group. He contributed a lot to the mathematical and realistic considerations and to setting up the mathematical model. He also played a significant role in documenting and presenting the product to the entire class. It should be noted that he worked out of interest and learned from his class mates and applied the new knowledge. For example, in the *School time* task (Maaß and Mischo 2011) no data were given and the students asked to verify the claim, “Most of the year is spent at school”. Sami was dominant in analysing the situation, finding out relevant data and in defining concepts such as: most of the year, school time, home time, etcetera. He was also dominant in setting up the mathematical model and in the documentation.

Another example that reflects his dominance at all stages of the solution is the *Volleyball* task. This task was based on a task of Lesh and colleagues (Zawojewski et al. 2003) and it was given after Sami and other students’ request to have a task that dealt with sport. The task is a complex task requiring decisions on relevant factors and how to weight them taking into account quantitative and qualitative data. In this task, the students were requested to divide 15 players into three groups.

As in the previous tasks, Sami took on the role of the organizer. He suggested that first of all each one of them look individually at the data and divide the players into three groups and then they decide together on the final solution.

Sami looked at the overall players’ points, chose the three players with high scores and listed each in a separate group, same for the three weakest players with low scores. The rest of the players, he divided among the three groups taking into account the opinion of the coach. For this dividing, he used mapping using three different marks, 1-2-3 to distinguish three categories, as can be seen in Fig. 12.3.

At the end of the individual work they moved on to set up a group model. At this stage Sami noticed that one of the students made a division according to the order of players in the given table without taking into account any of the players’ data. Sami explained to this student that strong and weak players must be mixed because otherwise, there will be a weak team that will lose all the time and a strong team that will win all the time. Sami suggested that he would show to his group his solution and they would make changes if it was necessary. As a result of a dispute between the members of the group about the dividing of the players, he drew a new table and wrote down the names of the players after he obtained the consent of all members of the group.

In the presentation in front of the class, he noticed that other groups calculated the sum of the points for each player and used it as a tool to compare between the players. Sami was excited and told his group mates, “This way is easier than ours, we can know easily which player is strong and which is weak, the dividing of the players in this way will be surely fair”.

As described above in this task, which was the last one in the sequence, Sami had a significant role in *analysing the situation, setting up a mathematical model,*

مطويات عن اللاعبين الذين سيشاركون في مخيم كرة الطائرة.

توصية وتقييم المدرب	العدد الإجمالي بالقفل إلى الأعلى بالمستويات	عدد النقاط التي جمعها كل لاعب خلال لمر 6 لعب					اسم اللاعب	
		لعبة 6	لعبة 4	لعبة 3	لعبة 2	لعبة 1		
ممتاز وجدي	26 سم	2	1	4	5	3	رامي	1
مربع	26 سم	2	3	4	3	2	مبارك	3
متحبط	26 سم	0	4	3	2	1	يوسف	1
مربع	30 سم	5	5	6	5	6	جادي	1
خفيف الحركة	24 سم	2	1	1	2	3	مسلم	2
ممتاز وجدي	29 سم	5	4	5	7	6	روبي	3
مربع	26 سم	2	0	5	4	3	ليزر	2
ممتاز	30 سم	6	6	6	5	4	شوم	1
مربع جدا	30 سم	7	6	6	6	5	وسام	2
خفيف الحركة	26 سم	3	4	5	4	3	راشد	1
خفيف الحركة	27 سم	6	5	4	2	0	موسى	2
ممتاز وجدي	29 سم	6	5	4	5	6	رامي	3
مربع	25 سم	3	4	2	1	2	صباح	3
مربع ممتاز	30 سم	7	6	7	6	6	يونس	3
ممتاز وجدي	27 سم	5	0	8	1	5	بطل	2

Fig. 12.3 Sami’s mapping during dividing the players into 3 groups with the annotations on the right indicating the assigned group number of the player in that row

reasoning, validation and documentation, as well, his ability to learn from his group mates stood out in this task.

To check if the development of the modelling competencies in the post-test were not affected by the identity between the post-test and the pre-test, a similar task *Farm visit* with complicated data and different activities was given to the students. Sami’s results for this task (see Fig. 12.4) were identical to the results of the post-test.

### 12.3.3 Sami’s Progress in Mathematical Knowledge

Beside Sami’s development of modelling competencies there was also a development in his mathematical knowledge as can be seen in the results of his post-tests for mathematical knowledge in multiplication and fractions: Fig. 12.5 presents Sami’s pre-test answer to a question that diagnoses student conception of an array by asking the students to determine the number of small rectangles in the figure. Figure 12.6 presents his post-test result for the same question. It is important to note that in the pre-test Sami used a counting strategy to find the number of rectangles in the figure.

القرار	التوقيت	بالتوقيت	ملاحظة
لا أريد أن أصعب نفسي لأشياء أخرى	9:00	8:30	كرة قدم
أفضل ألعاب الماء	10:00	9:00	الغاب باهية
لألعاب الماء الممتلئة	10:30	10:00	الغاب ليع
لأنني أريد أن أصعب نفسي لأشياء أخرى	11:00	10:30	شدة العمل
أفضل كبروا إن شاء	11:30	11:00	كرة سلة
لأنني سريع	12:30	11:30	مباراة الرغبي

Activity	Starting time	End	Decision
Soccer	8:30	9:00	Because I want to play another game
water games	9:00	10:00	I like water games
Inflatable games	10:00	10:30	Inflatable games are fun
Rope games	10:30	11:00	I play with friends
Basketball	11:00	11:30	I like basketball
Running race	11:30	12:30	I am fast

Fig. 12.4 Sami’s work on the modelling competencies post-test

كم تربيعة كهذه 62 يوجد في الرسم؟

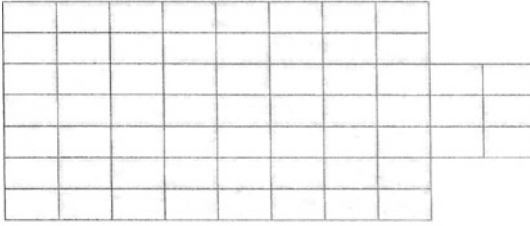
8	7	6	5	4	3	2	1		
16	15	14	13	12	11	10	9		
24	23	22	21	20	19	18	17	58	57
32	31	30	29	28	27	26	25	60	59
40	39	38	37	36	35	34	33	62	61
48	47	46	45	44	43	42	41		
56	55	54	53	52	51	50	49		

Fig. 12.5 Sami’s answer in the multiplication pre-test

In the post-test, he used his multiplication knowledge to figure out the number of rectangles.

Another example from the Fraction pre- and post-tests is given in Figs. 12.7 and 12.8. In these figures, we can see Sami’s answer to the question: “Given 2 identical containers  $\frac{1}{2}$  of the first and  $\frac{2}{3}$  of the second are filled with oil. Is it possible to transfer all the oil from the first to the second?”

كم تربيعة كهذه 62 يوجد في الرسم؟



$$7 \times 8 = 56 + 6 = 62$$

Fig. 12.6 Sami's answer in the multiplication post-test

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

Fig. 12.7 Sami's answer in the fraction pre-test



$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{2}{3} = \frac{4}{6}$$



4/6 of the second container is full with oil

yes it is enough and remain 1/6



3/6 of the first container is full with oil

Fig. 12.8 Sami's answer in the fraction post-test

In the pre-test, Sami wrote a multiplication exercise with the two numbers without understanding and analysing the situation. He did not give any reasoning for his solution and did not attribute any realistic significance to its result. On the other hand, in the post-test, he demonstrated a deeper understanding of the situation, which helped him to set up a realistic mathematical model.

Since the class continued learning according to the regular curriculum, the improvement in Sami's mathematical knowledge could be a result of the regular work rather than an effect of the task sequence. Therefore, the performance of Sami and the other eight students with mathematical difficulties was compared with a similar fifth grade taught by the same teacher. This class served as a control group and included 10 students with learning difficulties in mathematics. While the performance of the nine students in the modelling group increased in both concepts, the 10 students in the control group exhibited minimal knowledge development of both concepts.

## 12.4 Discussion

Our research examined the effect of a modelling task sequence on the development of modelling competencies and mathematical knowledge among students with difficulties in mathematics. In this chapter, we focused on one student, on Sami's development. Sami was one of the seven students for whom we found a significant development in modelling competencies and in mathematical knowledge. His mathematics teacher described him, before the beginning of the research, as a passive and unmotivated student in mathematics lessons. The teacher's description was compatible with the characteristics of students with difficulties as described by Peled (1997), Ginsburg (1997) and Bachor and Crealock (1986).

At the beginning of the process, Sami was passive and his role in the group work was insignificant. Soon afterwards, Sami started to use his daily life experiences in making realistic considerations suggesting factors that might be taken into account in a given situation. As the work progressed, he became active in organization of the group work and also started making mathematical suggestions with regard to possible representations or calculations. Close to the middle of the process, Sami became an active participant in working with the group at all stages of the modelling process, from the *analysis of the situation* through its *mathematisation* and *validation*.

In addition, Sami's experience with modelling tasks changed his work habits. He changed his own norms with regard to the time one is expected to devote to solving a problem, and especially the time spent on simplifying and understanding the situation, a stage that is crucial for building a sound mathematical model (Schoenfeld 1992). It is interesting to note that the development of the competency of *simplifying and understanding the situation* is also reflected in extending the time devoted for reading and analysing mathematical problems given in the mathematical knowledge post-test. This change might have been one of the factors affecting his mathematical knowledge development. He also began attributing importance to the reasons behind

his own decisions as is seen in both the modelling competencies post-test (Fig. 12.4) and mathematical knowledge post-test (Fig. 12.8).

Not less important was the effect of these changes on Sami's self-image, he saw himself as an active partner, initiator and decision maker, as he expressed in the interviews at the end of the process:

"I was an effective member in the group, in deciding what to choose and how to calculate".

"I had an important role in the group, they listened to my ideas".

"I enjoyed the activities because it was much more than just solving exercises, I could express my opinions and help the group making decisions".

"I started to like geometry"

"I learned useful things so I can manage a store when I grow up".

"I am the king of math".

What caused and triggered all these changes in Sami's knowledge and work habits?

Modelling tasks seem to have raised Sami's interest and encouraged him to become an active participant in the group's work. The nature of the modelling tasks made the situation accessible, but their effect went much beyond that. The problem situation also facilitated the understanding of the mathematical structures that were associated with it.

What this means is that the development of Sami's modelling competencies and mathematical knowledge as well as his shift in motivation and participation were actually a result of the difference in the instructional approach. If Sami managed to undergo these changes following the introduction of a new type of task together with new problem-solving norms, it means that the source of his difficulties, to begin with, was not some cognitive deficit.

## 12.5 Conclusion

Sami's case, supports the more general claim that a significant part of the students who experience failure in mathematics should not be labelled as mathematically disabled. Their low achievements in mathematics might be attributed to instruction that is not adequate for them (Ginsburg 1997; Reusser 2000). Like Sami and his peers, they might be helped by making a curricular and instructional change that involves instruction that is more appropriate and meaningful for them.

Thus, despite the intuitive tendency to avoid giving students with difficulties complex problems, this experience with modelling tasks with these particular students seems to have opened powerful learning opportunities for students weaker in mathematics. It facilitated the development of modelling competencies and the development of mathematical knowledge, thus helping them develop the ability to cope with situations they might encounter in their life.



## Appendix 1

Task: *Fun day*

The school management organized a fun day for the students.

The fun day starts at 8:30 and ends at 12:30 pm and includes activities and shows.

Please plan your sequence of shows/activities.

Comments:

1. There is more than one start time for every activity.
2. It is permissible to take a break up to half an hour only during the fun day.
3. You do not have to participate in all the activities.

Attached below, names, starting time and duration of activities.

## Appendix 2

*Cookie bakery*

The “Magic Bakery” sells chocolate chip cookies in two types of packages: packages of 4 cookies, and packages of 6 cookies.

Imagine that you work at this bakery and people come to buy a quantity of cookies (up to 100 cookies), you have to give them their exact order as soon as possible, if it is possible. Also, you should know how to serve it: number and type of packages.

Try to find a way to help you to provide any order efficiently.

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