

Chapter 10

Adopting the Modelling Cycle for Representing Prospective and Practising Teachers' Interpretations of Students' Modelling Activities



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Abstract Teachers play an important role in determining how students work on modelling activities. In the current study, practising and prospective teachers engaged with modelling activities to develop their ability to identify students' modelling process. The study sought to answer the research question as to how the teachers' participation in modelling affects their interpretation of students' modelling activity. Data included two sets of participants' reports on their observations of a video-recorded modelling activity carried out by a group of five sixth-grade students, pre and post participation in four modelling activities. The findings indicate that prior to engaging with modelling activities, most participants described the students' modelling as linear, noting only the final mathematical model and mathematical results. After participating in the activities, most of the practising teachers' reports and a third of the prospective teachers's reports identified cyclical processes in the modelling.

Keywords Modelling · Modelling process · Modelling cycle · Practising mathematics teachers · Prospective mathematics teachers

10.1 Introduction

Teachers play a pivotal role in fostering modelling among their students (Cai et al. 2014). Teachers' intervention may help students adopt strategies that facilitate the construction of situation models while engaging in modelling activities (Leiß et al. 2010). Effective intervention is related to teachers' subject matter knowledge and

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pedagogical content knowledge during modelling (Doerr and English 2006). For example, the way teachers think about modelling potentially influences the suggestions and ideas raised by their students while engaging with modelling activities (Borromeo Ferri and Blum 2010). Indeed, acquaintance with the modelling cycle can guide teachers towards effective intervention while their students engage with modelling activities (Blum and Leiß 2005). In addition, teachers' knowledge about the modelling process can contribute to their awareness of students following different modelling paths (Borromeo Ferri 2010). As well, this can assist teachers in identifying student blockages in transitioning between stages in the modelling process (Stillman et al. 2007).

Despite the importance of the teacher's role while students engage in modelling activities, studies (e.g. Borromeo Ferri and Blum 2010) indicate that teachers' interventions are mostly intuitive and not constructive. In addition, teachers' professional knowledge about modelling is limited (e.g. Frejd 2012; Lamb and Visnovska 2013). The importance of the role of teachers and their limited knowledge about modelling have led various researchers to suggest that teachers should take courses in modelling (for practising teachers: Maaß and Gurlitt 2011; Mischo and Maaß 2013; for prospective teachers: Anhalt and Cortez 2016; Bukova-Güzel 2011). Some researchers (e.g. Altay et al. 2014; Çetinkaya et al. 2016; Kang and Noh 2012; Shahbari 2018; Tan and Ang 2016) have suggested that practising and prospective teachers should engage in modelling activities as learners in order to become qualified in modelling.

Subsequently, the study proposed is an intervention in which practising and prospective teachers engage with modelling activities as learners. Our aim is to examine whether such an intervention has an impact on the abilities of practising and prospective teachers to identify their students' modelling process. Specifically, we seek to enhance participants' abilities to identify the actions between the phases in the modelling process and to recognize the cyclic nature of their students' modelling processes. We focus on this identification because each transition between phases in the modelling process is considered to be a possible source of blockage (Galbraith and Stillman 2006). Hence, such knowledge is critical for developing strategies to overcome student difficulties while transitioning from one step to the next (Çetinkaya et al. 2016).

We examine teachers' identification of students' modelling processes by adopting the modelling cycle proposed by Blum and Leiß (2005) (elaborated later). This modelling cycle enabled us to use a visual means to describe the cognitive analysis of a modeller's modelling process.

10.2 Theoretical Background

10.2.1 Modelling

Modelling is defined as the two-way process of translating between the real world and mathematics (Blum and Borromeo Ferri 2009). Modelling activities involve the partial, ambiguous or undefined information about a situation (English and Fox 2005) that learners working in small groups need to mathematize in ways that are meaningful to them (Doerr and English 2003). While traditional word problems require only one interpretation of a problem and hence, demand limited mathematical thinking (English 2003), engagement in modelling activities involves iterative cycles of translation, description, explanation and prediction of data outcomes and solution paths (Lesh and Doerr 2003).

The literature describes modelling cycles in various ways (e.g. Borromeo-Ferri 2006). In the current study, we use an adaptation of the modelling cycle proposed by Blum and Leiß (2005), who used a visual means to describe the cognitive analysis of modelling activity. Accordingly, cognitive activity is divided into phases and actions. The phases in our adaptation¹ include a situation model, a realistic model, a mathematical model, mathematical results, and realistic results. [The real situation phase is not included in the analysis for this study]. The actions consist of understanding the problem and simplifying a situation model (combining two actions from Blum and Leiß 2005 following our simplification of the stages); mathematizing, which leads to constructing a mathematical model; applying mathematical procedures that yield mathematical results (i.e. working mathematically); interpreting these mathematical results with respect to the real-world situation; and validating these results with reference to the original situation. If the results are unacceptable, the cycle begins again.

10.2.2 Teachers' Knowledge About Modelling

Teachers consider modelling to be difficult (Blum and Borromeo Ferri 2009). Several types of difficulties with modelling activities encountered by teachers have been identified: difficulty in discussing features of different models (Lamb and Visnovska 2013); lack of adequate knowledge about modelling (Çetinkaya et al. 2016; Chan 2013); and limited experience with the notion of mathematical modelling in mathematics education (Frejd 2012). Conversely, teachers' subject matter and pedagogical content knowledge influence how they interact with their students while participating

¹Editors' note: This simplification reduces the representation of the Blum and Leiß (2005) cycle to that of Maaß (2006, p. 115) if Maaß's "real world problem" is taken to be equivalent to "situation" as used in the diagrams that follow in this chapter and "simplifying" including understanding. Kaiser and Stender's representation of the modelling cycle (2013, p. 279) is an extension of Maaß (2006) where these equivalences have been represented explicitly.

in modelling activities (Doerr and English 2006). Teacher awareness of modelling processes is naturally very important to the support they provide their students in developing modelling competencies (Blomhøj and Kjeldsen 2006). This awareness is also crucial for effectively mediating students' group work during modelling activities (Blum and Leiß 2005). The importance of teachers' understanding of modelling in implementing a modelling approach in their own classroom cannot be underestimated (Kaiser and Maaß 2007).

Teachers' knowledge about modelling activities can be developed through their active engagement in modelling activities (Kang and Noh 2012). Hence, several researchers (e.g. Bukova-Güzel 2011; Mischo and Maaß 2013) have proposed interventions in which practising and prospective teachers engage in modelling activities as learners. Besides having an impact on teachers' development of modelling competencies (e.g. Kaiser 2007), these interventions affect practising and prospective teachers in other ways as well, including teachers' pedagogical knowledge. The teachers develop an understanding of the nature of mathematical modelling, of the relationship between mathematical modelling and meaningful understanding, and of the nature of mathematical modelling tasks (Çetinkaya et al. 2016). Other studies have noted that an intervention made teachers aware of their changing roles in interacting with their students, including a focus on listening and observing, and on asking questions for understanding and clarification (e.g. Doerr and English 2006). Furthermore, Kaiser and Schwarz (2006) reported on affective aspects, such as changes in teachers' beliefs about mathematics.

The current research seeks to examine the effect of engaging in a sequence of modelling activities on the development of pedagogical knowledge among practising and prospective teachers. The study attempts to answer the following research questions:

1. How do practising and prospective mathematics teachers with no prior modelling experience interpret students' modelling activities?
2. From a cognitive perspective, does participation in a sequence of modelling activities change the ways in which practising and prospective mathematics teachers interpret students' modelling activities? If yes, to what extent? Is the effect the same both for prospective and for practising teachers?

10.3 Method

10.3.1 *Participants and Procedure*

Thirty-four practising mathematics teachers and 49 prospective mathematics teachers participated in the current study. The practising teachers taught grades 1–6 at primary schools and were studying towards a master's degree at a college of education. The prospective teachers were studying at a college of education to become primary school mathematics education teachers and were in their second of four years of

study. All participants reported no prior experience with mathematical modelling. The participants were enrolled in a ‘problem solving’ course taught by the first author. Each group (practising and prospective teachers) took the course separately. The courses differed in structure and were similar only in the four lessons during which participants worked on the modelling activities that are the focus of the current study.

The participants were shown a 70-min video recording that documented five sixth-grade students working on a modelling activity known as the *Sneaker activity* from Doerr and English (2003). After watching the video and reading the transcript, participants were asked to write a report describing the work of the five sixth-graders on the *Sneaker activity* from the beginning to the end of their engagement with the task. Participants were instructed to emphasize the cognitive aspects only and to avoid other aspects, such as emotional ones. This first report (R1) was completed and submitted to the lecturer before the participants themselves engaged in a sequence of four modelling activities. After submitting R1, over the course of four lessons the participants worked on four modelling activities. Participants remained in the same group for each activity. Work on each of the four modelling activities took place once a week for approximately 90 min per session. The participants were then asked to watch the same *Sneaker activity* video and write a second report (R2) describing the students’ work on the activity. In total, two sets of 46 reports were collected: 20 reports submitted by practising teachers and 26 reports by prospective teachers.

10.3.2 Modelling Activities During the Intervention

As mentioned, in the time between submitting R1 and R2 the participants worked on a sequence of four activities designed by the researchers. Because these activities are not the focus of the current study, we describe them only briefly.

The first activity was the *Summer Camp activity* (Shahbari and Tabach 2017). Participants were asked to choose the summer camp they deemed most suitable, to suggest a means of choosing suitable camps for the coming years, and to write a letter explaining their decision. The summer camp activity was presented via four tables providing information about six camps, with each table referring to several components, such as dates, transportation, food, and cost of each camp; types and number of entertainment activities at each camp; and parents’ evaluations and ranking of the camps for the previous year.

The second activity was the *Flower activity*, in which an art teacher planned to recreate a picture of a flower (the image of the flower was given) through the participation of all 524 students at the school. In the re-creation, the students were required to wear yellow, green, or brown clothing in accordance with their location in the original picture. The prospective and practising teachers were requested to write a letter to the art teacher explaining how to enlarge the picture so that all the students could participate, how to place the students in the schoolyard, and how many students should be wearing clothing in each color (yellow, green, and brown).

The third activity was the *Toothpaste activity* (Shahbari and Tabach 2016). The participants were told that the opening of their toothpaste tube had been enlarged, and they were asked to write a letter describing how their toothpaste consumption from this tube might have changed compared to the original tube.

The fourth activity, the *Good Teacher activity* (Shahbari and Tabach 2017), also comprised four tables describing ten teacher candidates by providing different information, such as age, performance in practicum work, evaluation during an interview, and more. The participants were asked to choose the most suitable candidates for a teaching position, suggest a means of choosing suitable candidates for the coming years, and write a letter explaining their decision.

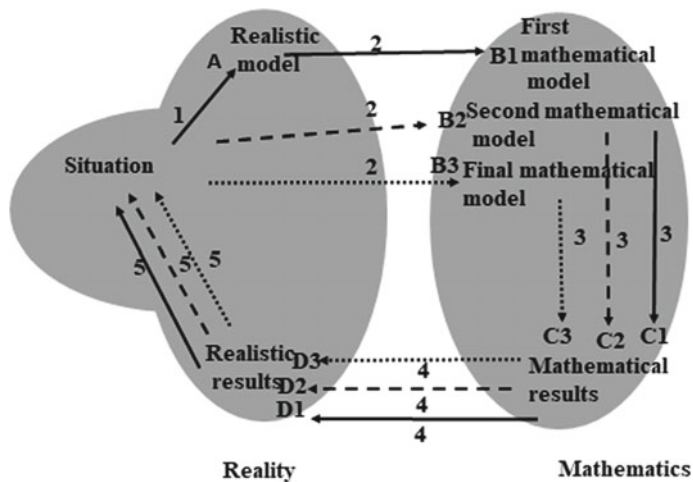
10.3.3 Sneaker Activity

The students who engaged in the *Sneaker activity* (Doerr and English 2003) were in the sixth grade. Their mathematical achievements, as reported by their mathematics teacher, differed. They were selected on a voluntary basis. The students were required to work as a group and answer the questions posed in the activity.

The *Sneaker activity* began with the opening question: ‘What factors are important to you for when you buy a pair of sneakers?’ After students discussed the factors they deemed important, they were given a list of ten factors (brand, size, comfort, fashion, purpose, grip, colour, quality, style, and cost) that are considered important in buying a pair of sneakers. The students were then asked to determine how to order these factors in deciding which pair of sneakers to purchase. Solving this assignment involved generating an ordered list of factors. For the current study, the students were given two additional lists of the same factors that were ranked differently than their list. They were told that these lists belonged to two other groups. In the original activity, each group in the class has its own list, so the number of lists is equal to the number of groups. In the current study we prepared two other lists such that all three lists were different. The final assignment was for the students to build a single list representing the priorities of all three lists.

10.3.4 Analyses of Students’ Modelling Activity

The solution path of the group of five sixth-graders in the *Sneaker activity* was analysed using our adaptation of the modelling cycle of Blum and Leiß (2005) and our analyses described in a visual diagram (see Fig. 10.1). The numbers in Fig. 10.1 indicate the modelling actions: (1) *understanding and simplifying*; (2) *mathematising*; (3) *applying (i.e. working mathematically)*; (4) *interpreting the mathematical results*; (5) *validating*. The letters indicate the modelling phases: (A) *real model*; (B) *mathematical model*; (C) *mathematical results*; (D) *realistic results*. There were three mathematical models (B1–B3), three different sets of mathematical results (C1–C3),



The first modelling cycle (action and phases): (1, A, 2, B1, 3, C1, 4, D1, 5)	—————>
The second modelling cycle (actions and phases): (2, B2, 3, C2, 4, D2, 5)	- - - - ->
The third modelling cycle (actions and phases): (2, B3, 3, C3, 4, D3, 5)>

Fig. 10.1 Researchers’ analyses of the solution path of the sixth-graders’ activity using an adaptation of the modelling cycle of Blum and Leiß (2005)

and three sets of realistic results (D1–D3). The combination of letters and numbers (e.g. B1) in Fig. 10.1 indicates the number of the modelling cycle during each phase.

Analysis of the *Sneaker activity* indicates that the sixth-graders’ solution included three modelling cycles, as indicated by the three types of arrows (see Fig. 10.1). The first modelling cycle included (1) interpreting the situation; (2) mathematising—quantifying the factors and trying to connect between the three lists, eliciting the first mathematical model (B1), dividing the factors into two groups of five factors each that contained the most important and the least important factors, and then ranking each group by using frequencies; (3) applying the frequency model to the situation to yield mathematical results (C1); (4) interpreting the mathematical results to yield realistic results (D1); and (5) validating the realistic results obtained by applying the frequency model. This is represented in Fig. 10.1 using solid arrows (→). The validating process revealed the following problem: Some factors were among the first five factors on some lists and among the last five factors on other lists. This problem led the students to begin a second modelling cycle.

The steps in the second modelling cycle were: (1) returning to the main situation; (2) mathematising—dealing with the number of factors and eliciting the second mathematical model (B2) using the average; (3) applying the average model to elicit mathematical results (C2); (4) interpreting the mathematical results in the situation, yielding the realistic results (D2) of applying the average model; and (5) validating the realistic results of applying the average model. This is represented in Fig. 10.1

Table 10.1 Sample analysis of participant's report

Participant's report	Researchers' analyses
"First the students read the problem and try to discover what is required of them... the students understand what the problem is about"	The participant pointed to the students' simplification and understanding of the action. (Action 1)
"the students understand the need for searching and organizing one list that represent the three lists ... we must integrate the three lists"	The participant's description identified the real model of the situation. (Phase A)
"they applied several mathematical strategies, they give each factor a value"	The participant reported on the mathematisation process in the third modelling cycle. (Action 2)
"after a long discussion they agreed to use the average as a factor in the three lists... the average of the value that each factor gets in the three lists ... and in the case of equal averages they decided to use estimation, ... average and estimation was the strategy they used"	The participant reported on the final mathematical model (B3)—the use of average and estimation
"the students get answers by using the strategy, they get a value for each factor..., the size is ten, the colour is eight... the average of size is 9.66"	The participant reported on applying the mathematical model and the mathematical results (C3): "the biggest average is 9.66"
"they ranked the factors ... the first and most important is size"	The participant reported on the interpretation of the mathematical results and the realistic results (D3): "the most important is size"

using dashed arrows (- ➔). This cycle revealed that some factors have the same average, leading the students to return to the situation for a third modelling cycle.

The third modelling cycle included the following steps: (2) mathematizing—dealing with the number of factors and eliciting the final mathematical model (B3) through the use of average and estimation; (3) applying the final model to elicit mathematical results (C3); (4) interpreting the mathematical results in the situation to elicit realistic results (D3); and (5) validating the realistic results obtained from applying the final model. This is represented in Fig. 10.1 using dotted arrows (...➔).

10.3.5 Data Analysis of the First and Second Reports

The reports of the practising and prospective teachers were analysed by the authors. All reports were examined to see whether the participants referred to each of the modelling phases (A–D) and modelling actions (1–5) and were then compared to the researchers' analyses, as shown in Fig. 10.1. Table 10.1 provides examples from the participants' reports (left column, translated into English by the researchers) and the researchers' analyses (right column).

After coding each report, a visual representation was created for each report to facilitate identification of changes between the first report set (R1) and the second report set (R2) and to characterize the changes in the group. Finally, we assigned each report to one of three levels. Reports that included only one modelling cycle were assigned to the first (lowest) level; reports that included two modelling cycles were assigned to the second (middle) level; and reports that included three modelling cycles assigned to the third level. Each level was divided into different sub-levels according to the identified phases and actions.

10.4 Findings

First, we visually present the participants' descriptions of the students' modelling process in the two sets of reports. We arranged our findings in the order of their complexity, from less to more complex. We then discuss the modelling phases and actions in all of the reports.

10.4.1 *Participants' Descriptions of Students' Modelling Process in R1 and R2*

Our analysis of the practising and prospective teachers' reports yielded nine different categories of analysis of the solution path (Fig. 10.2a–i) describing the students' solution paths. The visual representation of one was presented in Fig. 10.1 and the other 8 are presented in Fig. 10.2a–h.

At the first and lowest level of complexity, the students referred to only one modelling cycle. For this level, we identified three sub-levels (a–c) in the participants' reports. Sub-level (a) represents the simplest description in which the participants referred to presentation of the situation and the final mathematical model (B3) while ignoring all the other modelling phases. At the next sub-level (b), the participants' description in the report considers the situation interpretation, describes the real model (A) of combining the three lists into one list, considers the mathematical work of quantifying the factors, and discusses the relations between them. In the report, the participants also consider the final mathematical model (B3)—the use of average and estimation—and describe the mathematical results (C3) elicited by applying the final mathematical model. In the third sub-level (c), the participants' reports considered all the phases and actions of the third modelling cycle from sub-level (b), and in addition considered the realistic results (D3) obtained from interpreting the mathematical results to reality.

Table 10.2 provides visual descriptions of the three sub-levels of the first level, in which the students referred to one modelling cycle only. In addition, it details the numbers of teachers producing a report of the students' modelling process that

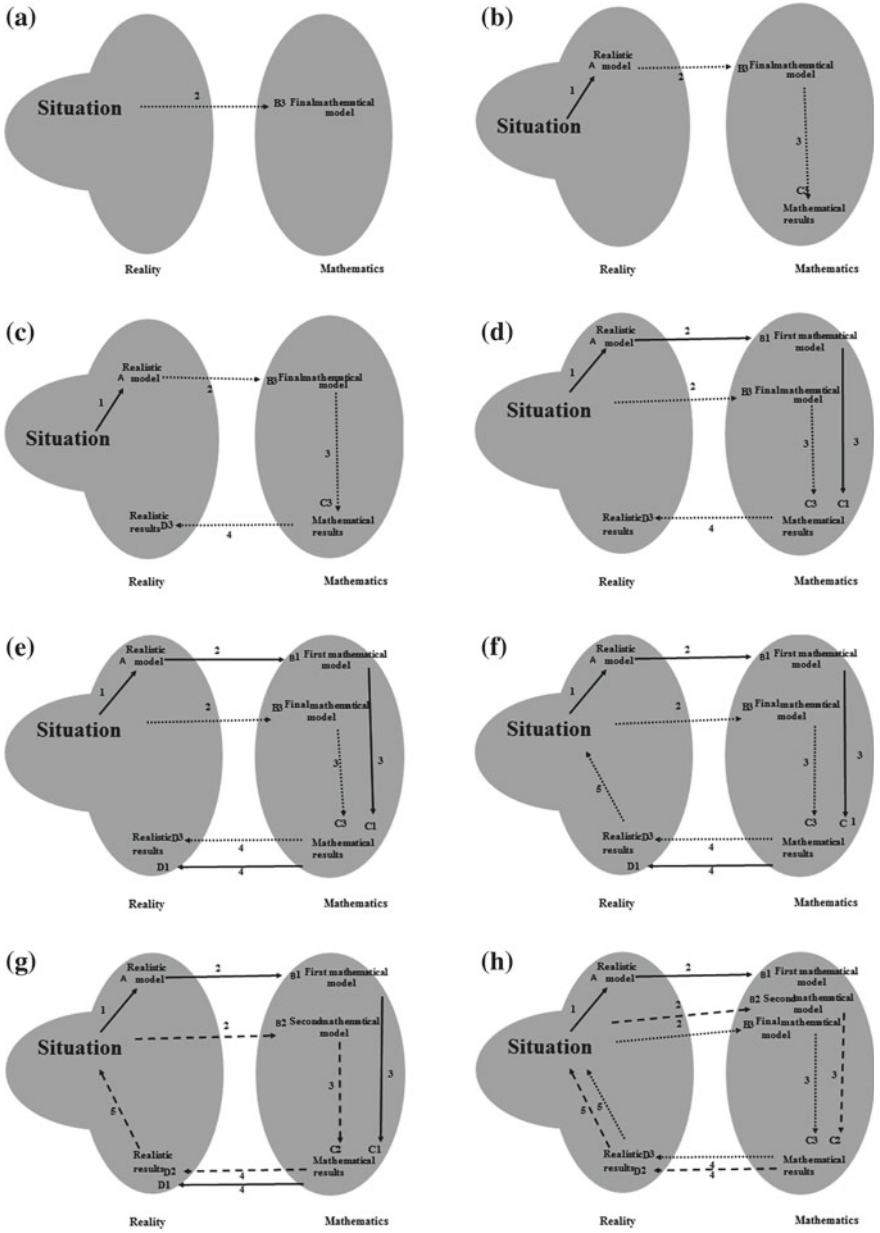


Fig. 10.2 Eight visual representations of the teacher analyses

Table 10.2 Distribution of the analyses in the reports of practising teachers (N = 20) and prospective teachers (N = 26) for the first level: sub-levels (a–c)

Groups	R1	R2	Visual representation
Practising teachers	8	1	<p>a</p>
Prospective teachers	10	3	
Practising teachers	7	–	<p>b</p>
Prospective teachers	12	7	
Practising teachers	2	–	<p>c</p>
Prospective teachers	3	8	

was categorised in this way for both R1 and R2. For example, the reports of eight practising teachers and 10 prospective teachers were classified at the lowest sub-level (a) for their initial report. After the intervention this number reduced to one and three respectively.

At the second level of complexity, the participants’ reports included descriptions of two modelling cycles. Three of these descriptions considered the first and the third modelling cycles without describing the second modelling cycle, while one description considered the first and the second modelling cycles without reporting the third

modelling cycle. This second level was separated into four different sub-levels (d–g). Sub-level (d) represents participants' reports that considered the first modelling cycle, which included working mathematically and deriving the first mathematical model by dividing the factors into two groups of five factors each—the most important and the least important; and considered the third modelling cycle [as in sub-level (c)], including the mathematical work, the final mathematical model (B3), the mathematical results (C3) obtained from applying the final model and the realistic results (D3). At sub-level (e) the reports indicate that the participants recognise that the task solvers worked through the first and third modelling cycles, but do not recognise any validating processes. At sub-level (f), the reports considered the first and third modelling cycles but did not consider the validating process of the first modelling cycle. At the sub-level (g), the reports point to recognition of the first and second modelling cycles but do not consider the validating process for the first modelling cycle. The visual descriptions of the four sub-levels of the second level are given in Table 10.3, and the data for the participants whose reports were classified this way.

At the third and highest level of complexity, the participants' reports included descriptions of the three modelling cycles. This level included two sub-levels: (h) and (i). At level (h), the reports show that the participants identified the second and third modelling cycles, considered all the phases and actions, but identified the mathematical work only in the first modelling cycle. Finally, sub-level (i) is the expert model, similar to the researchers' description in Fig. 10.1. At this sub-level, the reports included description of all the phases and actions in the three modelling cycles. In other words, the participants' reports considered the whole modelling process as elaborated in our explanation of the students' modelling activity in the method section. Table 10.4 shows the visual descriptions of the sub-levels of the third level, and the data for the participants whose reports were classified this way.

The data in each of Tables 10.2, 10.3 and 10.4 show the changes in the descriptions of practising and prospective teachers from the first to the second reports. Table 10.5 summarizes the distribution of the three levels of description between the first and the second reports.

The results in Table 10.5 indicate that by the second reports, the two groups were able to observe more modelling cycles. In addition, in the second reports a higher proportion of practising teachers observed more modelling cycles than the prospective teachers.

The analyses of the participants' first reports (R1) according to the specific phases and actions indicate that their attention was focused mainly on the third modelling cycle, as all the first reports (R1) included descriptions of the final mathematical model. In addition, three-fifths of the practising teachers' first reports (R1) considered the mathematical results of applying the final model (i.e. visual representations b, c, e, and h) and three-fifths of the reports considered the realistic model (i.e., visual representations b, c, e, and h). More than three-fifths of the prospective teachers' first reports considered the mathematical results of applying the final model (i.e., visual representations b, c and e). Little attention, however, was devoted to the modelling phases or to actions related to the first two modelling cycles. In contrast, the descriptions in the second reports (R2) pay attention to the three modelling cycles relative

Table 10.3 Distribution of the analyses in the reports of practising teachers (N = 20) and prospective teachers (N = 26) for the second level: sub-levels (d–g)

Groups	R1	R2	Visual representation
Practising teachers	–	4	<p>d</p>
Prospective teachers	–	–	
Practising teachers	2	–	<p>e</p>
Prospective teachers	1	–	
Practising teachers	–	4	<p>f</p>
Prospective teachers	–	–	
Practising teachers	–	1	<p>g</p>
Prospective teachers	–	–	

Table 10.4 Distribution of the analyses in the reports of practising teachers (N = 20) and prospective teachers (N = 26) for the third level: sub-levels (h) and (i)

Groups	R1	R2	Visual representation
Practising teachers	1	3	
Prospective teachers	–	3	
Practising teachers	–	7	i Expert model: similar to researchers’ analysis (see Fig. 10.1)
Prospective teachers	–	5	

Table 10.5 Summary of classification by level

Level	Practising teachers (N = 20)		Prospective teachers (N = 26)	
	R1	R2	R1	R2
First level	17 (85%)	1 (5%)	25 (96%)	18 (69%)
Second level	2 (10%)	9 (45%)	1 (4%)	–
Third level	1 (5%)	10 (50%)	–	8 (31%)

to the modelling phases and actions in them. More attention was devoted to the first and third mathematical modelling cycles than to the second modelling cycle. The results also show that compared to other actions, the least amount of attention was directed toward validating processes in the three modelling cycles. The findings obtained from the analyses of the two reports among the two groups indicate that the practising teachers identified more phases and actions in both the first and the second sets of reports.

10.5 Discussion and Conclusion

The current study examined how participation in four modelling activities affected how practising and prospective teachers interpret students’ modelling activities. The main findings indicate that the modelling cycle adapted from Blum and Leiß (2005)

for this study made it possible to closely monitor the modelling phases and actions emerging from practising and prospective teachers' interpretations of students modelling activity. The use of this adapted modelling cycle to visualize the interpretations of practising and prospective teachers enabled us to monitor which specific phases and actions were noted or ignored and to identify the changes in their interpretation.

The findings indicate that before participating in the modelling activities, the practising and prospective teachers were unable to identify and document the students' entire modelling process. Most of the descriptions in the first reports considered the final mathematical model and the mathematical results of applying this model, while overlooking the realistic results and the validating process. In addition, most of the first reports disregarded the second and third mathematical modelling cycles and the modelling phases and actions related to these cycles. The practising and prospective teachers emphasized the final model without considering the first and second modelling cycles, indicating that they considered the solution path to be linear (typical of school problems). In other words, both the practising and the prospective teachers expected to see a specific computational solution rather than a more general strategy, as discussed by Doerr and English (2006). Furthermore, the practising and prospective teachers may have emphasized the final mathematical model because they were expecting it to be the result of the students' work. This observation is in line with the findings of Blum and Borromeo Ferri (2009), who reported that teachers impose their preferred solution through their intervention while students engage in modelling activities.

The active participation of practising and prospective teachers in modelling activities contributed to their awareness of the modelling phases and actions and of the processes by which mathematical models progress. Findings from our analysis of the second reports indicate that after their participation, more practising and prospective teachers considered the three modelling cycles and the cyclic process of the mathematical models' progress. These results are in line with those of Tan and Ang (2013), who reported that experience with modelling activities enhanced teachers' knowledge of different elements in modelling process phases. Our findings show that the validating process was the least recognised action described by the practising and prospective teachers in both sets of reports. It is important to note that this process is considered to be the most difficult for students when undertaking modelling activities, because in a 'regular' classroom activity it is the teacher who is responsible for the correctness of the solutions (Blum and Borromeo Ferri 2009).

Finally, the findings indicate that practising and prospective teachers who participate in modelling activities can develop modelling lenses for themselves that find expression in the way they consider their students' modelling. These modelling lenses can help teachers observe and monitor the modelling process more precisely. In addition, the findings indicate that the effect of engaging in a sequence of modelling activities was greater among the practising teachers. The difference between the two groups may be attributed to the fact that practising teachers have more comprehensive knowledge about students' problem-solving processes. They are likely to have gained some of this knowledge from their general experiences teaching math-

ematics. Another explanation for the differences can be attributed to the practising teachers being participants in higher degree studies, namely, a master's degree.

In light of the study findings, we recommend the use of the modelling cycle of Blum and Leiß (2005), or our adaptation of this,² to describe from a cognitive perspective how practising and prospective teachers interpret students' modelling activity. The use of the modelling cycle enabled us to monitor the modelling phases and actions that emerged when the practising and prospective teachers interpreted the students' modelling activity and to observe their professional development. In addition, we recommend integrating courses about mathematical modelling into professional teachers' programs in which teachers actively engage in modelling activities as participants. Based on the results of this study for prospective teachers, we recommend that dealing with interpretation of students' modelling activity be postponed until they have gained experience as practising teachers.

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²Editors' note: Please note other cycles such as Kaiser and Stender (2013, p. 279) capture this modification and so would also be appropriate.

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