Chapter 7
Assessment and Structure of Secondary Students' Personal Meaning Related to Mathematics

Maike Vollstedt and Christoph Duchhardt

Abstract Students are in need of meaning when dealing with mathematics in a school context. To describe meaning, objective and personal facets need to be distinguished. This paper focuses on students' perspectives in describing the concept of personal meaning as personal relevance of a mathematical concept or action. To conceptualise personal meaning for quantitative studies, a survey was developed that makes 16 different personal meanings assessable with reliable scales. Results from cluster and correlation analyses generally support the underlying theory of personal meaning. Exploratory factor analyses suggest two meta-factors structuring the different personal meanings into those showing an orientation to mathematics and to social inclusion, respectively. Future research intends to investigate the relationship between personal meaning and affective psychological constructs like motivation and interest.

Keywords Personal meaning in mathematics · Orientation to individual · Orientation to mathematics · Orientation to social inclusion · Survey construction · Item response theory (IRT)

7.1 Introduction

The aim for meaning in education has been pervasive for many years and meaningful learning is assumed to be a fundamental drive (Biller, 1991) as well as one of the major goals (Vinner, 2007) of education. Hence, one of the challenges of (mathematics) education is to find convincing responses to the quest for meaning as well as to develop learning environments that foster meaningful learning. In addition, it is important to study personal meaning to comprehend how students relate mathemat-

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ical contents to their biography so as to better understand their learning processes (Meyer, 2008). Yet even when only the field of mathematics education is considered, the notion of meaning is complex and multifaceted. This article elaborates on a facet that considers the perspectives of the students and asks what is personally relevant for them when they are involved with mathematics in a school context. Vollstedt (2011c) names this facet *personal meaning* (c.f. also Vollstedt, 2010a, 2011a). In a former qualitative study (see Sect. 7.2.2, cf. Vollstedt, 2011c), 17 different personal meanings were reconstructed from interviews with secondary students. Subsequently, a survey was constructed with the aim to assess those different personal meanings. This paper reports on the study in which (1) the survey was tested whether it is a reliable instrument for the assessment of those 17 personal meanings; and (2) the theoretical model of personal meaning as presented in Vollstedt (2011c) was empirically tested on a larger data set.

7.2 Theoretical Framework

It is generally accepted that humans have a need for meaning (e.g. Biller, 1991; Bruner, 1991; Frankl, 1988; Heine, Proulx, & Vohs, 2006; Vinner, 2007). Accordingly, the question for meaning is dealt with in very different scientific communities among which philosophy may be the most fundamental one (Kilpatrick, Hoyles, & Skovsmose, 2005b). Skovsmose (2016) gives a brief overview of philosophers that developed theories of meaning with a paradigmatic interpretation from Plato in antiquity to Frege and Wittgenstein in the 19th and 20th centuries. In addition, the question for meaning in life is intensively discussed in different fields of psychology (e.g. Frankl, 1984; Heine et al. 2006; Reber, 2018; Schnell, 2011).

In general, Heine, Proulx, and Vohs characterise meaning from a psychological perspective as relation between people, places, objects, and ideas:

> In simple terms, meaning is what connects things to other things in expected ways—anything and any way that things can be connected. Meaning is what connects the people, places, and things around oneself: hammers to nails, cold to snow, mothers to daughters, or dawn to the rising sun. Meaning connects elements of the self: thoughts, behaviours, desires, attributes, abilities, roles, and autobiographical memories. Meaning is what connects people to that which lies beyond the self: the people, places, and things that surround them. Meaning can come in as many forms as there are ways to relate these elements of perception and understanding (2006, p. 90).

In the basic assumptions of their Meaning Maintenance Model (MMM) they propose that humans seek meaning in three domains: the external world (combining the beliefs about the outside world), the self (combining beliefs about oneself to arrive at a coherent self across time as well as across roles and contexts), and the self in relation to the outside world (the desire be part of a coherent cultural worldview). Hence, the need for meaning is related to all aspects of human life and, thus, it is only reasonable to assume that children and adolescents also strive for meaning at school. Although the meaning of life and meaningful learning may constitute different notions, they may well relate to the same source (Vinner, 2007). Accordingly, meaning also plays an important role in education (Reber, 2018). In the following, the focus will thus be on meaning in education with special attention to mathematics education.

7.2.1 Aspects of Meaning

The fundamental role of meaning in education is indisputable and the term *meaning* is used in a very big share of the papers that have recently been published in mathematics education journals.[1] However, hardly any paper does consider meaning as a central focus of study so that one can still argue that the notion of meaning has not yet received the focus that it deserves in mathematics education (Skovsmose, 2016; Thompson, 2013). One of the reasons why meaning is not discussed more may be related to a difficulty inherent in the concept itself: it is hard to capture conceptually. This may also be the reason why there is yet no commonly accepted definition for the term *meaning* in the field of mathematics education (Kilpatrick, Hoyles, & Skovsmose, 2005c; Thompson, 2013). On the contrary, there is a rich diversity of meanings for *meaning*. A resulting problem is that different interpretations of the term *meaning* are often used synonymously, which—strictly speaking—are not synonymous at all: a global understanding in contrast to different facets which each focus on a certain aspect of meaning.

Based on the discussions within the BACOMET-group, Kilpatrick et al. (2005c, pp. 12–15) present different facets of meaning of a mathematical concept content *X*, which are heavily linked: the meaning of *X* from a content perspective (as a content to be taught and as a content of reference from mathematical theory), the meaning of *X* within different spheres of practice (e.g. in scientific mathematics, in applications, or in the mathematics classroom), and the meaning of *X* from the perspectives of the different individuals involved in its construction (the student's vs. the teacher's point of view). They conclude:

> These views are actually different meanings of meaning insofar as different methodological tools are needed to explore them, different theoretical frameworks, etc. They insist on several different dimensions of meaning: psychological, social, anthropological, mathematical, epistemological or didactical. But all these dimensions must not be seen as isolated, one from the other. In fact they constitute a system of meanings whose interactions shape what may be seen as *the* meaning of a mathematical concept (2005c, pp. 14–15).

Another approach to defining meaning comes from an interdisciplinary group of educational scientists (Birkmeyer, Combe, Gebhard, Knauth, & Vollstedt, 2015). In

[1]The authors have exemplarily checked all original articles (except editorials, call for papers, book reviews etc.) that were published in 2018 for the journals *Educational Studies in Mathematics* (ESM) and *ZDM Mathematics Education*. From 49 articles published in ESM, 42 refer at least once to one facet or the other of meaning; for ZDM, 81 out of 93 articles refer to meaning. This means that more than 85% of the articles deal with meaning in one way or the other. [We want to give credit to Rudolf Sträßer who gave us the idea of counting articles in the first place].

a theoretical article, they present ten theses to build up a theoretical, conceptual, and pragmatic basis for describing the constitution of meaning. They relate meaning to cognition and affect: For the individual learner's acts of consciousness, meaning denotes a dimension that—apart from the realms of experience and action—focuses on a sphere of self-assuring clearance and clarification in the process of learning. The achievements of the consciousness with respect to giving meaning, as well as its affective embedment, generate effects of meaningfulness in learning processes, which are more or less distinct or can be experienced as such. Hence, it is also a matter of an inner psychic experience of meaning. This is neither sensation only nor thinking without emotion, and neither pure and isolated cognition nor knowledge that is independent from consciousness.

Following this description, the global concept of meaning has a dialectical relation to psychological as well as cognitive aspects. Both aspects contribute to the development of one's own identity: when something is meaningful to an individual, the content somehow makes sense for him or her (in terms of sense-making and understanding) and she or he gains orientation from it (in terms of understanding oneself and the development of one's own identity) (Birkmeyer et al., 2015). Hence, meaning is something other than pure sense-making of the content as it additionally relates the content to the individual's identity and biography. This relation is elaborated in the description of the relational framework of the study (see Sect. 7.2.2.1 below).

The differentiation between cognitive and affective aspects of meaning also becomes obvious when one considers that "even if students have constructed a certain meaning of a concept, that concept may still not yet be 'meaningful' for him or her in the sense of relevance to his/her life in general" (Kilpatrick et al., 2005c, p. 14). The first kind of meaning mentioned in the quotation involves a cognitive interpretation as the construction of a concept's meaning and usually involves sense-making processes. The second kind of meaning, however, is of affective nature as the relevance of the concept is related to the personal life of the individual. Generally speaking, two very different aspects of meaning can be distinguished here, namely "those relating to relevance and personal significance (e.g., 'What is the point of this for me?') and those referring to the objective sense intended (i.e., signification and referents). These two aspects are distinct and must be treated as such" (Howson, 2005, p. 18). In line with Howson's distinction, Vollstedt (2010a, 2011c) uses the term *personal meaning* to denote the first aspect of relevance and personal significance. *Personal meaning* denotes the personal relevance of a mathematical procedure, content, or the people involved in the learning process for an individual, in our case mostly a student of mathematics. Key questions in this area include: What is personally relevant for me when I am involved with mathematical contents? Why should I learn this? How do the contents relate to my own biography? In addition, we suggest to further differentiate Howson's second facet of the intended objective sense of a mathematical concept into collective meaning and innermathematical meaning. *Collective meaning* on the one hand denotes the relevance of a mathematical procedure or content for a certain group of people in contrast to an individual. This group of people can be characterised by a set of shared beliefs about the use of mathematics e.g. in terms of application in a certain profession, in life, in other scientific areas etc. The key question in this field is: What is the relevance of a mathematical procedure or content for certain professions and applications in other scientific areas? *Innermathematical meaning* on the other hand denotes the relevance of a mathematical procedure or content without a relation that refers to something else than mathematical theory. Key questions include: What is the meaning of a certain mathematical theorem for other areas of mathematics? Why are some theorems judged as being more important than others (e.g. fundamental theorems)? How could criteria of relevance look like? In addition to this distinction, other differentiations between different facets of meaning are possible. Reber (2018), for instance, discusses *objective* and *subjective* meaning in the context of meaning in life and making school meaningful. In his understanding of objective meaning, he enlarges Metz's (2013) understanding of meaning as the pursuit of "'the good, the true, and the beautiful' ... as a rough way of referring to certain kinds of moral achievement, intellectual reflection, and aesthetic creation" (p. 5). Reber understands objective meaning as "knowledge about meaning (roughly corresponding to the true), caring (good) and aesthetic experience (beautiful) [and further includes] meaning from creating identity, maintaining tradition or from agency" (2018, p. 3). With reference to Wolf (2010), he finally makes the following distinction: "Objective meaning is related to actions and parts of life that provide meaning without regard of whether a person herself considers these parts as meaningful. Subjective meaning is the personal experience of meaningfulness" (2018, p. 3). Personal meaning understood as personal relevance of a mathematical procedure, content, or the people involved in the learning process for the individual is, thus, per se subjective. In addition, objective meaning can also become personally relevant and, hence, subjective and personal.

7.2.2 Personal Meaning

It can be seen that the term *meaning* is multifaceted and hard to describe. As the focus of this study is on the students' points of view, the following elaborations are restricted to the field of personal meaning and are characterized in line with Howson's (2005) previously discussed distinction by personal relevance or significance. Accordingly, Vollstedt's (2011c) definition of a mathematical procedure, content, or the people involved in the learning process being personally meaningful for an individual when it is/they are personally relevant for him or her is adopted. The individual then shows a special relation to the object, action of person in question and relates it/him/her to his or her own biography (Meyer, 2008). Thus, the focus on the learner's perspective is emphasized. Accordingly, this paper does not focus on what might be meaningful from a normative or domain-specific perspective, but—on the contrary—the aspects students judge to be meaningful for them. As Kilpatrick et al. (2005c) pointed out (see above), these do not necessarily have to (but may) be the same.

Kilpatrick et al. (2005c) also link meaning to aspects important to the student him- or herself personally, being a student in the school environment, and being involved in a bigger system of education and relationships:

> Some students find it pointless to do their mathematics homework; some like to do trigonometry, or enjoy discussions about mathematics in their classrooms; some students' families think that mathematics is useless outside school; other students are told that because of their weakness in mathematics they cannot join the academic stream (p. 9).

Among others, this quotation also relates meaning to aspects such as purpose or usefulness, interest and motivation, the need for relatedness and friendship, as well as the students' performance and perspectives on their lives. All these can become relevant for the construction of personal meaning in a school context. When taking the personal relevance of an object or action as a defining criterion for personal meaning, all of these instances can be taken as examples for this concept. Thus, the rich diversity of personal meaning is maintained when thinking of it as personal relevance.

But is there a difference between personal meaning and other affective concepts such as interest, motivation, values, goals etc.? To approach this question, Heine et al.'s (2006) basic propositions shall be considered. They propose that human beings are meaning-makers who are able to create relations where they do not naturally exist and who create mental representations of expected relations that link elements of their external world, elements of the self, as well as elements of the self to the external world and vice versa. Those elements are to be integrated into existing relational structures so that a feeling of congruity is caused. This feeling, in turn, motivates people to interpret or, if necessary, reinterpret the elements so that they are consistent with their mental representations. The overall goal is establish "a sense of normalcy and coherence in their lives" (Heine et al., 2006, p. 89). Heine et al. conclude that "the one account that is parsimonious across each and every instance is that people are striving to affirm coherent structures of expected relationships" (2006, p. 102), i.e. meaning. Thus, the need for meaning and a coherent system of meaning is forming the basis for the development of a coherent sense of self. Thus meaning forms the basis of the other affective constructs and can be assumed to be their generating force. In addition, the idea of meaning is very broad and covers lots of aspects. When taking over the students' perspective and asking what is personally relevant for them, also ideas become relevant which are also important in other psychological theories such as values, goals, or motivation. Thus, there is a very close relationship between meaning and the other constructs so that a considerable amount of overlap can be assumed between the concepts and no sharp line of division can be drawn. Empirical studies are needed to further elaborate on the finer structure and relation between those concepts. First empirical results that help to discuss these issues can be found on the one hand by e.g. Mitchell (1993) who considers meaningfulness as one out of five catch or hold facets respectively that generate situational interest (meaningfulness and involvement constitute the two hold facets whereas group work, computers, and puzzles constitute the three catch facets). Suriakumaran and colleagues, on the other hand, have a deeper look at the structural relationship between meaning and motivation. From a theoretical perspective, they propose meaning as a generating source for the development of values, which, in turn, are crucial for the development of motivation if one follows Expectancy-Value-Theory (Wigfield, Tonks, & Klauda, 2016; cf. Suriakumaran, Duchhardt, & Vollstedt, 2017). To enable future studies that shed some more light on these issues, the survey that was developed in this study may be used.

7.2.2.1 Construction of Personal Meaning

So how is personal meaning constructed? The model of personal meaning (Vollstedt, 2011c) adopted for this study consists of a relational framework involving different concepts from mathematics education (e.g. mathematical beliefs), educational science (with a special focus on educational experience and learner development, e.g. developmental tasks), and educational psychology (e.g. learning motivation, academic self-concept, basic needs; see Fig. 7.1). The framework describes a (learning) situation in which an individual is dealing with subject content. The context of this learning situation does not only consist of the plain subject context; rather both the situation in the classroom and the individual's prior knowledge and experience are relevant for the construction of personal as well as objective meaning (Kilpatrick et al., 2005c). Mercer (1993) states:

> What counts as context for learners … is *whatever they consider relevant*. Pupils accomplish educational activities by using what they know to make sense of what they are asked to do. As best they can, they create a meaningful context for an activity, and the context they create consists of whatever knowledge they invoke to make sense of the task situation. (pp. 31–32, italics in original)

Therefore the students decide what information and experiences are relevant for them, to deal with a given task. These are, however, subject to cultural influences as culture has a strong impact on the way learning takes place in any learning situation (see Vollstedt, 2010b, 2011a, 2011b, 2011c for the impact of culture and context on personal meaning). According to Mercer (1993, p. 43; see also Birkmeyer et al., 2015), learning in the classroom depends both on culture and context as it is "(a) culturally saturated in both its content and structure; and (b) accomplished through dialogue which is heavily dependent on an implicit context constructed by participants from current and past shared experience."

Also, according to Vollstedt's (2011c) relational framework, certain preliminaries are relevant that constitute parts of the individual's personality and identity, i.e. personal background and personal traits respectively. Aspects of the personal background include an individual's cultural and family background. Skovsmose (2005) agrees that making mathematics education meaningful is essential. I also agree that meaningfulness has to do with the cultural background of the students, but I would argue that meaningfulness has much to do with another dimension as well, namely the foreground of each student (p. 6).

He defines the complex notion of the foregrounds of a person as follows:

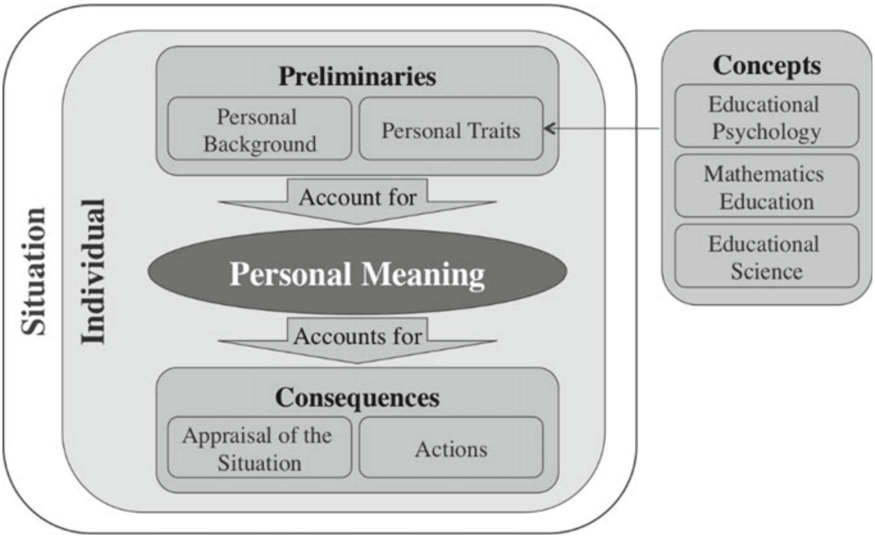

Fig. 7.1 Relational framework of personal meaning (Vollstedt, 2011c)

> On the one hand, a foreground is formed through the possibilities, tendencies, propensities, obstructions, and barriers, which the social context provides for a person. One can therefore see the foreground as structured through economic, political, cultural, and discursive parameters…. On the other hand, one should not think of the foreground of a person as a simple objective affair. The foreground is formed as much through the person's *experiences* and *interpretations* of possibilities, tendencies, propensities, obstructions, and barriers (2014, p. 5, italics in original).

Thus, the foreground of a person is not just objectively given to him or her, but rather is a personal interpretation of former experiences and future possibilities (Skovsmose, 1994, 2005, 2014). Intentionality is another important part of foreground (Skovsmose, 2016).

The foreground, then, is one important aspect of the individual's personal traits together with various concepts from for example, mathematics education, educational science, and educational psychology. Another central concept, which is strongly linked to the students' foreground, is developmental tasks (Havighurst, 1972). Meyer (2008) connects developmental tasks to personal meaning such that the developmental tasks posed by society are the engine of learning. He explains that the student who experiences meaning, or who has the feeling that the teacher demands things that are meaningless, argues from his or her biographical situation, anticipating his or her future in a particular way. Thus, the construction of personal meaning is the production of objective meaning of content with relation to the person itself. Meaningfulness is the expression of and simultaneous work on the development of competence and the construction of identity. Meaning builds a bridge towards the content, relates it to imagination, experiences, attitudes, and values, and relates it at the same time to the environment separate from teaching and school.

Other influential concepts that appear to play a decisive role for the construction of personal meaning are mathematical beliefs (Op 't Eynde & Verschaffel, 2002), different aspects of learning motivation (Krapp, 1999), academic self-concept (Marsh, 1986), and the three basic needs for autonomy, competence, and relatedness (Deci & Ryan, 2002).

Based on these preliminaries, the individual then constructs personal meaning with relation to learning content in the school context. Therefore, the construction of personal meaning is inherently subjective and individual. This means that every person constructs her or his own meaning with respect to a certain object or action. There is no given objective meaning, which can simply be adopted. Meaning cannot be endowed. Also, as the construction of meaning is not collective but individual; different students attending the same lesson can also construct different meanings for it. However, if a certain meaning is offered by the teacher or it is likely due to the context of the task, this meaning can be assimilated. Yet the individual is involved in the process of constructing a meaning before a certain personal meaning is generated.

In addition, personal meanings can, but do not have to be reflected on. On the one hand, this suggests that the process of construction of personal meaning may in some aspects be dominant in the situation so that one is aware of what is going on; the meaning enters consciousness. An aha-experience (Liljedahl, 2005), for instance, is an example of a meaning, which is dominant and conscious in the actual situation. On the other hand, meaning does not have to be conscious but can be constructed implicitly so that it is there without being dominant in the situation. From a constructivist perspective, Kilpatrick et al. (2005a, p. 137) state that in a learning situation, "the problem of construction of meaning itself is not really tackled. This is an evasive problem: It is difficult to know what each partner thinks; we can only hypothesise this by interpreting what they do and say." So, although it is not possible to directly ask for personal meaning, it is still possible to ask for it indirectly and for aspects or concepts related to it. Therefore, it is possible to hypothesise about personal meaning from interpreting interview data. The premise is that the construct of personal meaning is researchable with qualitative methods.

According to the framework in Fig. 7.1, the situation in which the student is involved is assessed depending on which personal meanings have been constructed and with relation to his of her personal background and personal traits, especially the foreground (Skovsmose, 2014). Depending on the appraisal of the situation, the different outcomes may then, accordingly, be followed by different actions that may have intentional or functional character. Thus, different actions can result from different personal meanings when dealing with mathematics. To conclude, the key characteristics of personal meaning are that (1) it is subjective and individual, (2) its construction is context bound, and (3) it can be reflected on, but does not have to.

7.2.2.2 Development of the Model of Personal Meaning

The model of personal meaning discussed here was developed in a former qualitative study (Vollstedt, 2011c) that was embedded in the Graduate Research Group on

Educational Experience and Learner Development at the University of Hamburg, Germany. The main focus of this research was on the learners' perspective on their educational process. Taking this perspective, Vollstedt (2011c) demonstrates that mathematical content and learning mathematics contain a productive range of aspects that may be personally relevant for the students.

The model resulting from the study was developed from 34 guided interviews conducted with students from the lower secondary level in Germany and Hong Kong[2] (17 interviews per country with students from grade 9 or 10 aged 15–16). The guided interviews began with a sequence of stimulated recall (Gass & Mackey, 2000) in which the students were shown a video abstract of five to ten minutes from the last lesson they attended. Their task was to utter and reflect on the thoughts they had when having attended the lesson. The interviews then tackled various topics inspired by the relational framework of personal meaning (see above) and lasted for about 35–45 min. Sample questions were for instance: How did you like this mathematics lesson? What was especially interesting? What feelings do you relate to mathematics lessons? Why do you learn mathematics? What can mathematics be used for? (cf. Vollstedt, 2011c for the detailed interview guide) The data were coded following Grounded Theory (Strauss & Corbin, 1990; see Vollstedt, 2015 for a detailed description of the coding process) and theoretical saturation (Strauss & Corbin, 1990) was reached. In the course of the analysis process after the last two interviews, no new categories were developed and the relationships among the categories seemed well established and validated. Also, from a theoretical point of view, the theory could be judged as thick for this age group. This, however, does not mean that the theory may not require subsequent revision. Although the theory of personal meaning may be corroborated by future research, it may well be that it can also be elaborated further (Vollstedt, 2015).

In total, 17 different personal meanings were reconstructed from the data. Those personal meanings were then further developed into seven types according to theoretical perspectives. The typology was constructed in relation to the level of the orientation to mathematics and the self of each personal meaning (cf. Fig. 7.2; see Vollstedt, 2011a, 2015).

As this model was developed from the analysis of 34 student interviews, there was the need to examine if the theory of personal meaning is supported empirically with a larger sample.

[2]The aim of the study was on the one hand to develop an empirically grounded model of personal meaning. On the other hand, another research question tackled the role of the learning culture the students are accustomed to. Thus, the study was conducted in Germany and Hong Kong serving as examples for two very different learning cultures such as the Western and the Confucian Heritage Culture (CHC). Similarities and differences could be worked out for the students from the two cultures (cf. Vollstedt, 2010b, 2011a, 2011b, 2011c for more information).

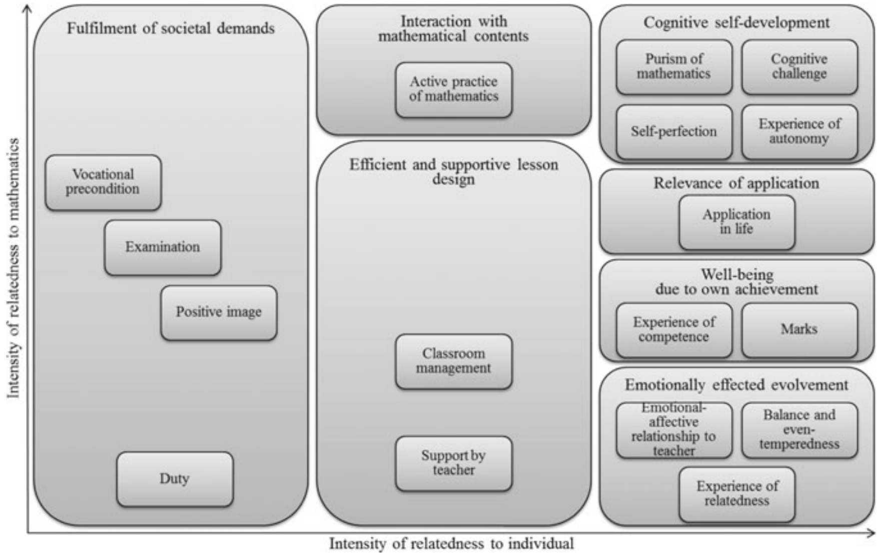

Fig. 7.2 The typology of personal meaning with relation to the intensity of relatedness to mathematics and the individual consisting of the 17 personal meanings. A slight change was made with respect to the personal meaning *Efficiency* from (Vollstedt, 2011a), which was changed to *Classroom management* in this study. See Sect. 7.3.1 for details

7.2.2.3 Research Questions

The preceding section highlighted that the concept of personal meaning is both multifaceted and highly relevant for learning mathematics in a school context. One interesting question in this field is what structural relations can be found between the concept of personal meaning and the different kinds of motivation (cf. Suriakumaran et al., 2017). To investigate this relationship, as well as other research questions, and in studies with larger sample sizes, a survey is needed. As quantitative empirical research in this field is still in its infancy, this paper addresses some initial research questions that focus on the assessability of personal meaning, descriptive results from a pilot study, and the validation of a model for personal meaning. All questions relate to the previously described model for personal meaning (Vollstedt, 2011c).

Assessability of personal meaning. Vollstedt's (2011c) typology for personal meaning was reconstructed in a qualitative interview study and thus it is currently based on a small number of cases. In order to further investigate personal meaning and its relation to adjacent constructs, personal meaning need to be evaluated for its ability to be quantitatively assessable. Hence the first research question addressed in this paper is:

1. Is it possible to construct a psychometrically sound survey the scales of which assess the different personal meanings?

Descriptive results. Based on Vollstedt's (2011c) data, no reliable conclusions can be drawn about the frequency of the different personal meanings among students. A potential direction for future research could investigate whether students from the same class tend to have similar preferences for personal meanings compared to students from different classes. This leads to the next questions:

2. Which personal meanings are especially popular among students aged 15–16?
3. To what extent does the fact that students are nested in different classes explain any variance in the different personal meanings?

Validation of the theoretical model of personal meaning (Vollstedt, 2011c). The arrangement of the different personal meanings in relation to their levels of orientation to mathematics and to the self as well as the resulting typology (cf. Fig. 7.2; Vollstedt, 2011c) were solely based on theoretical considerations. This motivates the following questions:

4. What is the correlational structure of the different personal meanings? Do the personal meanings within a type of personal meaning correlate highly?
5. Is it possible to identify meta-factors that structure the empirical results with relation to the students' different personal meanings? Can these meta-factors be associated with the dimensions from Vollstedt's (2011c) typology that were developed from a theoretical perspective (relatedness to mathematics and the relatedness to the individual)?

7.3 Methods

One of the aims of this study was to develop and test a survey containing scales for the 17 different personal meanings introduced in Vollstedt (2011c). Item response theory was chosen for primary data analysis, as it allows model assumptions to be checked and scale qualities to be evaluated. To validate the theoretical model previously described, a high-dimensional model was specified, in which each personal meaning was included as a dimension of its own. As a second step, factor analytic techniques were then applied to the resulting correlation matrix in order to uncover higher order (meta-)factors underlying these personal meanings. In the following sub-sections, the survey design as well as the sample will be described. Then, the analysis steps summarised above will be explained in detail.

7.3.1 Instrument

Pillath (2011) and Dahms (2012) undertook the first attempts to develop a survey to assess the different kinds (Pillath) and types (Dahms) of personal meaning. Both attempts did not provide reliable scales, but, together with a re-analysis of Dahms' data, provided a sound basis for revision. A fully revised survey was then developed by the authors in collaboration with Neruja Suriakumaran and three masters students (cf. Büssing, 2016; Schröder, 2016; Wieferich, 2016). Its objective was to provide 17 scales for the 17 different personal meanings reconstructed in Vollstedt (2011c). At one stage, a theoretical revision was necessary, as the personal meaning *Efficiency* from Vollstedt (2011c) combined two aspects: efficient classroom management and students' efficient ways of working, which proved not assessable with one scale. Instead, items addressing the latter aspect were merged with *Active practice of mathematics* and *Experience of autonomy*. The remaining scale was consequently renamed *Classroom management*. The resulting instrument was used in the current study. Table 7.1 shows the operationalization of the personal meanings.

7.3.2 Data Collection and Sample

Prior to the main study, cognitive labs were carried out in order to ensure that the items were formulated clearly and thus understood in the intended way. In March 2016, the survey was finally used in a study with students ($N = 193$) aged 15–16 from four Northern German schools: two higher secondary schools (*Gymnasium*) and two lower secondary schools (*Oberschule*). Although this was a convenience sample, it also approximately mirrors the situation in Germany, where roughly 30–50% of the students (depending on the federal state) attend higher secondary schools, whereas the others attend some lower secondary school. Here, systems and designations vary considerably between federal states. The two lower secondary schools of the study provide a good cross section of the wide range of different lower secondary schools in Germany. The participating students (51.8% girls) were from seven classes in grade nine ($n = 119$), and four classes in grade ten ($n = 74$). They had one regular school lesson (45 min) to fill in the paper and pencil survey. Cognitive labs and data collection were part of three masters theses (Büssing, 2016; Schröder, 2016; Wieferich, 2016).

7.3.3 Data Analysis

Data management and analyses were conducted in R (R Core Team, 2015).

7.3.3.1 Scaling of Personal Meanings

In order to answer the question whether assessing personal meanings with a survey yields scales with good psychometrical properties, item response theory (IRT, de Ayala, 2013) scaling methods were used. For every personal meaning, a partial credit model (PCM; Masters, 1982; Masters & Wright, 1997) was fitted to the data.

Table 7.1 Operationalization of personal meanings: number of items and example items

Personal meaning	Number of items	Example item(s)
Active practice of mathematics (Act)	13*	I like to deal actively with mathematics. Working actively on tasks helps me understand the content
Application in life (App)	16*	Learning mathematics is important to me because in doing so you learn for life. Tasks referring to everyday life make me occupy myself more deeply with the mathematical content
Balance and even-temperedness (Bal)	5	I like it when we can finally relax after a strenuous working phase in mathematics
Classroom management (Cla)	9	It is important to me that at the beginning of mathematics class, the teacher summarizes the previous lesson
Cognitive challenge (Cog)	7	I get bored when mathematical content is too simple
Obligation (Obl)	7	I do mathematics mainly because it is a compulsory subject
Emotional-affective relationship to teacher (Emo)	6	It is important to me that I feel valued by my teacher
Examination (Exa)	8	I do mathematics in order to pass exams and tests
Experience of autonomy (Aut)	8	Working autonomously is important to me to when it comes to mathematics
Experience of competence (Com)	7	I do mathematics because my learning success makes me feel good
Experience of relatedness (Rel)	8	It is important to me that there is a positive classroom climate during mathematics classes
Marks (Mar)	5	I do mathematics in order to be proud of my marks
Positive impression (Pos)	8	It is important to me that my family can be proud of my mathematics performance
Purism of mathematics (Pur)	7	In its purity, mathematics is uniquely beautiful to me
Self-perfection (Per)	8	I work on many mathematics tasks because becoming better and faster matters to me

(continued)

Table 7.1 (continued)

Personal meaning	Number of items	Example item(s)
Support by teacher (Sup)	6	It is important to me that my teacher communicates my progress in mathematics to me
Vocational precondition (Voc)	5	I do mathematics because I need it for my desired profession

Note Agreement to statements was given on a four-point Likert scale ranging from *strongly disagree* to *strongly agree*
*The large number of items is due to the fact that one of the research questions in Büssing (2016) and Schröder (2016) was to investigate the sub-facets of these personal meanings. Their results are, however, not relevant for this paper. Still, two exemplifying items are given to illustrate the different facets of the respective personal meanings

The PCM is a generalization of the Rasch model (Rasch, 1960) to items with more than two ordered response categories, like Likert scale items. Within the PCM, an item can be described by transition parameters pertaining to two adjacent response categories (de Ayala, 2013; Wu & Adams, 2007). These parameters describe points on the underlying latent variable scale where the respective two category characteristic curves intersect. The item location parameter, defined as the arithmetical mean of an item's transition parameters, is one way to gauge that item's "difficulty". Model parameters were determined with marginal maximum likelihood (MML) estimation using the R package TAM (Kiefer et al., 2015). Thereby, the mean of the trait distribution was constrained to zero. Missing values were not recoded in any way.

Fit of the PCM models was evaluated on both item and scale level. Given the relatively small sample size, item fit was evaluated based on the *t*-values of the weighted fit mean-square (Infit; Wu & Adams, 2007). Infit is an IRT statistic that describes how well the observed data match the IRT model predictions for a single item. The expected value of a perfectly fitting item is 1, with Infit values >1 indicating less discrimination than predicted (underfit). If an item showed significant underfit, i.e. $t(\text{Infit}) \geq 1.96$, it was removed from the scale. Overfitting items ($t(\text{Infit}) \leq -1.96$), i.e. items that discriminated too well, were only removed if additionally Infit <0.8 (Wu & Adams, 2007). After removing an item, the model was refitted, possibly leading to an iterative procedure. On scale level, EAP/PV reliability and the variance of the trait distribution were considered. EAP/PV reliability is an IRT measure of person parameter accuracy; its size is comparable to Cronbach's α.

In the sample of this study, the students were nested in classes. One crucial question with clustered data like these is whether observations from the same cluster are more similar to each other than random observations. This is expressed by the intraclass correlation coefficient (ICC). An ICC substantially greater than 0 implies that observations cannot be considered independent, possibly necessitating more advanced methods–multilevel modelling–in further analyses. In addition, it provides insight into the personal meanings themselves: To what extent is the class level responsible for the construction of personal meaning? Hence, computation of ICCs for the personal meanings was considered both informative from a theoretical perspective and essential for further work with the scales. To do so, for each personal meaning person parameters–Warm's (1989) weighted likelihood estimates (WLE)–were extracted. Then, using the R package lme4 (Bates et al., 2015), they were used as dependent variable in a simple random intercept model with class as the only predictor. Comparison of estimated intercept variance and residual variance yielded the ICC.

7.3.3.2 Structure of Personal Meanings

The correlational structure between those personal meanings that could be assessed reliably was investigated in two steps. First, a high-dimensional PCM that included the different personal meanings as separate dimensions was specified. Thereby, items showing misfit in the prior unidimensional analyses were not included. Again, MML estimation implemented in the R package TAM (Kiefer et al., 2015) was used. However, for computational reasons, quasi Monte Carlo integration (González et al., 2006; Pan & Thompson, 2007) with 2000 × (number of dimensions) nodes was employed instead of Gaussian integration. This model yielded latent correlations between the personal meanings.

For better readability and interpretability, a hierarchical cluster analysis (Kaufman & Rousseeuw, 1990) using Ward's method (Ward, 1963) was applied to the resulting correlation matrix. Thereby, the distance between two personal meanings was defined as 1—the latent correlation between those two dimensions).

In a second, exploratory step, the correlation matrix was analysed using exploratory factor analysis (EFA; Thompson, 2004). As multivariate normality of the personal meanings could be assumed, maximum likelihood extraction of factors as implemented in the R package psych (Revelle, 2016) was applied. EFA was preferred over principal components analysis as the objective of this step was to uncover larger meta-factors underlying the various personal meanings rather than data reduction. It was not the primary goal to make these supposed meta-factors measurable in a psychometric sense. Instead, it was intended to compare resulting meta-factors from EFA with the dimensions on which Vollstedt's (2011a) typology is based.

The number of extracted factors was determined by parallel analysis (Horn, 1965), scree plot inspection (Cattell, 1966), and the Kaiser criterion (Guttman, 1954; Kaiser, 1960). As explained above, no attention was paid to criteria that try to optimize the resulting measurement model, like χ^2, Root Mean Square Error of Approximation (RMSEA), or Bayesian Information Criterion (BIC; Schwarz, 1978). There was no reason to expect uncorrelated meta-factors, so an oblique rotation using the promax algorithm was applied. Significance of coefficients and correlations between the meta-factors was determined using a bootstrap analysis with 5,000 samples.

7.4 Results

In Vollstedt (2011a), 17 personal meanings could be reconstructed from interview data. For subsequent quantitative research involving personal meaning, it was considered necessary to assess these constructs by means of a survey. As this had not been accomplished prior to this study, the structural relationships between the different personal meanings as postulated in Vollstedt (2011a) had not yet been investigated quantitatively.

Results related to research question 1, i.e. whether assessing personal meaning by means of survey scales is possible, as well as descriptive results concerning the different personal meanings (cf. research questions 2 and 3), are presented in the first section. The second section addresses structural relationships between the different personal meanings: First, the correlational structure of the personal meanings is investigated (cf. research question 4). Building on that, meta-factors resulting from an EFA (cf. research question 5) are described.

7.4.1 Scaling of Personal Meanings

Table 7.2 shows the results from the (iterative) scaling process for each personal meaning. For most scales, only up to two items were excluded, which was considered uncritical for scale validity. The exceptions were *Obligation* and *Classroom management*, where three of the initial seven items and six of the initial nine items, respectively, had to be removed. In the first case, a close inspection of that personal meaning showed that the remaining items still cover its content validly. In the case of *Classroom management*, however, the remaining three items only focused on a very narrow facet of that personal meaning. Consequently, the whole scale was excluded from further analyses.

Overall, the psychometric properties of the remaining scales were good: As can be seen, estimated variances ranged from 0.64 to 4.56, with most values around or above 1. Only those for the personal meanings *Application in life*, *Experience of relatedness*, *Positive impression*, and *Emotional-affective relationship to teacher* were somewhat low. Scale reliabilities ranged from an acceptable 0.68 to a very good 0.86.

Intraclass correlations of the scales were very low, ranging from 0 to 0.055. Only for *Application in life* and *Experience of relatedness* did class membership account for more than 5% of the variance.

Table 7.3 shows descriptive results characterizing the popularity of the scales after badly fitting items (and the scale for *Classroom management*) had been removed. Values pertaining to item means are in the original Likert scale metric ranging from 0 (expressing strong disagreement with the scale) to 3 (expressing strong agreement),

Table 7.2 Scaling results per Personal Meaning: number of remaining items, variance of underlying construct, reliability, and intraclass correlation (ICC)

Personal meaning	Number of remaining items (initial number)	Variance	EAP/PV reliability	ICC
Active practice of mathematics (Act)	12 (13)	0.86	0.83	0
Application in life (App)	16 (16)	0.64	0.84	0.055
Balance and even-temperedness (Bal)	4 (5)	1.07	0.68	0.017
Classroom management (Cla)	3 (9)	– (Scale removed)–		
Cognitive challenge (Cog)	6 (7)	0.94	0.75	0.008
Obligation (Obl)	4 (7)	4.40	0.85	0
Emotional-affective relationship to teacher (Emo)	6 (6)	0.74	0.69	0.027
Examination (Exa)	6 (8)	1.25	0.78	0
Experience of autonomy (Aut)	7 (8)	0.79	0.73	0
Experience of competence (Com)	7 (7)	0.94	0.75	0
Experience of relatedness (Rel)	8 (8)	0.67	0.73	0.052
Marks (Mar)	5 (5)	1.10	0.73	0.024
Positive impression (Pos)	7 (8)	0.71	0.72	0
Purism of mathematics (Pur)	5 (7)	4.56	0.86	0.011
Self-perfection (Per)	8 (8)	0.87	0.77	0
Support by teacher (Sup)	6 (6)	1.39	0.77	0
Vocational precondition (Voc)	4 (5)	3.34	0.82	0.023

whereas location parameters are in the IRT logit metric, where low values indicate that it is easy to agree with an item.

Item means varied considerably over all scales. There was quite strong disagreement (0.62) for the item “As it is fun, I occupy myself with mathematical tasks or contents in my leisure time.” (*Active practice of mathematics*) and quite strong agreement (2.62) for the item “It is important to me that my teacher explains the contents

Table 7.3 Descriptive results per personal meaning: minimum, maximum, and mean of item means as well as mean of location parameters

Personal meaning	Minimum of item means	Maximum of item means	Mean of item means	Mean of location parameters
Active practice of mathematics (Act)	0.62	2.26	1.58	−0.27
Application in life (App)	1.30	2.00	1.64	−0.23
Balance and even-temperedness (Bal)	1.51	2.35	1.96	−0.76
Cognitive challenge (Cog)	0.73	1.88	1.21	0.41
Obligation (Obl)	1.55	1.85	1.70	−0.41
Emotional-affective relationship to teacher (Emo)	1.81	2.24	2.00	−0.83
Examination (Exa)	1.41	2.38	1.87	−0.68
Experience of autonomy (Aut)	1.28	2.06	1.70	−0.44
Experience of competence (Com)	1.23	2.44	1.84	−0.59
Experience of relatedness (Rel)	1.44	2.28	1.98	−0.73
Marks (Mar)	1.68	2.10	1.83	−0.60
Positive impression (Pos)	0.70	1.94	1.29	0.43
Purism of mathematics (Pur)	0.90	1.20	1.00	1.44
Self-perfection (Per)	0.72	2.13	1.48	0.04
Support by teacher (Sup)	1.55	2.62	2.03	−1.05
Vocational precondition (Voc)	1.12	1.84	1.58	−0.33

Notes All Likert scales were coded from 0 (*strongly disagree*) to 3 (*strongly agree*). Negatively worded questions were recoded. Items removed during the scaling procedure were not included well to me." (*Support by teacher*). For some scales (like *Obligation*) the item means did not vary considerably, whereas for other scales (like *Self-perfection*) they did, indicating scales of different homogeneity.

7.4.2 Structure of Personal Meanings

The 16-dimensional PCM fitted to the data of the remaining personal meaning scales yielded the latent correlations. Overall, correlations ranged from −0.79 to 0.85. (In general, sizes of correlation coefficients should be interpreted against the background of the theories involved. Having said that, following the frequently cited book by Cohen (1988), correlation coefficients could be classified according to their absolute value as small (>0.1), medium (>0.3), or large (>0.5), respectively.) For clarity of presentation, a hierarchical cluster analysis was used to detect groups of personal meanings in which personal meanings are similar to each other (with regard to correlations). Then, the correlation matrix was ordered accordingly. Figure 7.3 presents the correlational structure of the personal meanings in different ways: The upper part shows results from the cluster analysis, whereas the lower part presents the correlation matrix arranged accordingly.

Both results of the cluster analysis and correlational patterns indicate two clusters of personal meaning. The smaller cluster comprises the personal meanings *Balance and even-temperedness*, *Obligation*, *Emotional-affective relationship to teacher*, *Support by teacher*, and *Experience of relatedness*. Those are precisely the personal meanings that Vollstedt (2011b) characterized as having a low level of orientation towards mathematics (cf. Fig. 7.2). Amongst themselves, these personal meanings show positive correlations ranging from 0.22 to 0.73 with an average of 0.50. The larger cluster comprises the remaining personal meanings. Within this second cluster, all correlations are positive, too, with an average of 0.60.

Two sub-clusters stand out as particularly homogeneous: correlations between *Marks*, *Positive impression*, and *Experience of competence* range from 0.67 to 0.82, and correlations between *Purism of mathematics*, *Cognitive challenge*, *Active practice of mathematics*, and *Experience of autonomy* range from 0.74 to 0.85. Additionally, between personal meanings from this latter sub-cluster and the personal meanings *Balance and even-temperedness* and *Obligation*, correlation are substantially negative, ranging from −0.49 to −0.79 with a mean of −0.65.

Prior to the subsequent exploratory factor analysis, it was checked whether the data were suitable for this procedure. The Kaiser-Meyer-Olkin measure (KMO; Kaiser, 1970), KMO = 0.91, confirmed very good sampling adequacy. So did measures of sampling adequacy for individual personal meanings, which ranged from 0.79 to 0.96. Bartlett's test of sphericity, $\chi^2(120) = 3534.4$, $p < 001$, strongly indicated that correlations were sufficiently high to carry out an EFA.

All three criteria employed to determine the number of factors clearly indicated that two meta-factors should be retained. Table 7.4 shows the main results of factor analysis after oblique rotation. After inspection of substantial pattern and structure

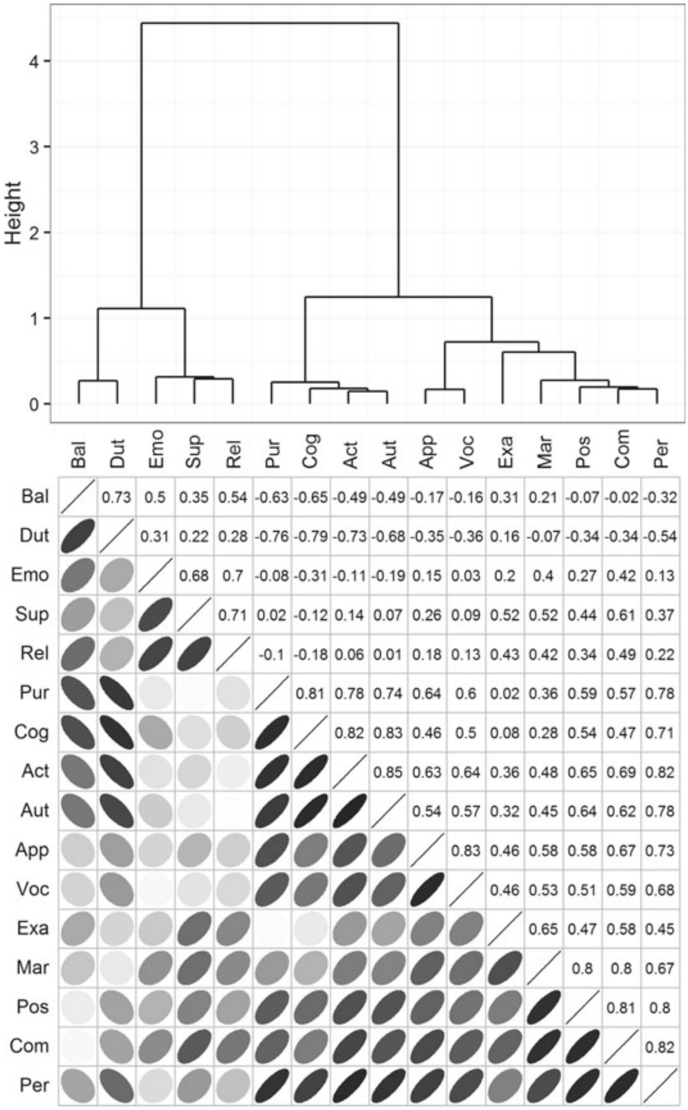

Fig. 7.3 Relations between personal meanings. *Lower part*: Latent correlations between personal meanings computed in a 16-dimensional model given as numeric values (above diagonal) and represented with an ellipse depicting the shape of a bivariate normal distribution with that correlation (below diagonal). Personal meanings are ordered according to a hierarchical cluster solution (see upper part). *Upper part*: Results of hierarchical clustering of the correlations between the personal meanings

Table 7.4 Summary of promax-rotated factors for 16 personal meanings (n = 193): pattern coefficients, structure coefficients, and communalities

Personal meaning	Pattern		Structure		h^2
	Orientation towards mathematics	Orientation towards social inclusion	Orientation towards mathematics	Orientation towards social inclusion	
Bal	**−0.71**	**0.64**	**−0.59**	**0.50**	0.74
Dut	**−0.86**	0.39	**−0.79**	0.22	0.77
Emo	−0.27	**0.74**	−0.13	**0.69**	0.55
Sup		**0.81**		**0.80**	0.64
Rel	−0.18	**0.79**		**0.75**	0.59
Pur	**0.91**	−0.09	**0.89**		0.81
Cog	**0.94**	−0.24	**0.90**		0.86
Act	**0.91**	0.06	**0.92**	0.24	0.85
Aut	**0.89**		**0.89**	0.18	0.79
App	**0.60**	0.35	**0.67**	**0.46**	0.56
Voc	**0.61**	0.25	**0.66**	0.37	0.49
Exa	0.13	**0.66**	0.26	**0.69**	0.49
Mar	0.35	**0.72**	**0.49**	**0.79**	0.74
Pos	**0.60**	**0.52**	**0.70**	**0.64**	0.75
Com	**0.57**	**0.65**	**0.70**	**0.77**	0.90
Per	**0.81**	0.34	**0.88**	**0.50**	0.89

Notes Personal meanings are arranged according the cluster analysis introduced in Fig. 7.3. For ease of interpretation, non-significant coefficients are suppressed and coefficients with absolute value >0.4 appear in bold

coefficients the two meta-factors were identified. As can be seen most clearly from the pattern coefficients, in a positive way the personal meanings *Cognitive challenge*, *Purism of mathematics*, *Active practice of mathematics*, *Experience of autonomy*, and *Self-perfection* (i.e. becoming a better or smarter person by being involved with mathematics) contribute most to the first meta-factor, whereas *Obligation* as well as *Balance and even-temperedness* contribute negatively to it. Therefore, it was interpreted as *Orientation towards mathematics*. The second meta-factor is best described by the personal meanings *Support by teacher*, *Experience of relatedness*, *Emotional-affective relationship to teacher*, and *Marks*. It was hence interpreted as *Orientation towards social inclusion*. The correlation of 0.2 between the two meta-factors was low, but significant with confidence interval [0.1, 0.28].

Fit indices show that the above meta-factor solution makes a poor measurement model: Neither the Comparative Fit Index (CFI) of 0.836, nor the Tucker-Lewis Index (TLI) of 0.776 or the RMSEA of 0.186 were even close to acceptable. Extracting measurable meta-factors was, however, not the goal of this study. On the contrary, this finding indicates that there is a lot more to personal meaning than two meta-factors can express.

7.5 Discussion

In this study, a survey was developed to assess the students' personal meaning, understood as personal relevance of an object or action, when dealing with mathematics (Vollstedt, 2011c). Results relating to research question 1 show that the scales for the different personal meanings show all good psychometric properties, except for the scale *Classroom management*. Here, six out of nine items had to be removed to meet acceptable psychometric results, but the remaining three items did only cover one core aspect of *Classroom management* (summary of lesson contents at different points of time). Hence, this scale is not acceptable and is being revised and tested in an ongoing study (cf. Suriakumaran et al., 2017).

When looking at the preferred personal meanings of the students (research question 2), results show that on the one hand *Balance and even-temperedness*, *Emotional-affective relationship to teacher*, *Experience of relatedness*, and *Support by teacher* matter much to students, as do *Examination*, *Marks*, and *Experience of competence* on the other hand. Interestingly, the first group combines personal meanings of a social nature that are located in the bottom right corner of Vollstedt's (2011c) typology (low relatedness to mathematics, high relatedness to individual, see Fig. 7.2). The second group then groups personal meanings with medium relatedness to mathematics that all belong to the realm of performance and achievement. The personal meanings least preferred are *Cognitive challenge*, *Purism of mathematics*, *Self-perfection*, and *Positive image*. The first three all belong to the type *Cognitive self-development* (high relatedness to mathematics and individual) and combine aspects that value the beauty of mathematics, being challenged by difficult mathematical tasks and, thus, being able to become better and develop oneself. This facet may relate to *Positive image* (low relatedness to individual, medium relatedness to mathematics), which denotes that it is relevant for the students that other people value them and get a positive impression of them.

Results related to the third research question show that personal meaning seems to be indeed personal as belongingness to a certain class usually accounts for less than 5% of the variance in the respective scale. The ICC was just slightly bigger than that for *Application in life* (0.055) and *Experience of social relatedness* (0.052). These results are rather surprising. One would have expected bigger effects since personal meanings are supposed to be dependent on the context, which is highly influenced by the teacher and his or her way of teaching. However, this study only researched eleven classes from four schools, which is quite a small number for conducting multilevel analyses. This should be seen as a limitation, and results should be interpreted with the appropriate caution.

The correlational structure between the different personal meanings of a certain type (research question 4) shows that there are close relations within all types. The personal meanings from the type *Cognitive self-development* correlate highly (all >0.7) so that one could suggest to group them together and assess them in one scale instead of four. However, Schröder's (2016) model comparisons show that the four-dimensional model suits the data considerably better than the one-dimensional one. It follows that these personal meanings should be best assessed as dimensions of their own. In analogy, Wieferich's (2016) results show that the same holds true for the two personal meanings from *Well-being due to own achievement*, which show a correlation of 0.8, and the three personal meanings from *Emotionally effected evolvement*, the correlations between which range from 0.5 to 0.7. Within *Fulfilment of societal demands*, however, there are only substantive correlations between *Vocational pre-condition*, *Examination*, and *Positive image* ranging from 0.46 to 0.51. *Obligation* is the only personal meaning that does not totally fit, showing correlations ranging from −0.36 to 0.16. This could be explained using the Organismic Integration Theory from Self-Determination Theory of Motivation (Deci & Ryan, 2002): *Obligation* seems to be the only personal meaning that is characterized by external regulation whereas all other personal meanings seem to be related to some level of self-regulation ranging between introjected and intrinsic motivation. To conclude, the correlational structure of the types broadly agrees with Vollstedt's (2011c) typology. Again, the relatively small sample size for the complex multidimensional IRT-model constitutes a certain limitation of this study.

Results pertaining to the fifth research question finally indicated two meta-factors that structure the personal meanings with respect to *Orientation towards mathematics* and *Orientation towards social inclusion*. At first glance there is an obvious relation between the mathematical meta-factor and one of the dimensions that formed the basis of Vollstedt's (2011c) typology: *Relatedness to mathematics*. The second meta-factor (*Orientation towards social inclusion*) does, however, not correspond with the second dimension from the typology (*Relatedness to individual*). It rather describes what becomes personally relevant to students when it is not the subject that matters, i.e. social inclusion. Thus, the two meta-factors both seem to describe the two directions of *Relatedness to mathematics*: a high relatedness to mathematics corresponds with an *Orientation towards mathematics* whereas a low relatedness goes along with an *Orientation towards social inclusion*. The correlation of 0.2 between the two meta-factors indicates that they are not excluding counterparts.

In addition, fit indices imply that the two meta-factor model is not an adequate measurement model for the data. For this study this is considered a positive result as this means that Vollstedt's (2011c) typology of personal meaning cannot be reduced to just these two meta-factors indicating orientation towards mathematics and social inclusion, respectively.

To conclude, this paper could show that it is possible to assess 16 personal meanings in a survey. Main results from correlation analyses generally support Vollstedt's (2011c) type structure. In addition, multilevel analyses suggest that personal meaning is personal whereas EFA indicates that math matters for the construction of personal meaning.

7.6 Further Perspectives

As could be seen from the results presented above, the subject contents matter with being either very valued or quite unimportant, in which case social inclusion is put forward. However, Vollstedt's (2011c) second dimension (*Relatedness to the individual*) could not yet be related to the data. This gap might be closed drawing on affective psychological constructs related to subject contents or the learning of mathematics such as motivation, interest, mathematical beliefs, self-efficacy etc. As these constructs may have an influence on the continuum of the individual's affectedness towards (the learning of) mathematics, they may also indicate the relatedness of the individual to subject contents or the learning of mathematics. In line with Vollstedt's (2011c) theoretical framework, this may result in the construction of different personal meanings showing a stronger or weaker relatedness to the individual, i.e. different values on the second dimension of Vollstedt's (2011c) typology. The relationship between personal meaning and the affective psychological constructs will be investigated in future studies (cf. e.g. Suriakumaran et al., 2017).

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