

Chapter 12

Maths Adds up



Maite Gorriz and Santi Vilches

Abstract In this chapter, talking from our professional experience in Secondary Mathematics Education in Catalonia, we will first describe the essential elements of working with interdisciplinary activities and knowing how to carry them out from an organizational point of view. Subsequently, we will describe in detail two examples in which good learning outcomes were achieved. Both examples have arisen from the necessity to attend to all the capacities of the students in low secondary school (12–14 years old) and to give them the possibility to develop their own creativity. Furthermore both examples have an extension activity for students in upper secondary school (16–18 years old) where it is possible to attend to their abstract capacity and connect all their knowledge appropriate to their age. Furthermore, we have chosen these two cases because both are very close to the reality of the students and both have an important background: to the desire for, and valuing of, a critical attitude. In the concluding section, we define some features of these interdisciplinary activities which can be extrapolated to any educational centre.

Keywords Interdisciplinary activities · Creative activities · Context · Assessment · Cooperative learning

12.1 Introduction: A New Approach to Teaching Mathematics

Mathematics has always been the spine of knowledge that has allowed every scientific, technological and artistic discipline to develop and grow. Let's think, as some examples, about the physical laws, the chemical formulations, the technological structures, the musical scales, the proportions in art, the philosophical logic or

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the syntactic structures in linguistics. Let's even think about the harmony, beauty and emotional transmission. Mathematics has such a creative power that even if you wanted to escape from its theoretical hermeticism and try to make an abstract art piece, completely free and without following any method, you could still describe this with some theory about chaos or randomness.

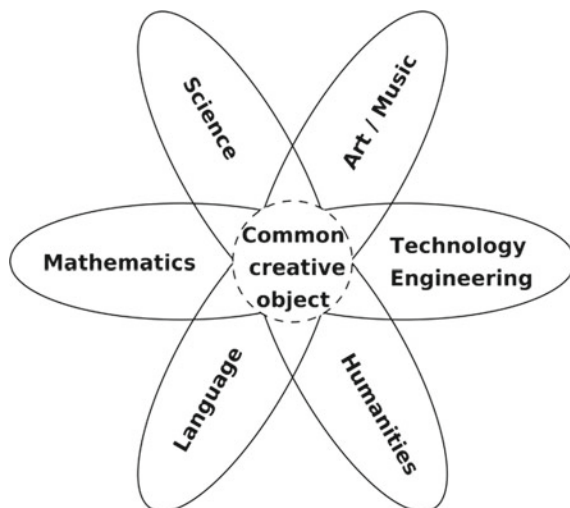
Contrary to fact, mathematics has often been taught disconnected from reality, as something independent. In some periods, especially for example during the 70s when an obsessive Bourbakism made mathematical teaching totally abstract, it did not have any connection at all. Luckily, some contributions like the ones Freudenthal has made, have provided new pedagogical tendencies. Tendencies which are closer to an approach of mathematics for everybody.

Nowadays the reality of teaching is quite complex. On the one hand, we can still find many teachers who teach mathematics in an ideal school where calculation processes have a huge importance. On the other hand, in our experience, we find teachers who incorporate some research on their didactic activities and somehow improve the mathematical learning. These two models coexist under the tradition. This is possible because in our country teachers have the responsibility for their own teaching model (so, "Everyone thinks his or her way is best"), and in the social perception that the mathematics we learn at school has no use. A social perception which is enriched by the new technological reality in which any calculation, even symbolic ones, can be done by anyone using simple but powerful Android applications.

A proposal in order to help mathematics restore its role as the spine of knowledge is to create new didactic methodologies. Ones that take into account the new technological reality in which students use maths to develop their creative potential in any of the knowledge's disciplines. So as to reach this goal, we suggest the learning of mathematics using interdisciplinary activities as the starting point.

In the next section, we will present our experience of the interdisciplinary work in which we have been involved as part of our role as high school mathematics teachers. In particular, we will detail two examples that have been developed in different schools and have been taught from low secondary level through to upper secondary level. We started in interdisciplinary work because we needed to attend to different students' capacities and the first aspect to solve was language problems. So, we started working with language teachers in a wide context and the necessity naturally appeared to involve more subjects: science, arts, humanities, etc. Our examples give a flavour of the range of interdisciplinary work we engage with. And finally, the concluding section describes some of the main points of view that have been developed from this experience so far.

Fig. 12.1 Different specialists develop the educational curriculum around a final outcome



12.2 Interdisciplinary Activities: Form and Requirements

12.2.1 *Form: Different Specialists Develop the Educational Curriculum Around a Final Outcome*

Throughout our professional career, we have carried out interdisciplinary activities in different ways. We consider extremely important not only designing the interdisciplinary activity but also knowing how to put it in practice. So, we propose coordination between teachers based on the educational curriculum of each subject. The idea is to define a final outcome, a “common creative object” which has to be built (Miettinen, 1998), one for each student, and design a common working guideline for the students as the starting point to work on curricular content within every subject (in each class). That is, each specialist teacher works in his/her subject with a common guideline in order to guide each student to build a common creative object from his/her subject. For example, in the interdisciplinary activity “FX Thales”, the common creative object is to create a short film with special effects. The subjects involved are Language, Technology and Mathematics. The language teacher works with the script, the technology teacher works with production of the short film and the maths teacher works on special effects using Thales Theorem (Figs. 12.1 and 12.2).

The advantages of this form are that the learning process is close to current social necessities where jobs often involve interdisciplinarity, team work and the need to improve. This kind of learning is achieved through the common working guideline for every subject, a cooperative group methodology and with an assessment as part of the activity. Furthermore, this form accepts that teachers do not necessarily need to be specialists in more than one subject. For instance, in Catalonia, a teacher only needs



Fig. 12.2 A frame of “We don’t need to call a tow truck”. An example of a short film. 13-years old interdisciplinary activity. INS Pla Marcell, 2017

one specialization in order to get into Secondary Education. It is obvious (to us) that a good teacher needs to be able to teach under a multidisciplinary perspective. However, we would argue, only a specialist is capable of properly structuring students’ learning in a particular way, as Watson and Barton (2011) suggest. This is especially clear in mathematics. It is difficult for a non-mathematician to understand how a teenager’s brain makes a step forward to abstraction. On many occasions, abstract or geometric procedures, like Thales theorem, are taught as a pure and rigid algorithm and there is a lack of concern in how a teenager perceives these procedures.

12.2.2 Requirements of Interdisciplinary Activities

Interdisciplinary activities promote teacher collaboration (St. Clair & Houghs, 1992). From our experience, to work with interdisciplinary activities, the starting point must be coordination between teachers of different subjects. In this initial coordination, two things are needed: a reflection about the curricular content that has to be taught in each subject and the defining of a single common outcome that each student has to achieve. “Common” meaning that this outcome is the same in every subject that is part of the activity. Experience shows that defining this common outcome helps specify the curricular content of each subject.

This initial coordination between teachers has to lead to a working guideline for students which will lead to the achievement of the outcome by each one of them.

This working guideline needs to be the same in every subject involved. However, each part of it will be done by the specialist teachers. This way, each teacher will be in charge of helping students learn essential topics of the corresponding subject within his or her own professional didactic criteria.

According to Van den Heuvel-Panhuizen and Drijvers (2014) “realistic situations are given a prominent position in the learning process”. We propose that the common outcome must be defined really well and must be feasible and close to students, so that the achievement of the outcome is meaningful for them from the beginning. This guarantees students’ motivation and their involvement in the learning process.

In Realistic Mathematics Education, Freudenthal (1973) argued “students should be active participants in the educational process”. So, we propose that each student develops their own creative ability in accomplishing the outcome and constructing their own learning process. In order to reach this goal, students’ task lies on writing a report describing the whole process. This way, the teaching and learning process adjusts to each student’s abilities. And this is how we take care of student diversity.

It is essential to work in groups, cooperating and using heterogeneous teams. Although each student writes his or her own report about the process and creates his or her own final outcome as an individual work, a general methodology of cooperative work is suggested in order to enhance learning using role assignment. Every student will later analyse their classmates’ outcomes, enriching their own knowledge by others’ ideas. When these ideas are their own, or from their classmates, we find that motivation is highest and both interest and learning increase. And we assess the learner’s creative work on open-ended innovative tasks and their capacity to work together (Howes, Kaneva, Swanson, & Williams, 2013).

The teacher role is that of a facilitator. Teachers should be trained to help in developing new ideas, adapting to each situation and finding tools to help students to achieve their goals. Technology should have a key role in facing and achieving new situations. For example, using photography or video to visualise new mathematics concepts or using functions or statistic structures in some specific software to solve higher level problems. So, teacher’s knowledge about his or her subject and about didactics of his or her subject are the key to a proper orientation of the learning process of students.

From understandings of assessment for learning (Black, Harrison, Lee, Marshall, & Wiliam, 2004), assessment must be integrated within the learning process, answering to the following question: “How can I improve my creation?” It is not the outcome that matters most but the way to achieve it. This is why the aspects that need to be considered during the assessment are: the language of the reports from a formal point of view, the mathematical content that helps accomplishing the desired goal, and every curricular aspect from each subject involved. Constant research for improvement and error correction with teachers’ help is essential for the assessment. An important part of the whole learning process will consequently be providing students with the necessary tools to evaluate and improve their professional productions in the future. Assessment means learning to improve and, therefore, learning to evaluate. It includes ‘self-assessment’ in the self-regulation process (Panadero & Alonso-Tapia, 2013).

The last requirement for interdisciplinary activity is coordination between the teachers involved in the process in order to do a final assessment of the interdisciplinary activity once it is finished.

We have outlined what in our experience are the most important aspects of interdisciplinary work in our context, we now turn to look in more detail at two cases focused on particular tasks which we have carried out in recent years. We have chosen the following cases because they are familiar to teenagers. When you see a group of teenagers in the street, they are drinking a soft-drink and listening to music. Aren't they? So, our task as teachers, would be to develop an activity around a "package of 33 cl, like a can" and around "music" from different subjects. Furthermore, another important aspect is that this activity should provide students with value and encourage a critical attitude. For example, why don't companies manufacture packages following ecological criteria in spite of commercial criteria? How are emotions involved in choosing a package or in listening to music?

12.3 Case Studies

12.3.1 *Case Study 1: Create Your Own Package (12–14-Year-Olds)*

This case study engaged students in designing a package for an object of unspecified shape but with a volume of 33 cl. The original idea was to study geometry while avoiding boring exercises of volume calculation from a book. We asked ourselves: how can students understand volume (3 dimension) with pictures (2 dimension) in an unreal situation? On the other hand, which volumes can be familiar to a teenager? The answer was "building a can". So, we decided to study the volume contents in the geometry curriculum on the basis of students' own packages of 33 cl. This way we can have as many "real volumes" as students in the class. After this first idea, we (maths teachers) saw that arts and language teachers were facing the same problem in their classes when dealing with the unfolding 3-dimensional figures and advertisement language respectively: they needed a real situation, too. So, this was the starting point: we needed to work together!

12.3.1.1 Context, Curriculum and Assessment

This interdisciplinary activity has been developed in the following centres during the specified periods of time and with different subjects involved (Table 12.1).

In each school, the activity is first organised with a meeting of the various subject teachers to decide the educational curriculum which they will work with. So, the educational curriculum could change every school year. Table 12.2 is an example from school year 2015–2016.

Where Physics has been involved, in Montserrat Roig High school and Marta Estrada High school, the educational curriculum has included Archimedes' principle, mass, density, force and pressure.

The common outcome, as already stated, is a package of 33 cl. Initially, this package had to be designed for soda. However, when Science became part of the interdisciplinary activity it was agreed that it only needed to be a package for some food product. In this task, the creative process of the students is of great importance, and each subject too.

Each subject determined some specific goals:

Maths	The mathematical challenge lies in calculating the volume of different figures and adjusting the measures to a set volume. In maths class, students will make the initial design, the necessary calculations, and will develop their geometrical competence.
Science	In this case the challenge lies in deciding which food item will be in the package and which nutritional information the tag has to include. To do so, students have to analyse some tags from different products and think about the consumption of some products taking the food pyramid into consideration. They will also have to study the energetic value of the food item they have chosen.
Physics	Here, Archimedes' principle is used so as to check the final volume of the object. Notions of mass, density, force and pressure of different food produce inside a package are also researched.

Table 12.3 Where, who, when and how are involved in the interdisciplinary task “Creating musical instruments”

Centre	Involved students	School year	Involved subjects
El Sui high School (Cardedeu, Barcelona)	15 students	2011–2012, 2012–2013	Maths (and Statistics), Technology, Music, Geography
Arquitecte Manel Raspall high School (Cardedeu, Barcelona)	96 students	2014–2015	Maths (and Statistics), Technology, Music
Marta Estrada high School (Granollers, Barcelona)	60 students	2016–2017	Maths (and Statistics), Music, Physical Education, Linguistics (Catalan and English), Arts
Vilamajor high School (Sant Pere de Vilamajor, Barcelona)	96 students	2017–2018	Maths (and Statistics), Music, Arts, Technology and History

Table 12.1 Where, who, when and how are involved in the interdisciplinary task “Create your own package”

Centre	Students involved	School year	Subjects involved
El Sui High School (Cardedeu, Barcelona)	92–96 students	2010–2011, ..., 2016–2017	Maths, Art, Linguistics
Montserrat Roig High School (Sant Andreu de la Barca, Barcelona)	110 students	2012–2013, ..., 2016–2017	Maths, Art, Linguistics, Physics
La Sinya High School (Parets del Vallès, Barcelona)	60 students	2013–2014, ..., 2016–2017	Maths, Art, Science
Arquitecte Manel Raspall High School (Cardedeu, Barcelona)	96 students	2015–2016, 2016–2017	Maths, Art, Linguistics, Science, Technology, Music
Marta Estrada High School (Granollers, Barcelona)	60 students	2016–2017	Maths, Art, Linguistics, Science, Physics
Pla Marcell High School (Cardedeu, Barcelona)	95 students	2016–2017	Maths, Linguistics, Technology

Table 12.2 Educational curriculum worked in Arquitecte Manuel Raspall High school, 2015–2016

Subject	Educational curriculum
Mathematics	Volumes. Unfolding figures
Art	Tag design, technical drawing, plane representation systems
Linguistics	Advertising vocabulary
Science	Food pyramid
Technology	Materials
Music	Music and society

- Art** Students will make the artistic design of the tag. In some grades the unfolding 3-dimensional figures have been studied too.
- Technology** The challenge lies in studying the possible materials with which the package could be made, considering their features and deciding which one they will use. The package will be created at the technology laboratory.
- Linguistics** In this case the challenge lies in making an advertisement to promote the product they have created.
- Music** Adding music to the advertisement.



Fig. 12.3 Heterogeneous working groups

Teachers then create the working guideline with the activities from their subject, following the agreed common cooperative work guidelines. It is highly important to keep in mind that the interdisciplinary activity “Create your own package” is not done after the subject activities, for example, on volume in a mathematics class. It is itself the activity on volumes (or the linguistic activity on advertising language, etc.). The guideline contains everything necessary to carry out this interdisciplinary activity. The same happens for each subject involved.

Cooperative work guidelines define that each heterogeneous group of 4 or 5 students represents a team from a fictitious food company in charge of creating a new product line. They will have to decide which product to put in the package (Science) and design a corporate logo to add to this one (Art). They can choose or not choose homogeneous shapes within the team members (Mathematics). They will have to present their commercials together (Linguistics and Music) in order to promote the new product line. They will also have to help each other in cooperative work but, even though they team up, each student must contribute with their own individual task in creating their own package (Fig. 12.3).

At the beginning of the interdisciplinary activity, teachers from each subject explain the activity, the challenges students need to overcome in that subject in relation to the final object and the assessment criteria are shared by students and teacher. Each subject is studied using the one and only working guideline during the assigned hours within the usual schedule. In every case, the interdisciplinary activity “Create your own package” was carried out during the same weeks in each subject involved.

Students are asked to produce a report for each of the subjects as part of their learning process assessment. Each subject gives the guidelines to fulfil the report within the working guideline. The working guideline explains how to write the report about each student's process so that it can become a tool of assessment. This tool allows students to modify their learning process so as to improve it. The final object and the chosen procedure will also be taken into account according to each student's skills. This assessment will be used in order to motivate students in a more personal way. Last but not least, calculations about others' packages and a group reflection about the new product line created will also be part of the assessment. Each one of these items will be taken into account to assess students' acquired knowledge. This whole process and reflection will enable also a teachers' assessment and students' grades analysing every possible item which can be improved in their learning process. Grades will be agreed with students themselves individually and will be used to inform the families about their progress. It will be considered as another learning item for students.

12.3.1.2 The Activity in Mathematics

In Mathematics, the teacher observes, reflects and takes part in assisting the students to make steps toward abstraction. The teachers will guide students so that they are able to create their package according to their own abilities. Therefore, we can have some students working with simple prisms (just a cube, for example) and some others designing highly complex shapes. Regarding the volume calculations for the package, some students can work with small shapes to fill in the cube using the Voxcad program (see Fig. 12.4).

Other students use the Archimedes principle to make sure their calculations are correct in the laboratory. Others do calculations using abstract formulae. And some use the dynamic geometry software GeoGebra. Dynamic geometry allows students to have a different vision of Maths and helps enormously in developing their creative processes (see Fig. 12.5).

Moreover, teachers help students to develop their creative abilities in asking them to freely create the new packaging. This "freedom principle" regarding work is seen in every subject (advertising, tag design...) but it is important for students to realise how big the creative power of Mathematics is. The only restriction in the task is the final volume the package must have. This restriction makes students find connections between measures, shapes and volume (Fig. 12.6).

Each student is asked to calculate the volume of every package created by their classmates in order to practise and consolidate different techniques and formula about volume calculation. This way they won't be working with exercises detached from reality or with exercises that can be found in any book. They will be working with activities based on their classmates' productions and which consists of analysing a physical and tangible object. This helps them to notice and reflect on any mistakes that they make in the calculations.

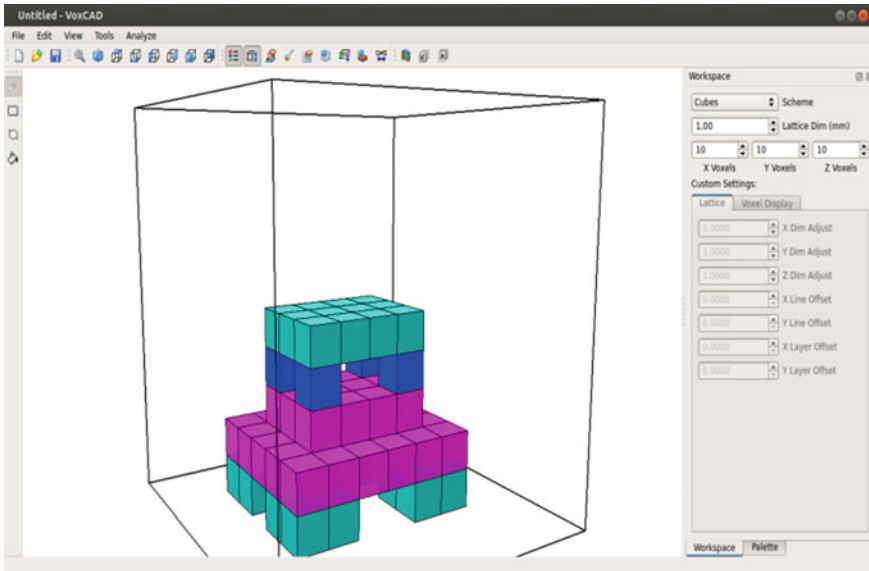


Fig. 12.4 A student's voxelcad solution

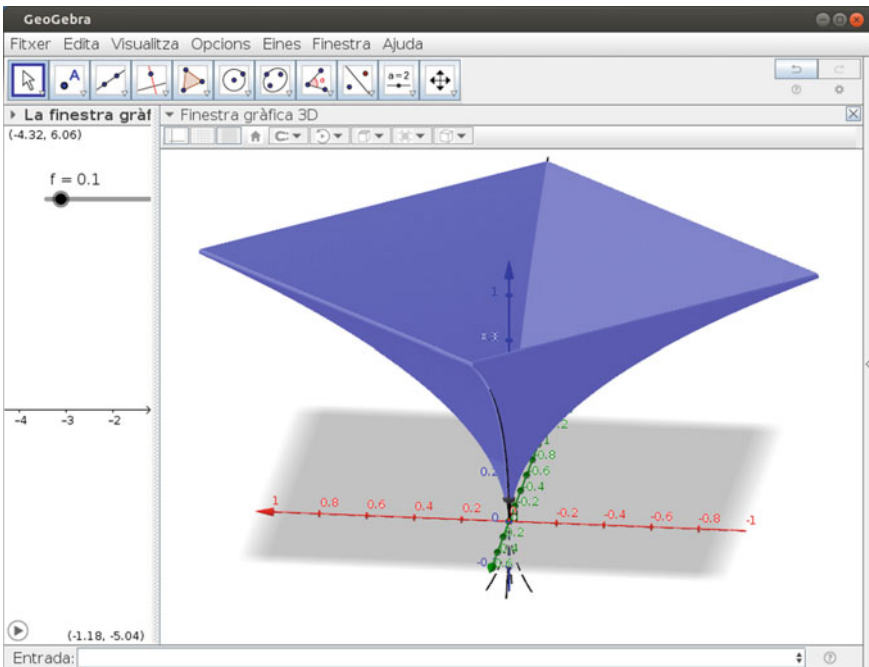


Fig. 12.5 A student's GeoGebra research

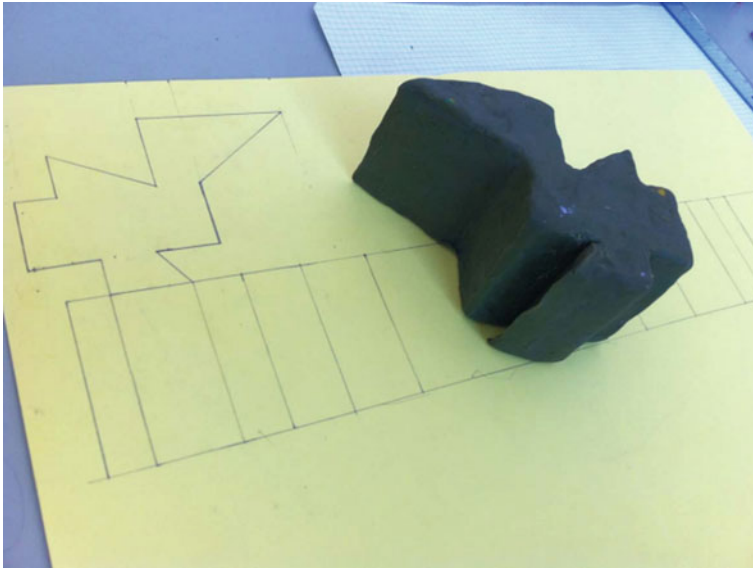


Fig. 12.6 A student's creative object

We would like to remark on two Mathematical problems that have appeared and the solution we have used. The first relates to the Pythagorean theorem. The students didn't know the Pythagorean theorem before the activity was carried out. Students who wanted to create a pyramid or cone needed this theorem in order to know which measures of the flat design of the package were required. This problem was solved using GeoGebra or drawing the triangle and then measuring its unknown sides. The second problem relates to irregular shapes. Some students created packages with a totally irregular base. In order to solve this problem, the "covering with spaghetti" technique was used so as to calculate the base's area. This consists of covering the whole shape with spaghetti and then adding up each piece of spaghetti's area (see Fig. 12.7). It is an approximate integral technique.

Here we explain the results of the interdisciplinary activity "Create your own package" in detail from the perspective of the mathematical complexity and difficulty of the final object. We specifically discuss the objects created at our own school in 2015–2016.

The activity involved 96 students and 6 teachers. Students were divided into 4 classes with heterogeneous groups of 4 members in each. 10.5% of the 96 students had learning difficulties.

Students created their packages with the following results:

- (i) 3% of the students didn't create any package due to serious discipline or absenteeism problems. These students didn't have any learning difficulties.



Fig. 12.7 Using an approximate integral technique

- (ii) 42% of the students created their packages using a basic shape. In these cases, the volume calculation consisted of calculating the base's area and multiplying it by its height (including cylinders or prisms with a rectangular or polygonal base).
- (iii) 22% of the students created packages formed by two or more basic shapes such as a cube and a cylinder.
- (iv) 26% of students created figures which required a complex calculation of its volume or construction. This included, for example, pyramids, cone's trunks, spheres or semi-spheres.
- (v) The final 7% constructed completely irregular figures which required a highly difficult volume calculation. Considering the students' ages and knowledge, this meant only making an approximation, using tools such as GeoGebra (Fig. 12.8).

12.3.1.3 Impact of the Activity

In order to evaluate the students' mathematical knowledge, they also had to complete a test in which they were asked to calculate the volume of a given three-dimensional geometrical figure. These were the results:

- (i) 7% of the students were not able to solve the problem.
- (ii) 15% made some serious mistake.
- (iii) 25% had some difficulty or made a small mistake.
- (iv) 52% calculated the volume correctly.



Fig. 12.8 Examples of student packages (top left, basic; top center, mixed; top right, complex calculation, bottom, irregular figure)

Elsewhere, in Assessment Board of the Educational System¹ for example, there are some indications that students at the schools which engage in these interdisciplinary activities do better than average in Catalan standardized tests at age 16, but this requires further research to substantiate.

In general, teachers assessed the overall experience in a positive way. Teachers said that students had a higher involvement when they worked with their own creative production and with “real objects”. For instance, a comment by a science teacher in *Arquitecte Manel Raspall* High school, is very illustrative. She wrote in her final assessment report of the activity “Create your own package” 2015–2016: “I don’t really know if this activity improves students’ results. But what I do know is that it is an activity which gives the opportunity to connect many things and this fact makes students think and reflect, which is always a good thing”.

In relation to the coordination in an interdisciplinary activity, teachers felt it was useful knowing what teachers from other subjects were doing, because they could then make teaching much more rational through finding out common knowledge. It was also seen to help avoiding unnecessary repetitions and having different approaches to the same content. However, the majority of teachers involved men-

¹http://csda.gencat.cat/ca/arees_d_actuacio/avaluacions-consell/.

tioned that the activity demanded a large amount of meeting hours with each of the other teachers involved, and they did not always have availability for this. This was probably the biggest concern when carrying out this interdisciplinary activity.

However, from the assessment point of view, teachers agree easily about suggestions for improvement in the formative assessment of each student and their marks because methodology are similar and there are some common assessment criteria in each subject involved in the interdisciplinary activity.

12.3.1.4 Continuing the Activity

“Create your own package” is an interdisciplinary activity for 12–14 year-old students but we have extended this activity from a different point of view for 16–18 year-old mathematics students. In this case, it is not an interdisciplinary activity, as there are not more subjects involved, but there are connections with physics, technology and social aspects. It is really interesting to design packages with students who know derivative functions and have the tools to work with the curricular content of integrals. We work with the design of a one-litre-capacity package (bottles, jars, decanters, etc.) using the dynamic geography program GeoGebra 3D and the modification of different parameters of polynomial functions as the starting point. In fact, the activity consists only in designing a package and seeing it in a virtual environment. It’s impossible to build the outcome till now. However, in Pla Marcell High school we are working with engineering teachers because they have bought a 3D-printer. So, we are preparing a new interdisciplinary activity together for the next course. We will see!

The most important thing for us is to continue this interdisciplinary activity in each of the schools where it has been put in practice, although subjects may vary depending on the school year.

12.3.2 Case Study 2: Creating Musical Instruments (11–13-Year-Olds)

This case study engaged students in creating musical instruments. The original idea was to study feelings which are produced by music using mathematical or statistical methods. Music teachers listened to all kind of music with teenagers and it was a good activity because students were completely hooked. So, music teachers wanted to do some activity more deeply. On the other hand, maths teachers wanted to join mathematics and music in a single subject in order to study progressions, integer numbers, powers and roots, functions, especially logarithmic and exponential ones, and statistics. So, maths teachers and music teachers started to work together but we needed to define which feelings could be involved. This way, language teachers joined us. And finally, could it be possible to perform a concert where students played their own instruments?

Table 12.4 Educational curriculum put into practice in Marta Estrada High school, 2016–2017

Subject	Educational curriculum
Mathematics	Progressions, integer numbers, power and roots Statistics: average and diagrams
Music	Music and feelings
Physical Education	Mime
Linguistics	Objective and subjective text. Vocabulary about feelings Descriptive text. Idioms
Arts	Colours and feelings

12.3.2.1 Context, Curriculum and Assessment

This interdisciplinary activity has been developed in the centres during detailed periods of time and with different subjects involved (Table 12.3).

As in the previous case, in each school, the activity began with a meeting which brought together teachers of different subjects, in order to decide the educational curriculum they would develop. Table 12.4 is an example from school year 2016–2017.

When Technology has been involved, in El Sui High school and Arquitecte Manuel Raspall High school, the educational curriculum included materials and tools. Also, Geography was studied at El Sui High School using different instruments and musical scales from different cultures as the starting point.

The common outcome here is a performance where students play instruments created by themselves, while communicating feelings. In order to achieve the common outcome, students have to aim for the following specific goals in each subject:

- Maths** The mathematical challenge lies in calculating progressions and distinguishing arithmetic ones from geometric ones. In arithmetic progressions students go deeper into integer numbers and in geometric progressions students go deeper into powers and roots. After that, students argue about the musical scale and the relation between notes and size and shapes of “musical instruments”.
- Statistics** The statistical challenge lies in using diagrams in order to assess feelings. Each student experimentally measured the necessary length of a tube so that it could generate a specific frequency (using a digital instrument tuner). Studying the average will give us the chance to experimentally determine each tub’s length and prove that it adjusts to a geometrical progression. Statistical parameters which enable us to define which emotion a specific music creates are also studied. We work using spider diagrams and the averages came from the votes of the listeners to the music.
- Music** In this case the challenge lies in listening to different kinds of music and deciding which feeling will be involved. While studying how



Fig. 12.9 Preparing their performance playing their own musical instruments. Marta Estrada High School, 2016–2017

to listen to music, emotions (defined at language class) awakened by different kinds of music were analysed (using statistical tools). Acoustic foundations of the instruments students made were studied and musical interpretations with these instruments were prepared.

- | | |
|--------------|---|
| Physical Edu | Mime is the tool to identify the different traits of the transmission of feelings through the expression. |
| Language | The challenge lies in defining feelings and producing text expressing them by idioms, poetry, etc. |
| Art | Students will make the artistic work with colours and feelings. |
| History | The importance of the music in the ancient history: culture, religion and tradition. |

Teachers then create the working guideline with the activities from their subject and define a cooperative work where each heterogeneous group of 4 or 5 students have to choose a feeling (Linguistics) and prepare their performance (Physical Education and Art) playing (Music) their musical instruments (Maths) about that feeling. At Marta Estrada High school, the musical instruments have been built in Maths sessions because the Technology teacher has not been involved.

Pan flutes made of PVC tubes and idiophones made of cardboard tubes were created in El Sui High School. In Arquitecte Manuel Raspall High School they also made tambourines, rain sticks and drums. And in Marta Estrada High school they also made sophisticated idiophones (Fig. 12.9) and maracas (yellow object in Fig. 12.9).

Assessment is totally integrated within the learning process due to the fact that if calculations were wrong, the instruments would not sound properly. Furthermore, students assess the performance of each other group of students using statistical

diagrams in order to evaluate feelings. Finally, they share opinions about the performance and feelings.

12.3.2.2 The Activity in Mathematics

In Mathematics, the first step is to study progressions and relate them to musical scale. It's easy to start with arithmetic progressions. From a curricular point of view, Maths teachers were interested in connecting this activity to order of operations with integers. So, for instance, a detailed study of the numerical values of different successions made it possible to contextually justify multiplication rules. On the one hand, “doing” was connected to the positive sign and “undoing” to the negative one. On the other hand, going up was positive and going down was negative. In this way, in a numerical sequence, undoing a down meant going up. Which means that negative (undoing) of a negative (down) is positive (going up).

Then, we continue with geometric progressions and the measures of the “tubes”. In particular, one of the goals of this activity was to experimentally check if the measure of the pan flute's tube had to be a geometrical progression of common ratio twelfth root of 2 (well-tempered scale). Surprisingly, it is like this with the big cardboard tubes used to create the idiophones but not with the small PVC tubes used to create pan flutes. The lack of a physics specialist made it impossible to analyse the reason why. However, an activity about measuring the necessary length of a cello string to create a tempered scale was added. The presence of the geometrical progression of common ratio twelfth root of 2 was verified with this experiment. This subject became an interesting discussion with the students and created a big interest in doing experiments and numerical testing.

At the same time, students calculate averages when they are needed. For example, at Arquitecte Manuel Raspall High school each student made their own calculations about the length of the tub to make the pan flute and shared the results. Average was calculated and 5% of the values were far away from the average so they had an implied mistake. These values were checked or rejected. Furthermore, spider diagrams are studied in order to evaluate feelings. Those diagrams will be used in Music class and they will be the tool to assess their colleagues' performances (Fig. 12.10).

12.3.2.3 Impact of the Activity

At Arquitecte Manuel Raspall High school, in later internal tests meant to detect the routine knowledge about integers operations, results were surprisingly good. More than 80% of the students were detected to properly solve this kind of exercise. While in other courses where this interdisciplinary activity wasn't carried out, the level of achievement was under 50%. However, this requires further research to substantiate.

Besides, the use of spider diagrams to do the emotions study was easily achieved by students. Later on they used them to make reflections on how to improve different kinds of self-assessment in tutorial sessions and in other subjects like social sciences.

Fig. 12.10 Playing her pan flute. Arquitecte Manuel Raspall High school, 2014–2015



At Arquitecte Manuel Raspall High school, the assessment made by teachers was highly positive, especially for the technology and music teachers. On the one hand, technology teachers were able to work the “technological process” part of the curriculum with a final outcome that was more than just the typical object that students bring home to decorate. On the other hand, music teachers felt more valued since students showed more interest in the music subject than usual. It is also important to mention the mathematics teachers’ satisfaction when improving integer calculation as a consequence of this interdisciplinary activity. At Marta Estrada High school, in a coordination meeting, language teachers (Catalan and English) said that it has been very useful to simultaneously study linguistic aspects like definitions, idioms and poetry because students have compared both languages spontaneously. At Vilamajor High School, like in Marta Estrada High School, the most important thing in the assessment by teachers (and students) was the final concert using their own instruments. Furthermore, in Vilamajor was a public performance playing flutes and singing while showing the process of construction in the technology lab (Fig. 12.11).

12.3.2.4 Continuing the Activity

“Creating musical instruments” is an interdisciplinary activity for 11–13 year-old students and we have continued with this activity from a different point of view for 16–18 year-old mathematics students. In that case, the math teacher is the only teacher



Fig. 12.11 Final outcome: A public performance of music using their own flutes and showing their construction in technology lab, 2018

involved because in Upper secondary education curricula, Music is not included in the Technological branch. The same way Maths is not included in Arts branch.

Students analyse the musical scale, which sequence it belongs too, and they find out the exponential function. Then the Maths teacher asks students which music they could make if instead of using the exponential function they used other functions (like the parabola, trigonometric functions, ...). In this activity, students must find the hypothetical music frequencies that different functions would create and program them using PureData, a software program that gives them the option to listen to music played with a freely programmed musical scale. This interdisciplinary activity makes students think about the features of every basic function using such a ludic and dynamic aspect as musical interpretation is.

Finally, another activity about musical scales is “The Vincenzo Galilei rebellion: when music was part of mathematics”. It has been carried out in the university for the last three years, as part of a mathematical talent development program called “Crazy for Mathematics”, targeted at 16–17 year-olds and it is guided by the authors. This activity presents to the students different musical scales along history from a Maths point of view. Then, students discover that it is impossible to have perfect musical scales mathematically because if we accept the harmonics, we will have an imperfect interval. In the opposite way, if we have perfect intervals, we will not respect the harmonics. So we, as Math teachers, propose to students to create a new scale using mathematical criteria. Finally, students have to perform a composition using their



Fig. 12.12 Showing their musical scale research by Pure Data in “The Vincenzo Galilei rebellion: when music was part of mathematics”, 2016

own scale (Fig. 12.12). In the last edition of this activity, on February 2017, students’ assessment of this activity averaged 9.4 out of 10.

12.4 Conclusions

In this final section we would like to present the conclusions from the experience of our interdisciplinary activities’, including those described above, in which we have been engaged with over the last seven school years, both directly as teachers working with our own students, and as trainers of other teachers who have carried them out at their own schools.

First of all, interdisciplinary activities promote a learning improvement in students, not only in the mathematical skills but also in other key competencies. (We believe that they also improve skills in other subjects even though we cannot prove it). It is not possible to establish a direct connection with the improvement in national tests though since many factors have an influence.

In Mathematics, interdisciplinary activities allow us to give a new role to mathematical teaching, which promotes creative abilities in students and makes mathematics more than just a calculating tool. Notably, every student takes part in his or her own active learning process due to several reasons. Firstly, because the interdisciplinary activity offers to students a consistency between the subjects which are involved.

Furthermore, the activity is about a reality close to students (cans, music, feelings, etc.) and therefore it engages students. Secondly, because collaborative work forces students to be part of the group since the other members themselves make them work. And finally, because they see the task as suitable to their cognitive and creative abilities. On the other hand, this kind of activity includes an assessment that ensures students are active from the beginning with producing the report and achieving the final outcome.

Co-ordinated interdisciplinary activity allows teachers to know what teachers from other subjects are doing and helps them make teaching much more rational in finding out common knowledge. It also helps avoiding unnecessary repetitions and having different approaches to one and the same content. However, the activity demands a big amount of meeting hours with each of the other teachers involved, and they do not always have availability for this. This is probably the biggest concern when carrying out this interdisciplinary activity. Another difficulty is how to enrol teachers to work within an interdisciplinary activity. From our experience, it's a "seduction problem": we have always collaborated with friendly teachers.

Personally, we have enjoyed a lot working with interdisciplinary activities because students are more cheerful. So, their learning improves not only in Maths but also in other subjects involved. Furthermore, we have gained from interdisciplinary activities because we have taught collaboratively with teachers from other subjects and, as a result, we have improved our teaching.

12.5 We Encourage You to Try It

Three variables constitute the instructional core (City, Elmore, Fiarman, & Teitel, 2009), that is the teacher, the learner and the contents. These variables are interdependent and in constant motion. Thus, a change in any of these variables has an impact on others and on the final result. A student's academic success results from the quality and frequency of the interconnections between student, teacher and content (Salavert, 2015). So, a competent teacher should achieve each student to be aware of four aspects about their own learning: what should they learn, why should they learn that, how should they do it and how should they improve it. Students should achieve the outcome in their learning process if teachers propose them a creative, closer and motivating challenge. In conclusion, students lead their own learning because they know the purpose of their learning through competence activities (Gorriz & Nuñez, 2013).

In this chapter, we have proposed these activities which encourage students to reflect on their progress by sharing what they have learned and by asking questions about what they did not understand and teachers use this ongoing formative feedback to inform their guidance in a continuous, differentiated and enriching loop. Furthermore, these activities allow to show knowledge as interdisciplinary from Maths. Teachers from different subjects find a common final object to develop students learning. All of us think that "it's better to work with the same activities in a different

way (from students creativity) rather than to work with different activities in the same way” because students motivation is essential.

These activities allow both the curriculum and teaching to develop around the learners’ own interests and these emphasise students engagement in their formative assessment. These proposal is coherent with the new project of law about assessment in Catalonia which is according to the Competence based Curriculum (decree 187/2015). However, Assessment Decree project would had been published a few month ago but he Spanish government has paralysed the regional government of Catalonia from October 2017.

From July 2017, Innovation and Training department of Ministry of Education of Catalonia is promoting a training program about competence assessment (formative assessment and selfassessment) and we, the authors of this chapter, are coordinating the training in Maths as a subject. In this training program, we promote interdisciplinary activities and encourage Maths teachers to find partnerships in other subjects in order to encourage learners to take control of and regulate their own learning in a creative environment.

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