Chapter 1
Notes to the Reader

The present book covers the multivariate version of the theory of Generalized Locally Toeplitz (GLT) sequences, also known as the theory of multilevel GLT sequences. In addition, the book presents some emblematic (multidimensional) applications of this theory in the context of the numerical discretization of Partial Differential Equations (PDEs).

The generalization of the theory of GLT sequences from the univariate case addressed in [22] to the multivariate case addressed here is essentially a matter of technicalities, which results in the technical nature of the present volume. *We therefore recommend that, before going into this book, the reader give a reading to* [22, pp. 1–3] *in order to call to mind the motivations behind the theory of (unilevel and multilevel) GLT sequences,* which will not be repeated here for the sake of conciseness. When reading [22, pp. 1–3] in a multidimensional perspective, the GLT sequences and the Differential Equations (DEs) mentioned therein should be understood as multilevel GLT sequences and PDEs, respectively.

After going through [22, pp. 1–3], we encourage the reader to try reading this book according to the scheme suggested in the preface, which consists in reading Chaps. 6 and 7 first, and then coming back to fill the gaps (if necessary or wanted).

When reading the present book, it is advisable that the reader have at hand the first volume [22], for at least two reasons. First, [22] is cited many times throughout the book. Secondly, several "multivariate proofs" from Chaps. 2–5 are essentially the same as their corresponding "univariate versions" from [22], and we recommend that the reader compare them with each other so as to learn the way in which the multilevel language (especially, the multi-index notation) allows one to transfer many results from the univariate to the multivariate case. Roughly speaking, this transfer process is carried out through a sort of "automatic procedure" consisting in turning some letters (n, i, j, x, θ, etc.) in boldface ($\boldsymbol{n}$, $\boldsymbol{i}$, $\boldsymbol{j}$, $\mathbf{x}$, $\boldsymbol{\theta}$, etc.). Finally, we remark that, as highlighted in the preface, the first volume [22] is an *essential prerequisite* to this second volume. In addition to [22], the other necessary prerequisite for reading this book is a basic knowledge of multidimensional integro-differential calculus (partial derivatives, multiple integrals, etc.).