



# Improved Time-Resolved MRA Using $k$ -Space Deep Learning

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**Abstract.** In dynamic contrast enhanced (DCE) MRI, temporal and spatial resolution can be improved by time-resolved angiography with interleaved stochastic trajectories (TWIST) thanks to its highly accelerated acquisitions. However, due to limited  $k$ -space samples, the periphery of the  $k$ -space data from several adjacent frames should be combined to reconstruct one temporal frame so that the temporal resolution of TWIST is limited. Furthermore, the  $k$ -space sampling patterns of TWIST imaging have been especially designed for a generalized autocalibrating partial parallel acquisition (GRAPPA) reconstruction. Therefore, the number of shared frames cannot be reduced to provide a reconstructed image with better temporal resolution. The purpose of this study is to improve the temporal resolution of TWIST using a novel  $k$ -space deep learning approach. Direct  $k$ -space interpolation is performed simultaneously for multiple coils by exploiting spatial domain redundancy and multi-coil diversity. Furthermore, the proposed method can provide the reconstructed images with various numbers of view sharing. Experimental results using in vivo TWIST data set showed the accuracy and the flexibility of the proposed method.

**Keywords:** Dynamic contrast enhanced MRI · Parallel imaging  
Deep learning

## 1 Introduction

DCE-MRI is useful for the diagnosis of stroke or cancer because it provides information on the physiological characteristics of the tissue by imaging the flow of the contrast agent [16]. In particular, TWIST [11] imaging gives improved temporal and spatial resolution thanks to its highly accelerated acquisition. In TWIST, the high frequency regions of the  $k$ -space from multiple temporal frames should be combined to obtain uniformly sub-sampled  $k$ -space data so that GRAPPA [4] can be applied to reconstruct the data. However, the temporal resolution of

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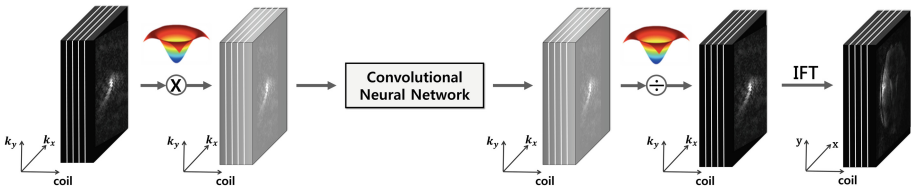
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TWIST is not a true one due to the view sharing of several temporal frames. In addition, since the  $k$ -space sampling patterns are designed for GRAPPA reconstruction, the number of view sharing is fixed after the data acquisition.

In our previous works [3], we proposed to improve temporal resolution of TWIST via  $k$ -space interpolation using ALOHA [8, 12, 14] which synergistically combines parallel MRI (pMRI) and CS-MRI. However, since the multiple matrix factorization is essential for applying ALOHA, the computational cost for the reconstruction of 4-dimensional TWIST imaging was too expensive. In addition, if the number of view sharing is not enough, the spatial resolution can be degraded. Therefore, new approach is required to overcome this limitation.

This paper aims at enhancing the temporal resolution of TWIST imaging by reducing the number of view sharing using deep learning. Furthermore, we proposed the algorithm that can generate reconstructed images at multiple number of view sharing to exploit the trade-off between spatial and temporal resolution. For our purposes, we need to deal with two major technical issues. First, unlike most of the deep learning approaches for MR reconstruction [6, 10, 13, 15, 17], our deep network needs to learn the  $k$ -space interpolation kernels for reconstruction at various number of view sharing. Second, with reduced view sharing, the reconstructed images using GRAPPA cannot be regarded as ground-truth data, so there is no label data for learning.

Based on the recent mathematical finding of the link between a deep convolutional neural network and a data-driven decomposition of Hankel matrix [18], here we propose a  $k$ -space deep network using the basic idea of ALOHA for parallel MRI [9], which is implemented in the  $k$ -space domain by stacking multi-coil  $k$ -space data along the channel direction of the network as shown in Fig. 1.



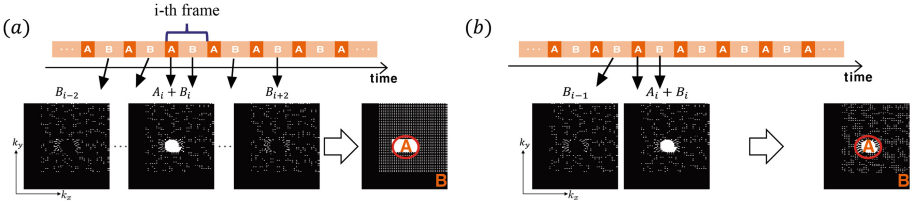
**Fig. 1.** An overall scheme of  $k$ -space deep learning for parallel MRI. IFT represents inverse Fourier transform.

Another major contribution of this paper is that our network learns the  $k$ -space interpolation relationship between the minimum number of  $k$ -space samples and completely sampled  $k$ -space data from GRAPPA reconstruction to address the lack of ground-truth data. As will be shown in later, this approach allows the trained network to provide accurate reconstruction results at various number of view sharing, since the network is trained to learn the Fourier domain features rather than image domain ones.

## 2 Theory

### 2.1 Problem Formulation

In TWIST, the center of  $k$ -space data (A region in Fig. 2) is more frequently sampled than the periphery of  $k$ -space data (B region in Fig. 2). Since it reduces the total number of samples for each frame, the reduced acquisition time is required. However, high frequency  $k$ -space data from several frames should be shared to make one time frame due to the strongly subsampled high frequency  $k$ -space data. Therefore, the actual temporal resolution of TWIST imaging is determined by the number of view-sharing.



**Fig. 2.** The center and periphery of  $k$ -space are denoted by A and B, respectively. (a) Standard view sharing scheme for 2D GRAPPA reconstruction, and (b) an example of reduced view sharing scheme.

There are different types of view sharing. For example, as shown in Fig. 2(a), one type of view sharing is specifically designed for 2-D GRAPPA reconstruction, where high frequency regions of five time frames ( $B_{i-2}, \dots, B_{i+2}$ ) are combined to provide a 2-D uniform sub-sampled  $k$ -space data with downsampling factor of three and two along  $k_x$  and  $k_y$  directions, respectively.

Unlike the standard TWIST view sharing scheme, we are interested in using various number of reduced view sharing. For example, the number of view sharing can be reduced to two frames as shown in Fig. 2(b). GRAPPA cannot be applied to this irregular sampling pattern, so we proposed a multi-coil deep learning approach to reconstruct the  $k$ -space data.

### 2.2 From ALOHA to Deep Neural Network

ALOHA [9, 19] was developed based on the duality between the sparsity in image domain and the low-rankness of associated Hankel matrix in the  $k$ -space domain. In addition, for parallel MRI, there exists the  $k$ -space inter-coil annihilating filter relationship [9]:

$$\hat{g}_i \otimes \hat{s}_j - \hat{g}_j \otimes \hat{s}_i = 0, \quad \forall i \neq j, \quad (1)$$

where  $\hat{g}_i$  and  $\hat{s}_i$  denote  $k$ -space data of the  $i$ -th coil and the spectrum of the  $i$ -th coil sensitivity map, respectively. This relationship in (1) leads to the low-rank

property of the following extended Hankel structured matrix [9]:

$$\mathbb{H}_{d|P}(\widehat{\mathbf{G}}) = [\mathbb{H}_d(\widehat{\mathbf{g}}_1) \cdots \mathbb{H}_d(\widehat{\mathbf{g}}_P)] \quad (2)$$

where

$$\widehat{\mathbf{G}} = [\widehat{\mathbf{g}}_1 \cdots \widehat{\mathbf{g}}_P] \in \mathbb{C}^{N \times P}$$

with the  $k$ -space measurement  $\widehat{\mathbf{g}}_i = [\widehat{g}_i(\mathbf{k}_1) \cdots \widehat{g}_i(\mathbf{k}_N)]^T$ , and  $\mathbb{H}_d(\widehat{\mathbf{g}}_i)$  is a Hankel matrix constructed from  $\widehat{\mathbf{g}}_i$  with  $d$  denoting the matrix pencil size.  $P$  denotes the number of coils. Therefore, the missing elements of  $k$ -space data can be recovered using low rank Hankel matrix completion approaches [2, 5]:

$$\begin{aligned} (MC) \quad & \min_{\widehat{\mathbf{Z}} \in \mathbb{C}^{N \times P}} \text{RANK } \mathbb{H}_{d|P}(\widehat{\mathbf{Z}}) \\ & \text{subject to } \mathcal{P}_A[\widehat{\mathbf{g}}_i] = \mathcal{P}_A[\widehat{\mathbf{z}}_i], \quad i = 1, \dots, P, \end{aligned} \quad (3)$$

where  $\mathcal{P}_A$  is the downsampling operator  $\mathcal{P}_A : \mathbb{C}^N \rightarrow \mathbb{C}^N$  defined as

$$[\mathcal{P}_A[\widehat{\mathbf{x}}]]_j = \begin{cases} [\widehat{\mathbf{x}}]_j & j \in A \\ 0, & \text{otherwise} \end{cases}. \quad (4)$$

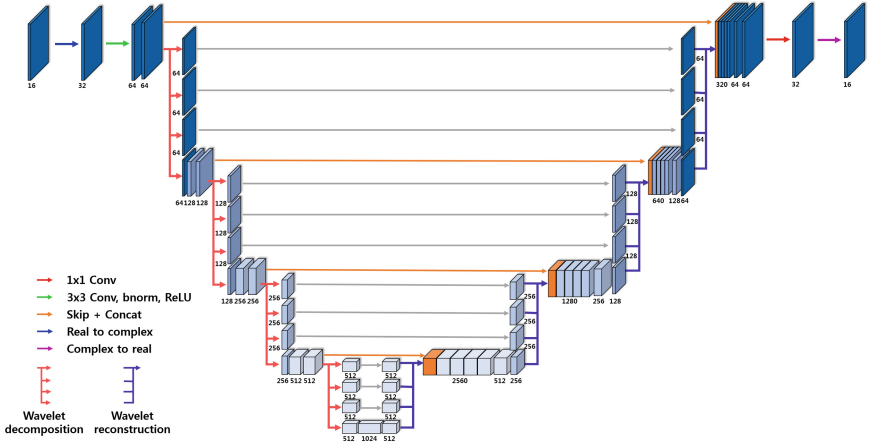
However, this approach needs a relatively expensive computational cost for matrix factorization.

Recently, our group proposed  $k$ -space deep learning approaches for accelerated MRI [7] based on the observation that the Hankel matrix in the weighted  $k$ -space domain is low-ranked so that deep neural network can be efficiently implemented. By extending this idea, we apply the deep learning for the multi-channel  $k$ -space data by stacking the multi-coil  $k$ -space data along the channel direction of the network input.

### 3 Method

Four sets of in vivo 3D DCE data for carotid vessel imaging were acquired with a TWIST sequence using Siemens 3T Verio scanners. The scanning parameters for two sets were as following: repetition time (TR) = 2.5 ms, echo time (TE) = 0.94 ms,  $159 \times 640 \times 80$  matrix, 2.5 mm slice thickness, 16 coils, and 37 temporal frames. For other two sets, the acquisition parameters were same as above, expect for 1.2 mm slice thickness and 30 temporal frames. The sampling pattern of data sets is illustrated in Fig. 2(a). Only 63% of data was acquired due to the partial Fourier. The downsampling factor was three and two along  $k_x$  and  $k_y$  direction, respectively. Among four patient data sets, three patient data sets were used for training and validation. We used the remaining one patient data set for test. The input  $k$ -space data for network is the  $k_x$ - $k_y$  slice along  $z$  direction and temporal frames.

We employed the tight-frame U-net [18] thanks to its capability of preserving of the details of image. To deal with complex-valued multi-channel  $k$ -space



**Fig. 3.** Network architecture of tight-frame U-net.

data, we divide the complex-valued  $k$ -space data into real and imaginary channels similar to [7]. The interpolated  $k$ -space data can be formed from the real and imaginary channels as shown in Fig. 3. We implemented the network using TensorFlow library [1].

## 4 Result

Figure 4 showed the subtracted maximum intensity projection (MIP) images for test data. The temporal frames were selected to illustrate the propagation of the contrast agent. In the proposed method, we generated the reconstructed images using same neural network at various number of view sharing (VS). The raw data in Fig. 4 is obtained by directly apply inverse Fast Fourier Transform (FFT) to the  $k$ -space data without view sharing, which provide the true temporal resolution.

In the GRAPPA reconstruction, the contrast agent was suddenly propagated from the  $T = 10$  frame to  $T = 11$  frame as shown in Fig. 4. Since the degradation of temporal resolution can be caused by the combination of multiple temporal frames, the flow of contrast agent can be quickly changed only between one frame. In the reconstructed images with  $VS = 2$  using the proposed method, the dynamics of the contrast agent is correctly demonstrated. As shown in Fig. 4, the degree of temporal blurring in  $T = 11$  frame can be captured depending on the number of view sharing. The results of proposed method with  $VS = 5$ , which is same to the conventional method, provided very similar spatial and temporal resolution to the GRAPPA reconstruction.

Furthermore, the computational cost of the proposed method is more efficient than that of GRAPPA and ALOHA. The proposed method can produce the result only in 0.029s, which is several order of magnitude faster than GRAPPA and ALOHA.

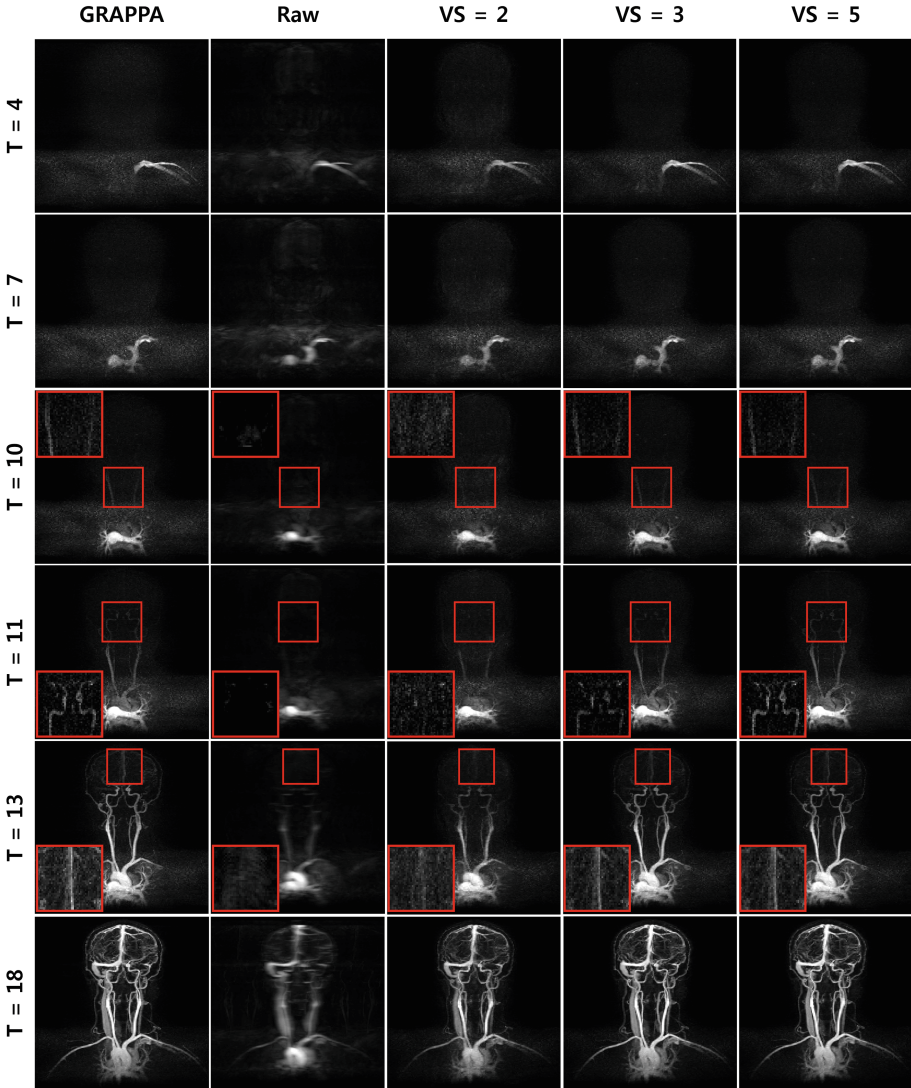


Fig. 4. Subtracted MIP results of GRAPPA, raw data and the proposed methods for various number of view sharing. VS stands for the number of view sharing.

## 5 Conclusion

In this paper, to enhance the temporal resolution of TWIST imaging and to develop an algorithm that generates reconstruction results at various sliding window size, we proposed a novel  $k$ -space deep learning algorithm for parallel MRI. Our  $k$ -space deep network can exploit the redundancies along the coil and image domain. The experimental results showed that one trained network

can provide multiple reconstruction results with various spatial and temporal resolution by changing the number of view sharing for the network input. We believe that the proposed method suggests a significant new research direction that can extend the clinical applications.

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