Chapter 14

In Conclusion

We have come full circle in a study of stochastic Lagrangian models of turbulent diffusion. We began with Brownian motion as the classic stochastic process and Kolmogorov's hypotheses on the small-scale properties of turbulence. We ended with Kolmogorov's refined hypotheses and fractional Brownian motion. Along the way, we learned about the Markov and Wiener processes, integration of stochastic differential equations, Gaussian and non-Gaussian probability distribution functions, the related Langevin and Fokker–Planck equations, the well-mixed criterion, nonuniqueness of models in more than one dimension, boundary conditions, and parameterization of turbulence statistics for use as model inputs.

For turbulent diffusion in the vertical dimension, the mathematical theory appears to be quite complete (especially for Gaussian turbulence), and input parameterizations are available for *some* of the height-and-stability regimes (e.g., the surface layer). There is a scarcity of high quality turbulence statistics for use in formulating generalized inputs to the models. Specifically, data are needed for use in developing parameterizations that give *continuous* values of model inputs over the *entire* height versus stability parameter space of the atmospheric boundary layer.

There has been some progress in modeling diffusion in two-dimensional Gaussian turbulence, but only a limited amount in multidimensional, non-Gaussian turbulence. These subjects are virtually unexplored.

More work must be done on boundary conditions.

We close with some remarks on "verification." We can test our models of turbulent diffusion by comparing numerical results with 1) analytical solutions and 2) high quality data from experiments in wind tunnels and at selected sites in the field. Can we "verify" these models? Oreskes et al. (1994) answered this question in the negative because natural systems are open, not closed, and model results are not unique. Therefore, all we can hope for is partial confirmation that our models are a satisfactory approximation to incompletely understood natural phenomena.

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