Erich L. Lehmann's Work on the Philosophy of Statistics

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Erich Lehmann's work on the foundations of probability and statistics is discussed in this chapter but his contributions in this area are dispersed over other chapters in this volume, particularly the chapter on his work on the history of classical statistics. The books Theory of Point Estimation (TPE) and Testing Statistical Hypotheses (TSH) also contain implicit and explicit ideas on the foundations of statistics. As expected, the Neyman-Pearson-Wald school is dominant in these works. In addition, Lehmann (2008) and the soon-to-be-published *Fisher, Neyman, and the Creation of Classical Statistics*, contain many of Erich's thoughts on philosophical issues and the historical breakthroughs that have impacted and provided directions for the future of the discipline.

Erich commented in various occasions that he did not involve himself too much in the debate of the foundational issues of statistics. Thus, in DeGroot (1986), after having been asked if he thought about the foundations of probability, he stated: "I don't really think very much about foundational questions." In Lehmann (2008), responding to the self-posed question *What is my outlook?* Erich writes:

> *Foundational issues have not been an active interest of mine, so I find it easiest to describe my attitude with a number of (rather superficial) comments ...*

His work, however, demonstrates otherwise. Erich believed in the frequentist interpretation of probability but also understood that such a perspective is not applicable in some situations (see, e.g. page 188, Lehmann (2008)), and in Lehmann (2001) he discusses some of the philosophical shortcomings of such an interpretation.

Erich was a strong believer in the Neyman-Pearson-Wald school of optimality, while recognizing its limitations. Thus, for example, in Lehmann (1985), Erich writes:

> *Complete reliance on optimality, on the other hand, suffers from some drawbacks.*

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In particular, optimality considerations achieve solutions but only in simple situations, and such solutions may lack robustness and other desirable properties.

There are at least three recurring themes in his work on foundational issues: (i) model selection; (ii) frequentist statistical inference; (iii) Bayesian statistical inference; and (iv) exploratory data analysis. Model selection has been discussed in Erich's contributions to the history of classical statistics. Therefore, attention will be focused on (ii), (iii), and (iv).

It is interesting that the likelihood principle is not discussed in any of Erich's writings, as far as I have been able to ascertain, except for a brief reference in Lehmann (2008) to Berger and Wolpert (1984). This is a bit puzzling in light of the fact that one of his former Ph.D. students at Columbia University published, see Birnbaum (1962), a long treatise on the likelihood principle. The reader must surmise that Erich, rather than expressing his disagreement with the likelihood principle, decided not to discuss it.

Erich viewed the trichotomy (ii), (iii), and (iv) as being ordered by the level of model assumptions made. Thus, (iv) does not make any assumptions of an underlying probability model, and lets the data speak for itself. The frequentist approach, on the other hand, relies on a probability model to carry out an evaluation of the statistical procedures under consideration, while the Bayesian approach introduces the additional component of the prior distribution. None of these three approaches is perfect, however. Exploratory data analysis, except in very limited cases, cannot determine if the observed effects in the analysis are indeed real without the use of a probability model, and the Bayesian paradigm has to deal with the selection of the prior distribution and the sensitivity of the solution to such a choice of prior. On the other hand, the frequentist Neyman-Pearson-Wald approach runs into problems in its search for a procedure that minimizes the risk uniformly (in the parameter). Since this is not possible, then attention has to be restricted to a class of procedures satisfying some additional principles none of which are always satisfactory. Against this background of lack of satisfaction with any given approach, Lehmann (1985) and Lehmann (1995) develop ideas that bridge the chasm that has been created by the heated philosophical debates. For example, Lehmann (1985) discusses how the Neyman-Pearson can contribute to the exploration of underlying data structure and its relation with Bayesian inference. In Lehmann (1995), Erich continues with this line of thought and writes that:

> *In practice, the three approaches can often fruitfully interact, with each benefiting from considerations of the other points of view.*
> *It seems clear that model-free data analysis, frequentist and Bayesian model-based inference and decision making each has its place. The question appears not to be – as it is often phrased – which is the correct approach but in what circumstances each is most appropriate.*

The reader learns to appreciate Erich's balanced view on foundational issues – he discusses both the benefits and the shortcomings of the various approaches to inference and interpretations of probability.

Erich did not like confrontation and, given the tone of some of the published work on the philosophy of statistics, must have preferred to express his views in a low-key and conciliatory manner – but see Lehmann (1990) for an exception to this statement.

Erich writes, in Lehmann (2008), that his original position was solidly in the frequentist camp. With time, however, his position was somewhat influenced by classical Bayesian ideas, but remained unconvinced by the radical Bayesian position. Thus, he writes, in Lehmann (1995), that *bridge building to the "radical" [Bayesian] position is more difficult*. Erich never defined what he meant by a radical Bayesian position, but the reader may surmise that he was referring to a Bayesian paradigm that insists on the elicitation of a prior distribution at all costs. Thus, in Lehmann (2008), he writes:

> *However, it seems to me that the strength of these beliefs tends to be rather fuzzy, and not sufficiently well defined and stable to assign a definite numerical value to it. If, with considerable effort, such a value is elicited, it is about as trustworthy as a confession extracted through torture.*

In the end, Erich's work reflects a philosophy that indicates that no single approach can be totally satisfactory. Rather than debating their differences, Erich proposed that a fruitful approach can be obtained from the bringing together of ideas from (ii), (iii), and (iv) – with (iii) serving as a bridge that connects all three.

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