Murray Rosenblatt and cumulant/higher-order/poly spectra

By David R. Brillinger

The bispectrum, or third order spectrum, of a stationary process has been around at least since the early 50s, for example it appeared in the paper Tukey (1953). It was studied in some detail in John Van Ness's thesis, “Estimates of the Bispectrum of Stationary Random Processes”, supervised by Murray Rosenblatt. A 1965 Annals of Mathematical Statistics paper, "Estimation of the bispectra", Rosenblatt and Van Ness (1965) followed.

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There is a Soviet history extending the concept. In particular Shiryaev (1989) writes: "In the late 1950s and early 1960s Kolmogorov suggested to his pupils V. P. Leonov and A. N. Shiryaev a series of problems related to the issue of nonlinear analysis of random processes (in particular, in radio technology) which brought about the techniques of calculating cumulants under nonlinear transformations, and the development of the theory of spectral analysis of high-order moments of stationary random processes."

I first met Murray in New Jersey in 1963. He was then consulting at Bell Telephone Laboratories Murray Hill under a Brown University program. In particular he was developing properties of a cepstrum estimate, the cepstrum being the Fourier transform of the log of a second-order spectrum. It had been developed as a tool for estimating how deep explosions, as opposed to earthquakes, were sited.

The study in the polyspectra paper, Brillinger (1965), started out as a young academic having fun. Independently of the Russian work I had found that cumulants were a pertinent tool. They were a multilinear operator with the property that if for a vector-valued variate some subset of its entries was independent of the complementary subset, then the joint cumulant of all was zero. This property takes one directly to a definition of mixing for general stationary processes. Perhaps the Russians already knew that type of result. At one point Tukey had mentioned the word polyspectra, and a connection was made. The 1965 paper surely led to my getting to collaborate with Murray later when we were both in London. Ady and Murray had come to London on a Guggenheim Fellowship in the Fall of 1965. On meeting Murray again I was surely a bit disrespectful going right up to Murray and saying something like, ``How about we do some work and write a paper together?" Murray replied with a remark like ``Sure.", and the papers Brillinger and Rosenblatt (1967a, b, c) resulted.

The first, Brillinger and Rosenblatt (1967a,) summarizes the work to be presented in (1967b) and (1967c). It is noteworthy for also seeking to develop a tensor-like notation for joint cumulant spectra after encouragement from John Tukey.

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Brilllinger and Rosenblatt (1967b) sets down some asymptotic theory of estimates of a cumulant spectrum developed under an asymptotic independence or mixing assumption. The estimate is based on an empirical Fourier transform of a long stretch of data. A higher order periodogram is defined and smoothed to form the estimate. An important insight in the work is to base the estimates on the Fourier transforms evaluated at the Fourier frequencies. Large sample means and variances of the estimate were developed, with uniform error bounds. In developing the results substantial use was made of the theorem in Leonov and Shiryaev (1959). The mixing condition involved the absolute summability of the time domain joint cumulant functions. Asymptotic normality was shown by proving that the standardized cumulants of order greater that 2 tended to those of the normal, which is determined by its moments.

Brillinger and Rosenblatt (1967c) concerns properties and interpretations of cumulant spectra as well as details of computation and a scientific example. The thorny problem of aliasing is addressed with fundamental domains for the second, third and fourth order cases presented. Estimates of the second, third and fourth order spectra are computed for a long series of monthly relative sunspot numbers. A fast Fourier transform is employed in the computations.

Murray was an academic role model for me as my career progressed. He was someone to ask to write recommendation letters and to provide advice concerning the professional sides of academic life. For example, I remember being quite concerned when I found an error in a paper that I had published. Murray's reaction was, "Is that the first one?" There have been others since.

References.

D. R. Brillinger, "An introduction to polyspectra", Ann. Math. Statist. 36 (1965) 1351-1374.

D. R. Brillinger and M. Rosenblatt: "Asymptotic theory of estimates of kth order spectra," Proc. Nat. Acad. Sci., 57 (1967a), 206-210.

D. R. Brillinger and M. Rosenblatt: "Asymptotic theory of estimates of kth order spectra," Spectral Analysis of Time Series, John Wiley (1967b), 153-188.

D. R. Brillinger and M. Rosenblatt. "Computation and interpretation of kth order spectra," in Spectral Analysis of Time Series, John Wiley (1967c), 189-232.

M. Rosenblatt: "Remarks on higher order spectra," in Multivariate Analysis, Academic Press (1966), 383-389

M. Rosenblatt: "Cumulants and cumulant spectra," in Handbook of Statistics, vol. 3 (eds. D. Brillinger and P. Krishnaiah), (1983), 369-382.

M. Rosenblatt and J. W. Van Ness: "Estimation of the bispectra," Ann. Math. Stat., 36 (1965), 1120-1136.

A. N. Shiryaev, "Kolmogorov: life and creative activities." Ann Probab. 17 (1989) 866-944.

J. W. Tukey: "The spectral representation and transformation properties of the higher moments of stationary time series." Reprinted in The Collected Works of John W. Tukey, Vol. 1 (Edited by D. R. Brillinger), 165-184. Wadsworth, Belmont.

J. Van Ness "Estimates of the Bispectrum of Stationary Random Processes" 1964 Ph.D. Dissertation, Brown University.