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Perturbations of Order $(\delta R)^2$

Brown, we recall, in commencing his computation of the inequalities due to the direct and indirect actions of the planets and the figures of the Earth and Moon, proposed to neglect, *pro tem*, quantities of the order $(\delta R)^2$. With the initial computation now completed, he undertook, in Chapter 14 of his *Theory of the Motion of the Moon*, an investigation to discover whether any of these second-order perturbations were non-negligible.

The complete disturbing function for all actions other than those dealt with in the main problem was

$$\delta R = R(r' + \delta r' + \delta^2 r', V' + \delta V' + \delta^2 V', z' + \delta z') + \sum (R_P + R_E + R_e).$$

Here $\delta^2 r'$, $\delta^2 V'$, $\delta z'$ are the terms of the second order in the motion of the Sun, and $R(r', V', z')$ is the disturbing function for the Sun's action; R_P, R_E, R_e are, respectively, the parts of the disturbing function for the actions of a planet, the figures of the Earth and Moon, and the motion of the ecliptic. Having derived $\delta^2 R$ from the above expression, Brown examined in detail eight possible second-order results, and found most of them negligible. Two that were not were the indirect effects of solar terms with arguments $4M - 7T + 3V$ and $3J - 8M + 4$; they yielded the terms

$$\delta^2 w_1 = +0''.04 \sin(152^\circ + 119^\circ.0 t_c) + 0''.84 \sin(41^\circ.1 + 20^\circ.2 t_c),$$

where t_c is the number of centuries since 1850. Among additions due to periodic perturbations of the solar and planetary coordinates in ΣR_P due to the Earth's action, Brown found a single term with a sensible coefficient; it had the argument $\ell + 3T - 10V$, and a period of 1900 years. The yield in included terms was small; its main result was to establish that other possible terms in $\delta^2 R$ were negligible.

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