(origins or supply),

$$\sum_i x_{ij} = b_j, \quad j = 1, \dots, n$$

(destinations or demand), where $x_{ij} \geq 0$ and $\sum a_i = \sum b_j$, which is called the *balance condition*. The assignment problem arises when $m = n$ and all a_i and b_j are 1.

If all a_i and b_j in the transposed problem are integers, then there is an optimal solution for which all x_{ij} are integers (*Dantzig's theorem on integral solutions of the transport problem*).

In the assignment problem, for such a solution x_{ij} is either zero or one; $x_{ij} = 1$ means that person i is assigned to job j; the weight c_{ij} is the utility of person i assigned to job j.

The special structure of the transport problem and the assignment problem makes it possible to use algorithms that are more efficient than the **simplex method**. Some of these use the *Hungarian method* (see, e.g., [4], [5, Chapt. 7]), which is based on the König–Egervary theorem (see **König theorem**), the method of potentials (see [5], [1]), the *out-of-kilter algorithm* (see, e.g., [3]) or the transportation simplex method.

In turn, the transportation problem is a special case of the network optimization problem.

A totally different assignment problem is the **pole assignment problem** in control theory.

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M. Hazewinkel

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ASYMPTOTIC INVARIANT OF A GROUP – A property of a **finitely-generated group** G which is a **quasi-isometry** invariant of the **metric space** (G, d_A), where d_A is the **word metric** associated to a finite generating set A of G (cf. also **Quasi-isometric spaces**). This definition does not depend on the choice of the set A, since if B is another finite set of generators of G, then the metric spaces (G, d_A) and (G, d_B) are quasi-isometric.

The theory of asymptotic invariants of finitely-generated groups has been recently brought to the foreground by M. Gromov (see, in particular, [2] and [3]). As Gromov says in [3, p. 8], 'one believes nowadays that the most essential invariants of an infinite group are asymptotic invariants'. For example, amenability (cf. **Invariant average**), hyperbolicity (in the sense of Gromov, cf. **Hyperbolic group**), the fact of being finitely presented (cf. **Finitely-presented group**), and the number of ends (cf. also **Absolute**) are all asymptotic invariants of finitely-generated groups. It is presently (1996) unknown whether the Kazhdan T-property is an asymptotic invariant. For an excellent survey on these matters, see [1].

A few examples of algebraic properties which are asymptotic invariants of finitely-generated groups are: being virtually nilpotent, being virtually Abelian, being virtually free.

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A. Papadopoulos

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ASYMPTOTIC OPTIMALITY of estimating functions – Efficient estimation (cf. **Efficient estimator**) of parameters in stochastic models is most conveniently approached via properties of estimating functions, namely functions of the data and the parameter of interest, rather than estimators derived therefrom. For a detailed explanation see [3, Chapt. 1].

Let $\{X_t \colon 0 \leq t \leq T\}$ be a **sample** in discrete or continuous time from a stochastic system taking values in an r-dimensional Euclidean space. The distribution of X_t depends on a parameter of interest θ taking values in an open subset of a p-dimensional Euclidean space. The possible probability measures (cf. **Probability measure**) for X_t are $\{\mathsf{P}_\theta\}$, a union of families of models.

Consider the class $\mathcal{G}$ of zero-mean square-integrable estimating functions $G_T = G_T(\{X_t \colon 0 \leq t \leq T\}, \theta)$, which are vectors of dimension p and for which the matrices used below are non-singular.

Optimality in both the *fixed sample* and the *asymptotic* sense is considered. The former involves choice of an estimating function G_T to maximize, in the partial order of non-negative definite matrices, the information criterion

$$\mathcal{E}(G_T) = (\mathsf{E}\nabla G_T)'(\mathsf{E}G_T G_T')^{-1}(\mathsf{E}\nabla G_T),$$

which is a natural generalization of the **Fisher amount of information**. Here ∇G is the $(p \times p)$-matrix of derivatives of the elements of G with respect to those of θ and

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prime denotes transposition. If $\mathcal{H} \subset \mathcal{G}$ is a prespecified family of estimating functions, it is said that $G_T^* \in \mathcal{H}$ is *fixed sample optimal* in $\mathcal{H}$ if $\mathcal{E}(G_T^*) - \mathcal{E}(G_T)$ is nonnegative definite for all $G_T \in \mathcal{H}$, θ and P_θ. Then, G_T^* is the element of $\mathcal{H}$ whose dispersion distance from the maximum information estimating function in $\mathcal{G}$ (often the likelihood score) is least.

A focus on asymptotic properties can be made by confining attention to the subset $\mathcal{M} \subset \mathcal{G}$ of estimating functions which are martingales (cf. **Martingale**). Here one considers T ranging over the positive real numbers and for $\{G_T\} \in \mathcal{M}$ one writes $\{\langle G \rangle_T\}$ for the *quadratic characteristic*, the predictable increasing process for which $\{G_T G_T' - \langle G \rangle_T\}$ is a martingale. Also, write $\{\overline{G}_T\}$ for the predictable process for which $\{\nabla G_T - \overline{G}_T\}$ is a martingale. Then, $G_T^* \in \mathcal{M}_1 \subset \mathcal{M}$ is *asymptotically optimal* in $\mathcal{M}_1$ if $\overline{\mathcal{E}}(G_T^*) - \overline{\mathcal{E}}(G_T)$ is almost surely nonnegative definite for all $G_T \in \mathcal{M}_1$, θ, P_θ, and $T > 0$, where

$$\overline{\mathcal{E}}(G_T) = \overline{G}_T' \langle G \rangle_T^{-1} \overline{G}_T.$$

Under suitable regularity conditions, asymptotically optimal estimating functions produce estimators for θ which are consistent (cf. **Consistent estimator**), asymptotically unbiased (cf. **Unbiased estimator**) and asymptotically normally distributed (cf. **Normal distribution**) with minimum size asymptotic confidence zones (cf. **Confidence estimation**). For further details see [1], [2].

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C.C. Heyde

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