Appendix E
Functions of Normal Random Variables

If $Y_j \sim N(0,1)$, $j = 1, \ldots, J$, with $Y_1, \ldots, Y_J$ independent, then

$$Z = \sum_{j=1}^{J} Y_j^2 \sim \chi_J^2, \qquad (E.1)$$

a chi-squared distribution with J degrees of freedom and $E[Z] = J$, $var(Z) = 2J$.

If $X \sim N(0,1)$, $Y \sim \chi_d^2$, with X and Y independent, then

$$\frac{X}{(Y/d)^{1/2}} \sim T(0,1,d), \qquad (E.2)$$

a Student's t distribution with d degrees of freedom.

If $U \sim \chi_J^2$ and $V \sim \chi_K^2$, with U and V independent, then

$$\frac{U/J}{V/K} \sim F(J,K), \qquad (E.3)$$

the F distribution with J, K degrees of freedom.