Pascal's Triangle

Techniques: TAB (tabulate): conversion of numeric values to strings for output.

Pascal's triangle is composed of rows of integers, each row containing one more integer than the row above. The digit 1 forms the apex. On lower rows, each integer is calculated as the sum of the two integers above it. See Figure 19 and 20 for program and results.

The integers correspond to the coefficients found in the binomial expansion

$$(1+1)^T = 1 + \frac{T}{1} + \frac{T(T-1)}{1.2} + .. + \frac{T!}{I!(T-I)!}$$

where I is the term number starting with I = 0 at the left, and T is the power to which 2 is raised, and also the row number with T = 0 the apex. Note that each term can be found by multiplying its predecessor by:

$$(T-I+1)/I$$

In this program we write ten rows, with T increasing from 0 to 9 (see line 10). In 15, we work out where to start printing the first integer. Each line is going to start half an integer space to the left of the one above. Each integer needs four character positions - a space and three digits. So we have to begin each row with a 'TAB' of as many spaces as the preceding row less half the allowance of four characters: hence the expression

$$(2*(9-T))$$

in line 15. On the first row, which corresponds to T = 0, the TAB will be 18.

In 20 we set the first value to be printed as 1, then start printing the rows. At 40 we compose a string V$ as two spaces followed by the alphabetic representation of V. Then we take just the right-hand four characters and print them, preventing a new line by following the PRINT with a semicolon.

At 50 we compute the next integer in the current row by the recurrence relation shown, and continue: but when we have finished the row (when I reaches one more than the row number T), we force a couple of line feeds, and go on to the next row.

It would be an interesting exercise to calculate the numbers by the method mentioned first - i.e. examining the line/row above and picking up the two numbers whose sum the new number should be. Before doing this it is instructive to prove that

$$\frac{T!}{(T-I)!I!} = \frac{(T - 1)!}{(T-1-(I-1))!(I-1)!} + \frac{(T - 1)!}{(T-1-I)!I!}$$

i.e. that any term can be calculated as the sum of the two terms on the preceding row T-1, with term numbers I and (I-1).

It is also advisable to see how one can extract numeric values from the screen - see the section describing GIP the General Input Program, which contains a routine for doing this as part of a 'clear program' step.