M. M. Schiffer, Explorer

Steven R. Bell

My favorite objects of complex analysis and potential theory for a finitely connected region in the plane are the Bergman kernel and projection, the Szegő kernel and projection, the Garabedian kernel, the Poisson kernel, the Ahlfors map, the zeroes of the Szegő kernel, solution operators to the inhomogeneous Cauchy–Riemann equations, the harmonic measure functions and their derivatives, Green's function and operator for the Laplacian, and the Neumann kernel and operator for the Laplacian.

I have often felt envious of the pleasure that Max Schiffer must have taken in uncovering relationships between these objects. He and his collaborators were the first mathematicians to set foot on this continent of geometric analysis, and they very quickly charted out the main attractions. Name any two of the objects above, and you will find that Schiffer had a hand in finding a simple and beautiful relationship between them and then had the further pleasure of extending it all to finite Riemann surfaces.

After spending years looking for hidden valleys that Schiffer and his co-explorers might have missed, I think I know what drove Schiffer to come up with so many of his gems of analysis. The tantalizing relationships between these objects have often led me to feel just inches away from getting a stranglehold on one of them, that there is a formula for the object almost as simple and concrete as on the unit disc; but, alas, terms involving at least one abstractly defined object appear in the formula, so there is more to do. The origins of these results always trace back to papers of Schiffer and clearly follow in his tradition. In fact, I am now fairly certain that Schiffer was on the same quest that I have followed so fruitlessly these past years, namely, to lay hands concretely on Green's function of a multiply connected domain in the plane.

I never met Max Schiffer, but I have read so many of his papers and books that I feel as if I know the man. I wish he were here today. I'd like to be able to tell him about exciting developments in the theory of quadrature domains and double quadrature domains which lead me to believe that success in our long-term quest for a formula which expresses Green's function in finite terms may, finally, lie just around the corner.

P. Duren and L. Zalcman (eds.), *Menahem Max Schiffer: Selected Papers Volume 1*, Contemporary Mathematicians, DOI 10.1007/978-0-8176-8085-5_7,