A Unified Approach to Accessibility in 5-axis Freeform Milling Environments.

Gershon Elber (gershon@cs.technion.ac.il)
Department of Computer Science
Technion, Israel Institute of Technology
Haifa 32000, Israel

Elaine Cohen (cohen@cs.utah.edu)
Department of Computer Science
University of Utah
Salt Lake City 84112, U.S.A.

Keywords: Multi-axis machining,Accessibility,Visibility,Non-linear constraints, Symbolic computation.

Abstract: This paper presents a unified view that allows one to consider the global accessibility question in the context of multi–axis machining, as well as globally quantify and resolve it. No constraints are imposed on the possible orientations of the tool during the machining process neither are any inflicted on the shape of the freeform surface. Being global, the presented method augments and complements contemporary multi–axis tool positioning and verification schemes that were developed in recent years for 5-axis machining; schemes that identify and resolve the gouging problem on a point by point basis for each individual point–location along the tool path.

Given a freeform surface to be machined, the orientation of the tool at each surface location, and a check surface, the outcome of the proposed method is a global dichotomy that provides the regions of the freeform surface that are accessible without gouging into the check surface.

1. Introduction

The advantages of multi–axis machining are quite significant. Faster machining and a nicer finish makes multi–axis machining a prime selection when 5-axis machining is feasible. Unfortunately, it is far more difficult to employ 5-axis machining compared to 3-axis modes. These difficulties stem mainly from the more complex and quite unpredictable motion that is introduced by multi–axis machining. The 5-axis toolpath introduces many difficult questions regarding the accuracy, validity, and accessibility of the generated motion commands.

The problem of accessibly, or the ability to verify and possibly correct gouging into the machined surface or even into other surfaces, is apparently the most fundamental hindering factor in the broad use of 5-axis machining. In 3-axis machining, the orientation of the tool is fixed and hence one is able to reduce the problem of accessibility in 3-axis machining to the problem of hidden surface removal of the same scene from a direction collinear with the tool axis [5, 12]. The fact that the tool has a finite thickness can be compensated for by offsetting all the check surfaces by the radius of the tool. The use of Z-buffer [10] based hidden surface removal techniques to verify and

The original version of this chapter was revised: The copyright line was incorrect. This has been corrected. The Erratum to this chapter is available at DOI: 10.1007/978-0-387-35392-0_40

G. J. Olling et al. (eds.), *Machining Impossible Shapes*

correct the 3-axis toolpath, is a common practice in many contemporary computer aided manufacturing algorithms [18, 19, 21].

Solutions to the accessibility, verification, and correction problems in 5-axis machining modes are not many. The problem of tool accessibility can be subdivided into two almost independent questions. The first weighs possible local gouging and optimal tool placement, taking into account only the *tip* of the tool. The second problem considers the detection of global gouging, by testing for possible collisions between the entire tool as well as the tool's holder, and other surfaces.

Given a surface location, the problems of local gouge prevention and selections of optimal tool orientations attracted the attention of several researchers with some successful results. Solutions to this local problem were found by forming a polygonal approximation to the surfaces [17] and/or employing iterative numerical solutions [3]. Other similar attempts exploited highly populated nets of points or normal vectors to approximate the surface [13]. In [14, 16], the effective curvature of the tool at the contact point is derived from its silhouette and is matched against the curvature of the surface.

The global interference problem has been investigated by employing volumetric representations. For example, in [11], an approach that is based on unidimensionally elongated volumetric strips is proposed for 5-axis NC verification.

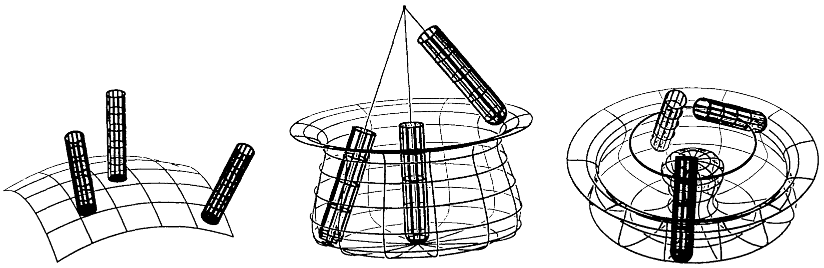
Research to gain insight into the visibility and accessibility problems in freeform surfaces have yielded several fruits, in recent years [5, 12]. The notion of visibility maps that are derived from the Gauss map [4] of the surface was introduced in [1, 2] for polyhedral models and for freeform surface in [8]. In [15, 20], these maps are used for determining the accessibility of a tool, given its point location, exploiting the subdivision and convex hull properties of the freeform Bézier and NURBS surface representations to compute directional bounds.

All the above surveyed methods resolve the accessibility problem on a point by point basis. Given a point location of the tool, these methods verify and possibly correct the tool orientation. Nonetheless, in this work we are interested in the ability to resolve this problem for the entire surface:

> **GAP** *(Global Accessibility Problem): Given a surface $S(u,v)$, a vector field $O(u,v)$ that prescribes the orientation of the tool at each (u,v) location on the surface, and a check surface $K(s,t)$, find all regions in $S(u,v)$ that are $O(u,v)$-accessible with respect to $K(s,t)$.*

Providing a global solution has many advantages over a point by point local validation and correction approach. First, the solution is independent of the selected toolpath as it solves the general accessibility problem that is not specific to some given toolpath. Further, by considering the global picture, global algorithms can be potentially made more accurate and efficient. When a toolpath is validated on a point by point basis, no guarantee can be made on possible gouging while in *between* the verified contact point locations. In contrast, this guarantee is fundamental in global algorithms and, in fact, is cost free once a global algorithm is employed. Interestingly enough, in [15], the global accessibility question is formulated as a "Machining Feasibility Checking" (MFC) problem. Nonetheless and while recognizing its importance, no hints whatsoever for the solution of the MFC problem are actually provided in [15].

Assume an NC machining operation with an orientation that follows the normal of the surface. If the freeform surface is convex, every point above the surface could be projected along a single normal that contains that point. In [7], this unique mapping allowed the reduction of this specific accessibility problem in [7] to the 3-axis visibility and hidden surface removal problem.



(a) (b) (c)

Figure 1. The orientation of a tool in 5-axis machining modes can be prescribed, for example, to follow the normal of the surface (a), go through a point (b), or go through a (circular) curve (c). Several tool positions are shown.

Nevertheless, the problem of accessibility in 5-axis modes is more general. The orientation of the machining tool need not follow the normal of the surface as in Figure 1 (a), but can be arbitrarily prescribed. For example, the toolpath may follow a toolpath orientation *through a point* as in Figure 1 (b). Here, the infinite line of the tool axis is coerced to go through both the contact location with the machined surface and some prescribed point(s). This machining mode might be useful in machining cavities with negative slopes (see Figure 1 (b)). Similarly, one might enforce a machining operation with the toolpath orientation *through a curve* (see Figure 1 (c)). Clearly, other possibilities for the selection of the orientation of the tool may be employed. In this context, the regular hidden surface removal problem in computer graphics [10] is a special GAP problem for which $O(u, v)$ is fixed and follows the viewing direction.

In the ensuing discussion, we assume that both the surface to be machined and the check surface are regular C^1 continues surfaces. Also, we assume that the check surface is closed and it has already been offset by an amount equal to the tool radius. Hence, we can consider the accessibility problem as an intersection problem between a line along the axis of the machining tool and the (offset of) the check surface.

In this work, we consider a zero radius tool because one can always reduce the problem of a tool of radius r to a tool of zero radius and an offset check surface by radius r [7]. Moreover, in the ensuing discussion, we will concentrate on global gouge detection away from the tip of the tool, complementing local accessibility methods [3, 14, 13, 16, 17]. This paper is organized as follows. Section 2 surveys the necessary background. In Section 3, we present the proposed algorithm while some results are depicted in Section 4. Finally, we conclude in Section 5.

2. Background

Denote the freeform surface to be machined by $S(u, v)$ and let $K(s, t)$ be the check surface. In [7], two conditions were proposed toward the detection of the boundary of the accessible surface (See Figure 2 (a)):

$$\begin{aligned} \left\langle n^k(s,t), n^s(u,v) \right\rangle &= 0, \\ \left\langle n^k(s,t), S(u,v) - K(s,t) \right\rangle &= 0, \end{aligned} \tag{1}$$

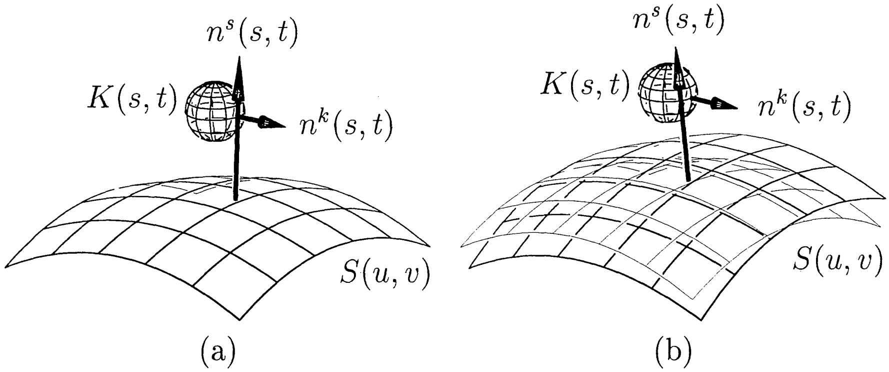


Figure 2. Assume an NC machining operation with orientations following the normal of the surface. On the boundary between the accessible and inaccessible regions in (a), the normal of $S(u,v)$, $n^s(u,v)$, must be perpendicular to the normal of the $K(s,t)$, $n^k(s,t)$. Similarly, the vector $S(u,v) - K(s,t)$ must be perpendicular to $n^k(s,t)$. As is shown in (b), these two conditions are insufficient as they do not coerce $n^s(u,v)$ to be collinear with $S(u,v) - K(s,t)$. The original surface in (a) is rotated at the contact location point in (b) along the $n^k(s,t)$ vector. The original surface is shown in (b) in thin lines.

where $n^k(s,t)$ and $n^s(u,v)$ are the normal fields of $K(s,t)$ and $S(u,v)$, respectively, and are not necessarily normalized.

Equations (1) were not employed in [7] toward a solution of the GAP for several reasons. First and foremost, and as pointed out in [7], the constraints that these two equations impose are insufficient. The vector of $n^s(u,v)$ is not constrained to be in the direction of $S(u,v) - K(s,t)$. The scene in Figure 2 (b) was derived from Figure 2 (a) by rotating $S(u,v)$ at the machined point along the vector $n^k(s,t)$. Hence, $n^s(s,t)$ and $n^k(s,t)$ remain perpendicular, yet $n^s(s,t)$ is no longer collinear to vector $S(u,v) - K(s,t)$.

Moreover, Equations (1) are restricted to a 5-axis machining mode with a tool orientation that follows the normal of the surface. Finally, since the approach requires one to find a solution for a four variate problem, one should expect this approach to be slow.

For all these reason and since a more efficient approach has been proposed in [7] for the restricted case of convex surfaces machined with a tool orientation along the normal of the surface, the potential of Equations (1) was not deeply explored in [7]. Nevertheless, in Section 3, we revisit this approach and define the sufficient conditions for the solution. In addition, we extend this approach for arbitrary type of machined surfaces and arbitrary tool orientations, providing a unified global solution to all 5-axis accessibility questions.

3. Algorithm

Denote the arbitrary tool orientation at surface point $S(u,v)$ by $O(u,v)$. That is, given a surface location $S(u,v)$, $O(u,v)$ prescribes the tool orientation there. For example, for the orientation scheme proposed in Section 1, one can get:

$$\begin{aligned}
O(u,v) &= n^s(u,v), && \text{for an orientation along the normal of the surface,} \\
O(u,v) &= P - S(u,v), && \text{for an orientation through a point } P, \\
O(u,v) &= C(u) - S(u,v), && \text{for an orientation through a curve } C.
\end{aligned} \tag{2}$$

Assume $O(u,v)$ is contained in the tangent plane of $S(u,v)$ at some location. Since $S(u,v)$ is regular, for every (u,v), either

$$O(u,v) \times \frac{\partial S(u,v)}{\partial u} \neq 0, \tag{3}$$

or

$$O(u,v) \times \frac{\partial S(u,v)}{\partial v} \neq 0. \tag{4}$$

Alternatively, if $O(u,v)$ is contained in the tangent plane of $S(u,v)$ at no location, both conditions (3) and (4) would hold for the entire domain.

Therefore, and without any loss of generality, assume $O(u,v) \times \frac{\partial S(u,v)}{\partial u} \neq 0$. Then, we define,

$$\begin{aligned}
O_1^n(u,v) &= O(u,v) \times \frac{\partial S(u,v)}{\partial u}, \\
O_2^n(u,v) &= O(u,v) \times O_1^n(u,v),
\end{aligned} \tag{5}$$

as two vector fields that span the plane orthogonal to $O(u,v)$, in $\mathbb{R}^3$.

Armed with these vector fields, we are ready to define the necessary and sufficient conditions for the boundary of the accessible regions of $S(u,v)$:

$$\begin{aligned}
\mathcal{F}_1(u,v,s,t) &= \langle O_1^n(u,v), S(u,v) - K(s,t) \rangle, \\
\mathcal{F}_2(u,v,s,t) &= \langle O_2^n(u,v), S(u,v) - K(s,t) \rangle, \\
\mathcal{F}_3(u,v,s,t) &= \left\langle n^k(s,t), S(u,v) - K(s,t) \right\rangle.
\end{aligned} \tag{6}$$

The three functions of Equations (6) are the result of symbolic differentiation, differencing, and multiplication operations over piecewise rational functions, operations that are closed for this domain of piecewise rationals [6]. Then,

LEMMA 1. *A (u,v,s,t) point would be on the boundary between a $O(u,v)$–accessible and $O(u,v)$–inaccessible regions of $S(u,v)$ if and only if the following constraints are simultaneously satisfied:*

$$\begin{aligned}
\mathcal{F}_1(u,v,s,t) &= 0, \\
\mathcal{F}_2(u,v,s,t) &= 0, \\
\mathcal{F}_3(u,v,s,t) &= 0.
\end{aligned} \tag{7}$$

Proof. See [9] ■

These are necessary and sufficient conditions. We are completely constraining the tool's orientation while also forcing the contact point at the check surface to provide tangent plane continuity. We are presented with three Equations (7) in four unknowns (u,v,s,t). Hence, the solution is expected to have one degree of freedom or is a univariate. This curve indeed delineates the $O(u,v)$–accessible region in $S(u,v)$ from the inaccessible regions.

3.1. Solving the Equations

The presumably simple set of constraints (7) can be difficult to solve. The set contains three non linear equations in four unknowns. Favoring a robust solution, we choose to employ a subdivision based method, that is intermixed with a numerical improvement stage.

DEFINITION 1. *A* bounding cone, $\mathcal{C}$, *contains the prescription of a unit length axial direction* $\mathcal{V}$ *and an angular span of* α. *All unit length vectors with a deviation of less than* α *from unit vector* $\mathcal{V}$ *are considered bounded by* $\mathcal{C}$.

Hence, for a given arbitrary unit vector $\mathcal{W}$:

$$\mathcal{W} \in \mathcal{C} \quad \Leftrightarrow \quad \langle \mathcal{W}, \mathcal{V} \rangle < \cos(\alpha).$$

Cones bounding all possible directions in the vector fields can be computed for the three vector fields that are represented using NURBS, of,

$$\begin{aligned} O_1^n(u,v) &\subset \mathcal{C}_1(\alpha_1, \mathcal{V}_1), \\ O_2^n(u,v) &\subset \mathcal{C}_2(\alpha_2, \mathcal{V}_2), \\ n^k(s,t) &\subset \mathcal{C}_3(\alpha_3, \mathcal{V}_3), \end{aligned}$$

by examining the control meshes of these vector fields and exploiting the Convex Hull property of the NURBS representation, the representation we employed throughout this work. A similar cone can be computed for the vector field of $S(u,v) - K(s,t)$, as,

$$S(u,v) - K(s,t) \subset \mathcal{C}_4(\alpha_4, \mathcal{V}_4),$$

by, again, examining all possible *differences* between the control points of $S(u,v)$ and $K(s,t)$ and exploiting the Convex Hull property of the representation.

Then, the constraints of Equations (7) can be rewritten as,

$$\begin{aligned} \langle \mathcal{C}_1(\alpha_1, \mathcal{V}_1), \mathcal{C}_4(\alpha_4, \mathcal{V}_4) \rangle &= 0, \\ \langle \mathcal{C}_2(\alpha_2, \mathcal{V}_2), \mathcal{C}_4(\alpha_4, \mathcal{V}_4) \rangle &= 0, \\ \langle \mathcal{C}_3(\alpha_3, \mathcal{V}_3), \mathcal{C}_4(\alpha_4, \mathcal{V}_4) \rangle &= 0. \end{aligned} \tag{8}$$

LEMMA 2. *The inner product of two vector fields bound by the two cones* $\mathcal{C}_i(\alpha_i, \mathcal{V}_i)$ *and* $\mathcal{C}_j(\alpha_j, \mathcal{V}_j)$ *might yield a zero result only if the following holds,*

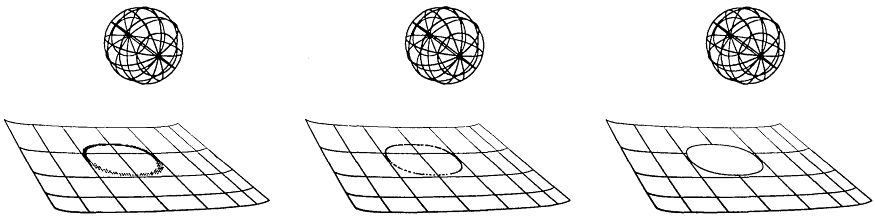
$$\frac{\pi}{2} - \alpha_i - \alpha_j < \langle \mathcal{V}_i, \mathcal{V}_j \rangle < \frac{\pi}{2} + \alpha_i + \alpha_j, \tag{9}$$

while recalling that $\mathcal{V}_i$ *and* $\mathcal{V}_j$ *are unit length vectors.*
Proof. See [9] ■

A subdivision algorithm to simultaneously resolve Equations (7) is now readily available. Subdivide both surfaces in either one of u, v, s or t until either the subdivided patches are sufficiently small or until one of the three conditions of Equations (8) can never be satisfied, as can be tested via Equation (9).

Figure 3 (a) shows the set of points that were extracted using the presented approach for a simple case of an almost flat surface $S(u,v)$ and a spherical check surface $K(s,t)$, and an orientation that follows the normal of the surface.

One can continue the subdivision process until the size of the patch is sufficiently small. Nevertheless, the process can be quite slow. Instead, we resort to a second numerical step once a rough approximation has been robustly computed using the subdivision algorithm and with a relatively large τ.



(a) (b) (c)

Figure 3. A view of a spherical check surface and an almost flat surface to be machined along the normal of the surface. In (a), the set of points, extracted from the presented subdivision process, is shown. In (b), the same set is presented after employing the numerical improvement stage. Finally, in (c) the inaccessible circular interior area is trimmed away.

Let the gradients of each of the three Equations (7) be denoted by,

$$\nabla_i(u,v,s,t) = \left(\frac{\mathcal{F}_i(u,v,s,t)}{\partial u}, \frac{\mathcal{F}_i(u,v,s,t)}{\partial v}, \frac{\mathcal{F}_i(u,v,s,t)}{\partial s}, \frac{\mathcal{F}_i(u,v,s,t)}{\partial t} \right), \quad \text{for } i = 1,2,3.$$

The numerical stage includes first order Newton Raphson iterations. That is, we assume a first order approximation to each of the three Equations (7), at the current (u, v, s, t) solution and follow the gradient until the equation equals zero in the first order approximation.

In practice, one is not required to guarantee the numerical convergence of all the sampled points from the subdivision process. A majority of samples would be sufficient, and this relaxation can tremendously improve the speed of the numerical stage as it is no longer required to be robust. In all the examples presented in Section 4, over 90% of the points provided by the subdivision process were successfully improved by the numerical step, in very few iterations, while the rest of the points were simply purged away. This, with a fairly inaccurate subdivision tolerance in the order of 10^{-2} or even larger, and for a unit size model. An improvement of two orders of magnitudes on the maximal error could be typically gained using about five numerical iterations. A numerical tolerance of 10^{-4} is typically sufficient for NC applications and was used in all the examples shown in Section 4. Needless to say, the numerical computation took less than 10% of the time compared to the subdivision approach and well justified the subdivision–numerical hybrid approach.

4. Examples and Results

Given the set of points on the boundary between the accessible and inaccessible regions, one can chain them into curves and trim the original surface $S(u, v)$ using these (trimming) curves. This exact process is used throughout the examples of this section.

Figure 3 (c) shows the result of chaining together the points that result from applying the subdivision stage shown in Figure 3 (a) and the numerical improvement stage shown in Figure 3 (b). Once the trimming curves are constructed, the surface is subdivided into different regions, each should be tested for its accessibility.

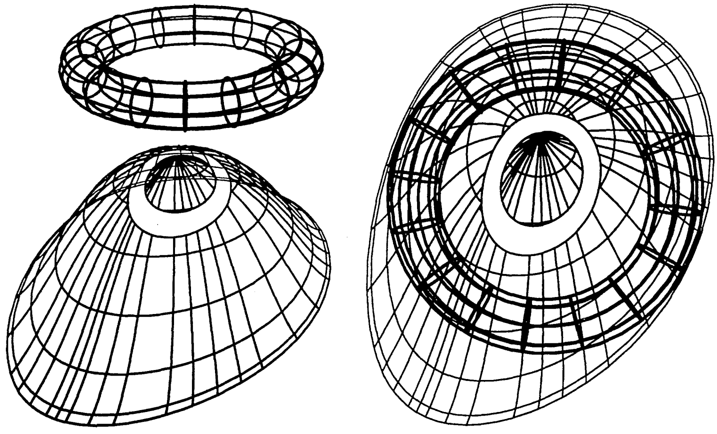


Figure 4. Two views of a toroidal check surface and bump like surface to be machined along the normal to the surface. The inaccessible regions are trimmed away.

Figure 4 consider a more realistic freeform pocket, with a toroidal check surface. The proposed accessibility testing scheme can clearly handle convex check surfaces. However, this scheme also support check surfaces of hyperbolic shape with virtually no changes. Concave surface, on the other hand, would create superficial edges that are of no interest. Hence, cavities on the check surface must be eliminated before the accessibility test takes place. See [6] for a trichotomy of freeform surfaces into convex, concave and saddle regions.

A single check surface can affect completely disjoint regions in the surface to be machined, provided that the normals of these disjoint regions are all pointing toward the check surface. Figure 5 shows one such example for a rounded cube (minus one face) and a spherical check surface at its center.

While common, the tool orientation need not follow the normal of the surface. In Figure 6, three examples of machining through a point are considered, for three different locations of the prescribed (through) point. Once the new orientation field $O(u, v)$ is derived, the accessibility algorithm as is presented in this work can be employed, unmodified.

All examples in the section were computed in time complexities starting from a minute or two for simple cases like Figure 3 and up to couple of hours for highly complex examples like Figure 5, all on a high end workstation. As stated already, the subdivision process consumed about 90% of the total computation time.

5. Conclusions

This work presents a solution to the accessibility questions of NC machining in 5-axis configurations. The approach presented as part of this work is capable of answering the fundamental global question of what regions of a surface can be machined from an arbitrarily prescribed orientation field and not gouge into the given check surface.

Throughout this work, the assumption was made that both $S(u, v)$ and $K(s, t)$ are C^1 continuous. Further, $K(s, t)$ was assumed to be a closed object. These assumptions

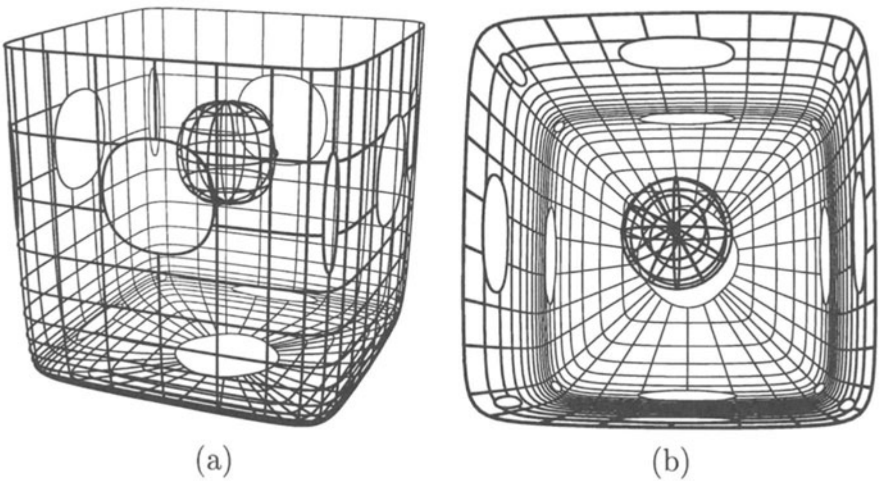


Figure 5. Two views of a spherical check surface and a rounded box surface to be machined (minus one face) along the normal to the surface. The inaccessible regions are trimmed away. Seventeen (!) disjoint regions are eliminated in this example.

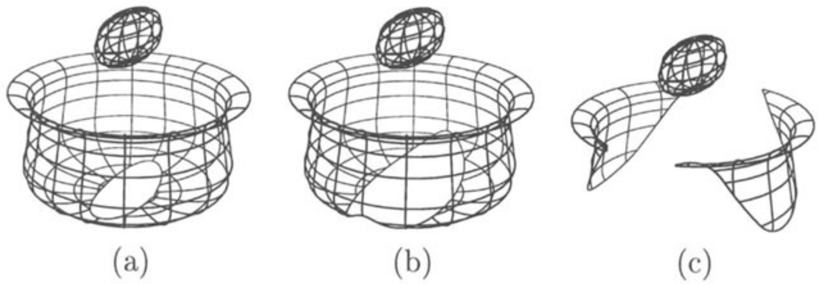


Figure 6. Three point locations above an elliptical check surface and cavity like surface to be machined through a point p. In (a), p is location way above the check surface. In (b), p is closer to the check surface, while in (c) p (shown in the figure) is just above the check surface.

can be relaxed in several ways. First, both surfaces can be piecewise C^1 continuous. That is, one can allow a finite number of C^1 discontinuities, locations were the surfaces would be split at. However, if $K(s,t)$ is piecewise C^1 and/or is not a closed object, the *boundary* curves of $K(s,t)$ must also be taken into considerations, as an extremum where the tool can start to gouge into $K(s,t)$.

Clearly, the next question to explore is, given the additional degrees of freedom, can one automatically modify the orientation in gouging situations and prevent the gouging from occurring?

A different way to approach this last question might be through the quest for all the gouge free orientations for all locations on the freeform surface $S(u,v)$. Given a surface $S(u,v)$ and a check surface $K(s,t)$, the outcome would be a four variate trimmed accessibility hyper-surface field of the form of $\mathcal{A}(u, v, \alpha, \theta)$, where α and θ span the trimmed hemisphere of accessible orientations above the tangent plane of surface $S(u, v)$.

Finally, one should recall that the special case of machining through a point brings us back to the hidden surface removal problem in computer graphics, this time for a perspective view. In other words, we are essentially providing as part of this work, a solution to the hidden surface removal problem, this time for perspective views. Further, the 3-axis machining mode which is equivalent to the hidden surface removal problem in orthographic view is also a special case of the presented approach of this work, this time with a constant orientation field $O(u, v) \equiv O_0$.

References

1. L. L. Chen and T. C. Woo. Computational Geometry on the Sphere With applications to Automated Machining. Technical Report No. 89-30, Department of Industrial and Operations Engineering, University of Michigan, August 1989.
2. L. L. Chen, Shuo-Yan Chou, and T. C. Woo. Separating and Intersecting Spherical Polygons: Computing Machinability on Three-, Four- and Five-Axis Numerically Controlled Machines. ACM Transaction on Graphics, Vol. 12, No. 4, pp 305-326, October 1993.
3. B. K. Choi, J. W. Park and C. S. Jun. Cutter Location Data Optimization in 5-axis Surface Machining. Computer Aided Design, Vol 25, No. 6, pp 377-386, 1993.
4. M. P. DoCarmo. Differential Geometry of Curves and Surfaces. Prentice-Hall 1976.
5. G. Elber and E. Cohen. Hidden Curve Removal for Free Form Surfaces. Computer Graphics, Vol 24, No. 4, pp 95-104, Siggraph, August 1990.
6. G. Elber and E. Cohen. Second Order Surface Analysis Using Hybrid Symbolic and Numeric Operators. Transactions on Graphics, Vol. 12, No. 2, pp 160-178, April 1993.
7. G. Elber. Accessibility in 5-axis Milling Environment. CAD, Vol 26, No 11, pp 796-802, November 1994.
8. G. Elber and E. Cohen. Arbitrarily Precise Computation of Gauss Maps and Visibility Sets for Freeform Surfaces. The third ACM/IEEE Symposium on Solid Modeling and Applications, SLC, Utah, pp 271-279, May 1995.
9. G. Elber and E. Cohen. A Unified Approach to Accessibility in 5-axis Freeform Milling Environments. Center for Intelligent Systems Tech. Report, CIS 9815, Computer Science Department, Technion. http://www.cs.technion.ac.il/Reports.
10. J. D. Foley et al. Computer Graphics, Principles and Practice, Second Edition. Addison-Wesley Systems Programming Series, Jul. 1990.
11. Sujit Verma. Simulation of Numerically Controlled Milling machines. Master Thesis, University of Utah, 1995.
12. C. Hornung, W. Lellek, P. Rehwald, and W. Strasser. An Area-Oriented Analytical Visibility Method for Displaying Parametrically Defined Tensor-Product Surfaces , Computer Aided Geometric Design, Vol. 2, pp 197-205, 1985,
13. R. B. Jerard and R. L. Drysdale Methods for Geometric Modeling, Simulation and Spatial Verification of NC Machining Programs. Proceedings of the Third IFIP Workshop on Geometric Modeling. Rensselaerville, NY, USA, June 1990.
14. C. G. Jensen and D. C. Anderson. Accurate Tool Placement and Orientation for Finish Surface Machining. Proceedings of the Symposium of Concurrent Engineering. ASME Winter Annual Meeting, 1992.

15. Yuan-Shin Lee and Tien Chien Chang. 2-Phase Approach to Global Tool Interference Avoidance in 5-axis Machining. Computer Aided Design, Vol 27, No 10, pp 715-729, October 1995.
16. Yuan-Shin Lee and Tien Chien Chang. Admissible Tool Orientation Control of Gouging Avoidance for 5-axis Complex Surface machining. Machining. Computer Aided Design, Vol 29, No 7, pp 507-521, 1997.
17. S. Li and R. B. Jerard. 5-axis Machining of Sculptured Surfaces with a Flat End Cutter. Computer Aided Design, Vol 26, No 3, pp 165-178, 1994.
18. T. Saito and T. Takahashi. NC machining with G-buffer method. Computer Graphics, Vol 25, No. 4, pp 207-216, Siggraph, July 1991.
19. T. W. Spencer. Scan Line Rendering for 3-axis NC Toolpath Generation, Simulation, and Verification. Technical Report CSE-TR-43-90, Department of Electrical Engineering and Computer Science, University of Michigan, January 1990, U.S.A.
20. Y. J. Tseng and S. Joshi. Determining Feasible Tool-approach Directions for Machining Bézier Curves and Surfaces. Computer Aided Design, Vol. 23, No. 5, pp 367-378, June 1991.
21. D. Zhang, A. Bowyer. CSG Set-Theoretical Solid Modeling and NC Machining of Blend Surfaces. The Second Annual ACM Symposium on Computational Geometry, ACM, 1986.