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Comprehensive Queueing Analysis for a Partial Buffer System with Discrete Markovian Arrival Processes

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ABSTRACT

In this paper, we evaluate the performance of a partial buffer system with finite capacity, deterministic service time and multiple sources. Each source is modeled by a Discrete Markovian Arrival Process (D-MAP). We evaluate the queueing system for several traffic composition and different sizes of the shared buffer area. Various traffic types are considered, including VBR (Variable Bit Rate) sources with periodical or negative exponential correlation functions and CBR (Constant Bit Rate) traffic with fixed interarrival cell emission. The probability distribution of the waiting time and the cell loss probability of each source are determined.

Keywords

D-MAP, Queueing System, Partial Buffer, Priority, Correlation Function, Cell Delay Variation, Cell Loss Probability, Waiting Time Distribution.

1. Introduction

In ATM networks, the AAL (ATM Adaption Layer) was standardized to adapt different kind of traffics to the cell transfer mode at the network edges. Former evaluations has shown that different traffic types also require accommodated mechanism inside the network to garantee the individuel QoS parameters (Herrmann, 9/94). Due to the necessity of accomplishing these requirements, many mechanism are currently subjects of researchers studies. Because of its simplicity for implementation one of the most favourite candidate is the partial buffer mechanism.

In this paper we use the D-MAP to imitate different types of sources. The D-MAP was introduced in (Blondia, 1989) and is the discrete-time version of the class of continous-time Markovian Arrival Process which was used in (Maglaris, 1988) for example. In our model we use the D-MAP for VBR-, CBR- and non-bursty-sources for different traffic composition. Each scenario consists of two priority and two non-priority sources to estimate the performance measurements.

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The remainder of this paper is organized as follows. In section 2 we define the different types of sources considering in our model. The queueing model is specified and analyzed in section 3. In section 4, the numerical results are shown. Concluding remarks are given in section 5.

2. Characterisation of the traffic sources

Throughout this paper, each source is modeled by a D-MAP with m-phases. An arrival of a cell is presented by a type 1 transition between the m-phases of the D-MAP. In addition, there are also type 2 transitions, which don't cause arrivals. Both types of transitions are defined by (m x m)-matrices **D** and **C**, respectively. The elements c_{ij} and d_{ij} represent the transition probability from phase i to phase j (i; j = 0; 1;.....; m-1).

2.1 VBR traffic sources

Besides traffic intensity, the most important parameter of VBR traffic is its burstiness behaviour. With (1) we are able to determine the autocorrelation coefficient of the number of arrivals.

$$corr[A_n, A_{n+h}] = \frac{\underline{\pi} \cdot \mathbf{D}(\mathbf{C} + \mathbf{D})^{h-1}\mathbf{D} \cdot \underline{e} - \lambda^2}{\lambda(1-\lambda)} \tag{1}$$

For the derivation of equation (1) we refer to (Herrmann, 9/94).

VBR traffic sources with periodical correlation function

Equation (1) shows that corr $[A_n, A_{n+h}]$ is only h-periodical, if the expression $\underline{\pi}\mathbf{D}$ $(\mathbf{C} + \mathbf{D})^{h-1}$ $\mathbf{D}$ $\underline{e}$ has this behaviour. To adhere the burstiness characteristic, the transition probability matrix (**C** + **D**) of the underlying Markov chain has to be periodical. As in (Herrmann, 8/94) the matrix of cyclic shifting was choosen.

The resultant periodical correlation function is

$$\text{corr}\left[A_n, A_{n+h}\right] = \frac{m\underline{e}\,\mathbf{D}(\mathbf{C}+\mathbf{D})^{h-1}\mathbf{D}\,\underline{e} - (\underline{e}\,\mathbf{D}\,\underline{e})^2}{m\underline{e}\,\mathbf{D}\,\underline{e} - (\underline{e}\,\mathbf{D}\,\underline{e})^2} \tag{2}$$

VBR traffic sources with negative exponetial correlation function

As in (Blondia, 1992) is shown, this behaviour can be modeled by a discrete-time birth-death Markov process, which is a special case of the D-MAP. The Markov process consists of (M+1)-phases i (i = 0; 1; . . .;M) , which are related to the discrete levels of iA cells/s. During the sojourn times of a phase the cell rate is constant which results to fixed interarrival times between phase transitions.

With reference to (Blondia, 1992) the parameter α, resp. β, is the probability that an active, resp. silent, on/off source becomes silent, resp. active, in the next time unit. d denotes the arrival time between consecutive cells of the same on/off source. The autocorrelation coefficient can be calculated with equation (3).

$$corr[A_n ; A_{n+h}] = e^{-(\alpha+\beta)h.} \quad (3)$$

2.2 CBR traffic sources

CBR traffic, which consists of a fixed emission sequence with the period T, can be modeled as D-MAP which underlying Markov chain (**C**+**D**) is periodical with T states. Thus for a CBR traffic source we can use the same Matrix of cyclic shifting which was already considered for VBR traffic sources with periodical correlation function.

Due to the random delay in the customer equipment and in each multiplexing stage of an ATM network, the initally periodic cell stream is altered by introducing Cell Delay Variation (CDV). Following the CDV tolerance τ is defined by the jitter cell stream T_{CDV} and the original constant time period T.

$$\tau = T_{CDV} - T \quad (4) \qquad E[\tau] = a\sum_{i=1}^{T} (1-a)^{i-1}(i-T) \quad (5)$$

We consider the CDV effect in our model by using the matrix $\mathbf{D}_{CDV}$ = Diag $(d_1;d_2;......;d_T) \cdot$ (**C**+**D**). For simplicity, let $d_i = a$ for i = 1;....; T-1.

2.3 Superposition of several traffic sources

During a time slot several ATM cells can arrive at each output queue of an ATM node, due to the fact that cells of different inlets are destined to the same outlet. Therefore the resulting arrival process **Q** consists of the superposition of n D-MAPs. When we use the properties at the Kronecker product as demonstrated in (Graham, 1981) the transition probability matrix **Q** can be decomposed.

$$\mathbf{Q} = \mathbf{D}_0 + \mathbf{D}_1 + \mathbf{D}_2 + \cdots + \mathbf{D}_n \text{ ; whereby}$$

$$\mathbf{D}_0 = \mathbf{C}^{(1)} \otimes \mathbf{C}^{(2)} \otimes \mathbf{C}^{(3)} \otimes \cdots \otimes \mathbf{C}^{(n)}$$

$$\mathbf{D}_1 = \mathbf{D}^{(1)} \otimes \mathbf{C}^{(2)} \otimes \mathbf{C}^{(3)} \otimes \cdots \otimes \mathbf{C}^{(n-1)} \otimes \mathbf{C}^{(n)} + \mathbf{C}^{(1)} \otimes \mathbf{D}^{(2)} \otimes \mathbf{C}^{(3)} \otimes \cdot \otimes \mathbf{C}^{(n-1)} \otimes \mathbf{C}^{(n)} + \cdots + \mathbf{C}^{(1)} \otimes \mathbf{C}^{(2)} \otimes \mathbf{C}^{(3)} \otimes \cdot \otimes \mathbf{D}^{(n-1)} \otimes \mathbf{C}^{(n)} + \mathbf{C}^{(1)} \otimes \mathbf{C}^{(2)} \otimes \mathbf{C}^{(3)} \otimes \cdots \otimes \mathbf{C}^{(n-1)} \otimes \mathbf{D}^{(n)}$$

$$\mathbf{D}_n = \mathbf{D}^{(1)} \otimes \mathbf{D}^{(2)} \otimes \mathbf{D}^{(3)} \otimes \cdots \otimes \mathbf{D}^{(n)} \quad (6)$$

3. Queueing Model with Partial Buffer Sharing

The partial buffer sharing mechanism, which we are considered in our queueing model, was proposed in (Blondia, 1989, Maglaris, 1988). The traffc model, which is considered in this paper, is shown in Figure 1.

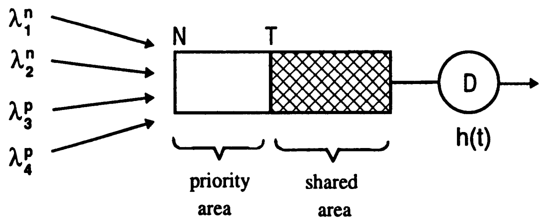

Figure.1 Partial buffer sharing mechanism

The buffer is partitioned by the threshold T in two areas. The lower section of the buffer, which is called shared area, accepts arriving cells of each class (i.e. priority and non-priority cells). On the contrary the buffer area above the threshold T can only be entered by cells with semantic priority. Therefore arriving non-priority cells will loss, when they observe a buffer occupany equal or greater than T. The whole system consists of N buffer spaces and additionally one space, which is provided by the server.

In this paper the service time h(t) of a cell is assumed to be constant and is equal to a time unit. The service discipline is FIFO. The transmission rate for each incoming linie i (i = 1,...,4) is equal to those of the outlet. Throughout this paper the arrival processes 1 and 2 represent non-priority cell streams, whereas the arrival processes 3 and 4 consist of priority cells (denoted by the index n and p in Figure 1, respectively). In case of a batch arrival the sequence of cell admission is random, i.e. we do not consider any enqueueing sequence priority in our model.

3.1 The embedded Markov Renewal Process at Departure instants

Next to we consider the occupancy of the system at departure instants τ_i (i = 0, 1, 2,...). We assume that departures and arrivals occur at the end of a time slot. Further an arriving cell should enter the system before the served cell leave the system (Arrival First). The resultant semi-Markov kernel $\mathbf{q}(t)$ is shown in Figure 2.

$$
\mathbf{q}(t) = \left[\begin{array}{ccccccccc|cccccc}
\mathbf{B}_0^s(t) & \mathbf{B}_1^s(t) & \mathbf{B}_2^s(t) & \mathbf{B}_3^s(t) & \mathbf{B}_4^s(t) & \mathbf{B}_5^s(t) & \mathbf{B}_6^s(t) & \mathbf{B}_7^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^s(t) & \mathbf{A}_4^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^s(t) & \mathbf{A}_4^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^s(t) & \mathbf{A}_4^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^s(t) & \mathbf{A}_4^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^s(t) & \mathbf{A}_4^s(t) & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^s(t) & \mathbf{A}_3^{sp}(t) & \mathbf{A}_4^{sp}(t) & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^s(t) & \mathbf{A}_2^{sp}(t) & \mathbf{A}_3^{sp}(t) & \mathbf{A}_4^{sp}(t) & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^s(t) & \mathbf{A}_1^{sp}(t) & \mathbf{A}_2^{sp}(t) & \mathbf{A}_3^{sp}(t) & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_0^p(t) & \mathbf{A}_1^p(t) & \mathbf{A}_2^p(t) & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{A}_2^p(t) & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{A}_1^p(t) & \mathbf{A}_2^p(t) \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{A}_0^p(t) & \sum_{n=1}^{2}\mathbf{A}_n^p(t)
\end{array}\right]
$$

Figure 2 Semi-Markov kernel of the system with partial buffer sharing
(N+1 rows and N+1 columns, T=8)

The elements of the submatrices are defined as follows:

$\left[\mathbf{A}_k^b(t)\right]_{ij}$: The joint probability that a departing cell leaves a non-empty system and during the time-interval t there are k arrivals. The indices i and j denote the phases of the arrival process at the beginning and the end of the time-interval, respectively. The index b refers to the relevant buffer area at which ´s´ and ´p´ denote that the transition occurs inside the shared area or inside the priority area, respectively. The index ´sp´ denotes that a transition happens from the shared area into the priority area.

$\left[\mathbf{B}_k^b(t)\right]_{ij}$: eqivalent defined as $\left[\mathbf{A}_k^b(t)\right]_{ij}$ but with the difference, that a departing cell leaves an empty system.

It is assumed, that the transitions $\left[\mathbf{B}_k^b(t)\right]_{ij}$ only occur inside the shared buffer area, i.e. threshold $T \geq 7$. By that $\left[\mathbf{B}_k^s(t)\right]_{ij}$ can be computed as follow:

$$\mathbf{B}_k^s(t) = \sum_{\omega=\mu}^{\nu} \mathbf{D}_0^{t-2}\, \mathbf{D}_\omega\, \mathbf{D}_{k-\omega+1}\,;\ \mu = \max[1;k-3];\ \nu = \min[4;k+1];\ 0 \leq k \leq 7 \qquad (7)$$

whereby $\mathbf{D}_0$, $\mathbf{D}_\omega$, $\mathbf{D}_{k-\omega+1}$ are defined as in (6) with n = 4. The calculation of matrices $\mathbf{A}_k^b$ (t) depends on the affected buffer areas. Transitions inside the shared buffer area without cell loss:

$$\mathbf{A}_i^s = \mathbf{D}_i \qquad ; i = 0; 1; 2; 3; 4 \qquad (8)$$

With reference to section 2.3 $\mathbf{D}_i$ is the superposition of D-MAPs, here n = 4.
The transition probabilities from the shared area into the priority area depend on the system state $\mathbf{x}_n$:

$\underline{\mathbf{x}}_n = \text{T-2}$:

$$\mathbf{A}_3^{sp} = \mathbf{A}_3^s + \frac{1}{2}\cdot\mathbf{A}_4^s \quad (9) \qquad \mathbf{A}_4^{sp} = \frac{1}{2}\cdot\mathbf{A}_4^s \quad (10)$$

$\underline{\mathbf{x}}_n = \text{T-1}$:

$$\mathbf{A}_2^{sp} = \mathbf{A}_2^s + \frac{2}{3}\cdot\left[\mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)} + \otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{3}\cdot\left[\mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{D}^{(4)} + \mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{6}\cdot\mathbf{A}_4^s \quad (11)$$

$$\mathbf{A}_3^{sp} = \mathbf{A}_3^s + \frac{1}{2}\cdot\mathbf{A}_4^s \quad (12) \qquad \mathbf{A}_4^{sp} = \frac{1}{6}\cdot\mathbf{A}_4^s \quad (13)$$

$\underline{\mathbf{x}}_n = \text{T}$:

$$\mathbf{A}_1^{sp} = \mathbf{A}_1^s + \mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{C}^{(4)} + \frac{1}{2}\cdot\left[\mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)} + \mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{2}\cdot\left[\mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)} + \mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{3}\cdot\left[\mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)}\right] + \frac{1}{3}\cdot\left[\mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] \quad (14)$$

$$\mathbf{A}_2^{sp} = \mathbf{C}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{D}^{(4)} + \frac{1}{2}\left[\mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)} + \mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{2}\left[\mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)} + \mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{2}{3}\left[\mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{C}^{(4)}\right] + \frac{2}{3}\left[\mathbf{D}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{C}^{(3)}\otimes\mathbf{D}^{(4)} + \mathbf{D}^{(1)}\otimes\mathbf{C}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{D}^{(4)} + \mathbf{C}^{(1)}\otimes\mathbf{D}^{(2)}\otimes\mathbf{D}^{(3)}\otimes\mathbf{D}^{(4)}\right] + \frac{1}{6}\cdot\mathbf{A}_4^s \quad (15)$$

$$\mathbf{A}_3^{sp} = \frac{1}{3} \cdot \left[\mathbf{D}^{(1)} \otimes \mathbf{C}^{(2)} \otimes \mathbf{D}^{(3)} \otimes \mathbf{D}^{(4)} + \mathbf{C}^{(1)} \otimes \mathbf{D}^{(2)} \otimes \mathbf{D}^{(3)} \otimes \mathbf{D}^{(4)} \right] + \frac{1}{2} \cdot \mathbf{A}_4^{s} \quad (16)$$

In the equations (9) - (16) the factors are based on the theory of combination and represent the ratio of valid cell arrival sequences to all possible cell arrival sequences for the considered event.

Transitions inside the priority area:

$$\mathbf{A}_0^{p} = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{C}^{(3)} \otimes \mathbf{C}^{(4)} \quad (17)$$

$$\mathbf{A}_1^{p} = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{D}^{(3)} \otimes \mathbf{C}^{(4)} + (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{C}^{(3)} \otimes \mathbf{D}^{(4)} \quad (18)$$

$$\mathbf{A}_2^{p} = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{D}^{(3)} \otimes \mathbf{D}^{(4)} \quad (19)$$

The transition probability matrix of the underlying Markov chain is obtained by consideration of the marginal values. Since the service time is identical with the time unit, $\mathbf{A}_k^b(t)$ in equations (8) - (19) are directly the marginal values. For the marginal values of $\mathbf{B}_k^s(t)$ we obtain:

$$\mathbf{B}_k^s = (\mathbf{I} - \mathbf{D}_0)^{-1} \sum_{\omega=\mu}^{\nu} \mathbf{D}_{\omega} \mathbf{D}_{k-\omega+1} \quad \mu = \max[1;k-3];\ \nu = \min[4;k+1];\ 0 \le k \le 7 \quad (20)$$

3.2 The stationary queue length distribution

For the computation of the stationary vector $\underline{\mathbf{x}}$, which denotes the number of cells and the phases of the arrival process after a departure instant, we follow the procedure is used in (Herrmann, 9/94). The vector $\underline{\mathbf{x}}$ can be partitioned as $\underline{\mathbf{x}} = (\underline{\mathbf{x}}_0, \ldots, \underline{\mathbf{x}}_N)$, whereby $\underline{\mathbf{x}}_k$ (k = 0,...,N) is a vector with components $\underline{\mathbf{x}}_{k,i}$ (i = 0;......; m_1 x m_2 x m_3 x m_4) being the probability that a departing cell leaves k cells in the system and i is the phases of the whole arrival process at the departure instants. The vector $\underline{\mathbf{x}}$ satisfy the equations (21).

$$\underline{\mathbf{x}}\, \mathbf{q} = \underline{\mathbf{x}} \qquad ; \underline{\mathbf{x}}\, \underline{\mathbf{e}} = 1 \quad (21)$$

To computate the waiting times and the loss probabilities of each arrival process, we need the system occupancy $\underline{\mathbf{y}}_r$ at arbitrary time instants:

$$\underline{\mathbf{y}}_0 = \frac{1}{E^{\#}} \underline{\mathbf{x}}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \quad (22) \qquad \underline{\mathbf{y}}_r = \frac{1}{E^{\#}} \left[\underline{\mathbf{x}}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \mathbf{D}_r + \underline{\mathbf{x}}_r \right] \quad (23)$$

; for $1 \le r \le 4$

$$\underline{\mathbf{y}}_r = \frac{1}{E^{\#}} \underline{\mathbf{x}}_r \quad ; \text{for } 5 \le r \le N \quad (24) \qquad E^{\#} = \underline{\mathbf{x}}_0 (\mathbf{I} - \mathbf{D}_0)^{-1} \underline{\mathbf{e}} + 1 \quad (25)$$

3.3 The Cell loss probabilities

The cell loss probabilitiy is defined by the ratio of the number of lost cells to the number of arriving cells in all. Following $P_{loss}^{(i)}$ denotes the cell loss probability of the arrival process i.

On principle an arriving non-priority cell, which observed a system occupancy, i.e.buffer occupancy including the one in service, equal or greater than the threshold T+1 is lost. Equivalent an arriving priority cell is lost, when the queue is wholly occupied at the arrival instant.

By noting that $\mathbf{D}_n^{[i;k]}$ is the probability that a batch of n cells arrives which includes cells of the streams i and k the loss probabilities can be calculated with the equations (26) - (27).

$$P_{loss}^{(i)} = \Big\{\frac{1}{4}\ \underline{y}_{T-2}\ \mathbf{D}_4^{[i]}\ \underline{e} + \frac{2}{4}\ \underline{y}_{T-1}\ \mathbf{D}_4^{[i]}\ \underline{e} + \frac{1}{3}\ \underline{y}_{T-1}\ \mathbf{D}_3^{[i]}\underline{e} + \frac{3}{4}\ \underline{y}_T\ \mathbf{D}_4^{[i]}\ \underline{e} + \frac{2}{3}\ \underline{y}_T\ \mathbf{D}_3^{[i]}\underline{e} + \frac{1}{2}\ \underline{y}_T\ \mathbf{D}_2^{[i]}\ \underline{e} + \sum_{\mu=T+1}^{N} \underline{y}_\mu \mathbf{E}(i)\Big\} / \underline{\pi}\cdot \mathbf{E}(i)\ \underline{e} \qquad (51)$$

$i = 1; 2$ with $\mathbf{E}(1) = \mathbf{D}^{(1)} \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes (\mathbf{C}^{(3)} + \mathbf{D}^{(3)}) \otimes (\mathbf{C}^{(4)} + \mathbf{D}^{(4)})$ and $\mathbf{E}(2) = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes \mathbf{D}^{(2)} \otimes (\mathbf{C}^{(3)} + \mathbf{D}^{(3)}) \otimes (\mathbf{C}^{(4)} + \mathbf{D}^{(4)})$

$$P_{loss}^{(i)} = \Big\{\frac{1}{2}\ \underline{y}_N \left[(\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{D}^{(3)} \otimes \mathbf{D}^{(4)}\right] \underline{e}\Big\} \Big/ \underline{\pi}\cdot \mathbf{E}(i)\ \underline{e} \qquad (53)$$

i = 3; 4 with $\mathbf{E}(3) = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes \mathbf{D}^{(3)} \otimes (\mathbf{C}^{(4)} + \mathbf{D}^{(4)})$ and $\mathbf{E}(4) = (\mathbf{C}^{(1)} + \mathbf{D}^{(1)}) \otimes (\mathbf{C}^{(2)} + \mathbf{D}^{(2)}) \otimes (\mathbf{C}^{(3)} + \mathbf{D}^{(3)}) \otimes \mathbf{D}^{(4)}$

3.4 The waiting time distributions

On condition that an arriving cell is not lost, the waiting time of a cell depends on the buffer occupancy at arrival instant and in case of a batch arrival additionally on the allocated position in the buffer. In general, the waiting time can only take on values, which are multiple of the service time h(t)=1. An arriving cell gets immediately service, when the system occupancy is zero or one at the arrival instant, due to the fact, that in a busy system the serviced cell leaving the system at that moment. For each arrival process we obtain the following expressions:

$$P\{W_i = \mu\} = \Big\{ (\underline{y}_\mu \cdot \mathbf{1}_{\mu=0} + \underline{y}_{\mu+1}) \cdot D_1^{[i]} \underline{e} + \frac{1}{2} (\underline{y}_\mu + \underline{y}_{\mu+1}) \cdot D_2^{[i]} \underline{e} + \frac{1}{3} (\underline{y}_{\mu-1} \cdot \mathbf{1}_{\mu>0} + \underline{y}_\mu + \underline{y}_{\mu+1}) \cdot \mathbf{1}_{\mu<T} \cdot D_3^{[i]} \underline{e} + \frac{1}{4} (\underline{y}_{\mu-2} \cdot \mathbf{1}_{\mu>1} + \underline{y}_{\mu-1} \cdot \mathbf{1}_{\mu>0} + \underline{y}_\mu + \underline{y}_{\mu+1}) \cdot \mathbf{1}_{\mu<T} D_4 \underline{e} \Big\} \Big/ \sum_{\mu=0}^{M-1} \{W_i = \mu\} \; ; 0 \le \mu < M; \quad i = 1, .., 4$$

$$M = T \text{ for } i = 1; 2 \text{ an } M = N \text{ for } i = 3; 4 \qquad (28)$$

In equations (28) the operator $\mathbf{1}_x$ is only equal to 1if x is true, 0 otherwise.

4. Numerical Results

In this section, we present numerical examples for various scenarios. The parameters N and h(t) of the queueing system, which was introduced in section 3, are constant in all scenarios.

Type	Poison traffic sources									
A_1	0.2									
A_2	0.25									
	VBR traffic sources with periodical correlation function									
	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
B_1	0	0.096	0.048	0.241	0.144	0.024	0.337	0.33	0.096	0.144
C_1	0	0.110	0.088	0.060	0.005	0.005	0.005	0.220	0.385	0.495
	VBR traffic sources with negative exponential correlation function									
	M	α	β	d						
D_1	10	0.079	0.042	17.5						
D_2	10	0.079	0.042	13.99						
	CBR traffic sources									
	T	d1		d2		d3		d4		d5
E_1	5	$1\cdot10^{-10}$		$1\cdot10^{-10}$		$1\cdot10^{-10}$		$1\cdot10^{-10}$		$1\text{-}10^{-10}$
E_2	4	$1\cdot10^{-10}$		$1\cdot10^{-10}$		$1\cdot10^{-10}$		$1\text{-}10^{-10}$		

Table1 Types of arrival processes

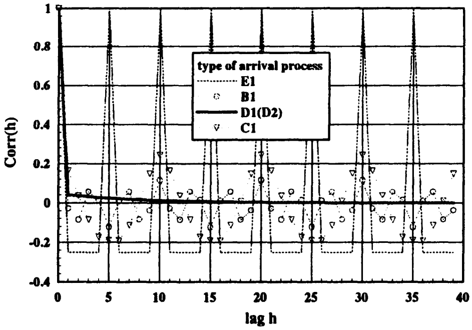

Figure 4 Correlation functions of the arrival processes

Scenario	λ_1^n	λ_2^n	λ_3^p	λ_4^p
1	E_1	B_1	A_1	A_1
2	E_1	D_1	A_1	A_1
3	E_1	C_1	A_1	A_1
4	E_1	A_1	A_1	A_1
6	E_2	D_2	A_2	A_2

Table 2 Scenarios with different traffic composition

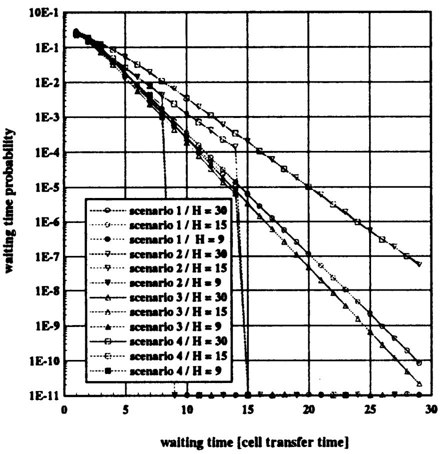

Figure 6 Waiting time distribution of Arrival process E_1

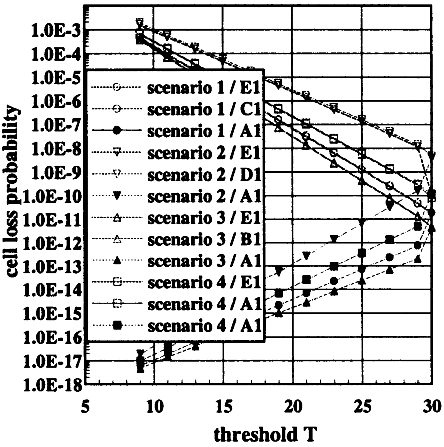

Figure 7 Comparison of cell loss probabilities scenario 2 and 6

We choose N = 30 (cells) and h(t) = 1, i.e. the service time is equal to a time slot. The types of arrival processes we consider throughout this section are shown in Table.1. In column 1 of this table, the different types are denoted by capital letters, at which the indices 1 and 2 mark processes with a mean arrival rate of $\lambda = 0.2$ or $\lambda = 0.25$, respectively. In Table 2, the various scenarios which we consider in our investigation are shown. The comparison of the first four scenarios discloses the influence of the correlation function in the arrival process No.2 on the QoS-parameters. The results are demonstrated in Figure 5 and Figure 6.

From the curves, we can derive that the cell loss probabilities and the waiting time distributions of the streams are essentially dependent on the correlation function in the arrival process No.2. With the negative exponential correlation function, the QoS parameters attain the adverse values. The sensitiveness of the QoS parameters from the correlation function is smaller when the threshold T is low. The results also show that threshold levels close by thewhole buffer capacity of the system are not a good policy, because the effect of the partial buffer mechanism can be compensated by the correlation function.

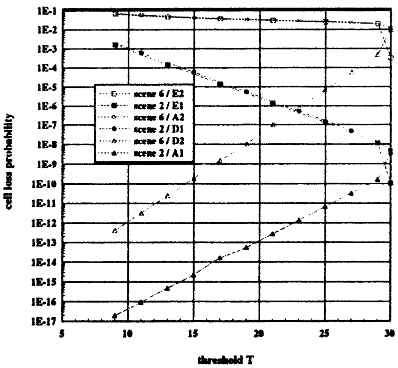

Figure 5 Cell loss probability of priorty and non-priority streams

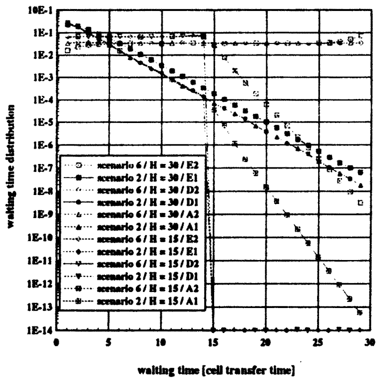

Figure 8 Comparison of the waiting time distribution scenario 2 and 6

To evaluate the influence of the offered load for the QoS parameters, we compare the numerical results of scenario 2 with those of scenario 6. We consider a total offered load of $\rho = 0.8$ or $\rho = 1.0$, respectively. Figure 7 and Figure 8 show that the cell loss probabilities and the waiting time distributions are strongly dependent on the total load. In the saturated case, only the cell loss probabilities of the priority cells can be sufficiently influenced on the threshold T. As we consider that the variation of the waiting time w influences the CDV on the large scale, the threshold T is also useful to control the CDV of time-sensitive cells, e.g. CBR traffic. In scenario 6 we obtain var[w] = 72,7 for T = 30 and var[w] = 18,9 for T = 15.

5. Conclusions

We demonstrated that the presented model of the D-MAP/D/1 queue with partial buffer mechanism allows a comprehensive analysis, including for heterogenous scenarios. The D-MAP allows to imitate different types of traffic sources. The cell loss probabilities and the waiting time distributions were investigated for each source. The numerical results disclosed the influence of the correlation functions in the arrival processes to the QoS paramters. It was shown that the partial buffer mechanism is able to minimize this influence when the difference between the QoS requirements is great, i.e. for small threshold levels.

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Biography

Dietmar Becker was born in Bensberg, Germany, on July 20, 1961. He received Dipl.-Ing. degrees in Communications and Electrical engineering from Cologne University, Hagen University, Germany in 1988 and 1994, respectively. From 1988 to 1989 he developed communicationssoftware at SAE Elekronik. From 1989, he spent five years at Bayer AG Leverkusen, where he was responsible for telecommunication projects. Since July 1995 he has been the managing director of the operation center of Plusnet, a subsidiary of Thyssen Telecom. His research interests include queueing theory and performance analysis of broadband communication systems. Mr.Becker is a member of IEEE, VDE and VDI.