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A Diffusion Cell Loss Estimate for ATM with Multiclass Bursty Traffic

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Abstract

We describe a diffusion approximation model for an ATM statistical multiplexer using the instantaneous return model approach (Gelenbe, 1975). Two Cell Loss Estimates are proposed for multiclass traffic. Our aim is to provide a novel conservative, accurate and computationally efficient method for predicting cell loss probabilities which we call the *Finite Buffer Diffusion Cell Loss Estimate (FBDCLE)* and *Infinite Buffer Diffusion Cell Loss Estimate (IBDCLE)*. We evaluate their accuracy by comparing them with simulation results using a wide variety of input traffic characteristics. In particular we test the model with traffic which is a mixture of different "On-Off" sources with varying loads. Both homogeneous and heterogeneous aggregated arrival processes have been taken into account. These comparisons, which include evaluations of the statistical confidence of the simulation runs, show that our model predictions are very close to the simulation results. In particular, FBDCLE is a conservative upper bound to cell loss ratio, while the other (IBDCLE) provides an accurate predictor which may slightly under-estimate or over-estimate cell loss.

Keywords

ATM network performance prediction, quality of service, queueing theory, diffusion model, call admission control, bandwidth allocation.

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1 INTRODUCTION

ATM provides a universal carrier service that can carry voice, data and video using the same cell transport arrangement. This technique allows complete flexibility in the choice of connection bit rate and enables the statistical multiplexing of variable bit rate traffic streams. On the other hand it also introduces a risk of overload, due to traffic variations which may cause network capacity to be exceeded. Overload is the main cause of cell loss and jitter in such systems. Thus the performance analysis of ATM multiplexers is critical to the design and analysis of appropriate control mechanisms for call admission, bandwidth allocation and bandwidth adaptation. Although much work has been done on the computation of cell loss ratios or probabilities which will result from a given ATM multiplexer in the presence of a given traffic (Kobayashi *et al.*, 1993) (Heffes *et al.*, 1986) (Sriram *et al.*, 1986) (Akimaru *et al.*, 1994), there is still much room for improvement in the methods used for finding computationally effective, fast and tight estimates of cell loss.

Typically, call admission and bandwidth adaptation controls use estimates of cell loss ratio for a given description of the incoming traffic at an ATM multiplexer or along a path traversing a series of multiplexers. For instance the call admission control policy used in IBM's ATM architectures (Guerin *et al.*, 1992) bases its bandwidth allocation conservatively using the minimum of two cell loss estimates: one based on equivalent bandwidth and the other on a Gaussian approximation of cell loss probability. Therefore more accurate estimates of cell loss probabilities will necessarily lead to better decisions for call admission. Thus it is important to be able to estimate cell loss ratios within a very wide range of variations ranging from 10^{-1} at the high end to less than 10^{-7} at the low end. It is important that the estimates obtained be conservative, i.e. that they be upper bounds, so that any bandwidth allocation based on these estimates does result in higher cell loss ratios. However, it is also essential that the estimate be a tight upper bound so that it will not result in the wasteful allocation of excessive bandwidth. Another consideration for any tool used for estimating cell loss is its computational cost. Many of the decisions making processes which use such estimates will have to be carried out in real time at low computational cost. Therefore our research aims at obtaining a tight, conservative and computationally effective method for estimating cell loss in an ATM multiplexer from given traffic characteristics. This paper uses diffusion approximations to contribute:

- a conservative cell loss ratio estimate we name FBDCLE (*Finite Buffer Diffusion Cell Loss Estimate*),
- and a tight estimate we call IBDCLE (*Infinite Buffer Diffusion Cell Loss Estimate*),

for superposed multiclass "On-Off" traffic. We use simulations to show the validity of FBDCLE and IBDCLE in the cell loss ratio range between 10^{-1} and 10^{-5}.

We describe the diffusion model in Section 2. In Section 3 and Section 4 we derive the FBDCLE and the IBDCLE. In section 5 we use the two estimates to compute cell loss ratios for multiple class "On-Off" traffic, and compare the analytical results with simulations for a wide variety of input traffic characteristics and different loads.

2 THE DIFFUSION MODEL

Diffusion approximations are continuous approximations to the discontinuous arrival and service processes in queueing models. They have long been used in queueing theory to model traffic and service. Their advantage is that they will generally result in computationally more tractable models of performance for more detailed traffic representations, that what can be obtained from a direct study of the corresponding discrete processes. In the past, two different approaches to diffusion approximations for queueing models have been proposed. In both cases whenever the queue length is non-zero and the maximum buffer capacity has not been attained, the queue length distribution is approximated by solving a partial differential equation. However the two methods differ according to the choice of boundary conditions. The simpler one uses reflecting boundaries (Kobayashi, 1974) (Kobayashi *et al.*, 1993) so that no probability mass accumulates at the boundaries. Clearly this approach will not be totally satisfactory if the boundaries themselves are very important to the process being modeled. The more sophisticated approach is based on the "instantaneous return process" (Gelenbe, 1975) (Gelenbe *et al.*, 1976) (Duda, 1986) which combines the partial differential equation formulation for the process *strictly* inside the boundaries, with a discrete state-space model at the boundaries themselves (Gelenbe, 1975). This leads to a more accurate model of the queueing behavior of the system when the load is low, or when the queue length is close to the maximum value allowed by a finite buffer.

Diffusion approximations require that the first two moments of the interarrival and service times be known. These can be directly deduced from measurements or from other traffic models, such as the "On-Off" model often used in the literature (Heffes *et al.*, 1986) (Sriram *et al.*, 1986). The diffusion approximation approach we take for an ATM multiplexer buffer of size B, considers a random process $\{X(t),\ t \geq 0\}$ to represent the buffer contents. In the open interval]0,B[(excluding the two boundaries) it is a continuous random variable with probability density function $f(x,t)$ defined as:

$$f(x,t)dx = Pr[x \leq X(t) < x + dx], x \in]0, B[, \tag{1}$$

while at the boundaries we have:

$$m(t) = Pr[X(t) = 0], \tag{2}$$

$$M(t) = Pr[X(t) = B]. \tag{3}$$

The parameters for the diffusion process inside in]0,B[are the "drift" or instantaneous average rate of change:

$$\mu = \lim_{\Delta t \to 0} \frac{E[X(t + \Delta t) - X(t) | X(t) \in]0, B[\,]}{\Delta t} \tag{4}$$

and the instantaneous variance of the change in X(t):

$$\alpha = \lim_{\Delta t \to 0} \frac{Var[X(t + \Delta t) - X(t) | X(t) \in]0, B[\,]}{\Delta t} \tag{5}$$

and α will depend on the variance of the interarrival and service times at the ATM multiplexer. Since the service time is constant due to the fixed length of the cells being transmitted, α will only depend on the variance of interarrival times. Assuming time-independent traffic characteristics, let the mean aggregate cell arrival rate to the buffer be λ and the multiplexer cell transmission rate be C, both given in cells per second. Then we will have:

$$\mu = \lambda - C. \tag{6}$$

In the instantaneous return process model, when queue length reaches the lower boundary of the interval at $x = 0$, it remains there for a random length of random time which we denote h. This time clearly represents a period when the buffer is empty, and it ends as soon as a cell arrives to the multiplexer. At that time, say τ, the process $X(t)$ will jump from $X(\tau) = 0$ to $X(\tau^+) = +1$. Similarly for the upper boundary at $x = B$ where the random time spent at the boundary will be denoted by H, while the jump of the queue length process will be from the value B to the value $B-1$ representing the end of a service or transmission epoch for a cell, resulting in a decrease of buffer length by 1. This behavior results in the following system of equations for the ATM multiplexer queue length process as derived in (Gelenbe, 1975) in steady state, where we have dropped the dependence on t:

$$-\mu\frac{\partial}{\partial x}f(x) + \frac{\alpha}{2}\frac{\partial^2}{\partial x^2}f(x) + \frac{m}{E[h]}\delta(x-1) + \frac{M}{E[H]}\delta(x-B+1) = 0 \tag{7}$$

$$\lim_{x\to 0^+}[-\mu f(x) + \frac{\alpha}{2}\frac{\partial f(x)}{\partial x}] = \frac{m}{E[h]}\lim_{x\to 0^+}\int f(x)dx = 0, \tag{8}$$

$$\lim_{x\to B^-}[-\mu f(x) + \frac{\alpha}{2}\frac{\partial f(x)}{\partial x}] = -\frac{M}{E[H]}\lim_{x\to B^-}\int f(x)dx = 0, \tag{9}$$

where $\delta(x)$ is the Dirac Delta function. Also the probabilities must sum to 1:

$$m + M + \int_{0^+}^{B^-} f(x)dx = 1 \tag{10}$$

These equations have a simple interpretation. Equation (7) represents the stationary behavior for the motion of the queue length process in the interval $]0, B[$, and the effect of the jumps of the process $X(t)$ from 0 and B into the interval. On the other hand (8) represents the depletion of the probability mass m at the lower boundary due to the jumps to +1 at the end of the holding time at the lower boundary, as well as the flow of probability mass from inside the interval $]0, B[$ towards the lower boundary. Equation (9) has a similar interpretation.

2.1 Queue length distribution of finite capacity

The above equations may be solved directly (Gelenbe, 1975) to obtain:

$$f(x) = \begin{cases} \Phi\ [1\text{-}e^{\gamma x}], & 0 < x \leq 1 \\ \Phi\ [e^{-\gamma}\text{-}1]e^{\gamma x}, & 1 \leq x \leq B-1 \\ \Phi\ [e^{\gamma(x-B)}\text{-}1]e^{\gamma(B-1)}, & B-1 \leq x \leq B \end{cases} \tag{11}$$

with m and M the probability masses at 0 and at B, respectly, at stationary state being:

$$m = -\mu E[h]\Phi, \tag{12}$$

$$M = -\mu E[H]\Phi e^{\gamma(B-1)} \tag{13}$$

where $\gamma = \frac{2\mu}{\alpha}$, and

$$\Phi = \frac{1}{(1-\mu E[h]) - (1+\mu E[H])e^{\gamma(B-1)}} \tag{14}$$

2.2 Queue length distribution of infinite capacity

If we consider a diffusion process on the whole non-negative real line, i.e. as if the queue length were infinite, with holding time h only at $x = 0$, we will have the following formula for an unbounded queue diffusion approximation model:

$$f(x) = \begin{cases} \Phi\ [1\text{-}e^{\gamma x}], & 0 < x \leq 1 \\ \Phi\ [e^{-\gamma}\text{-}1]e^{\gamma x}, & 1 \leq x \end{cases} \tag{15}$$

$$m = 1 - \Phi \tag{16}$$

$$\Phi = \frac{1}{(1-\mu E[h])} \tag{17}$$

In the following sections, we will derive the practical applications of diffusion approximation models both for bounded queue and unbounded queue:

- Finite Buffer Diffusion Cell Loss Estimate (FBDCLE);
- Infinite Buffer Diffusion Cell Loss Estimate (IBDCLE).

In order to make use of these diffusion models we will need to determine the parameters μ, α, $E[h]$ and $E[H]$ from the arrival and service characteristics of the ATM multiplexer. From engineering application viewpoint of diffusion approximation models, various strategies can be used to obtain $E[h]$ and $E[H]$. More detail will be presented when we derive FBDCLE and IBDCLE.

3 FINITE BUFFER ESTIMATE - FBDCLE

In general the distributions for the residence times of moderately complex finite capacity queueing models at the upper and lower boundaries 0 and B are unknown. Their characterization can be quite complex and depends on both the arrival process, the buffer size, and the service process. Thus we will have to calculate $E[h]$ and $E[H]$ in a heuristic but plausible manner.

3.1 Calculation of $E[h]$ and $E[H]$

If the arrival process can be approximated by a Poisson process with arrival rate λ it follows that $E[h] = \lambda^{-1}$. Since the arrival traffic to an ATM multiplexer is made up of many superposed sources, when the number of sources is large this approximation may be acceptable. In our simulations it turns out that this heuristic for $E[h]$ slightly underestimates the actual value for superposed "On-Off" sources.

Recall that the time for transmitting one cell is C^{-1}. Now assume that at instant t the transmission of a cell begins and that $X(t) = B - 1$. At some instant $t + Z$ before $t + C^{-1}$ another arrival occurs so that now $X(t+Z) = B$. Then H, the random variable representing the holding time at the upper boundary, has the following distribution:

$$Pr[H \leq v] = Pr[\frac{1}{C} - Z \leq v | Z \leq \frac{1}{C}] = \frac{Pr[\frac{1}{C} - Z \leq v \ and \ Z \leq \frac{1}{C}]}{Pr[Z \leq \frac{1}{C}]} \tag{18}$$

We make the simplifying approximation that the arrival process is Poisson of rate λ so as to complete the computation, on the basis that it is justified when the arriving traffic results from the superposition of many independent sources. Then

$$Pr[Z \leq \frac{1}{C}] = 1 - e^{-\frac{\lambda}{C}}, \tag{19}$$

and

$$Pr[\frac{1}{C} - Z \leq v \ and \ Z \leq \frac{1}{C}] = Pr[\frac{1}{C} - v \leq Z \leq \frac{1}{C}] = e^{-\frac{\lambda}{C}}[e^{\lambda v} - 1]. \tag{20}$$

Thus

$$Pr[H \leq v] = \frac{e^{\lambda v} - 1}{e^{\frac{\lambda}{C}} - 1}, \tag{21}$$

with density function

$$f_H(v) = \begin{cases} \frac{\lambda e^{\lambda v}}{e^{\frac{\lambda}{C}} - 1}, & 0 \leq v \leq \frac{1}{C} \\ 0, & elsewhere \end{cases} \tag{22}$$

We can now derive the estimate for the average holding time at the upper boundary:

$$E[H] = \int_0^{\frac{1}{C}} v f_H(v) dv = \frac{\frac{1}{C}}{1 - e^{-\frac{\lambda}{C}}} - \frac{1}{\lambda}. \tag{23}$$

Of course, the first and second moments of the interarrival times are also needed in order to compute the density function $f(x)$ and the probability masses m and M. However, these moments will be available from the practical measurement and the precise traffic characteristics we shall use and will be discussed later in Section 5.

3.2 Estimating the cell loss ratio

The long run cell loss ratio L is the proportion of cells lost at the entrance to the multiplexer due to buffer overflow, to total cells arriving to the multiplexer. It is the primary measure of interest in this study and it needs to be estimated both *accurately* and in a *conservative* manner. Thus what is needed is in fact a tight upper bound, rather than a relatively accurate value which may underestimate L. Clearly cells will be lost only when the buffer is full, i.e. when buffer length has attained size B, in which case all the arriving cells will be lost. Thus the cell loss ratio in steady state may be written as:

$$L = \lim_{t \to \infty} M(t) Pr[\ N(t, t+H) \geq 1 \mid X(t) = B\ ], \tag{24}$$

where $N(t, t+H)$ is the number of arrivals in the open interval $(t, t+H)$. If the arrival process is stationary in time and independent of buffer size, in steady state the expected cell loss ratio is:

$$L = M.Pr[\ N(t, t+H) \geq 1\ ]. \tag{25}$$

There are several difficulties with using this expression when one deals with real traffic, including the issue of estimating H and the probability of the number of arrivals in the interval when the buffer is full. However we do know that $H \leq \frac{1}{C}$. Thus we have found that L^*_{FB} given below, which we call the *Finite Buffer Diffusion Cell Loss Estimate (FBDCLE)*, is a useful and tight upper bound which yields cell loss ratio values which are within the same order of magnitude as the value measured from simulation with various forms of "On-Off" traffic:

$$L \leq L^*_{FB} = M.Pr[N(t, t + \frac{1}{C}) \geq 1]. \tag{26}$$

The quality of this estimate L^*_{FB} has been tested by simulation with a very wide variety of "On-Off" traffic models, as shown in the simulation results we present.

4 INFINITE BUFFER ESTIMATE - IBDCLE

As indicated previously, the exact average residence times $E[h]$ and $E[H]$ of the finite capacity queueing model at the upper and lower boundaries are not known in general and are difficult to obtain. Thus we consider an alternate formulation - infinite capacity queueing model where we only deal with the holding time at lower boundary $x = 0$. Now the key value used for estimating the cell loss probability will be the stationary probability that the diffusion process exceeds the value B:

$$P_B = Pr[X \geq B] \tag{27}$$

From (15) we estimate the buffer overflow probability P_B:

$$P_B = \Phi \frac{1}{\gamma}[1 - e^{-\gamma}]e^{\gamma B} \tag{28}$$

If $R(t)$ is the instantaneous cell arrival rate, then the new diffusion cell loss ratio estimate L is:

$$L = P_B \, \frac{E[(R(t) - C)^+]}{E[R(t)]} \tag{29}$$

since cell loss will only occur if the arrival rate is greater than the multiplexer's service capacity C whenever the buffer length is at least B.

4.1 Choice of $E[h]$

It is known that for the GI/GI/1 queue with arrival rate λ the average idle time $E[h]$ satisfies (Medhi, 1991):

$$E[h] \geq E[h]^* = \frac{1}{\lambda} - \frac{1}{C} \tag{30}$$

Thus we will approximate $E[h]$ by its lower bound $E[h]^*$, all other things being equal, the resulting probability P_B^* that the queue length exceeds B will be larger than real value P_B. This is because the process will be spending less time at $x = 0$ and therefore will be more likely to exceed B. This can also be easily proved by applying inequality of (30) into (28).

4.2 Estimating the cell loss ratio

The estimate L_{IB}^*, which we call the *Infinite Buffer Diffusion Cell Loss Estimate (IBDCLE)*, which in turn is obtained by replacing $E[h]$ by $E[h]^*$ in equation (28). IBDCLE will be:

$$L_{IB}^* = P_B^* \, \frac{E[(R(t) - C)^+]}{\lambda} \tag{31}$$

since $E[R(t)] = \lambda$ if $R(t)$ is stationary.

5 CELL LOSS ESTIMATES FOR "ON-OFF" MULTICLASS TRAFFIC

In this section we present the numerical and simulation results to evaluate the accuracy of FBDCLE and IBDCLE for a wide variety of "On-Off" traffic models. Much of the work on ATM traffic analysis and cell loss estimates is based on the "On-Off" traffic model and on the superposition of such traffic streams (Heffes *et al.*, 1986) (Sriram *et al.*, 1986). Thus it is of particular interest to evaluate the accuracy of our cell loss estimates (diffusion estimate) for this specific class of practically useful traffic models. In order to do so, we will first derive the appropriate traffic parameters to be used in the diffusion approximation.

5.1 The traffic model

Consider first a single user u whose traffic follows a simple "On-Off" behavior. This user u either sends traffic into the network at a constant peak rate R_u during the "On" period, or it sends no traffic at all during the "Off" period. The following notation describes this traffic model:

- R_u – peak traffic rate during the "On" period, $T_u = 1/R_u$;
- θ_u^{-1} – average length of the "Off" period;
- β_u^{-1} – average length of the "On" period;
- $a_u = \theta_u/(\beta_u + \theta_u)$ – source activity.

The duration of the successive On and Off periods are assumed to be independent, so that the cell arrival process from a single such source is a *renewal process*. The cell interarrival time will be denoted by Y_u, and let $F_u(x) = Pr[Y_u \leq x]$ so that (Heffes *et al.*, 1986):

$$F_u(x) = [(1 - \beta_u T_u) + \beta_u T_u(1 - e^{-\theta_u(x-T_u)})]U(x - T_u) \tag{32}$$

where $U(x)$ is the unit step function. The Laplace-Stieltjes transform (LST) of the interarrival time density is given by:

$$\tilde{f}(s) = \int_0^\infty e^{-sx} dF_u(x) = [1 - \beta_u T_u + \beta_u T_u \theta_u/(s + \theta_u)]e^{-sT_u} \tag{33}$$

The mean cell arrival rate of cells from source u is then:

$$\lambda_u = -1/\tilde{f}'(0) = 1/(T_u + \beta_u T_u/\theta_u) = a_u/T_u = a_u R_u \tag{34}$$

Let $A_u(t)$ denote the number of arrivals of cells of user stream u in the interval [0,t). Then the squared coefficient of variation of the interarrival time from source u is (Cox *et al.*, 1966) (Heffes *et al.*, 1986):

$$c_u^2 = \frac{Var[Y_u]}{E^2[Y_u]} = \frac{Var[A_u(t)]}{E[A_u(t)]} \tag{35}$$

which leads to (Heffes *et al.*, 1986):

$$c_u^2 = \frac{1 - (1 - \beta_u T_u)^2}{(\beta_u T_u + \theta_u T_u)^2}. \tag{36}$$

Since $E[A_u(t)] = \lambda_u t$, we can write (35) as:

$$\lim_{t\to\infty} \frac{Var[A_u(t)]}{t} = \lambda_u \frac{Var[Y_u]}{E^2[Y_u]} = \lambda_u c_u^2 \tag{37}$$

Now if the total arrival process to the ATM multiplexer results from the superposition of N uncorrelated "On-Off" sources of renewal type as discussed above, $A(t)$ the resulting counting process $A(t) = \sum_{u=1}^{N} A_u(t)$ has the obvious properties:

$$E[A(t)] = \sum_{u=1}^{N} E[A_u(t)], \tag{38}$$

$$Var[A(t)] = \sum_{u=1}^{N} Var[A_u(t)] \tag{39}$$

and

$$E[A(t)] = \sum_{u=1}^{N} \lambda_u t, \tag{40}$$

$$Var[A(t)] = \sum_{u=1}^{N} \lambda_u c_u^2 t \tag{41}$$

Let $D(t, t+\tau)$ denote the number of departures in an interval $[t, t+\tau)$ when the queue is non-empty. Note that if the multiplexer queue is non-empty, then the service or emptying process at the queue is independent of the arrival process. Thus we have:

$$E[X(t+\Delta t) - X(t) | X(t) > 0] = E[A(t+\Delta t) - A(t)] - E[D(t+\Delta t) - D(t)] \tag{42}$$

and

$$Var[X(t+\Delta t) - X(t) | X(t) \in]0, B[\,] = Var[A(t+\Delta t) - A(t)] + Var[D(t+\Delta t) - D(t)] \tag{43}$$

so that

$$\mu = \lim_{\Delta t\to 0} \frac{E[X(t+\Delta t) - X(t) | X(t) \in]0, B[\,]}{\Delta t} = \sum_{u=1}^{N} \lambda_u - C, \tag{44}$$

$$\alpha = \lim_{\Delta t\to 0} \frac{Var[X(t+\Delta t) - X(t) | X(t) \in]0, B[\,]}{\Delta t} = \sum_{u=1}^{N} \lambda_u c_u^2. \tag{45}$$

We now have all the parameters needed by the diffusion model described in Sections 2,3 and 4 when it is used for superposed "On-Off" traffic sources, and can use it to calculate the IBDCLE and FBDCLE formulae given in (26) and (31).

5.2 The distribution of the number of arrivals

In order to calculate the FBDCLE, the quantity $Pr[N(t, t+\frac{1}{C})]$ must be obtained. To do so, we will consider the general case of arrival traffic composed of multiple "On-Off" sources of K different *types*. Each source of the same type will have the same set of parameters, and N_k will be the number of k-type sources, each with the same peak traffic rate R_k, activity a_k. Notice that here we use the subscript k to denote a user type, rather than the subscript u to denote an individual user. The total number of users or sources is then $N = \sum_{k=1}^{K} N_k$. The average arrival rate of cells will then be:

$$\lambda = \sum_{k=1}^{K} a_k N_k R_k \tag{46}$$

Now let $Z_k(t)$ be the random variable denoting the number of sources of type k which are "On" at some time t. Since the sources are independent and stationary we have for large enough t that:

$$Pr[Z_1(t) = n_1, ..., Z_K(t) = n_K] = \prod_{k=1}^{K} \binom{N_k}{n_k} a_k^{n_k} (1-a_k)^{N_k - n_k} \tag{47}$$

On the other hand for small enough $1/C$:

$$N(t, t+\frac{1}{C}) = [Z_1(t)R_1 + ... + Z_K(t)R_K]/C, \tag{48}$$

so that:

$$Pr[N(t, t+\frac{1}{C}) \geq 1] = Pr[Z_1(t)R_1 + ... + Z_K(t)R_K] \geq C], \tag{49}$$

which can be computed from the distribution (47). For homogeneous traffic, i.e. when all sources are of just one type, we simply have $K = 1$ and:

$$Pr[N(t, t+\frac{1}{C}) \geq 1] = 1 - \sum_{n_1=1}^{int(C/R_1)} \binom{N_1}{n_1} a_1^{n_1} (1-a_1)^{N_1 - n_1}. \tag{50}$$

For the IBDCLE we need $E[(R(t) - C)^+]$ to be used in (31), which is computed for the superposed multiclass "On-Off" traffic as:

$$E[(R(t) - C)^+] = \sum_{n_1,...,n_K \geq 0} (n_1 R_1 + ... + n_K R_K - C)^+ Pr[Z_1(t) = n_1, ..., Z_K(t) = n_K] \tag{51}$$

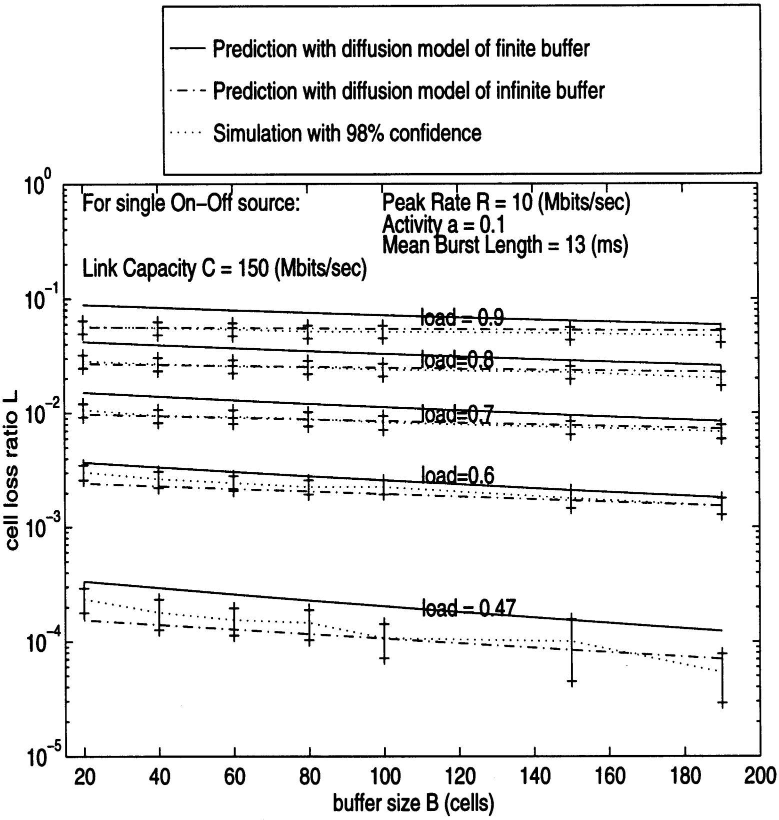

Figure 1 Cell loss probability vs. buffer size: comparison of simulation and DCLE for homogeneous sources under varying load (load = aggregate mean arrival rate /link capacity).

5.3 Comparison of numerical and simulation Results

In this section we present the numerical and simulation results to evaluate the accuracy of FBDCLE and IBDCLE. The validation of our new diffusion model is focused on the comparison of the cell loss probability predicted by the FBDCLE and IBDCLE and that obtained by simulations for a wide variety of "On-Off" traffic models. In our simulations, the runs were independently replicated 20 times, and each run included the transmission of 10^7 cells. Confidence intervals are calculated using the *Student* $- t$ distribution with 98% confidence so that the simulation results are of sufficiently high statistical quality. The resulting confidence intervals' width is also shown on the figures.

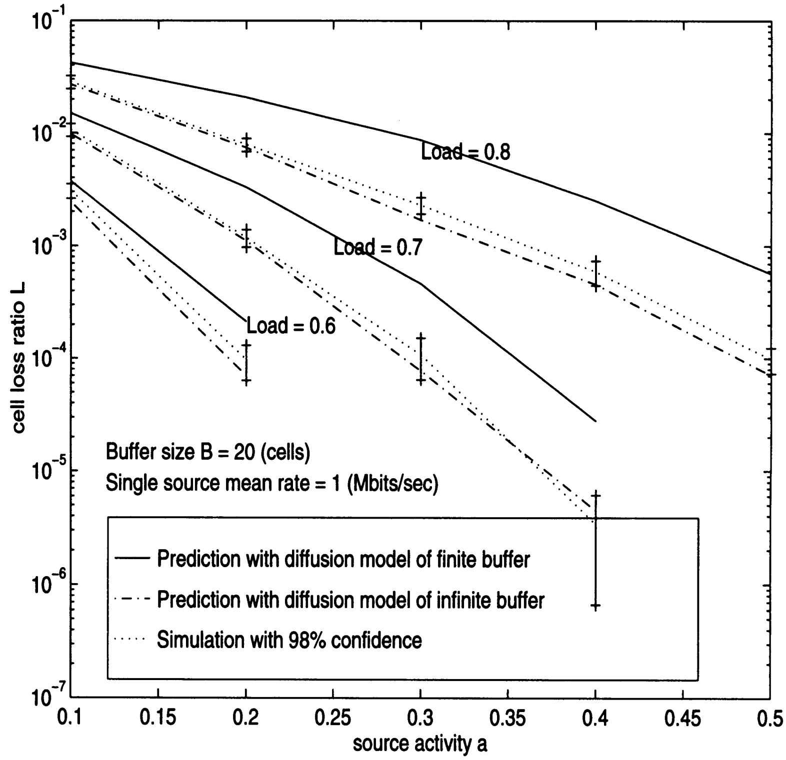

Figure 2 Cell loss probability vs. source activity (burstiness): comparison among sim ulations and analytical approach using DCLE for the homogeneous sources under variant load (load = aggregate mean rate /link capacity).

Figures 1 and 2 summarize the results for traffic with homogeneous sources.

In Figure 1 cell loss probability ($Pr[cell\ loss]$) is plotted versus buffer size B for different load, which is λ/C. The ATM multiplexer we consider here is a high speed link with link capacity $C = 150 Mbits/sec$ and there are a collection of homogeneous traffic sources which are very bursty with an activity value of $a = 0.1$, which means that it is at its peak value 10% of the time and is "Off" the rest of the time. Load is varied in Figure 1 simply by varying the number of sources. The results show that for cell loss ratio ranging from the high 10^{-5} to the 10^{-1} values, the FBDCLE (the solid line) provides a conservative upper bound, while the IBDCLE (the dashed and dotted line) is an accurate predictor which remains well

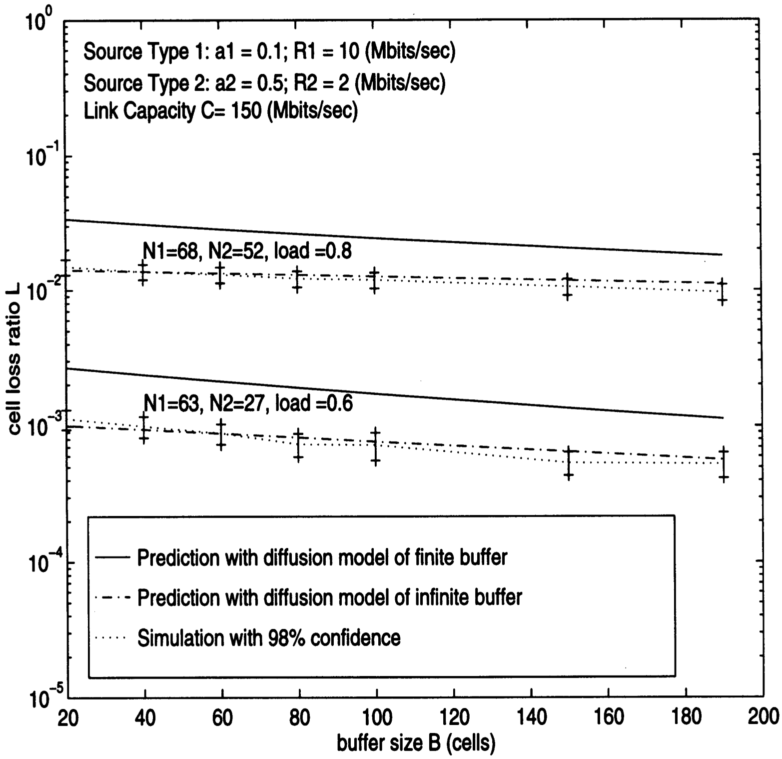

Figure 3 Cell loss probability versus buffer size: comparison between simulation and DCLE for heterogeneous sources with varying load (load = aggregate mean rate /link capacity).

within the confidence intervals. Simulation results are shown by the dotted lines while the 98% confidence intervals are vertical lines.

In Figure 2 similar results are observed when source activity a (or burstiness) is varied widely for different values of the load. Here each individual source generates cells at an average rate $\lambda_u = 1(Mbits/sec)$ and the buffer size is relatively small: $B = 20$ cells. Note that here we see that IBDCLE is an accurate predictor over cell loss ratio values ranging from 5×10^{-6} to 3×10^{-2}.

Figures 3 and 4 compare FBDCLE and IBDCLE with simulation under heterogeneous traffic. We have chosen two types of sources – more bursty sources with $a_u = 0.1$ and less

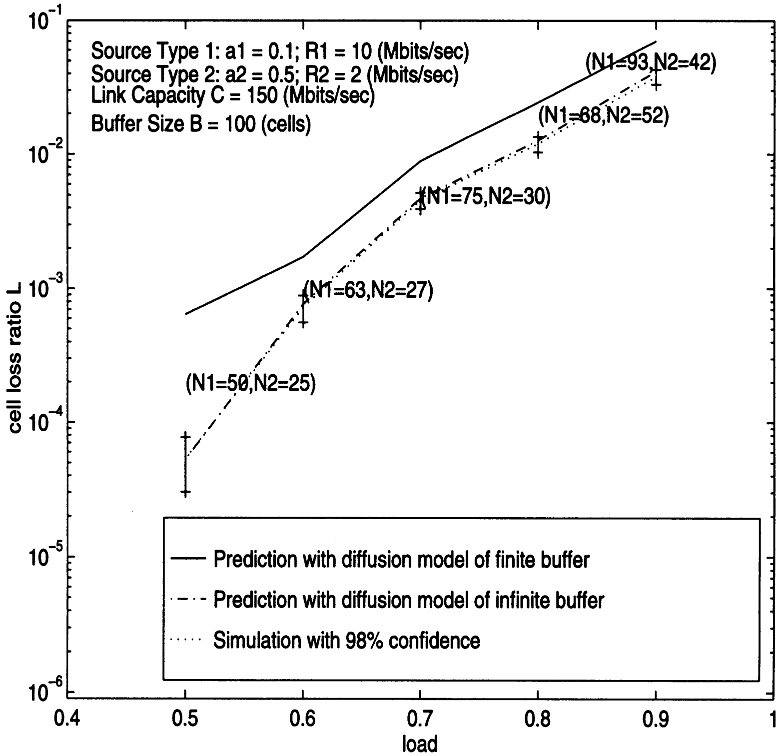

Figure 4 Cell loss probability versus load: comparison between simulation and the DCLE for heterogeneous sources (load = aggregate mean rate /link capacity).

bursty sources with $a_u = 0.5$. If N_1 and N_2 denote the number of sources with $a_u = 0.1$ and $a_u = 0.5$ respectively, and $N = N_1 + N_2$.

In Figure 3 we show matched results of simulations and the diffusion predictions for two different values of the load, and under different combinations of N_1 and N_2 with varying buffer size B. Note that the two classes are also characterized by two much different values of peak traffic rate: $R_1 = 10(Mbits/sec)$ and $R_2 = 2(Mbits/sec)$. Again we see that the FBDCLE (the solid line) gives a bounded estimate while IBDCLE provides a very accurate prediction (the dashed and dotted line).

In Figure 4 the cell loss probability is plotted versus traffic load for a fixed buffer size $B = 100$, the same two-class traffic as in Figure 3 and five different load values obtained by varying the mixture of class 1 and class 2 traffic. The simulation results, together with their confidence intervals, show once again excellent agreement with our infinite buffer estimate (IBDCLE) while the FBDCLE is again an upper bound, for cell loss ratio values going from 5×10^{-5} to 3×10^{-2}.

We conclude from these results, and from others which are available but which are not reported here because of space limitation, that the FBDCLE can be used for a very conservative estimate of cell loss, while IBDCLE is useful as an accurate predictor.

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