Interactive multiobjective optimization system NIMBUS applied to nonsmooth structural design problems

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Abstract

We shortly describe an interactive method, called NIMBUS, for multiobjective optimization involving nondifferentiable and nonconvex functions. We illustrate the functioning of NIMBUS by numerical examples in the area of structural design. We consider a beam with varying thickness and our aim is to find a thickness distribution in such a way that the resulting structure is as good as possible.

Keywords

Nonsmooth optimization, multiobjective optimization, structural optimization

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1 INTRODUCTION

In the solution process of optimal shape design and structural design problems we are often faced with optimization problems with several, not necessarily differentiable objective functions (see Mäkelä and Neittaanmäki, (1992), Mäkinen, (1989), and Miettinen and Mäkelä, (1993). However, methods applicable for solving nonsmooth multiobjective optimization problems appear rarely in the literature. Especially, the lack of numerically effective interactive solution methods is evident (as stressed in Miettinen, (1994)).

In this paper we shortly describe an interactive method, called NIMBUS, for multiobjective optimization involving nondifferentiable and nonconvex functions. It has been originally introduced in Miettinen, 1994, and Miettinen and Mäkelä, (1995). The motivations in the development have been numerical efficiency and easiness of use.

We illustrate the functioning of NIMBUS by numerical examples in the area of structural design. We consider a beam with a varying thickness and our aim is to find a thickness distribution in such a way that the resulting structure is as good as possible.

2 NIMBUS METHOD

Let us consider a multiobjective optimization problem of the form

$$\begin{array}{ll} \text{Minimize} & \left\{F_1(x), F_2(x), \ldots, F_k(x)\right\} \\ \text{subject to} & x \in S. \end{array} \tag{1}$$

Pareto optimality and weak Pareto optimality are used as optimality concepts. An essential part in the solution process of multiobjective optimization is played by a decision maker (DM).

The assumptions in the NIMBUS method are that

- All the objective functions are locally Lipschitz continuous.
- The feasible region S is convex.
- Less of any objective is preferred to more in the mind of the DM.

In the NIMBUS method, the idea is that the DM examines the values of the objective functions calculated at a current point x^h and divides the objective functions F_i, $i = 1, \ldots, k$, into up to five classes. Those classes are functions whose values

1. should be decreased ($i \in I^<$),
2. should be decreased down till some aspiration level ($i \in I^\leq$),
3. are satisfactory at the moment ($i \in I^=$),
4. are allowed to increase up till some upper bound ($i \in I^>$),
5. are allowed to change freely ($i \in I^\diamond$).

The DM is asked to specify the aspiration levels $\bar{F}_i$ for $i \in I^\leq$ and the upper bounds ε_i for $i \in I^>$. The difference between the classes $I^<$ and $I^\leq$ is that the functions in $I^<$ are to be minimized as far as possible but the functions in $I^\leq$ only till the aspiration level. Also weighting coefficients can be connected with the functions in the classes $I^<$ and $I^\leq$.

According to the classification and the connected information we form a new problem

$$\begin{array}{ll} \text{Minimize} & \left\{F_i(x)/w_i,\ \max_j \left[\max[F_j(x)/w_j - \bar{F}_j,\ 0]\right] \mid i \in I^<,\ j \in I^\leq\right\} \\ \text{subject to} & F_i(x) \leq F_i(x^h), \quad i \in I^= \\ & F_i(x) \leq \varepsilon_i, \qquad i \in I^> \\ & x \in S, \end{array} \tag{2}$$

where $\sum_{i \in I^< \cup I^\leq} w_i = 1$ and $w_i > 0$ for $i \in I^< \cup I^\leq$, $\bar{F}_i < F_i(x^h)$ for $i \in I^\leq$ and $\varepsilon_i > F_i(x^h)$ for $i \in I^>$.

This problem is solved by a multiobjective proximal bundle (MPB) method, where the multiple objective functions are treated individually without employing any scalarization. The method is capable of handling several nonconvex, locally Lipschitzian objective functions subject to nonlinear (possibly nonsmooth) constraints and it produces weakly Pareto optimal solutions.

The detailed algorithm of the NIMBUS method is the following.

1. Choose a starting point $x^0 \in S$ and calculate its weakly Pareto optimal counterpart x^1 by setting $I^< = \{1, \ldots, k\}$ and employing MPB. Set the iteration counter $h = 1$.
2. Ask the DM to classify the objective functions into $I^<$, $I^\leq$, $I^=$, $I^>$, and $I^\diamond$ at x^h such that $I^> \cup I^\diamond \neq \emptyset$ and $I^< \cup I^\leq \neq \emptyset$. If either of the unions is empty, go to step 9. Ask the DM for the possible weighting coefficients w_i^h, aspiration levels $\bar{F}_i^h$ and the upper bounds ε_i^h.
3. Calculate $\hat{x}^h$ by solving the problem (2) by MPB. If $\hat{x}^h = x^h$, ask the DM whether (s)he wants to try another classification. If yes, set $x^{h+1} = x^h$, $h = h+1$ and go to step 2. If no, go to step 9.
4. If the DM wants to see different alternatives between x^h and $\hat{x}^h$, set $d^h = \hat{x}^h - x^h$ and go to step 6. If the DM prefers x^h, set $x^{h+1} = x^h$, $h = h + 1$ and go to step 2.
5. Now the DM wants to continue from $\hat{x}^h$. Set $x^{h+1} = \hat{x}^h$, $h = h + 1$ and go to step 2.
6. Calculate P^h different vectors $x^h + t_j d^h$, where $t_j = \left(\frac{j-1}{P^h-1}\right)$ and $j = 1, \ldots, P^h$.
7. Produce weakly Pareto optimal criterion vectors from the above vectors, employing MPB (with $I^< = \{1, \ldots, k\}$).
8. Present P^h alternatives to the DM and let her or him choose the most preferred one among them. Denote the corresponding variable by x^{h+1} and set $h = h + 1$. If the DM wants to continue, go to step 2.
9. Solve the problem (3). Let the solution be $(\tilde{x}, \tilde{\delta})$. Stop. The final solution is $\tilde{x}$.

In the last step, the Pareto optimality of the final solution is guaranteed by solving an additional problem

$$\begin{array}{ll} \text{Maximize} & \sum_{i=1}^k \delta_i \\ \text{subject to} & F_i(x) + \delta_i \leq F_i(x^h) \text{ for all } i = 1, \ldots, k, \\ & \delta_i \geq 0 \text{ for all } i = 1, \ldots, k, \\ & x \in S, \end{array} \tag{3}$$

with respect to (x, δ). If x^h is not Pareto optimal, then the solution $\tilde{x}$ is. For clarity of notations, it has not been mentioned in the algorithm that the DM may check the Pareto optimality at any time during the solution process.

The NIMBUS method has been successfully applied to several problems of optimal control, like continuous casting of steel and optimal shape design (see Miettinen, (194), and Miettinen and Mäkelä, (1995)). Next, we utilize our method in the solution processes of a structural design problem.

3 MULTIOBJECTIVE OPTIMIZATION OF ELASTIC BEAMS

We consider an Euler-Bernoulli beam with a varying thickness. The structure is analyzed under three different load cases, namely static loading, free vibration and linear stability. Our aim is to find a thickness distribution in such a way that the resulting structure would be as good as possible. The state problem is discretized with the finite element method. We assume that the beam has a circular cross-sectional shape and we approximate the cross-sectional area by a piecewise constant function. Therefore, we state the following multiobjective optimization problem

$$\text{Minimize} \quad \{F_1(x), F_2(x), F_3(x)\} \tag{4}$$

subject to the discretized state problems

$$\begin{cases} \mathbf{K}(x)\,\mathbf{u} &= \mathbf{f} \\ \mathbf{K}(x)\,\phi &= \lambda\mathbf{M}(x)\,\phi \\ \mathbf{K}(x)\,\psi &= \mu\mathbf{B}\,\psi, \end{cases} \tag{5}$$

the volume constraint

$$V(x) \leq V_{max} \tag{6}$$

and the simple bounds for variables

$$x_{min} \leq x \leq x_{max}. \tag{7}$$

Here x is the vector of design variables, $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{M}$ is the mass matrix, $\mathbf{B}$ is the geometric stiffness matrix, and $\mathbf{u}$ is the nodal displacement vector. The eigenvalues λ and μ are the eigenfrequencies and the buckling load factors, respectively. The choice for objective functions F_i is the following:

$$\begin{array}{lll} F_1(x) & = \mathbf{f}^{\mathrm{T}}\mathbf{u}(\mathbf{x}) & \text{minimization of the compliance} \\ F_2(x) & = -\lambda_m(x) & \text{maximization of the } m\text{:th eigenfrequency} \\ F_3(x) & = -\mu_1(x) & \text{maximization of the buckling load.} \end{array} \tag{8}$$

We assume that the matrices in equations (5) are smooth functions of the design variable vector x. Then it is well-known that the displacement vector $\mathbf{u}$ depends smoothly on x. However, the situation is more complicated in the case of the eigenvalues. It can be shown that, in general, the eigenvalues are only directionally differentiable with respect to the design variables (Haug, Choi and Komkov, (1985)). Nonsmoothness of eigenvalues in structural optimization of beams has been recently studied by Rodrigues, Guedes and Bendsøe, (1995) and Seyranian, Lund and Olhoff, (1994).

Table 1 Intermediate alternatives of the third iteration

	F_1	F_2	F_3
1	0.36×10^{-2}	-4085.51	-47.21
2	0.36×10^{-2}	-4607.19	-46.81
[3]	0.39×10^{-2}	-5021.93	-43.51
4	0.43×10^{-2}	-5278.10	-38.33
5	0.50×10^{-2}	-5311.68	-31.99

4 NUMERICAL RESULTS

Next we will solve a design problem numerically to demonstrate the performance of the NIMBUS system in practical design problems.

The NIMBUS algorithm has been implemented in Fortran 77 and the test runs have been performed on an HP9000/735 (99MHz) computer. The implementation of the MPB routine (called MPBNGC) calls the quadratic solver QPDF4 derived in Kiwiel, (1986). After discretization the dimension of the problem is $n = 14$ and we have chosen $V_{max} = 1.0$, $x_{min} = 0.1$ and $x_{max} = 2.0$.

In the beginning the beam has a constant cross-sectional area, that is, the starting point is $x^0 = (1.0, 1.0, \ldots, 1, 0)$ and the corresponding criterion vector is $z^0 = (F_1(x^0), F_2(x^0), F_3(x^0)) = (0.52 \times 10^{-2}, -3804.06, -39.48)$. We start by minimizing all the objective functions simultaneously. This guarantees that we can begin the actual solution process from a (weakly) Pareto optimal solution. This time, z^0 cannot be improved and we begin our classification from $z^1 = z^0$.

At z^1, our primary goal is to decrease the value of the first objective function and thus $I^< = \{1\}$. Upper bounds are given to the others, $I^> = \{2,3\}$, by $\varepsilon_2^1 = -3000.0$ and $\varepsilon_3^1 = -39.0$. The solution obtained is $\hat{z}^1 = (0.36 \times 10^{-2}, -4085.51, -47.21)$. Notice that all of its components are better than those of z^1. This can be explained by the fact that we can only guarantee the local optimality of the solutions. The new solution is naturally selected for continuation without considering any intermediate alternatives, that is, $z^2 = \hat{z}^1$.

Next, we consider what happens when we decrease the value of the second objective. Thus, $I^{\leq} = \{2\}$ with $\bar{F}_2^2 = -5000.0$ and $I^{\diamond} = \{1,3\}$.

The result is $\hat{z}^2 = (0.50\times10^{-2}, -5311.68, -31.99)$. Because we did not restrict the values of the other objective functions in any way, we take a look at intermediate alternatives. A set of five candidates has been listed in Table 1.

From this table the third alternative is selected and we have $z^3 = (0.39\times10^{-2}, -5021.93, -43.51)$. The value of the first objective function is here satisfactory and we want to see whether we can further decrease the values of the second and the third function. The next classification is $I^< = \{2,3\}$ with $w_2^3 = 0.5$ and $w_3^3 = 0.5$. We let the first objective function change freely and $I^{\diamond} = \{1\}$. This setting produces $\hat{z}^3 = (0.38 \times 10^{-2}, -5023.04, -46.24)$ and we continue from it ($z^4 = \hat{z}^3$).

We would still like to see how the other objectives behave when we decrease the value of the second objective function. We classify $I^{\leq} = \{2\}$ with $\bar{F}_2^4 = -6000.0$ and $I^{\diamond} = \{1,3\}$.

Table 2 Intermediate alternatives of the fifth iteration

	F_1	F_2	F_3
1	0.38×10^{-2}	-5023.04	-46.24
[2]	0.39×10^{-2}	-5158.13	-45.09
3	0.40×10^{-2}	-5290.22	-43.70
4	0.41×10^{-2}	-5418.73	-42.09
5	0.43×10^{-2}	-5543.00	-40.28
6	0.44×10^{-2}	-5662.26	-38.30
7	0.47×10^{-2}	-5775.60	-36.18
8	0.49×10^{-2}	-5881.88	-33.92
9	0.52×10^{-2}	-5979.68	-31.57
10	0.56×10^{-2}	-6067.16	-29.13

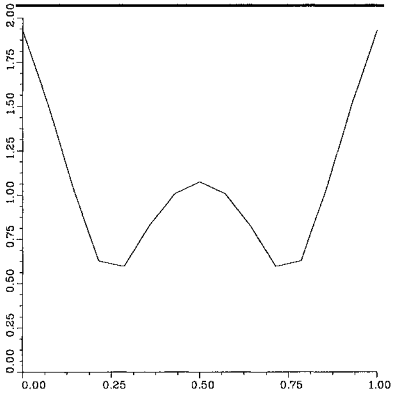

Figure 1 The cross-sectional area of the beam at the final solution.

This classification gives $\hat{z}^4 = (0.56 \times 10^{-2}, -6067.16, -29.13)$. Naturally, the value of the first objective function is too high and we consider ten alternatives listed in Table 2.

We select the second alternative as the final solution, that is, $z^5 = (0.39\times10^{-2}, -5158.13, -45.09)$. It is Pareto optimal. The cross-sectional area of the beam at the final solution is presented in Figure 1.

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