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On the superposition of a number of CDV affected cell streams

Sándor Molnár[1], Søren Blaabjerg[2] and Henning Christiansen[3]

[1]Department of Telecommunications and Telematics, Technical University of Budapest, Sztoczek u. 2, H-1111 Budapest, Hungary (e-mail: molnar@bme-tel.ttt.bme.hu)

[2]Ellemtel, Scheelevägen 17, S-223 70 Lund, Sweden

[3]Institute of Circuit Theory and Telecommunications, Technical University of Denmark, DK-2800 Lyngby, Denmark

Abstract

This paper considers the superposition of a number of Constant Bit Rate (CBR) cell streams which have been exposed to Cell Delay Variation (CDV). The case in which the CBR cell streams are affected by CDV in a single multiplexer is characterized by a diffusion model. Another and simpler model is used to characterize the effect of CDV after a number of multiplexing stages.

Queueing performance in which a number of such streams are superposed is obtained from the Beneš Result, and numerical examples supported by simulations are included for illustration.

Keywords

Cell Delay Variation, diffusion approximation, Beneš result

1 INTRODUCTION

The progress within the ATM technology has been so rapid and profound that no doubt exists whether ATM is going to be implemented or not. In the first applications, ATM will be used to realize virtual leased lines between large business customers, and some network providers will for strategic reasons probably introduce ATM and carry PSTN traffic by circuit emulation in their transit networks in order to be better prepared to support the large variety of services which ATM has the potential to support. It has also turned out that the ATM technology is very well suited for building LAN and for interconnection of LAN's [Mat93].

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Whether the applications for the ATM network is circuit emulation of 2 Mbit/s PSTN in a public transit network or it is interconnection of LAN's, constant bitrate services with strict requirements on cell delay variation will need to be supported. When the main application is data communication, the congestion control will probably be adaptive rate or window control and the ATM switches will be equipped with large buffers [DDJR91]. In order to guarantee the CBR traffic a satisfactory quality of service, delay priorities will need to be introduced, and the non-preemptive head of line polling mechanism is an efficient way to guarantee best possible delay performance to the high priority traffic [Kle76]. A further advantage with this head of line polling mechanism is that the delay performance of the high priority traffic can be accurately estimated by assuming that no low priority traffic is present. This is due to the short ATM cell size and the high speed of the links.

For many constant bitrate services like voice and circuit emulation, a major performance degradation is possible from variations in the delay of individual cells belonging to the same connection. This phenomenon called *Cell Delay Variation* has been a major issue for the standardization bodies, see [ITU] and ATM Forum. In the literature many contributions can also be found, see [Gro91], [Bla93], [Hue94] and [COSTI]. So far, most efforts have been made in describing a single CDV affected cell stream [Bla93], [BMS93], in dimensioning the UPC-algorithm (leaky bucket) [Hue94], [GBDR92], [CTF92], and deriving the effect CDV has on peak rate enforcement [GRo91], [Skl94].

The objective of this paper is to present a tractable analysis which gives accurate results for the queue length distribution of an ATM multiplexer receiving the data flow of a number of CDV affected CBR connections taking into account the correlation structure of the individual CDV affected streams. As we have just pointed out then this analysis applies both in a public ATM network with a traditional preventive congestion control as well as in a LAN environment in which the delay sensitive CBR traffic is protected by means of the head of the line delay priority mechanism.

In section 2 we present in short form a diffusion model by which it is possible to compute the number of departures[1] from a CDV affected CBR cell stream in a time interval both when the interval starts at an arrival and when the interval starts at an arbitrary point in time. In section 3 we consider a superposition of such streams and use the Beneš Result to get an accurate approximation for the queue length distribution. Thereby we avoid the usual approach where the complicated process under investigation is approximated by a renewal process. The approach is illustrated by numerical examples and simulations. In section 4 the CDV affected CBR cell stream is investigated after it has passed through a number of queues with interfering traffic. The queueing performance of a superposition of such streams is then investigated through the Beneš Result. Also here numerical examples and simulations are used to illustrate the approach.

1. When cells are entering a multiplexer with interfering traffic we call them arrivals and when they are leaving the multiplexer we call them departures even if they are arrivals to the next queue.

2 CHARACTERISTICS OF A CBR CELL STREAM AFTER PASSAGE OF A SINGLE MULTIPLEXING STAGE

In order to characterize the effect CDV has when a CBR stream is multiplexed with a background stream, we restrict our attention to a FIFO queue with deterministic service time receiving information from two cell arrival processes, a CBR cell arrival process with constant inter-arrival time T, and a background cell arrival process with arrival intensity λ (Figure 1). The time unit is chosen as the service time of one cell, and the load on the FIFO queue is $\rho = \lambda + 1/T$.

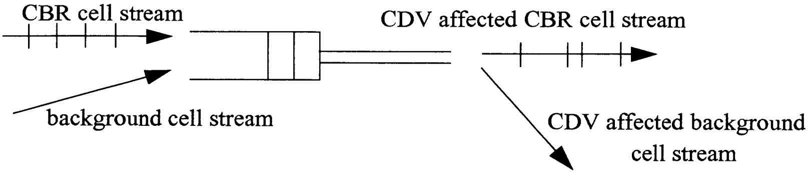

Figure 1 The FIFO model and the CDV affected cell stream.

Choose the time such that cell no. n is transmitted at time nT from the source.

Let W_n denote the waiting time of cell no. n. The sequence W_n is assumed stationary and the dependence between successive waiting times of CBR cells is assumed Markovian.

Define $\tau_n = nT + W_n$. Thus τ_n denotes the departure time[1] of cell no. n (See Figure 2). Also define the shifted interdeparture time of cell no. n as: $U_n = \tau_n - nT - \tau_0$.

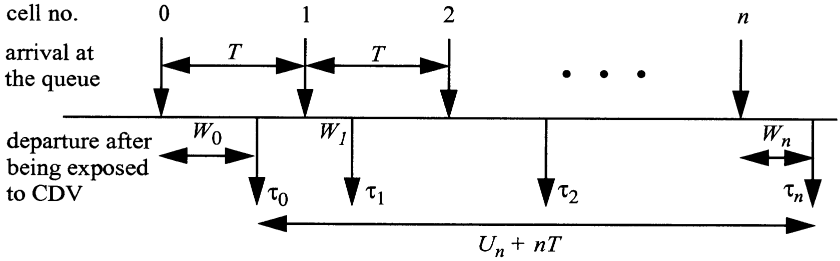

Figure 2 The shifted interdeparture time is to be seen as the difference between the actual departure time of cell n (τ_n) and the expected departure time ($\tau_0 + nT$).

In section 2.1 the case in which the load is smaller than 1 is considered and a diffusion analysis to characterize the CDV affected CBR stream after passage of the queue is applied. In section 2.2 the load of the multiplex is 1, and it is shown that in this case the CDV affected stream is well approximated by a renewal process and the distribution of the interdeparture time is derived.

1. Strictly speaking, τ_n is the time at which cell n starts service i.e. one time unit before it leaves the queue.

2.1 The diffusion approximation

It is possible by assuming a Markovian dependence between successive waiting times of the CBR cells at the multiplexer to derive exact results for most relevant quantities of interest, see [GRo91], but this approach is computationally demanding and the numerical complexity increases without bounds when the load of the multiplexer approaches 1 (see [Bla93]). The main difficulty comes from the fact the transition matrix describing the waiting time dependencies has a complex expression which to our knowledge can only be dealt with by numerical tools. To avoid these problems we shall apply a diffusion approach by which closed form expressions can be derived.

2.1.1 Virtual waiting time behaviour described by diffusion

The key idea in applying a diffusion model is to model the evolution of the queue length (or virtual waiting time) between CBR arrivals by a reflected Brownian motion.

Let $\tilde{W}_t$ denote the waiting time a fictitious observer would experience if he joined the diffusion queue at time t (the virtual waiting time at time t). The probability of $\tilde{W}_t \le x$ conditioned on $\tilde{W}_0 = y$ is, for a Brownian motion with drift m (assumed smaller than zero in order to ensure a stable queue), variance σ^2 and a reflection in zero, in section 2.8 in [Kle76] derived to be:

$$P\{\tilde{W}_t \le x | \tilde{W}_0 = y\} = \begin{cases} \Phi\left(\frac{x-y-mt}{\sigma\sqrt{t}}\right) - e^{2\frac{m}{\sigma^2}x}\,\Phi\left(\frac{-x-y-mt}{\sigma\sqrt{t}}\right) & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases} \tag{1}$$

where Φ denotes the standard Gaussian probability distribution.

The right hand side of (1) is a distribution function in x for all $y \ge 0$ and all $t > 0$, and it converges the exponential distribution with mean $-\sigma^2/2m$, independent of initial condition y, when t tends to infinity.

Both the arrivals of the CBR cells as well as the cells from the background process are modelled in accordance with the diffusion approach, i.e. the queue length is not increased by one at each CBR cell arrival. Instead these arrivals as well as background arrivals and departures are taken into account by a proper choice of drift and variance in the diffusion process as shown next.

The waiting time of CBR cell no. n, W_n is approximated by:

$$W_n = \tilde{W}_{nT} \tag{2}$$

that is *the waiting time that CBR cell no. n experience in the queue is approximated by the virtual waiting time in the diffusion queue at time nT.*

A disadvantage with formula (1) is that there is a positive probability that the waiting time between time t and time $t + u$ decreases more than u which of course in the original M+D/D/1 queue is impossible. If t is the time of CBR arrival no. n and $u = T$ then this would imply that CBR cell no. $n + 1$ would depart from the queue before cell no. n. This weakness is inherent to the model and therefore the model should only be used for values of T for which this probability is low, implying that T cannot be too small.

When the background traffic is Poissonian, the asymptotic behaviour for large queue length have been worked out in [BSi87]. They found that the distribution of the number of cells in the queue just prior to a CBR arrival has a geometric tail determined by the root z_∞ of $z^{T-1} = e^{\left(\rho - \frac{1}{T}\right)T(z-1)}$ outside the unit disk with smallest module. In our case with positive probability of a single arrival, [BSi87] shows that this root is unique, real and greater than one. The asymptotic slope of the waiting time therefore should be exponential with slope $s_\infty = ln(z_\infty)$.

By choosing the variance parameter σ^2 in the diffusion process properly, we are able to match the decay rates of the stationary waiting time distribution in the diffusion approximation with the correct one of the M+D/D/1 queue.

Since the diffusion decay rate is $-2m/\sigma^2$ we get:

$$\sigma^2 = -\frac{2m}{ln(z_\infty)} \qquad \text{and} \qquad m = \rho - 1 \tag{3}$$

where we as drift m have taken the same as in the M/D/1 queue.

If the background traffic is described by the Discrete time Markovian Arrival Process (DMAP) the superposition with the CBR source is also a DMAP and since the asymptotic decay rate of the DMAP/D/1 queue is found by computing the dominant root of a specific determinant function which involves the Perron Frobenious eigenvalue, see [GCa92] for details, the diffusion approximation is also applicable in this case only with the variance parameter changed. It should, however, be noted that the underlying Markovian assumption is no longer valid and that the accuracy of the diffusion process in this case therefore might suffer.

2.1.2 The probability distribution of the shifted interdeparture time

The shifted interdeparture time in the diffusion context is: $\tilde{U}_t = \tilde{W}_t - \tilde{W}_0$ (think of $t = nT$), and as shown in [Bla93] it is given as

$$\tilde{F}_t(x) = P\{\tilde{U}_t \le x\} = \int_0^{\infty} P\{\tilde{W}_t \le x+y \mid \tilde{W}_0 = y\}\, dP\{\tilde{W}_0 \le y\}$$

$$= \begin{cases} \dfrac{1}{2} + \dfrac{1}{2}\Phi\left(\dfrac{x-mt}{\sigma\sqrt{t}}\right) - \dfrac{1}{2}e^{\frac{2m}{\sigma^2}x}\,\Phi\left(-\dfrac{x+mt}{\sigma\sqrt{t}}\right) & \text{for } x>0 \\ \dfrac{1}{2}e^{-\frac{2m}{\sigma^2}x}\,\Phi\left(\dfrac{x-mt}{\sigma\sqrt{t}}\right) + \dfrac{1}{2}\Phi\left(\dfrac{x+mt}{\sigma\sqrt{t}}\right) & \text{for } x<0 \end{cases} \tag{4}$$

The interdeparture time distribution between cell no. k and $k+n$ is trivially obtained from the shifted interdeparture time distribution as

$$F_n(x) = P\{\tau_{k+n} - \tau_k \le x\} = P\{\tau_n - \tau_0 \le x\} = P\{\tilde{U}_{nT} \le x - nT\} = \tilde{F}_{nT}(x-nT)\,. \tag{5}$$

2.1.3 The number of departures in a window starting just after a departure

Consider an interval of the form $]\tau_j, \tau_j+t]$ starting just after the departure of cell no. j. Then

$$P\{N(\tau_j, \tau_j + t) \ge n\} = P\{\tilde{U}_{nT} + nT \le t\} = \tilde{F}_{nT}(t-nT) =$$

$$\begin{cases} \dfrac{1}{2} + \dfrac{1}{2}\Phi\left(\dfrac{t}{\sigma\sqrt{nT}} - \dfrac{(1+m)}{\sigma}\sqrt{nT}\right) - \dfrac{1}{2}e^{\frac{2m}{\sigma^2}(t-nT)}\,\Phi\left(-\dfrac{t}{\sigma\sqrt{nT}} + \dfrac{(1-m)}{\sigma}\sqrt{nT}\right) & \text{for } t>nT \\ \dfrac{1}{2}e^{-\frac{2m}{\sigma^2}(t-nT)}\,\Phi\left(\dfrac{t}{\sigma\sqrt{nT}} - \dfrac{(1+m)}{\sigma}\sqrt{nT}\right) + \dfrac{1}{2}\Phi\left(\dfrac{t}{\sigma\sqrt{nT}} - \dfrac{(1-m)}{\sigma}\sqrt{nT}\right) & \text{for } t<nT \end{cases} \tag{6}$$

by (4) where $N(\tau_j, \tau_j + t)$ denotes the number of departures in $]\tau_j, \tau_j+t]$.

2.1.4 The number of departures in an arbitrary window

Let t_0 denote an arbitrary point in time, and consider the interval $]t_0, t_0+t]$. According to a basic result in point process theory (see e.g. section 4.2 in [CLe66]) then

$$P\{N(t_0, t_0 + t) \ge n\} = P\{Y + X_2 + .. + X_n \le t\} = \frac{1}{T}\int_0^t (F_{n-1}(u) - F_n(u))\, du \tag{7}$$

in which Y denote the forward recurrence time, X_i denote the interdeparture time between cell no. i - 1 and i, $F_i(u)$ denotes the distribution function of $\sum_{j=1}^{i} X_j$, and $N(t_0, t_0+t)$ denotes the number of departures in $]t_0, t_0+t]$. Since $F_n(u) = P\{\tilde{U}_{nT} \le u - nT\} = \tilde{F}_{nT}(u - nT)$ is given in closed form in (6) we may obtain the probability distribution of the number of departures in an arbitrary window by integrating (6) with respect to t.

The result can be formulated as:

$$P\{N(t_0, t_0+t) \ge n\} = G_{n-1}(t) - G_n(t) \tag{8}$$

in which $G_n(t)$ for $t < nT$ is given as

$$\begin{aligned} G_n(t) = {} & \frac{\sigma^2}{(-4m)}\frac{1}{T}\Big\{ e^{-\frac{2m}{\sigma^2}(t-nT)}\,\Phi\Big(\frac{t}{\sigma\sqrt{nT}} - \frac{1+m}{\sigma}\sqrt{nT}\Big) - e^{\frac{2m}{\sigma^2}nT}\,\Phi\Big(-\frac{1+m}{\sigma}\sqrt{nT}\Big) \\ & - \Phi\Big(\frac{t}{\sigma\sqrt{nT}} - \frac{1-m}{\sigma}\sqrt{nT}\Big) + \Phi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big)\Big\} \\ & + \frac{1}{2T}\Big\{ (t-(1-m)nT)\,\Phi\Big(\frac{t}{\sigma\sqrt{nT}} - \frac{1-m}{\sigma}\sqrt{nT}\Big) + (1-m)nT\Phi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big) \\ & + \sigma\sqrt{nT}\varphi\Big(\frac{t}{\sigma\sqrt{nT}} - \frac{1-m}{\sigma}\sqrt{nT}\Big) - \sigma\sqrt{nT}\varphi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big)\Big\} \end{aligned} \tag{9.a}$$

and for $t > nT$, $G_n(t)$ is given as:

$$\begin{aligned} G_n(t) = {} & \frac{\sigma^2}{(-4m)}\frac{1}{T}\Big\{ e^{\frac{2m}{\sigma^2}(t-nT)}\,\Phi\Big(\frac{-t}{\sigma\sqrt{nT}} + \frac{1-m}{\sigma}\sqrt{nT}\Big) - e^{\frac{2m}{\sigma^2}nT}\,\Phi\Big(-\frac{1+m}{\sigma}\sqrt{nT}\Big) \\ & - \Phi\Big(\frac{-t}{\sigma\sqrt{nT}} + \frac{1+m}{\sigma}\sqrt{nT}\Big) + \Phi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big)\Big\} \\ & + \frac{1}{2T}\Big\{ ((1+m)nT - t)\,\Phi\Big(\frac{-t}{\sigma\sqrt{nT}} + \frac{1+m}{\sigma}\sqrt{nT}\Big) + (1-m)nT\Phi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big) \\ & + \sigma\sqrt{nT}\varphi\Big(\frac{-t}{\sigma\sqrt{nT}} + \frac{1+m}{\sigma}\sqrt{nT}\Big) - \sigma\sqrt{nT}\varphi\Big(-\frac{1-m}{\sigma}\sqrt{nT}\Big)\Big\} + \frac{t-nT}{T} \end{aligned} \tag{9.b}$$

where we have utilized the symmetry relation $\tilde{F}_t(x) + \tilde{F}_t(-x) = 1$ to obtain (9.b).

2.1.5 Index of dispersions for intervals

A commonly used method for studying second order properties of point processes is the index of dispersions either for interval or counts, see e.g. [SWi86]. In this context it is most convenient to work with the index of dispersions for interval, and it is defined as:

$$c_n^2 = \frac{n Var(\tau_n - \tau_0)}{E^2(\tau_n - \tau_0)} = \frac{Var(\tilde{U}_{nT})}{nT^2} \tag{10}$$

For a renewal process the index of dispersion for intervals (IDI) is independent of the n-parameter, and the deviation which the IDI of a given point process has from a horizontal line gives an indication of how far it is from a renewal process. From the diffusion approach it is possible to compute the index of dispersion. It gives

$$c_n^2 = \frac{\frac{\sigma^4}{2m^2}\left(2\Phi\left(-\frac{m}{\sigma}\sqrt{nT}\right) - 1\right) + (2\sigma^2 nT + m^2 n^2 T^2)\,\Phi\left(\frac{m}{\sigma}\sqrt{nT}\right) + \left(\frac{\sigma^3}{m}\sqrt{nT} + m\sigma(nT)^{\frac{3}{2}}\right)\varphi\left(\frac{m}{\sigma}\sqrt{nT}\right)}{nT^2} \tag{11}$$

2.2 When the load on the multiplex is one

Now, consider the case that the load of the multiplex is exactly 1. This case has been considered by both [BMS93] and [Kel93]. Here we shall apply the arguments from [Kel93]. The probability that the queue is empty can be neglected, and we get that the difference in waiting time between CBR cell no. i - 1 and CBR cell no i is

$$W_i - W_{i-1} = 1 + N_{\text{background}}(T(i-1), Ti) - T \tag{12}$$

where $N_{\text{background}}(s, t)$ denotes the number of arriving cells from the background stream in the time interval $]s,t]$

If the background stream constitutes a Poisson process, then $N_{\text{background}}(s, t)$ is a Poisson distribution with parameter $\lambda(t - s)$. From (12) we therefore derive the result that *the CDV affected CBR stream constitutes a renewal process with interdeparture time distribution* $1 + P(\lambda T) = 1 + P((1 - 1/T)T) = 1 + P(T - 1)$ where P denotes the Poisson distribution. The independence between different interdeparture times comes from the memoryless property of the exponential distribution in the Poisson process. The result implies that the squared coefficient of variation of the CDV affected stream is $(T-1)/T^2$.

If the background stream constitutes a renewal process with interarrival time distribution f, the CDV affected cell stream will no longer be a renewal process. The complication arises from the fact that the time from the CBR arrival until the first background arrival depends on the interarrival time between the previous background arrival and the present CBR arrival. However, if T is large, the number of background arrivals between two CBR arrivals will be large and since only the first interarrival interval has a distribution which differs from f, it will be an accurate approximation simply to use the equilibrium forward recurrence time independent of the previous background arrival.

Therefore, when the background process is a renewal process, when T is large, and when the load on the multiplex is 1, we may approximate the CDV affected cell stream with a renewal process with interdeparture time distribution $1 + N_{\text{background}}(T(i-1), Ti)$. Here, it should be noticed that it is in general difficult to obtain a closed form expression for $N_{\text{background}}(s, t)$.

By Taylor expanding the normal distribution function around 0 in formula (11) and utilizing the fact that $\sigma^2(\rho) \to 1 - 1/T$ when $\rho \to 1$, it is possible to prove that the IDI in the diffusion model approaches the correct value $\frac{T-1}{T^2}$ for all n, thus showing that the asymptotic matching is sufficiently powerful to ensure a correct IDI for any value of T.

3 THE SUPERPOSITION OF CDV AFFECTED CBR CELL STREAMS IN A SINGLE MULTIPLEXING STAGE

In this section we consider a multiplexer offered traffic from a number of CBR cell streams which prior to the arrival to the multiplexer have been exposed to CDV in a single multiplexing stage (see Figure 3).

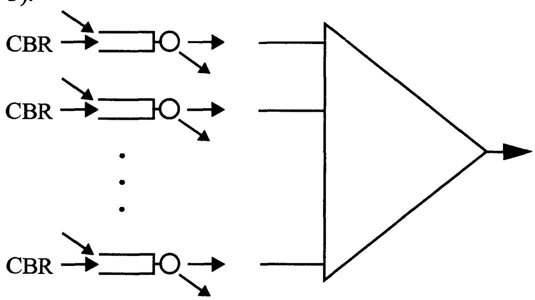

Figure 3 Superposition of CDV affected CBR cell streams.

For queues with constant service times it has turned out that the so-called Beneš Result, see [COST] section 5.3, is a very powerful tool to obtain either the exact queue length distribution or at least a tight approximation. The Beneš Result valid for stable queues ($\rho < 1$) offered a general stationary arrival process is:

$$P\{W_t > r\} = \sum_{n=1}^{\infty} P\{N(t-n, t) = n + r\} P\{W_{t-n} = 0 \mid N(t-n, t) = n + r\} \tag{13}$$

where W_t is the virtual waiting time at time t. The difficult term in the above expression is $P\{W_{t-n} = 0 | N(t-n, t) = n+r\}$. Applying the so-called "local load approximation", see section 5.3.2 in [COST] for details, we end up with:

$$P\{W_t > r\} \cong \sum_{n=1}^{\infty} P\{N(t-n, t) = n+r\} - \rho \sum_{n=1}^{\infty} P\{N(t-n, t) = n+r | \text{an arrival at } t-n\} \quad (14)$$

i.e. an expression containing only terms related to the arrival process.

In section 2, expressions for the number of departures in a window from a single CDV affected CBR cell stream was derived. Now a finite number of independent CDV affected CBR cell streams are multiplexed in the buffer. Therefore the distribution of the number of departures from the superposition can be found from a convolution of the individual distributions. The number of departures from a single stream is given in formula (6) and (8) in section 2. An approximation for the virtual waiting time distribution can now be obtained from (14).

3.1 Numerical results

To illustrate the approach we present two examples. In the first example 8 CBR sources are exposed to CDV in a single multiplexing stage (see Figure 3) and after that the 8 streams are offered to a FIFO with a load of 0.8 implying that the peak rate of the CBR sources are 0.1 corresponding to $T = 10$.

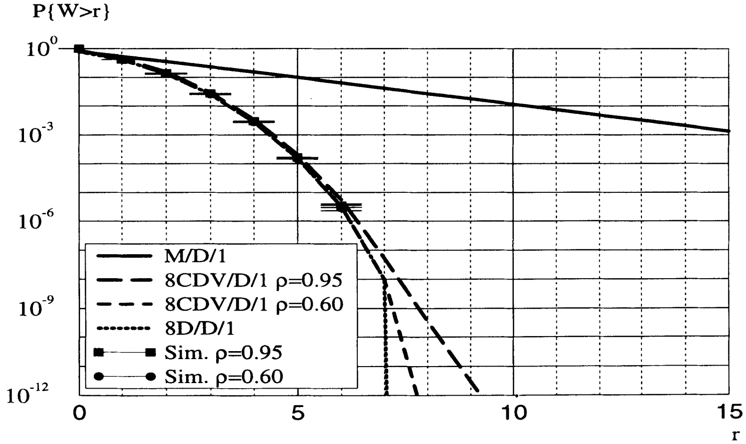

Figure 4 Comparison of the virtual waiting time distributions of 8CDV/D/1 queues (load=0.8).

The amount of CDV is controlled by the load of the interfering traffic on the CDV creating queues as described in section 2. For a load of 0.6 in the CDV creating queues the queueing result differs very little from the result of the 8D/D/1 queue while for a CDV creating load of 0.95 the difference become slightly larger. However, the delay performance is still much better than for the M/D/1 case.

A simulation has also been carried out and the analytically computed values fall within the confidence interval[1] over the entire range for which simulation is feasible.

In the second example 16 CDV affected CBR sources are multiplexed in a FIFO with a load of 0.8 implying $T = 20$.

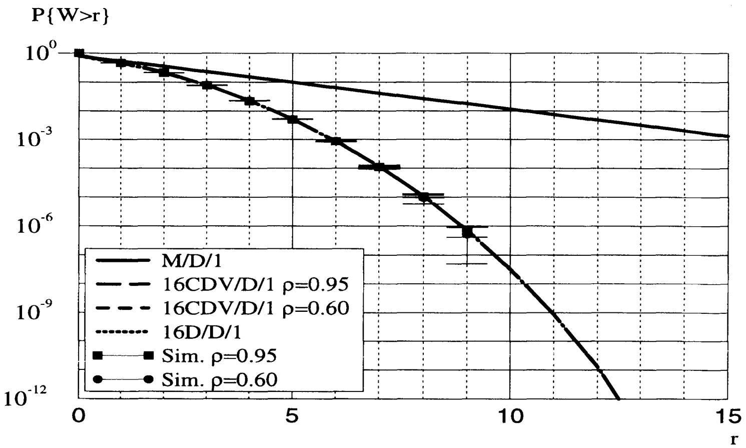

Figure 5 Comparison of the virtual waiting time distributions of 16CDV/D/1 queues (load=0.8). Notice that the 16D/D/1 curve and both 16CDV/D/1 curves visually coincide.

For this case the results are practically identical to the results for the 16D/D/1 queue even in the case with a CDV creating load of 0.95. For more numerical results see [MBl94]. Again simulations have been carried out and they support the analytical results.

An overall conclusion is that, *except for the case with multiplexing a very few high speed CBR cell streams, the effect of CDV in a single multiplexing queue is neglectable.*

1. Throughout the paper the simulation results are given with 95% confidence intervals.

4 CDV CHARACTERISTICS AFTER A SERIES OF MULTIPLEXING STAGES

The rather detailed diffusion model for characterizing the CDV after passage of a single queue becomes inappropriate when coming to the analysis of CDV after a series of queues. Instead a simpler model is needed. Here we shall give a brief description of the model presented in [BRe92], and apply the results to obtain the queueing performance of a number of CDV affected CBR cell streams.

Consider a discrete time queueing model consisting of M ATM queues in series. There are two types of arrivals to the system. First we have a reference connection entering the first queue and passing successively through all M queues. Secondly, for each queue k we have an interfering traffic entering queue k and immediately after service completion in queue k leaves the system. (See Figure 6.)

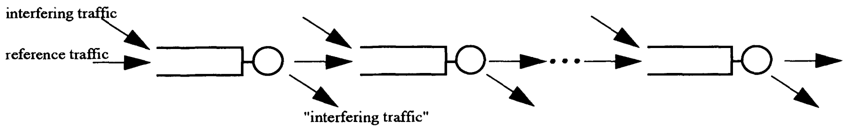

Figure 6 Queues in series with interfering traffic.

Let $\{X_n^{k-1}\}_n$ for $k = 1, ..., M$ be the sequence of interarrival times of the reference traffic to queue k. X_n^{k-1} is the interarrival time between reference cell no. n and $n + 1$ to queue k.

The interfering traffic is modelled as a batch Bernoulli process, that is the number of arrivals in successive times slots are independent identically distributed with common distribution f and generating function F. For simplicity it is assumed that the interfering traffic statistically is the same at each queue. Y_i^k denotes the number of interfering arrivals in timeslot i at queue k.

The output process of the reference connection from queue k $\{X_n^k\}_n$ is a function of the input processes $\{X_n^{k-1}\}_n$ and $\{Y_n^k\}_n$. However, to arrive at a tractable model, the following heavy traffic assumption will be imposed:

Heavy traffic assumption:

At each point in time the k'th queue is non-empty.

Under this assumption all slots between two adjacent reference cell arrivals consists of interfering cells, implying that

$$X_n^k = 1 + \sum_{i=1}^{X_n^{k-1}} Y_i^k \tag{15}$$

where for $i = 1, \ldots, X_n^{k-1}$ Y_i^k is the number of interfering arrivals in slot i. To arrive at (15) we have also assumed that the reference connection has priority over the interfering cells. A similar expression would be valid if the interfering cells had priority over the reference connection, see [Bre92] for more details.

(15) shows that the process $\{X_n^k, k = 0,..,M\}$ is a branching process with immigration. Furthermore we get

$$P_k(s) = sP_{k-1}(F(s)) \tag{16}$$

where $P_k(s)$ denotes the generating function of X_n^k.

From the general theory of branching processes it is possible to arrive at the following results:

Result 1: (Limit interdeparture time distribution - infinitely many queues)

Assume that $E(Y_i^k) < 1$. When $k \to \infty$, then $X_n^k \to X_n^\infty$ in distribution where the generating function of X_n^∞ satisfy $P(s) = \prod_{j=1}^{\infty} F_j(s)$ in which $F_j(s)$ is recursively defined by $F_1(s) = s$ and $F_j(s) = F(F_{j-1}(s))$.

Result 2: (Asymptotically a renewal process)

Let X_n^0 and X_{n+p}^0 be two possibly dependent interarrival times. Define:

$$(X_n^{k+1}, X_{n+p}^{k+1}) = \left(1 + \sum_{i=1}^{X_n^k} Y_i^k, 1 + \sum_{i=1}^{X_{n+p}^k} Y_i^{k,p}\right) \tag{17}$$

where Y_i^k and $Y_i^{k,p}$ are independent of each other and satisfying the assumptions of Result 1. Then

$$(X_n^k, X_{n+p}^k) \to (X_n^\infty, X_{n+p}^\infty) \tag{18}$$

in distribution, where $X_n^\infty, X_{n+p}^\infty$ are independent.

From these two results two important conclusions can be drawn:

1 When the reference cell stream has passed a sufficient number of queues the interdeparture time distribution has converged towards a limit distribution which only depends on the interfering traffic and not on the original characteristics of the reference stream except the rate.

2 When the reference stream has passed a sufficient amount of queues it becomes a renewal stream.

Section 6 in [BRe92] argues by an interpolation argument between the light traffic case (no interfering traffic) and the heavy traffic case that the Result 1 and 2 should also hold in moderate traffic with only the speed of the convergence decreased.

We now take a closer look on how the squared coefficient of variation of the interdeparture time changes as the cell stream passes through the network.

From (16) we get

$$E(X_n^{k+1}) = E(Y_i)(1+E(X_n^k)) \text{ and } Var(X_n^{k+1}) = (1+E(X_n^k))Var(Y_i) + E^2(Y_i)Var(X_n^k) \tag{19}$$

Consider the case where the reference connection is a CBR connection with rate $1/T = 1 - p$ and $Var(X_n^0) = 0$. Assume that the load on each of the queues are 1, and assume that the variance of the interfering stream $Var(Y_i) = p$ corresponding to Poisson traffic. From (19) it can be seen that

$$Var(X_n^k) = \frac{p}{1-p}\sum_{j=0}^{k-1} p^{2j} = \frac{p(1-p^{2k})}{(1-p)(1-p^2)} \tag{20}$$

For the limit case in which the number of queues to be passed is infinite and in the case with Poissonian interference, the CDV affected CBR cell stream will have a squared coefficient of variation

$$c^2 = \frac{Var(X_n^\infty)}{E^2(X_n^\infty)} = \frac{\frac{p}{(1-p)(1-p^2)}}{\frac{1}{(1-p)^2}} = \frac{p}{(1+p)} \tag{21}$$

An important implication of (21) is that *the squared coefficient of variation in the Poisson interfering case never exceeds 1/2.*

Consider the case in which the reference stream has interarrival time $T = 10$, corresponding to $p = 0.9$. Then Table 1 shows how the squared coefficient of variation of the reference stream varies as it passes through the queues of the network.

Table 1: The squared coefficient of variation as a function of the number of queues

c^2 ($T = 10$)	$M = 0$	$M = 1$	$M = 2$	$M = 4$	$M = 6$	$M = 10$	$M = 25$
computed	0	0.090	0.163	0.270	0.340	0.416	0.471
simulation	0	0.088	0.159	0.264	0.333	0.412	0.470

For the case $T = 20$, corresponding to $p = 0.95$ we also computed the values of the squared coefficient of variation of the reference stream as it travels through the queues of the network and the results were also verified by simulation as shown in Table 2.

Table 2: The squared coefficient of variation as a function of the number of queues

c^2 ($T = 20$)	$M = 0$	$M = 1$	$M = 2$	$M = 4$	$M = 6$	$M = 10$	$M = 25$
computed	0	0.048	0.090	0.164	0.224	0.313	0.450
simulation	0	0.047	0.089	0.162	0.220	0.308	0.444

Confidence intervals are not included in the Tables but ranges from ±0.001 to ±0.005 for all M.

As the Tables show then the analytical approach is in accordance with the simulation results for the investigated cases with load of 1. When the load is less than one, the increase in squared coefficient of variation as a function of the number of queues is slower.

For $M = 1$, the squared coefficient of variation is $\frac{p/(1-p)}{T^2} = \frac{(1-1/T)/(1/T)}{T^2} = \frac{T-1}{T^2}$ which is exactly the squared coefficient of variation of the CDV interdeparture time distribution given in section 2.2.

One outcome of the present approach is that the CDV affected cell stream should approach a renewal stream as the number of queues tends to infinity. A simulation was carried out to illustrate this effect. In Figure 7 the index of dispersions for intervals (IDI) are shown for the case of $T = 10$. The IDI was measured after passing through $M = 1, 2, 4, 6, 10$ and 25 queues.

For the case with a load of 0.6 the IDI have the same shape for all values of M, the curves are simply shifted upwards when M increases. Therefore we may conclude that the CDV affected cell stream has not approached a renewal process even after 25 queues. For the case with a load of 0.95 the picture is the same. However, here the curves are much closer to a renewal even for $M = 1$. For modelling purposes it is therefore only justified to use a renewal approximation when the load is close to 1.

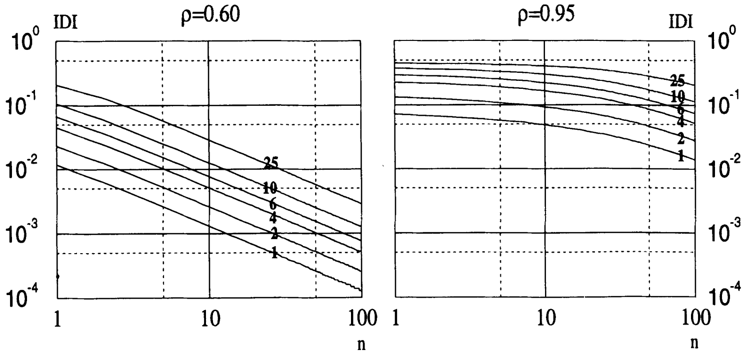

Figure 7 Index of dispersions for intervals (IDI) for two different loads.

5 THE SUPERPOSITION OF CDV AFFECTED CBR CELL STREAMS IN MANY MULTIPLEXING STAGES

Consider the superposition of N number of CBR cell streams which prior to the arrival to the multiplexer have been exposed to CDV in M number of multiplexing stages as shown in Figure 8. This queueing system is denoted by NCDVM/D/1.

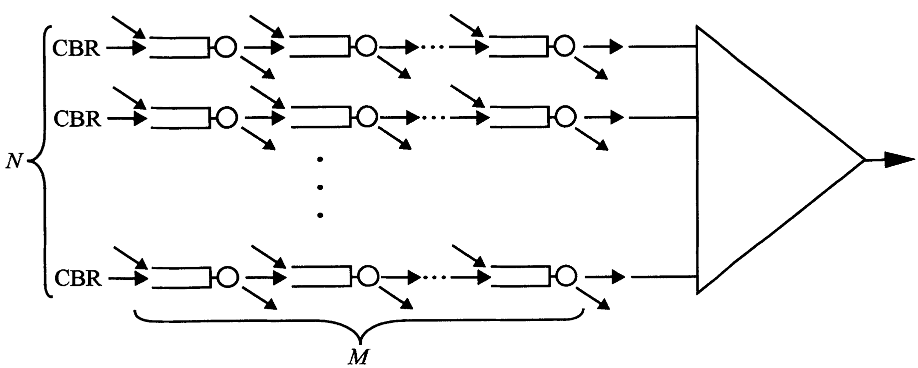

Figure 8 Superposition of CDV affected (after many multiplexing stages) CBR cell streams.

For queueing analysis the Beneš Result is used as in section 3, but the CDV affected cell streams are modelled as renewal processes with Erlang interarrival distributions chosen with the appropriate squared coefficient of variation which is obtained by the method presented in section 4.

5.1 Numerical results

As in section 3.1 we consider a scenario where 8 or 16 CBR cell streams, which have been exposed to CDV due to many multiplexing stages (all of them with load of 1), are multiplexed in a queue with a load of 0.8, see Figure 8.

From the values given in the Table 1 and 2 we approximate each CDV affected stream by renewal streams with an Erlang-n interarrival time distribution.

We have taken the values for n in case of $T = 10$ for $n = 11$ (corresponding to passage of a single queue), $n = 4$ (corresponding to passage of 4 queues), and $n = 2$ (corresponding to an infinite number of queues).

In case of $T = 20$ we have chosen $n = 21$ (corresponding to passage of a single queue), $n = 6$ (corresponding to passage of 4 queues), and $n = 2$ (corresponding to an infinite number of queues).

As in section 3, we have used the local load approximation of the Beneš Result for finding the virtual waiting time distribution.

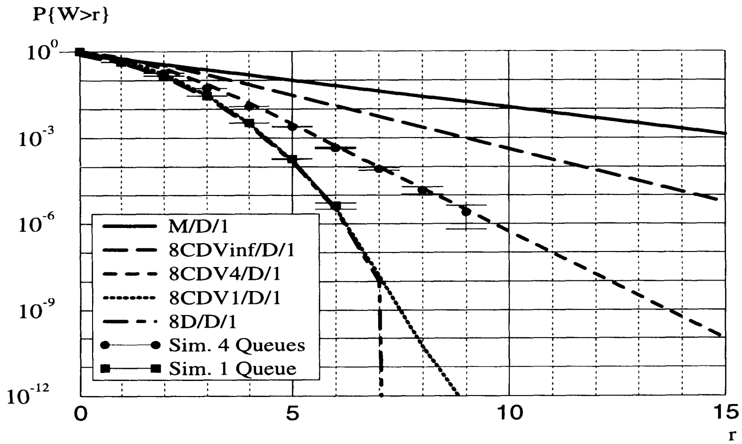

Figure 9 Comparison of the virtual waiting time distributions of 8CDV*M*/D/1 queues (load=0.8).

For the superposition of 8 and 16 cell streams we have simulated the cases where each CBR cell stream has passed through 1 queue and 4 queues respectively with interfering traffic. The analytical results are in agreement with the simulations.

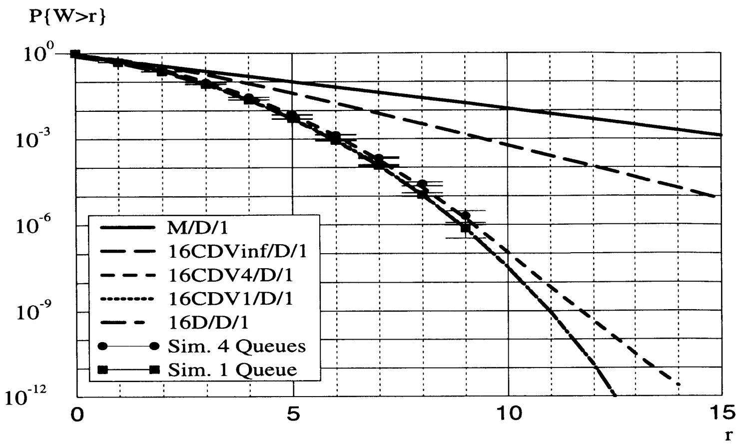

Figure 10 Comparison of the virtual waiting time distributions of 16CDV*M*/D/1 queues (load=0.8).

As the result shows then the effect of CDV becomes more and more apparent as the number of queues which the CBR streams has passed increases. It should also be noted that when *T* is large the effect of CDV becomes apparent only after passing through a considerable number of queues. For more numerical results see [MB194]. However, an important result of the model described in section 4 is that as long as we have Poissonian interference then the delay performance of the multiplexing of a number of CDV affected CBR cell streams will not be worse than the superposition of the same number of renewal processes with squared coefficients of variation 1/2.

6 CONCLUSION

In this paper we have investigated the effect of cell delay variation on the queueing performance when a queue is offered a superposition of CDV affected CBR cell streams. The approach has been verified by simulation. From the first and rather detailed model we conclude that the degree of CDV created in a single FIFO multiplexing stage has practically no influence on queueing performance at the next stage. From the second model which assumed heavy load in the interfering queues we conclude that in case of Poissonian interfering traffic the squared coefficient of variation of the individual cell streams will not exceed 1/2 implying that dimensioning approaches based on the simple M/D/1 model will be conservative.

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Sándor Molnár graduated in 1991 at the Technical University of Budapest, Faculty of Electrical Engineering, in Telecommunications. Since September 1991 he is working for his Ph.D. degree at the Department of Telecommunication and Telematics. His main interests include speech quality assessment, performance modeling and analysis of ATM networks and broadband telecommunications. Mr. Molnár is participating in the European project COST 242 on Methods for the Performance Evaluation and Design of Broadband Multiservice Networks.

Søren Blaabjerg received his Masters in Mathematics from Aarhus University in 1986. He was three years with Electromagnetics Institute of Technical University of Denmark working for RACE Atmospheric, and four years with Ellemtel Telecom System Labs in Lund Sweden doing performance analysis of ATM systems. In 1994 he returned to Technical University of Denmark working for the RACE Combine project. His interests includes mathematical modelling and performance analysis of Telecommunication networks.