

## Performance Study on ATM Adaptation Layer

Zhi Cui and Arne A. Nilsson

Center for Communications and Signal Processing,  
Department of Electrical and Computer Engineering, North Carolina State University,  
Raleigh, N.C. 27695-7914, U.S.A.

In an ATM network, the ATM Adaptation Layer(AAL) is used to support the connection between the ATM and non-ATM protocol layers. The AAL consists of two sub-layers: the Convergence Sub-layer(CS) and the Segmentation And Re-assembly Sub-layer(SAR). In the CS, a packet from higher layers is broken up into a number of sub-packets, which are further divided into ATM cells in the SAR sub-layer. The performance of the AAL is investigated through the analysis of a network of queues that is constructed based upon the functions of the AAL. Among those individual queues in this queueing network, we found the queue representing the SAR sublayer to be the most difficult and the most critical one to solve. The arrival process to this queue is assumed to be an  $IBP^{[x]}$  in order to capture the traffic burstiness property in ATM networks. In this paper, we focus on the analysis of this critical queue, with a more general queueing model,  $IBP^{[x]}/Geo/1/K$  queue. The mean waiting time, blocking probabilities, and the generating function of the interdeparture time distribution for this queue are presented. We also fit the departure process from this queue to a two-state MMBP. Using this MMBP as an arrival process of the downstream queue, the remaining queues in the network of queues can be easily analyzed.

Keyword Codes: C.2.2; C.4; I.6

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### 1. INTRODUCTION

The Asynchronous Transfer Mode (ATM) is a fast packet switching and multiplexing technique for broadband ISDN [1]. In an ATM network, all information ranging from narrowband voice and data traffic to broadband video traffic is transmitted in a fixed size "cell". Essential to the service offered by the new ATM networks is the ATM Adaptation Layer(AAL). As indicated in the ATM network protocol stack shown in Figure 1, the AAL, sitting above the ATM layer is used to support the connection between the ATM and non-ATM interfaces. The AAL consists of two sub-layers: the Convergence Sub-layer(CS) and the Segmentation And Re-assembly Sub-layer(SAR). In CS, the packet from higher layers is broken up into a number of sub-packets and CS Protocol Data Units(CS.PDUs) are formed. The CS.PDU is further broken up into 53 byte ATM cells, in the SAR sub-layer [2]. To study the performance of the AAL, a queueing network is

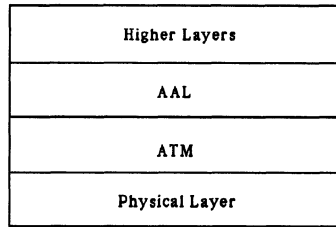


Figure 1. ATM Network Layers.

constructed based upon the functions of all ATM network layers, see Figure 2. At the source node, the queue SOURCE is used to generate the traffic of application packets from higher layers; the queue split1 is used to simulate the function of CS, which splits a packet from higher layers to several fixed size CS\_PDU:s; and the queue split2 is used to simulate the function of the SAR-sublayer, which converts the CS\_PDU to cells. Cells are transmitted through the ATM network. The network is assumed to have finite buffering capability and low cell loss probability. At the destination, the cells are reassembled into CS\_PDUs and packets at the reass1 and reass2 queue, respectively. In ATM networks, error control is performed on an end-to-end basis and retransmission should occur only at the CS\_PDU level. To evaluate the performance of this queueing network, the decomposition technique for analyzing large complex queueing system is used. The main idea of this technique is to decompose the queueing network into individual queues that will be analyzed independently and then recombined. This implies that it is very important to find accurate representations for the departure process from a queue since this process may be the arrival process to another queue.

In high speed networks, another important issue is the selection of a traffic model, because most of the traffic that an ATM network supports is highly bursty and the Poisson process (or in discrete time the Bernoulli process) is not a good choice for the traffic in such environments. In this paper, the packet arrival process to the split1 queue is modeled as an Interrupted Bernoulli Process (IBP), which captures the burstiness of the traffic. After the packet is segmented into several CS\_PDUs in the queue split1, the arrival process to the queue split2, which is the superposition of the bulk of CS\_PDU:s traffic and the retransmission traffic, can be approximated as an  $IBP^{[x]}$  [3], where the superscript [x] indicates bulk arrivals.

To simplify the queueing analysis, we study the network performance only at the CS\_PDU level. Thus, the constant service time for split2 queue, i.e., the time interval to transmit one CS\_PDU, is assumed to be the time slot.  $K$  is the finite buffer size of the queue split2. Therefore, the model that needs to be solved for the queue split2 is the  $IBP^{[x]}/D/1/K$

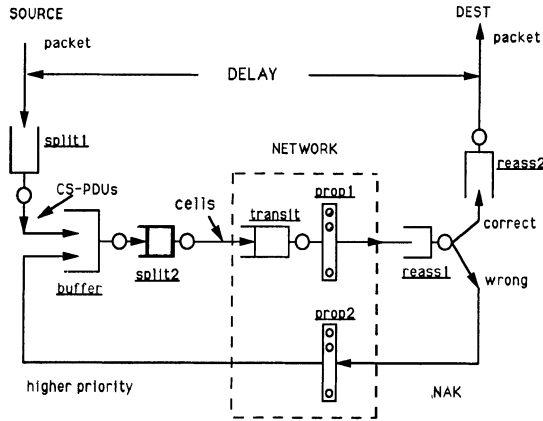


Figure 2. ATM Network Queueing Model.

queue, which is a special case of the  $IBP^{[z]}/Geo/1/K$  queue. Thus, in order to solve the queueing network by decomposition, solving the  $IBP^{[z]}/Geo/1/K$  queue becomes a critical step. Also, the departure process of this queue will be of importance for the analysis of the other queues. In this paper, we concentrate on the analysis of the  $IBP^{[z]}/Geo/1/K$  queue and its departure process.

The paper is organized as follows: In Section 2 we briefly describe the more general

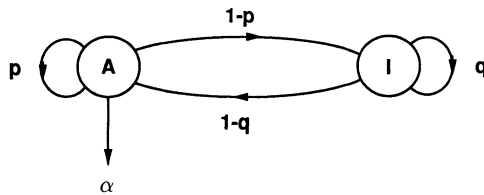


Figure 3. Markov chain of IBP.

queue,  $IBP^{[z]}/Geo/1/K$  queue. The queue length distributions and blocking probabilities

of both queues are obtained through a Markov chain analysis. In Section 3 the derivation of the generating function of the interdeparture time distribution for the  $IBP^{[x]}/Geo/1/K$  queue and the first four moments of the interdeparture time are presented. In Section 4 we report on the first four moments of the interarrival time distribution of an MMBP. The program "Interopt" [4] is then used to fit the departure process of the  $IBP^{[x]}/Geo/1/K$  queue to a two-state MMBP. It is verified that our fitting approach provides a satisfactory accuracy by comparing the four moments of the interdeparture time against the moments of the interarrival time in the MMBP. Using this MMBP as an arrival process of the downstream queue, the remaining queues can be easily analyzed.

## 2. THE QUEUE LENGTH DISTRIBUTIONS OF THE $IBP^{[x]}/GEO/1/K$ QUEUE

The IBP is a doubly stochastic Bernoulli process. It is governed by a discrete time Markov chain with two states, an active state and an idle state, shown in Fig. 3.  $IBP^{[x]}$  is an IBP with batch arrivals [5]. The batch size is assumed to be distributed geometrically, that is,  $g_n = (1 - p_1)p_1^{n-1}$ ,  $n \geq 1$ , where  $g_n = P[\text{batch size} = n]$ . We assume that this  $IBP^{[x]}$  process is the arrival process to a single server finite capacity queue with geometric service time with rate  $\sigma$ . In this system, if a bulk of sub-packet arrivals make the system full, only those sub-packets which find that the buffer is full are lost.

In order to obtain the steady state queue length distribution, we observe the system at the slot points and generate the embedded Markov chain at these points [6]. Since the process development is observed in the discrete time domain, several events (e.g., observing system, state change of IBP, arrival, and departure) can happen simultaneously. In this paper, the possible events are assumed to be processed in the following order: 1) departure, 2) observing, 3) state change, and 4) arrivals, as shown in Figure 4. In

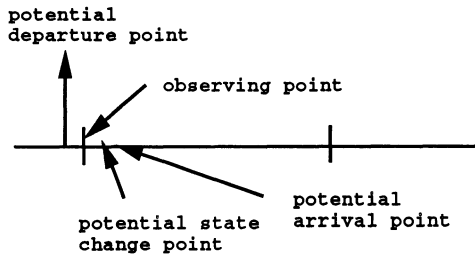


Figure 4. Potential Event Sequence.

the Markov chain, as shown in Figure 5, there are  $2(K+1)$  states denoted by  $(m,A)$  and

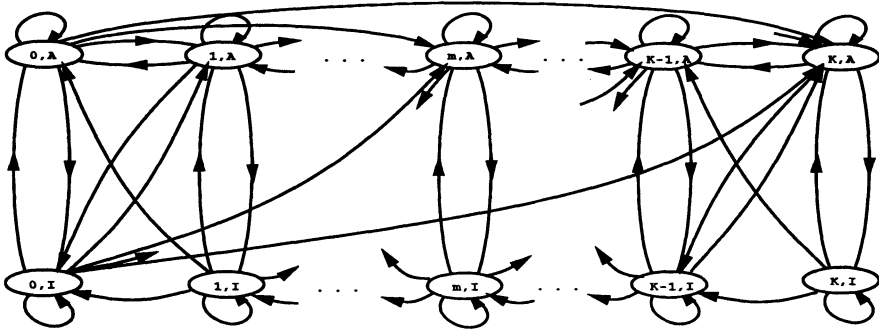


Figure 5. The Two dimensional Markov Chain of  $IBP^{[z]}/Geo/1/K$  Queue.

$(m,I)$  ( $m=0,1,\dots,K$ ), where  $(m,A)$  represents an active state of the arrival process with  $m$  sub-packets in the system;  $(m,I)$  represents an idle state of the arrival process with  $m$  sub-packets in the system. Note that  $K$  represents the total number of sub-packets permitted in this system, i.e., server and queue combined. The state changes in the Markov chain can only be caused by:

- 1) a state change between active and idle; or,
- 2) sub-packet batch arrivals; or,
- 3) sub-packet departure. Note that an arrival at the current slot cannot be served until the beginning of the next slot.

An example, see Figure 6, should provide some of the necessary details in terms of transition probabilities in the Markov chain.

By solving global balance equations of the Markov chain, the queue length distribution (i.e.,  $\text{Prob}[\text{number of sub-packets in the system} = n]$ ) of the  $IBP^{[z]}/Geo/1/K$  queue is obtained. Figure 7 shows the queue length distribution of the  $IBP^{[z]}/Geo/1/K$  queue with parameters  $p = 0.8$ ,  $q = 0.8$ ,  $\alpha = 0.8$ ,  $\sigma = 0.8$ , and  $K = 64$ .

As a special case, when the system service time is constant, i.e.,  $\sigma = 1$ , the  $IBP^{[z]}/Geo/1/K$  queue becomes an  $IBP^{[z]}/D/1/K$  queue. The result for the queue length distribution of the  $IBP^{[z]}/D/1/K$  queue is presented in Figure 8 when  $p = 0.8$ ,  $q = 0.8$ ,  $\alpha = 0.8$ , and  $K = 64$ .

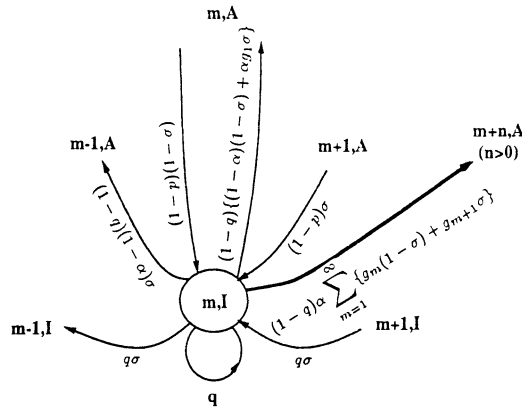


Figure 6. Transition probabilities of state (m,I).

In Figure 7 and 8, the peak values of the queue length distribution appear at system length = K-1 point when mean batch size is large. It makes sense because we observe the system at slot points which are right after the potential departure points. When the mean batch size is large, the system is very likely to be full all the time. At the other hand, when  $\sigma$  is big (here,  $\sigma = 0.8$  or  $1$ ), it is very likely to have a departure also. Thus, at our observing points, i.e. slot points, the probability of having K-1 CS\_PDU's in the system should be high.

By using Little's law, the mean waiting time can be easily obtained. Also, from the knowledge above, the blocking probabilities of the  $IBP^{[x]}/Geo/1/K$  queue and  $IBP^{[x]}/D/1/K$  queue can be derived as follows:

Let Prob(block) be the blocking probability of sub-packets;  $B_i[A]$  be the probability that the batch sub-packet arrivals see  $i$  sub-packets in the system and in the current slot the state of the Markov chain in the arrival process is active, in another word, it is the conditional probability that the batch arrivals see  $i$  sub-packets in the system and the current slot is in the active state given there are a batch of arrivals.

$$\begin{aligned}
 B_i[A] &= \frac{\pi_{i,A}p\alpha + \pi_{i,I}(1-q)\alpha}{\frac{1-q}{2-p-q}\alpha} \\
 &= \frac{(2-p-q)}{(1-q)} [\pi_{i,A}p + \pi_{i,I}(1-q)],
 \end{aligned}$$

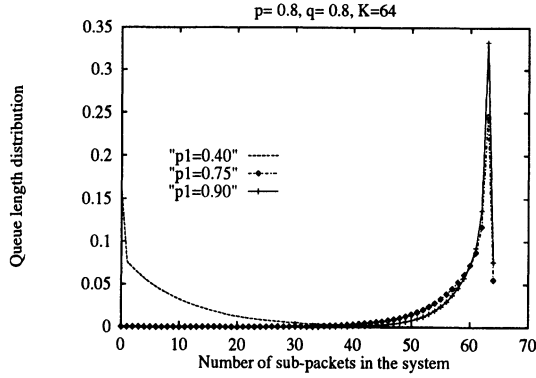


Figure 7.  $IBP^{[z]}/Geo/1/K$  Queue Length Distribution w.r.t. Different Mean Batch Size.

where  $\pi_{i,A}$  ( $\pi_{i,I}$ ) is the probability that there are  $i$  sub-packets in the system and the state of arrival process is active (idle). Thus,

$$\text{Prob}(\text{block}) = \frac{\sum_{i=0}^K B_i[A] \sum_{j=1}^{\infty} j \text{Prob}(\text{bulk size} = K + j - i)}{\sum_{i=0}^K B_i[A] \sum_{j=1}^{\infty} j \text{Prob}(\text{bulk size} = j)},$$

$$\text{Prob}(\text{bulk size} \geq j) = p_1^{j-1},$$

Finally,

$$\text{Prob}(\text{block}) = \frac{\sum_{i=0}^K [\pi_{i,AP} + \pi_{i,I}(1 - q)] p_1^{K-i}}{\sum_{i=0}^K [\pi_{i,AP} + \pi_{i,I}(1 - q)]}.$$

### 3. THE DEPARTURE PROCESSES OF $IBP^{[z]}/GEO/1/K$ QUEUE

Having developed a method to solve the queue length distributions of the  $IBP^{[z]}/Geo/1/K$  queue, in this section we use the results above to obtain the generating function of the interdeparture time for the  $IBP^{[z]}/Geo/1/K$  queue and the first four moments of the interdeparture time.

Let the random variable  $d$  be the interdeparture time of the  $IBP^{[z]}/Geo/1/K$  queue. We have

$$d = \begin{cases} t_s & \text{w.p. } 1 - d_0[A] - d_0[I] \\ t_s + R_A & \text{w.p. } d_0[A] \\ t_s + R_I & \text{w.p. } d_0[I] \end{cases} \quad (1)$$

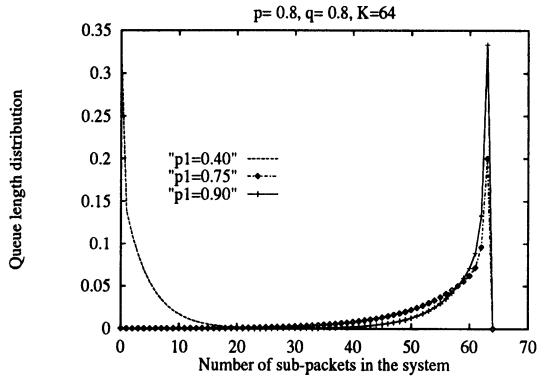


Figure 8.  $IBP^{[x]}/D/1/K$  Queue Length Distribution w.r.t. Different Mean Batch Size.

where

- $R_A$  is one slot plus the time interval from the current departure to the next arrival, given the current departure leaves the system empty and the next slot following this departure is in the active state.
- $R_I$  is one slot plus the time interval from the current departure to the next arrival, given the current departure leaves the system empty and the next slot following this departure is in the idle state.
- $t_s$  is the service time.
- $d_0[A]$  is the probability that the departing job leaves the system empty and the next slot following this departure is in the active state.
- $d_0[I]$  is the probability that the departing job leaves the system empty and the next slot following this departure is in the idle state.

In order to find  $d_0[A]$  and  $d_0[I]$ , it is very important to remember the assumption we made in the last section for the order of events at a slot boundary: 1) departure, 2) observing, 3) state change, and 4) arrivals.



For convenience, denote the state that at a slot point, with one departure, the system becomes empty, and the next slot is in the active state as  $W_A$ . Similarly if the next slot is in the idle state as  $W_I$ . Thus

$$d_0[A] = \frac{\text{Prob}(W_A)}{\text{Prob}(1 \text{ departure})}$$

$$d_0[I] = \frac{\text{Prob}(W_I)}{\text{Prob}(1 \text{ departure})}$$

Since the assumption has been made that an arrival at the current slot cannot be served until the beginning of the next slot, there are only four possibilities of getting to the state  $W_A$ : see Figure 9. The possibilities are

- a) at observing point E, the system is in the state (1,A); there is no state change in

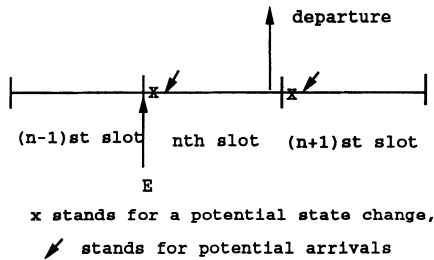


Figure 9. Event Sequence getting to state  $W_A$  or  $W_I$ .

the n-th slot; no arrivals in this slot; one departure happens at the end of the n-th slot; and the IBP arrival process stays in the active state in the (n+1)st slot.

b) at observing point E, the system is in the state (1,A); at the n-th slot the state of IBP is changed to idle; one departure happens at the end of the n-th slot; and the IBP arrival process changes to the active state in the (n+1)st slot.

c) at observing point E, the system is in the state (1,I); at the n-th slot the state of IBP is changed to active ; no arrivals in this slot; one departure happens at the end of the n-th slot; and the IBP arrival process stays in the active state in the (n+1)st slot.

d) at observing point E, the system is in the state (1,I); at the n-th slot the state of

IBP stays in the idle state; one departure happens at the end of the  $n$ -th slot; and the IBP arrival process changes to the active state in the  $(n+1)$ st slot. Thus,

$$d_0[A] = \frac{(p^2(1-\alpha) + (1-p)(1-q))\pi_{1,A} + (1-q)(p(1-\alpha) + q)\pi_{1,I}}{1 - \pi_0} \quad (2)$$

where,  $\pi_0$  is the probability that the system is empty.  $\pi_{1,A}$ ,  $\pi_{1,I}$ , and  $\pi_0$  can of course be obtained from the queue length distribution derived in the previous section.

Similarly, four possibilities exist for getting to the state  $W_I$ :

a) at observing point E, the system is in the state  $(1,A)$ ; there is no state change in the  $n$ -th slot; no arrivals in this slot; one departure happens at the end of the  $n$ -th slot; and the IBP arrival process changes to idle state in the  $(n+1)$ st slot.

b) at observing point E, the system is in the state  $(1,A)$ ; at the  $n$ -th slot the state of IBP is changed to idle; one departure happens at the end of the  $n$ -th slot; and the IBP arrival process stays in the idle state in the  $(n+1)$ st slot.

c) at observing point E, the system is in the state  $(1,I)$ ; at the  $n$ -th slot the state of IBP is changed to active; no arrivals in this slot; one departure happens at the end of the  $n$ -th slot; and the IBP arrival process changes to idle state in the  $(n+1)$ st slot.

d) at observing point E, the system is in the state  $(1,I)$ ; at the  $n$ -th slot the state of IBP stays in the idle state; one departure happens at the end of the  $n$ -th slot; and the IBP arrival process stays in the idle state in the  $(n+1)$ st slot.

Thus we have

$$d_0[I] = \frac{(1-p)[p(1-\alpha) + q]\pi_{1,A} + [(1-q)(1-\alpha)(1-p) + q^2]\pi_{1,I}}{1 - \pi_0} \quad (3)$$

and

$$1 - d_0[A] - d_0[I] = 1 - \frac{(1-p\alpha)\pi_{1,A} + (1-\alpha + \alpha q)\pi_{1,I}}{1 - \pi_0}$$

From the memoryless property of the geometric distribution, we have

$$R_A = \begin{cases} 1 & , w.p. \alpha \\ 1 + R_A & , w.p. (1-\alpha)p \\ 1 + R_I & , w.p. (1-\alpha)(1-p) \end{cases} \quad (4)$$

and

$$R_I = \begin{cases} 1 + R_I & , w.p. q \\ 1 + R_A & , w.p. (1-q). \end{cases} \quad (5)$$

It is found that

$$E[z^d] = E[z^{t^*}] ( 1 - d_0[A] - d_0[I] + E[z^{R_A}]d_0[A] + E[z^{R_I}]d_0[I] ) \quad (6)$$

where

$$E[z^{R_A}] = z(\alpha + E[z^{R_A}](1 - \alpha)p + E[z^{R_I}](1 - \alpha)(1 - p)) \tag{7}$$

$$E[z^{R_I}] = z(E[z^{R_I}]q + E[z^{R_A}](1 - q)) \tag{8}$$

$$E[z^{t_s}] = \frac{z\sigma}{1 - z(1 - \sigma)}. \tag{9}$$

By solving the equations 6- 9, the generating function of the interdeparture time distribution,  $E[z^d]$ , is obtained.

Now, we focus on the first four derivatives of the generating function of the interdeparture time. For simplicity, let us introduce the compact and obvious notation,  $D[z] = E[z^d]$ ,  $R_A[z] = E[z^{R_A}]$ ,  $R_I[z] = E[z^{R_I}]$ , and  $T_s[z] = E[z^{t_s}]$ .

The first four derivatives of the generating function  $D[z]$ , evaluated at  $z=1$ , can be calculated as follows

$$D^{(1)}[1] = T_s^{(1)}[1] + a, \tag{10}$$

$$D^{(2)}[1] = T_s^{(2)}[1] + 2T_s^{(1)}[1]a + b, \tag{11}$$

$$D^{(3)}[1] = T_s^{(3)}[1] + 3T_s^{(2)}[1]a + 3T_s^{(3)}[1]b + c, \tag{12}$$

$$D^{(4)}[1] = T_s^{(4)}[1] + 4T_s^{(3)}[1]a + 6T_s^{(2)}[1]b + 4T_s^{(4)}[1]c + e, \tag{13}$$

where

$$a = d_0[A]R_A^{(1)}[1] + d_0[I]R_I^{(1)}[1], \tag{14}$$

$$b = d_0[A]R_A^{(2)}[1] + d_0[I]R_I^{(2)}[1], \tag{15}$$

$$c = d_0[A]R_A^{(3)}[1] + d_0[I]R_I^{(3)}[1], \tag{16}$$

$$e = d_0[A]R_A^{(4)}[1] + d_0[I]R_I^{(4)}[1], \tag{17}$$

and

$$T_s^{(i)}[1] = \frac{i!(1 - \sigma)^{i-1}}{\sigma^i}, \tag{18}$$

$i = 1, 2, \dots$

$R_A^{(i)}[1]$  and  $R_I^{(i)}[1]$ , ( $i = 1, 2, 3, 4$ ), can be derived by the following computation:

First, rewrite Eq. 7 and 8 in matrix form as

$$S[z] \begin{bmatrix} R_A[z] \\ R_I[z] \end{bmatrix} = \begin{bmatrix} za \\ 0 \end{bmatrix} \tag{19}$$

where

$$S[z] = \begin{bmatrix} 1 - (1 - \alpha)pz & -(1 - \alpha)(1 - p)z \\ (1 - q)z & qz - 1 \end{bmatrix}. \tag{20}$$

Differentiate Eq. 19 and take  $z=1$ , we have

$$\begin{bmatrix} R_A^{(1)}[1] \\ R_I^{(1)}[1] \end{bmatrix} = S^{-1}[1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (21)$$

The  $i$ :th derivative can be written recursively as

$$\begin{bmatrix} R_A^{(i)}[1] \\ R_I^{(i)}[1] \end{bmatrix} = -iS^{-1}[1]B \begin{bmatrix} R_A^{(i-1)}[1] \\ R_I^{(i-1)}[1] \end{bmatrix} \quad (22)$$

$i = 2, 3, \dots$

where

$$S[1] = \begin{bmatrix} 1 - (1 - \alpha)p & -(1 - \alpha)(1 - p) \\ 1 - q & q - 1 \end{bmatrix},$$

and

$$B = \begin{bmatrix} -(1 - \alpha)p & -(1 - \alpha)(1 - p) \\ 1 - q & q \end{bmatrix}.$$

The departure process of the  $IBP^{[x]}/Geo/1/K$  queue has also been found through simulation. The simulation result indicates that the successive interdeparture times are slightly correlated, and the mean and second moment of the interdeparture time is 1.65 and 4.53, respectively when  $p = 0.4$ ,  $q = 0.5$ ,  $\sigma = 0.8$ ,  $p_1 = 0.4$ , and  $K = 64$ . Based on the Eq. 10- 22, the analytical results for the mean and second moment related to the given parameters are 1.65 and 4.54, respectively.

Similarly, as a special case, the departure process of  $IBP^{[x]}/D/1/K$  queue can be easily obtained by setting  $\sigma = 1$  in the departure process of  $IBP^{[x]}/Geo/1/K$  queue .

#### 4. MATCHING THE DEPARTURE PROCESS TO AN MMBP

In this section, we characterize the departure process of the  $IBP^{[x]}/Geo/1/K$  queue as an MMBP. Since simulation results reported on in the previous section indicate that the successive interdeparture times are correlated, neither a Bernoulli nor an IBP is a reasonable candidate for the departure process. Instead, we have selected the Markov Modulated Bernoulli process (MMBP), which can successfully capture both the burstiness and correlation properties, as a model for the departure process of the  $IBP^{[x]}/Geo/1/K$  queue and  $IBP^{[x]}/D/1/K$  queue.

##### 4.1. Markov Modulated Bernoulli process

The Markov Modulated Bernoulli process (MMBP) is a doubly stochastic point process whose arrival phase process for each slot is governed by an  $m$ -state irreducible Markov chain [7] [8]. The dwell time at phase  $i$  of the arrival phase process is geometrically distributed. We further assume that if the  $n$ -th slot is in state  $i$ , ( $i = 1, 2, \dots, m$ ), an arrival occurs according to a Bernoulli process with rate  $\alpha_i$ . The MMBP is characterized by

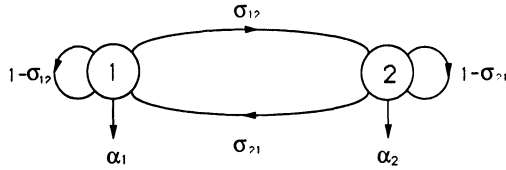


Figure 10. Markov Chain of MMBP.

the transition probability matrix  $Q$  and the  $m$  Bernoulli process rates  $\alpha_1, \alpha_2, \dots, \alpha_m$ . For simplicity we focus on a two state MMBP, as shown in Figure 10. We use the notation

$$Q = \begin{bmatrix} 1 - \sigma_{12} & \sigma_{12} \\ \sigma_{21} & 1 - \sigma_{21} \end{bmatrix},$$

$$\lambda = (\alpha_1, \alpha_2)^T,$$

and

$$\Lambda = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}.$$

The steady-state vector of the Markov chain  $\pi$  is such that

$$\pi = \pi Q.$$

Let

$$p_i = \text{Prob}(\text{the arrival comes from state } i \mid \text{there is an arrival})$$

$i = 1, 2,$   
we have

$$\begin{aligned} P &= (p_1, p_2) \\ &= \left( \frac{\pi_1 \alpha_1}{\pi_1 \alpha_1 + \pi_2 \alpha_2}, \frac{\pi_2 \alpha_2}{\pi_1 \alpha_1 + \pi_2 \alpha_2} \right). \end{aligned} \tag{23}$$

**4.2. Fitting the departure process of  $IBP^{[x]}/Geo/1/K$  queue to an MMBP**

From the description of an MMBP in the previous subsection, we know that the four parameters  $\sigma_{12}$ ,  $\sigma_{21}$ ,  $\alpha_1$  and  $\alpha_2$  are sufficient description for the process. Our approach is to match the first four moments of the interdeparture time for the  $IBP^{[x]}/Geo/1/K$  queue against the four moments of the interarrival time of an MMBP. By using the program “Interopt”, these four parameters can be derived from the four equations related to the four moments. The program “Interopt” uses a simulated annealing approach to extract unknown parameter values from nonlinear equations. We could of course use other moments for our fitting approach. Auto correlation coefficients of lag 1 and maybe higher could be used. In this investigation we did, however, decide to use as our matching equations the first four moments of the interdeparture time. The main reason for this was our observation from the simulation results that indicated that the correlation was not that significant.

For completeness we introduce in this section a condensed derivation of the MMBP interarrival time distribution. Let  $c_{n-1}$  and  $c_n$  be the (n-1)-th and n-th arrival, respectively,

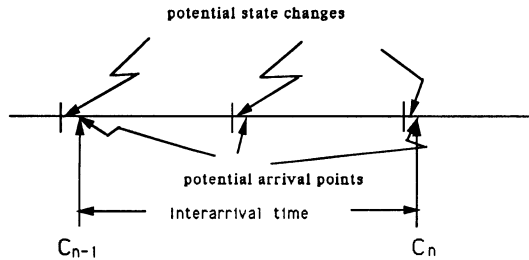


Figure 11. Interarrival time .

see Figure 11, and  $t_{n,i}$  be the time interval from any slot point to the next arrival given the state in the current slot is  $i$  ( $i = 1, 2$ ). Assume, as above, that the potential state switch point is at the beginning of each slot, and a potential arrival point is at the beginning of each slot immediately after the potential state switch point.

Let  $A_1[z]$  and  $A_2[z]$  denote the z-transforms of  $t_{n,1}$  and  $t_{n,2}$ , respectively. The following matrix equation for  $A_1[z]$  and  $A_2[z]$  is easily found.

$$[I - zF] \begin{bmatrix} A_1[z] \\ A_2[z] \end{bmatrix} = z \begin{bmatrix} (1 - \sigma_{12})\alpha_1 + \sigma_{12}\alpha_2 \\ (1 - \sigma_{21})\alpha_2 + \sigma_{21}\alpha_1 \end{bmatrix}, \quad (24)$$

where

$$F = \begin{bmatrix} (1 - \sigma_{12})(1 - \alpha_1) & \sigma_{12}(1 - \alpha_2) \\ \sigma_{21}(1 - \alpha_1) & (1 - \sigma_{21})(1 - \alpha_2) \end{bmatrix}.$$

$A[z]$ , the z-transform of the unconditional interarrival time is given as

$$A[z] = P \begin{bmatrix} A_1[z] \\ A_2[z] \end{bmatrix} \quad (25)$$

The  $k$ :th derivative of  $A[z]$  evaluated at  $z=1$  is

$$A^{(k)}[1] = P \begin{bmatrix} A_1^{(k)}[1] \\ A_2^{(k)}[1] \end{bmatrix}. \quad (26)$$

By differentiating Eq. 24  $k$  times and putting  $z = 1$ , we get

$$\begin{bmatrix} A_1^{(k)}[1] \\ A_2^{(k)}[1] \end{bmatrix} = k!(I - F)^{-k} F^{k-1} e \quad (27)$$

where  $e = [1, 1]^T$ , and  $k = 1, 2, 3, \dots$ . Thus, the first four moments of the interarrival time can be obtained. Matching these four moments against the first four moments of the interdeparture time derived in the previous section, we have four equations from which the four parameters  $\sigma_{12}, \sigma_{21}, \alpha_1$  and  $\alpha_2$  can be obtained.

Given  $p = 0.4$ ,  $q = 0.5$ ,  $\sigma = 0.8$ ,  $p_1 = 0.4$ , and  $K = 64$ , the first four moments of the interdeparture time of the  $IBP^{[z]}/Geo/1/K$  queue are found to be 1.65, 2.88909, 11.9571 and 73.8954. By using the matching approach above, the four parameters  $\sigma_{12}, \sigma_{21}, \alpha_1$  and  $\alpha_2$  are approximately obtained as 0.107774, 0.318488, 0.771349 and 0.129266, respectively. Thus the departure process of the  $IBP^{[z]}/Geo/1/K$  queue is characterized by an MMBP with above parameters.

In order to test the accuracy, the first four moments of the MMBP can be obtained by using the values of  $\sigma_{12}, \sigma_{21}, \alpha_1, \alpha_2$  and Eq. 26 and 27. They are 1.64201, 2.86897, 11.9054 and 73.1123, respectively. This example consequently indicates a satisfactory accuracy in our approach and this approach certainly can be applied to the departure process of  $IBP^{[z]}/D/1/K$  queue.

## 5. CONCLUSION

Queueing networks are very useful models for investigating the performance of communication systems. A simple way to study a queueing network is to approximately decompose it into individual queues that are analyzed independently and then recombined. This approach is used in this paper to indicate how a performance study of the

ATM Adaptation Layer can be done. To capture the traffic burstiness property in ATM networks, an  $IBP^{[z]}$  is used as the model for the arrival process of sub-packets. It turns out that one critical model that needs to be solved is an  $IBP^{[z]}/D/1/K$  queue. In this paper, the queue length distributions, the blocking probabilities, and departure processes of the  $IBP^{[z]}/Geo/1/K$  and  $IBP^{[z]}/D/1/K$  queues are studied by a Markov chain analysis. Since the simulation results indicates that the interdeparture times are correlated, a Markov-Modulated Bernoulli process(MMBP), which can capture both burstiness and correlation properties of traffic in high speed network, is used for modeling the departure process. A four moments fitting approach is used to match the departure processes of the  $IBP^{[z]}/Geo/1/K$  queue to a two state MMBP:s. Using this MMBP as an arrival process of the downstream queue, the remaining queues can be easily analyzed.

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