Differential Linear Logic and Processes

Thomas Ehrhard

Preuves, Programmes & Systèmes
Université Paris Diderot and CNRS

In Linear Logic, the tensor/par and plus/with dualities are lost when exponentials come in.

- The "?" modality is introduced by the *weakening* and *dereliction* rules, and *contraction* allow to contract two occurrences of a formula $?A$ (from an unique premise sequent) into a single one
- whereas the "!" modality can be introduced only by mean of a *promotion* rule.

By adding new rules for the "!" modality, one retrieves, in the exponential fragment, a duality and a symmetry similar to that of the multiplicative fragment. These new rules are

- *coweakening* and *cocontraction* which are new ways of introducing "!" formulas
- and *cocontaction*, which allow to contract two occurrences of $!A$ (from two different premise sequents) into a single one.

Corresponding reduction (cut-elimination) rules are added, which express operationally this new !/? symmetry. These reduction rules are semantically justified, when interpreting the new logical rules for "!" as standard operations on functions (in particular, codereliction corresponds to differentiation of a function at point 0 of a vector space). This extended linear logic is called Differential Linear Logic (DLL).

This new symmetry adds expressive power to linear logic. In particular, we show how a fragment of the π-calculus can be translated into *differential interaction nets* (a system if interaction nets where cells correspond to rules of DLL) and how the dereliction/codereliction reductions of this differential interaction net simulate the reductions of the process. Last, we present a simple denotational model of differential interaction nets, in a category of sets and relations. This model, which is also a natural model of the pure lambda-calculus (β and η), becomes therefore a model of the considered fragment of the π-calculus, and we explore some of its properties.

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