A Divide-and-conquer Strategy In Recovering Shape Of Book Surface From Shading

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Abstract. A strategy in solving the shape from shading problem for the shape and albedo recovery of book surfaces under the fully perspective environment is proposed. The whole recovery process is composed of three sequential steps : preprocessing, recovery of apparent shape, and ortho-image generation. A set of pure shade and albedo images are separated in preprocessing step. Implicit equations governing the shading and observation have been transformed into explicit ones. Direct and unique recovery becomes possible by combining the transformed ones and the recurrence relation. A feed-back recovery process using pure shade images is implemented as a practical algorithm in order to overcome self-shadows. Results of simulations and real experiments show the properness and acceptability of the proposed strategy and the implemented algorithms.

1 Introduction

The purpose of this paper is to propose a fast and reliable way in the shape recovery of book surface obtained under the fully perspective environment. The observing environment is composed of two light sources and one camera. We have assumed the constant albedo of row paper zone, the global variation of depth along one direction, Lambertian reflection characteristics[3], and the fact that the effect of interreflections exists only near the folded zone of book.

Two major difficulties should be considered in real situation. One is the obstacles such as non-constant albedos, self-shadows, interreflections[8][9], and the discontinuity of shading in folded zone. Another one is the effect of perspective observation which makes the problem implicit.

There have been researches on the shape recovery of book surface using a shaded scanner image[10][11][12] under the similar assumptions of ours, and on the approximated solution of the perspective shape from shading (SFS) problem[1][6][7]. The main contribution of this paper is on a strategy which makes it possible to recover the shape without iteration by resolving the implicitness of the perspective SFS problem, and on a way of separating albedo and shading from the observed shade images which enables it to overcome the self-shadows and the interreflections.

A divide-and-conquer strategy, which divides the complicated SFS problem into sequential and simple ones as is shown in Fig. 1, is proposed as the fun-

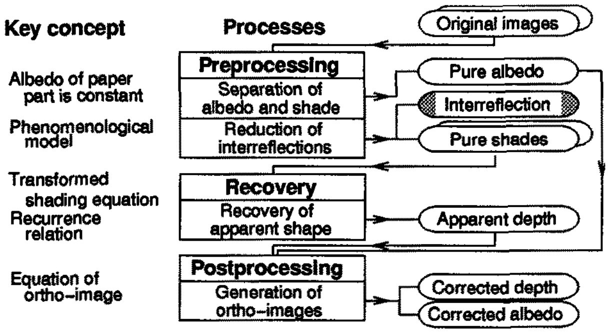


Fig. 1. Flow diagram of the whole process.

damental idea. The pure albedo and shade images are separated in the first stage. Our strategy divides the recovery problem into two for resolving the implicitness of the concerned equations. Therefore, the shape is recovered using separated shade images through two sequential steps.

This paper is arranged as follows. Mathematical problem and theoretical solution will be discussed in Sect. 2. Implementation of algorithms will be discussed in Sect. 3, and the results of simulations and real experiments will be shown in Sect. 4. Finally, conclusions will be done in Sect. 5.

2 Theoretical solution

2.1 Problem formulation

The orthographic projection of observation has been assumed by most of conventional SFS algorithms. However, in perspectively observed environments, both the world and pixel coordinate systems[4][5] should be considered simultaneously.

For the purpose of discrimination, (X, Y, Z) and (x, y, z) will be used for the world and pixel coordinate systems respectively. We will denote the coordinate of light source as (X_i, Y_i, Z_i), the height of camera as Z_o, and the local slope along X axis as p. Product of the departed strength of light rays and the albedo of surface will be denoted as $I_0\rho$.

Lambertian reflectance L_i, illuminated by a point light source and observed by a perspective pinhole camera, is described as

$$L_i = I_0\rho\frac{\cos\theta_{inc}}{D_i^2} = I_0\rho\frac{-\Delta_{Xi}p + \Delta_{Zi}}{(\Delta_{Xi}^2 + \Delta_{Yi}^2 + \Delta_{Zi}^2)^{\frac{3}{2}}\sqrt{1+p^2}}, \tag{1}$$

$$\Delta_{Xi} \equiv X_i - X, \qquad \Delta_{Yi} \equiv Y_i - Y, \qquad \Delta_{Zi} \equiv Z_i - Z,$$

where a coordinate (X, Y) which is unknown until the completion of shape recovery is represented by the equation of perspective observation

$$X = x(1 - \frac{Z}{Z_o}), \qquad Y = y(1 - \frac{Z}{Z_o}). \tag{2}$$

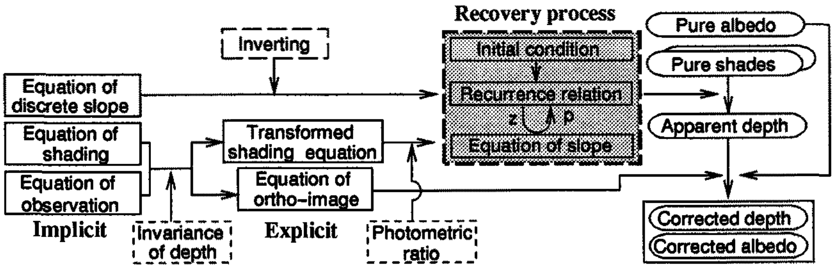


Fig. 2. Conceptual process of solving the perspective SFS problem.

Although the above equations look like containing five unknown parameters (X,Y,Z,$I_0\rho$,p) due to the implicitness of the equations, once a prior value of $Z(X,Y)$ becomes available, the values of (X, Y, p) can be calculated consequently. Therefore, the actual unknown parameters of this problem are only two, Z and $I_0\rho$, and the real difficulty lies in determining the depth prior to the slope.

2.2 Theoretical recovery process

The conceptual process of solving the problem is shown in Fig. 2. The key concept is based on the minimization of unknown parameters by transforming the implicit problem into the explicit one.

The proposed procedure provides three equations defined in the pixel coordinate system (detailed mathematical derivation is described in [2]).

1. Equation of slope

(p^D when two shade images are available, and p^S when single one is available)

$$p^D = \frac{L_1\delta_{Z2}d_1^3 - L_2\delta_{Z1}d_2^3}{L_1\delta_{X2}d_1^3 - L_2\delta_{X1}d_2^3}, p_i^S = \frac{B \pm \sqrt{B^2 - AC}}{A}, \qquad I_0\rho = 1, \tag{3}$$

$$\delta_{xi} \equiv X_i - x(1 - \frac{z}{Z_o}), \quad \delta_{yi} \equiv Y_i - y(1 - \frac{z}{Z_o}), \quad \delta_{zi} \equiv Z_i - z, \quad z \equiv z(x,y),$$

$$d_i \equiv \sqrt{\delta_{xi}^2 + \delta_{yi}^2 + \delta_{zi}^2}, \quad A \equiv \delta_{Xi}^2 - L_i^2 d_i^6, \quad B \equiv \delta_{Xi}\delta_{Zi}, \quad C \equiv \delta_{Zi}^2 - L_i^2 d_i^6.$$

2. Recurrence relation with initial condition

$$z(x,y) = z(x-2,y) + 2\epsilon\, p(x-1,y), \qquad \epsilon \approx 1, \tag{4}$$

$$z(0,y) \equiv 0, \qquad z(1,y) \equiv p(0,y).$$

3. Equation of ortho-image $\big(([X],[Y])$ denotes integral coordinate$\big)$

$$x = [X]\frac{Z_o}{Z_o - z}, \quad y = [Y]\frac{Z_o}{Z_o - z}, \quad z \equiv z(x,y). \tag{5}$$

Recovery of shape is done by the proposed process of bold-dotted box of Fig. 2. Initial condition provides the value of $z(0,y)$. This value is used for calculating the slope of this pixel. The slope is then used for calculating the depth of next pixel by recurrence relation. The unique and direct solution can be obtained by performing this loop.

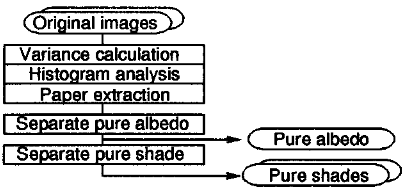


Fig. 3. Process of separating a set of pure albedo and shade images.

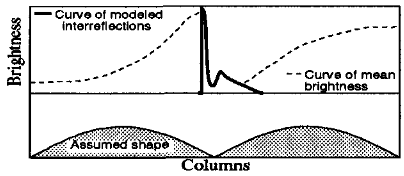


Fig. 4. Phenomenological model of global interreflection.

3 Implementation of algorithms

3.1 Separation of a set of albedo and shade images

The production of pure shade images is necessary for the reliable shape recovery of the self-shadow zone because of the requirement of constant albedo. Since row paper zone has constant albedo, a pure albedo $\rho(x,y)$ can be separated by dividing the sum of two observed images, I_1 and I_2, into the column-wise mean value, and hence a pure shade $L_i(x,y)$ can be separated consequently.

$$\rho(x,y) = \frac{I_1(x,y) + I_2(x,y)}{[I_1(x) + I_2(x)]_{paper}}, \quad L_i(x,y) = \frac{I_i(x,y)}{\rho(x,y)}, \quad i = 1,2. \tag{6}$$

The process is shown in Fig. 3. The extraction of paper zone is done through the local statistical analysis and the histogram analysis.

3.2 Reduction of the effect of interreflections

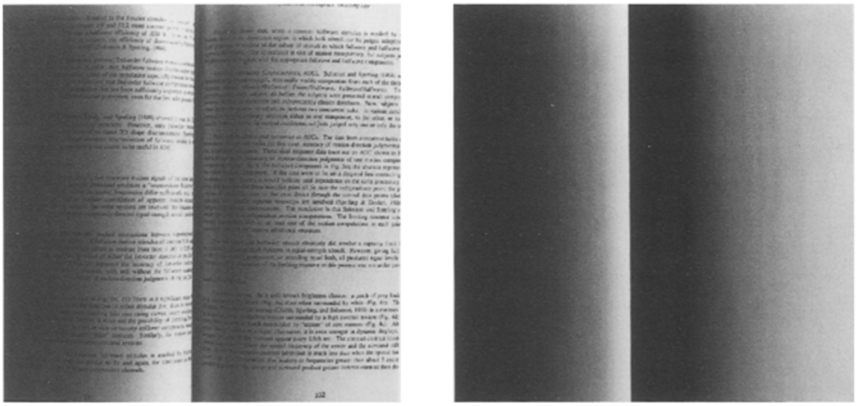
The effect of interreflections appeared mainly on the folded zone of book[12] is hard to remove due to the complexity of situation. A phenomenological model, which can be directly obtained from a shade image, is proposed for reducing the effect. In Fig. 4, the dotted curve represents the column-wise mean brightness of a separated pure shade image. It can be easily acknowledged that the brightness peak of folded zone (bold line) has been influenced by this effect.

The global model $L_i^G(x)$ is obtained by following the bold curve. The local effect $L_i^L(x,y)$, which is given by

$$L_i^L(x,y) \equiv a(x,y) L_i^G(x) + b(x,y), \tag{7}$$

is estimated by minimizing the fitting error of both sides of this equation, and is removed from each pure shade image.

One of the original shade images and a separated pure shade image are shown in Fig. 5. Although this preprocessing is based on a simple idea, we believe that the reliable and acceptable results can be obtained.



(a) Original shade image. (b) Separated pure shade image.

Fig. 5. Original shade image and separated one.
$\left(Z_o = 37cm,\ (X_i, Y_i, Z_i) = (\pm 47, 0, 26)cm,\ 1 \text{ pixel} = 0.48mm.\right)$

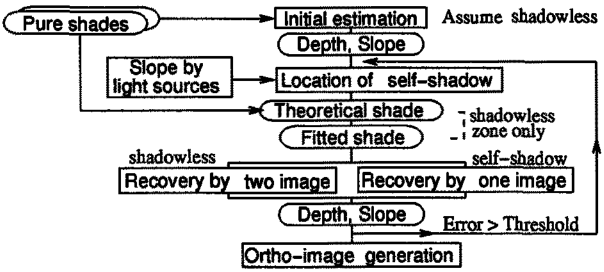


Fig. 6. Feed-back process proposed for shape recovery.

3.3 Shape Recovery

A feed-back process of Fig. 6 is implemented by enhancing the proposed theoretical process of Fig. 2 in order to overcome the self-shadows and the discontinuity of shading.

Initial shape is estimated using pure shade images by applying eq. (3) on the assumption of non-shadow everywhere. Using this shape, a zone over the line of incident light ray p_i^L is determined as a shadowless zone on the following condition

$$|p^D| \leq |p_i^L| + \theta_{thres}, \qquad p_i^L \equiv \frac{\sqrt{\delta_{Xi}^2 + \delta_{Yi}^2}}{\delta_{Zi}}, \quad \theta_{thres} > 0. \tag{8}$$

Within the shadowless zone, the pure shade image is fitted with the theoretical reflectance calculated using the recovered shape. In the next process, the same process as initial estimation is done on the shadowless zone. However p_i^S of eq. (3) is calculated using only one shadowless image when another image is self-shadowed. Combining the shape of both zones, shadowed and non-shadowed, the whole shape is obtained, and hence this recovered one is used for the refinement on the next iteration. On the folded zone of book, where no direct light reaches, the extrapolation of shape from the already recovered zone is done.

This feed-back process is repeated until the iteration error of shape recovery becomes acceptably small. In real images, this feed-back process becomes converged within few steps with respect to the acceptable iteration error.

Finally, eq. (5) is applied for generating the ortho-image by rectifying the deformations caused by the perspective observation.

Table 1. Results of simulations. (Z_o = 4000 pixels, (X_i, Y_i, Z_i) = ($\pm$9000, 0, 3000) pixels.)

Case	Lighting condition	Observing condition	Shadow condition	Inter-reflection	Error (%)
1	No	No	No	No	79.6
2	Yes	Yes	No	No	75.7
3	Yes	Yes	Yes	No	41.7
4	Yes	Yes	Yes	Yes	12.7

$$\text{Error} = 100 \times \frac{|Z_{real} - Z_{recovered}|}{Z_{real}}$$

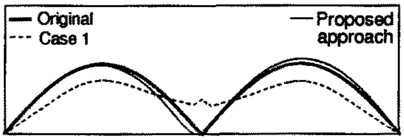


Fig. 7. Simulated results of shape recovery.

4 Simulations and experiments

Simulated shade images have been generated on the following conditions.
- Size : 512 × 512 with 45 pixels of maximum depth
- Rate of interreflections : 1.0 %
- Perspective shading and observation, self-shadows, and interreflections

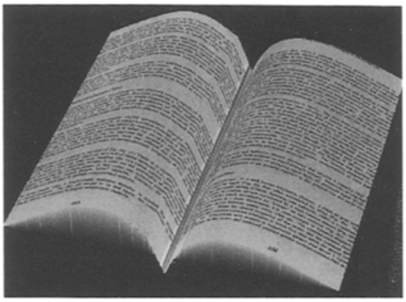
The results of simulations are shown in Table 1 and Fig. 7. While **Yes** means imposing the true condition, **No** means ignoring the condition during the recovery. The proposed approach (case 4) has provided smaller error than the conventional approaches which usually ignore perspective effects and self-shadows. It is analyzed that the error of case 4 has been caused by the extrapolation error and the deformation of observed shape during the perspective observation.

The ortho-image of albedo and an image of three dimensional perspective view as final results are shown in Fig. 8. Comparing with Fig. 5(a), the reliability of albedo separation and that of shape recovery can be acknowledged. Figure 9 shows another result of shape recovery. Although, in this case, the removal of interreflection had been insufficient owing to multiple peaks near folded zone, acceptable results have been obtained.

5 Conclusions

A strategy and algorithms in shape recovery of book surface from shading under the fully perspective environment is proposed. Following new concepts or approaches in solving the perspective SFS problem have been discussed.

(a) Ortho-image of albedo



(b) Three dimensional perspective view of the recovered book.

Fig. 8. Final results of the recovery.

- Transform of implicit SFS problem into explicit one
- Separation of the pure albedo and shade images
- Phenomenological model of interreflections
- Feed-back shape recovery overcoming self-shadows

Further researches shall be required about developing more reliable preprocessing algorithms and about finding another invariant or constraint for the application of the shape recovery of general object.

The approach proposed in this paper might be applicable to similar industrial applications because of efficiency in time and reliability of algorithm. For example, this technique is expected to be useful in producing the flattened image of rare books where the restriction of non-contact observation is highly required.

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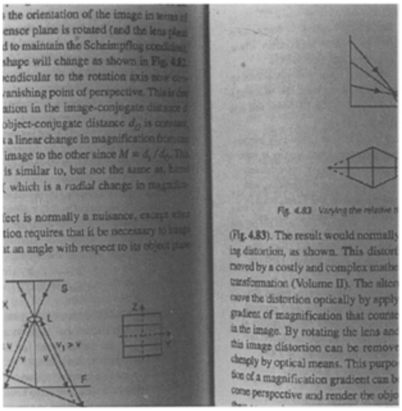
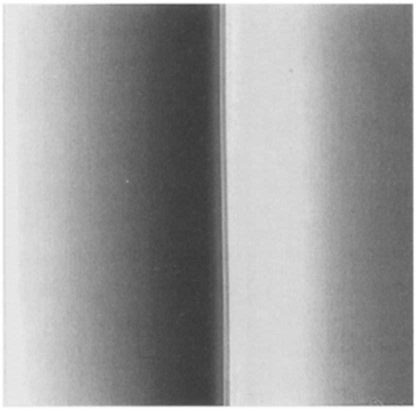
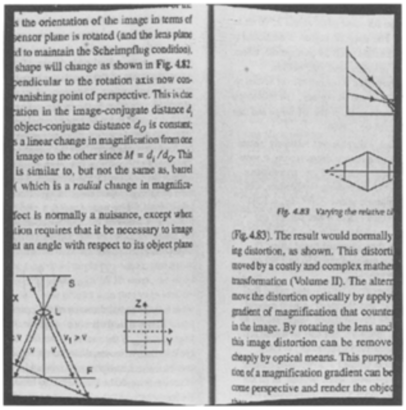


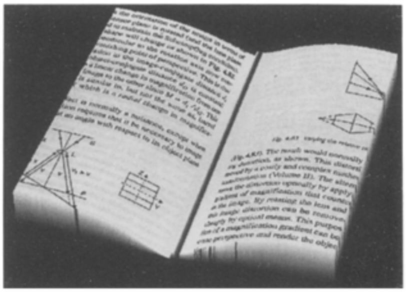
(a) One of observed shade image.



(b) Separated pure shade image.



(c) Ortho-image of albedo.



(d) Three dimensional perspective view.

Fig. 9. Another case of shape recovery.
$\big(Z_o = 29cm,\ (X_i, Y_i, Z_i) = (\pm 22, 0, 31)cm,\ 1 \text{ pixel} = 0.17mm.\big)$

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