Finding the Pose of an Object of Revolution

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Abstract: An algorithm able to locate an object of revolution from its CAD model and a single perspective image is proposed. Geometric properties of object of revolution are used in order to simplify the localization problem. The axis projection is first computed by a prediction verification scheme. It enables to compute a virtual image in which the contours are symmetric. A rough localization is done in this virtual image and then improved by an iterative process. Experiments with real images prove its robustness and its capability to deal with partially occluded contours.

1 INTRODUCTION

Among the visual features that can be extracted from an image, contours are a major source of information about the shape and the attitude of an object. Retrieving the object location from the contours detected in a single perspective image is not usually an easy task. The main difficulty consists in finding perspective invariants that enable to match model elements to image primitives.

In the case of polyhedric scenes, matching is greatly simplified, as straight lines are projected onto straight lines in the image. Thus linear ridges can be successfully used as invariants ([LOW-85], [DHO-89]).

The problem is more difficult when dealing with a world including curved objects, mainly because the primitives used to describe the model and the image curves are not necessarily of the same nature ([MAR-82]). Alignment of points ([HUT-87]) allows to deduce the location of an object as soon as three pairs of corresponding model- and image-points are found. But finding such pairs is not trivial.

In the case of Straight Homogeneous Generalized Cylinders (S.H.G.C.) ([MAR-82]), invariants like zero of curvature can provide matching pairs ([PON-89], [ULU-90]), [RIC-91], but the resultant methods are very sensitive to occlusion.

When no model is available, the location problem is under-constrained; Meridians can then be used to avoid ambiguity ([ULU-90]), but the extraction of meridians is not straightforward.

In this paper, we have chosen to deal with objects of revolution whose model is described by a generating curve. The matching difficulty is avoided by taking advantage of local symmetries (cf also [PON-89]).

Our localization method consists in three stages, that make full use of the symmetry properties:

- Algorithm (A_1) is devoted to the localization of the axis projection and to the creation of a virtual image in which the contours are symmetrical [GLA-91]. This step contributes to the simplification of the following computations.
- Algorithm (A_2) deals with the determination by any available means of a rough starting localization [DHO-90] (see section 3).
- Algorithm (A_3) uses the contours equations (section 2), in order to improve iteratively the location found with (A_2) (see section 4).

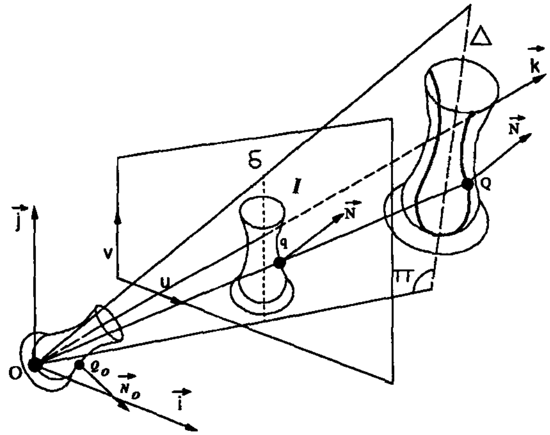
This work can be seen as an enhancement of a method presented at the last ECCV ([DHO-90]). In ([DHO-90]) the concept of the virtual image was still used to simplify (A_2), but its determination was not very accurate because it was only based on the knowledge of two image points resulting from a same object section; these chosen points were double points of the limb projection; their detection is often biased. In case of noisy or partially occluded contours, such a method fails. On the contrary, the processus (A_1) is quite robust to noise (due to the fact that it avoids brute accumulation schemes) and then provides a virtual image known with good accuracy.

Algorithm (A_2) has recently be improved; the reader can find in [LAV-90] the details of its new implementation. An iterative improving scheme has been added to the previous algorithm to reach accuracy (A_3). Its goal is similar to Kriegman & Ponce's work [KRI-90]. Respective advantages of the two methods are detailed in paragraph (4-2).

Implementation details and experimental results are given in section 5 (see also [GLA-92]).

2 CONTOURS EQUATION

In this paragraph, we derive the equations of the limb projections of a solid of revolution from the attitude parameters. To obtain simple equations, we only consider the attitudes for which the limb projections are symmetrical with respect to the vertical axis through the image center. Using algorithm (A_1) it is always possible from any brightness image of a solid of revolution to compute a virtual image having this property (see [GLA-91]).



Let $\Re$ be the frame $(O,\vec{i},\vec{j},\vec{k})$ where
- O is the optical center.
- $\vec{i}$ is parallel to the image lines.
- $\vec{j}$ is parallel to the image columns.
- $\vec{k}$ is orthogonal to the image plane.

Figure 1: Perspective projection of an object of revolution.

A limb point of a curved object is a surface point for which the tangent plane passes through the optical center of the camera.
Let R be the model of an object of revolution whose axis initially lies along $(0,\vec{k})$, described by its generating curve r(z) in $\Re$.
Let Q_O be a point of the model surface and $\overrightarrow{N_O}$ the normal to the surface at Q_O.
We have :

$$\overrightarrow{OQ_O}=\begin{pmatrix} x_O \\ y_O \\ z_O \end{pmatrix}=\begin{pmatrix} r.\mathrm{Cos}\theta \\ r.\mathrm{Sin}\theta \\ z_O \end{pmatrix} \quad \text{and} \quad \overrightarrow{N_O}=\frac{\partial\overrightarrow{OQ_O}}{\partial\theta}\times\frac{\partial\overrightarrow{OQ_O}}{\partial z_O}=\begin{pmatrix} n_{Ox} \\ n_{Oy} \\ n_{Oz} \end{pmatrix}=\begin{pmatrix} \mathrm{Cos}\theta \\ \mathrm{Sin}\theta \\ -r' \end{pmatrix} \quad \text{where} \quad \begin{cases} r=r(z_O) \\ r'=\frac{dr}{dz_O}(z_O) \end{cases}$$

To bring, from its initial pose, the model in an attitude producing a virtual image in which the contours are symmetrical, such as discussed previously, it must undergo a transformation ψ, composed of a rotation around $\vec{i}$ of angle α, followed by a translation $(b\vec{j}+c\vec{k})$.
Let us call p the attitude parameters vector: $p=(p_1,p_2,p_3)=(\alpha,b,c)$.

$$\begin{cases} Q_O \overset{\psi}{\Rightarrow} Q \\ \overrightarrow{N_O} \overset{\psi}{\Rightarrow} \overrightarrow{N} \end{cases} \qquad \overrightarrow{OQ}=\begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \overrightarrow{N}=\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

If f is the focal length and u,v the coordinates of perspective projection q of Q, we deduce from the limb equation $(\overrightarrow{OQ}\cdot\overrightarrow{N}=0)$:

$$\begin{cases} u=\dfrac{f.r.\mathrm{Cos}\theta}{r.\mathrm{Sin}\alpha.\mathrm{Sin}\theta+z_O.\mathrm{Cos}\alpha+c} \\ v=\dfrac{f.(r.\mathrm{Cos}\alpha.\mathrm{Sin}\theta-z_O.\mathrm{Sin}\alpha+b)}{r.\mathrm{Sin}\alpha.\mathrm{Sin}\theta+z_O.\mathrm{Cos}\alpha+c} \end{cases} \quad \text{where} \quad \mathrm{Sin}\theta=\frac{(c.\mathrm{Cos}\alpha-b.\mathrm{Sin}\alpha+z_O).r'-r}{(b.\mathrm{Cos}\alpha+c.\mathrm{Sin}\alpha)} \quad \text{for limb points} \quad (1)$$

Then, for a given section z_O of the model, the perspective projection of limb points can easily be computed.

3 ROUGH LOCALISATION IN THE VIRTUAL IMAGE (A_2)

Many processes are available to find an approximate value of attitude parameter vector p , especially when the contour detected in the image presents a reflectional symmetry. The weak perspective view assumption can be sufficient to treat the problem.

Moments, smoothness and compactness may help to reach an initial value of p, but such features are not reliable when a part of the contour is occluded.

When an object of revolution has a flat base, a part of an ellipse is seen in the image and can be used to find an approximate location of the object ([DHO-90]); extraction of the ellipse among contours points is often difficult, but reliable algorithms exist ([RIC-85]).

A prediction verification method using zero curvature points ([RIC-91]), recently generalized to any points of a symmetrical contour ([DHO-90] [LAV-90]) can also lead to an estimation of the localization, useful to initialize the iterative improvement described in the next section.

4 IMPROVEMENT OF THE LOCALISATION IN THE VIRTUAL IMAGE (A_3)

With a rough localization at hand, we can implement an iterative process that will improve it.

4.1 Description of the method

At this stage, we know an approximate location p_0 of the solid R, in a frame such that the perspective projection of the object limbs are symmetrical with respect to the vertical axis through the image center. Let C_0 be the calculated contour for this initial value p_0 (see section 2) and I be the contour observed in the virtual image.

The problem is now to decide of an association rule between the points belonging respectively to I and C_0, under the constraint that if p_0 is the good attitude, the rule must give the exact pairing.

Once this rule of association is chosen, distance between paired points can be computed (as well as partial derivatives if the association is not too complicated) and a process of minimization can be run.

The perfect rule is to associate each point $q_{C_0}(z_O)$ of C_0 to the point $q_I(z_O)$ of I corresponding to the same section z_O of the model; unfortunately matching points that way is not trivial (excepted for zero curvature points).

Another possible association is to connect each point q_{C_0} to the point q_I whose normal passes through q_{C_0} (see [PON-89b]), but here, the preliminary computation of a virtual symetrical image allows to choose a very simple rule of matching;

For each section z_O of the model, we compute $u(z_O)$ and $v(z_O)$ using equation (1) of section (2) and we compute the nearest contour point on the image line $v = v(z_O)$, i.e. the association is made horizontally.

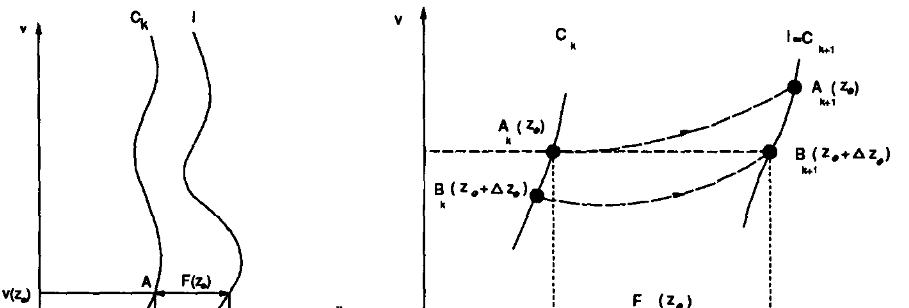


***Figure 2:** Notations: a) our distance, b) following an image point between two iterations.*

The image contour I is supposed given by a function $U(v)$. C_k is the calculated contour at iteration k.

We must find the vector p minimizing $D = \sum_{z_O} F^2(z_O) = \sum_{z_O} | U(v(z_O)) - u(z_O) |^2$ (see figure 2b).

The section of the model which is projected onto a given image line at iteration k, can change at iteration $k+1$. This variation of section can be calculated and taken into account in the equations involved in a Newton-Raphson minimization.
Let us consider two successive iterations $k, k+1$, providing an unknown variation Δp of the parameters vector, such that the perspective projection $A_k(z_O)$ of a limb point of section z_O becomes $A_{k+1}(z_O)$ at the next iteration.

It is possible to find the section $z_O+\Delta z_O$, and consequently the point $B_k(z_O+\Delta z_O)$ which will be transformed at iteration $k+1$ in a point $B_{k+1}(z_O+\Delta z_O)$ having the same ordinate v as point $A_k(z_O)$. In fact we can write:

$$\text{(2)}\qquad \Delta u=\left(\sum_{j=1}^{3}\frac{\partial u}{\partial p_j}\Delta p_j\right)+\frac{\partial u}{\partial z_O}\Delta z_O,\quad \Delta v=\left(\sum_{j=1}^{3}\frac{\partial v}{\partial p_j}\Delta p_j\right)+\frac{\partial v}{\partial z_O}\Delta z_O$$

and as we imposed $\Delta v=0$ for the measure of $F(z_O)$ between two iterations,

we can deduce $\Delta z_O=-\left(\sum_{j=1}^{3}\frac{\partial v}{\partial p_j}\Delta p_j\right)\Big/\frac{\partial v}{\partial z_O}$

Then (2) becomes: $\Delta u=\sum_{j=1}^{3}\left(\frac{\partial u}{\partial p_j}-\left(\frac{\partial v}{\partial p_j}\Big/\frac{\partial v}{\partial z_O}\right)\cdot\frac{\partial u}{\partial z_O}\right)\cdot\Delta p_j$

and as $\Delta F=\Delta u=F_{k+1}(z_O)-F_k(z_O)$. It comes $F_{k+1}(z_O)=F_k(z_O)+\sum_{j=1}^{3}\left(\frac{\partial u}{\partial p_j}-\left(\frac{\partial v}{\partial p_j}\Big/\frac{\partial v}{\partial z_O}\right)\cdot\frac{\partial u}{\partial z_O}\right)\cdot\Delta p_j$.

Now we solve for $F_{k+1}=0$ (the distance between C_k and C_{k+1} tends to zero)

thus $F_k(z_O)=-\sum_{j=1}^{3}\left(\frac{\partial u}{\partial p_j}-\left(\frac{\partial v}{\partial p_j}\Big/\frac{\partial v}{\partial z_O}\right)\cdot\frac{\partial u}{\partial z_O}\right)\cdot\Delta p_j$

and we obtain

$$|\,U(v(z_O))-u(z_O)\,|=-\sum_{j=1}^{3}\left(\frac{\partial u}{\partial p_j}-\left(\frac{\partial v}{\partial p_j}\Big/\frac{\partial v}{\partial z_O}\right)\cdot\frac{\partial u}{\partial z_O}\right)\cdot\Delta p_j \qquad (3)$$

4.2 Comparison with Kriegman & Ponce's work

Our method can be compared with Kriegman & Ponce's ([KRI-90]), that produces similar results with the same assumptions.

Kriegman & Ponce work with S.H.G.C. and do not compute any projection axis. They use elimination theory to precompute the implicit equation of limb projection, parametrized by the attitude parameter vector p and then run an iterative minimization scheme with another choice of the matching rule. This choice also involves a one-dimensional minimization for each point.

The fact that our method only applies to solids of revolution and not to general S.H.G.C. seems to be the main weakness of our work when compared to Kriegman & Ponce's (however, all the objects presented in their experiments are also solids of revolution [KRI-90]). The main advantage of our method is its ability to deal with objects whose generating curve is very complex (this do not seem possible in Kriegman & Ponce's work, due to the use of the elimination theory). Also it is clear in our approach that, at each step of the iterative process, the limb projection must be recomputed, which is not necessary for Kriegman & Ponce's, where the parametrized equation is at disposal (but very simplified). This fact is counter-balanced by the triviality of the rule of association that makes the iterations easier.

5 IMPLEMENTATION

5.1 Process

The contours issued from an image of an object of revolution are first extracted. The axis projection is computed by a procedure A_1; it enables to compute a rotation R_0 that must be applied to the camera frame to obtain a virtual image in which the contours are symmetrical.

A rough estimation of the parameter vector p is done by a procedure A_2 in this virtual image; this vector is used to compute the initial contour C_0 of our iterative scheme.

We think that the least squares B-Spline curve fitting provides a compact and faithful representation for smooth contours (see also [LAU-87]). Thus the right part of the image contour I (related to the axis projection) is approximated by a B-Spline curve (the left part could have been used to reconstruct occluded parts or to enhance accuracy).

The calculated contour C_k at iteration k is computed using the generating curve equation and equation (1) of section 2; internal loops of this contour are pruned by keeping, for a given image line, the farthest point from the axis projection. This operation avoids ambiguous associations in computing the distance $F(z_O)$.

C_k is then sampled to use only 20% of the contour points; for each of these points the derivatives involved in equation (3) and the corresponding $F_k(z_O)$ are computed ($F_k(z_O)$ is obtained by B-Spline interpolation). The resulting system is then solved by the classic Gauss elimination which gives the correction vector Δp;$p + \Delta p$ becomes the initial parameter vector for the next iteration.
$E_k = D/nbpointsfound$ is used as stopping error criterion.

Once convergence is reached, R_O^{-1} is applied to obtain the localization in the original frame.

5.2 Experimental results

Results for a vase are presented below. They show the robustness of the algorithm. The initial localization has been chosen farther than the one given by (A_2).

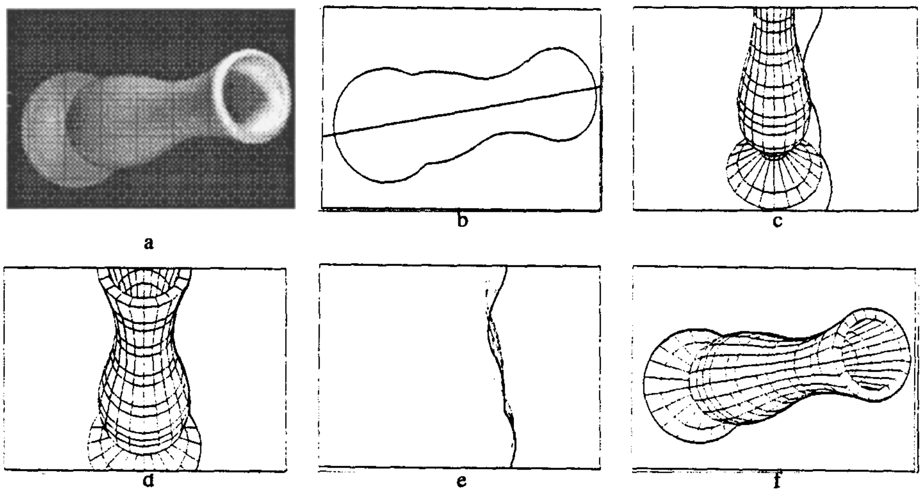


Figure 3 : Visualization of the different steps of the process:
a: the brightness image, b: the found axis projection (A_1),
c: the initial location in the virtual image (A_2), ***d: successive iterations*** $C_0..C_{10}$,
e: the found location in the virtual image with (A_3), ***f: localization in the original frame.***

6 CONCLUSION

We have provided a complete method able to find the pose of objects of revolution, with a great accuracy. The symmetry properties that result both from the geometric characteristics of such an object and from the perspective view assumption have given usefull cues to the localisation problem.

Positioning a revolution object needs to find five localization parameters. Computing the axis projection is an elegant way to get rid of two of them. A rough estimation of the three remaining ones can easily be done, using one among the many algorithms dedicated to this task. An iterative process can then be applied to improve these parameters.

The iterative process presented in section 4 is both simple and robust; it is able to deal with B-Spline curves representation. Elimination theory and implicit contour equations (comparison with Kriegman & Ponce's work) are avoided.

A contour point observed at a given iteration results from the projection of a given section. For a fixed image line, this section changes between two iterations. We have taken this variation of section into account in our way of pairing points between two successive iterations; such operation avoids the main problems of stability met in similar processes.

This precise localisation can be used as a preliminary step in modelling objects of revolution, as soon as a partial model is known (see [LAV-90]).

An extension of our algorithm to S.H.G.C. is forecast and will be soon available.

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