# DREAM$^2$S: Deformable Regions Driven by an Eulerian Accurate Minimization Method for Image and Video Segmentation

## Application to Face Detection in Color Video Sequences

Stéphanie Jehan-Besson[1], Michel Barlaud[1], and Gilles Aubert[2]

[1] Laboratoire I3S, CNRS-UNSA
2000, route des Lucioles 06903 Sophia Antipolis, France
{jehan,barlaud}@i3s.unice.fr

[2] Laboratoire J.A. Dieudonné, CNRS-UNSA
Parc Valrose 06108 Nice Cedex 2, France
gaubert@math.unice.fr

**Abstract.** In this paper, we propose a general Eulerian framework for region-based active contours named DREAM$^2$S. We introduce a general criterion including both region-based and boundary-based terms where the information on a region is named "descriptor". The originality of this work is twofold. Firstly we propose to use shape optimization principles to compute the evolution equation of the active contour that will make it evolve as fast as possible towards a minimum of the criterion. Secondly, we take into account the variation of the descriptors during the propagation of the curve. Indeed, a descriptor is generally globally attached to the region and thus "region-dependent". This case arises for example if the mean or the variance of a region are chosen as descriptors. We show that the dependence of the descriptors with the region induces additional terms in the evolution equation of the active contour that have never been previously computed. DREAM$^2$S gives an easy way to take such a dependence into account and to compute the resulting additional terms. Experimental results point out the importance of the additional terms to reach a true minimum of the criterion and so to obtain accurate results. The covariance matrix determinant appears to be a very relevant tool for homogeneous color regions segmentation. As an example, it has been successfully applied to face detection in real video sequences.

## 1 Introduction

Active contours are powerful tools for image and video segmentation. Since the original work on snakes [1], an extensive research has been performed that leads today to the use of "region-based active contours". Originally, active contours were boundary-based methods. Snakes [1], balloons [2] or geodesic active contours [3] are driven towards the edges of an image. The evolution equation is then computed from a criterion that only includes a local information on the boundary of the object to segment.

A. Heyden et al. (Eds.): ECCV 2002, LNCS 2352, pp. 365–380, 2002.

The key idea of region-based active contours, firstly proposed by [4,5,6,7], and further developed by [8,9,10,11,12,13], is to introduce a global information on the different regions to segment, in addition to the boundary-based information, to make the active contour evolve. However, it is not trivial to compute the evolution equation of the active contour that will make it evolve towards a minimum of a criterion including both region-based and boundary-based terms.

Recently, many papers have addressed this problem. Some of these works do not compute the theoretical expression of the velocity vector of the active contour but they choose the displacement that will make the criterion decrease [7,9]. Other works propose the computation of the velocity vector by reducing the whole problem to boundary integrals [6,8] or by using the level set method [10,11]. They then use the Euler-Lagrange equations to compute the evolution equation.

However, the information on the different regions, that we call here "descriptor" of the corresponding region, is generally globally attached to the region. Indeed this case arises for statistical descriptors such as the mean, the variance or the histogram of the region. The main drawback of previous works on region-based active contours is that they do not take into account this possible variation of the descriptors to compute the evolution equation of the active contour.

We propose here a general Eulerian framework for region-based active contours named DREAM$^2$S (Deformable Regions driven by an Eulerian Accurate Minimization Method for Segmentation). The main contribution of our work is to propose a theoretical framework based on shape optimization principles. With such an approach, we can readily take into account the variation of the descriptors that are globally attached to the evolving regions (named region-dependent descriptors). We show that the variation of these region-dependent descriptors during the evolution of the active contour induces additional terms in the evolution equation of the active contour. These additional terms have never been previously reported in the literature.

Some examples of unsupervised spatial segmentation using the variance of the regions show the importance of the additional terms for the accuracy of segmentation results.

For color images segmentation, we propose to use the covariance matrix determinant which appears to be a very relevant tool for homogeneous color regions segmentation. This tool is successfully applied to face detection on real video sequences.

The Eulerian framework DREAM$^2$S is described in Sec. 2. This framework is then applied to the unsupervised segmentation of homogeneous regions using the covariance matrix determinant in Sec. 3.

## 2 Setting a General Framework for Region-Based Active Contours

Here, we describe the equations addressing the issue of the segmentation of an image $I$ in two regions. An image $I(x, y)$ is a function defined for $(x, y) \in \Omega \subset$

$\mathbb{R}^2$, where $\Omega$ is the image domain. The image domain is considered to be made up of two parts: $\Omega_{in}$ the region containing the objects to segment and $\Omega_{out}$ the background region. Their common boundary is noted $\Gamma$ (see Fig.1).

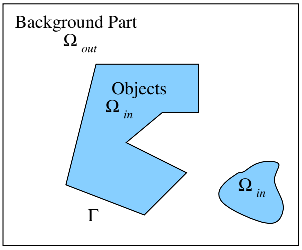



**Fig. 1.** The two regions of an image

We search for the partition of the image which minimizes the following criterion:

$$J(\Omega_{in}, \Omega_{out}, \Gamma) = \iint_{\Omega_{out}} k^{(out)}(x, y, \Omega_{out})dxdy + \iint_{\Omega_{in}} k^{(in)}(x, y, \Omega_{in})dxdy + \int_{\Gamma} k^{(b)}(x, y)ds \quad (1)$$

where $k^{(out)}$ is named the "descriptor" of the background region, $k^{(in)}$ the "descriptor" of the object region and $k^{(b)}$ the "descriptor" of the contour. A descriptor is a function that measures the homogeneity of a region. Most of relevant statistical descriptors depend themselves on the region (in that case they are called "region-dependent descriptors"). This arises when statistical features of a region are selected as region descriptors (for example the mean or the variance).

The purpose is then to make an active contour evolve towards a partition of the image, $(\Omega_{out}, \Omega_{in}, \Gamma)$, that minimizes the criterion (1). We propose here a new Eulerian proof for the computation of the evolution equation of the contour. We compute the Eulerian derivative of the criterion by using shape optimization principles. This method provides the advantage that the variation of region-dependent descriptors with the evolution of the active contour can be readily taken into account.

The computation of the evolution equation of the active contour is performed in three main steps:

1. Introduction of a dynamical scheme,
2. Derivation of the criterion using shape optimization principles,
3. From the derivative, computation of the evolution equation.

This general framework can be applied straightforward to any descriptors. This general framework has been notably applied to moving objects detection in video sequences acquired either with a static [14] or a mobile camera [15]. Descriptors may also be sequentially used to improve the final result as it has been proposed in [16].

### 2.1 Introduction of a Dynamical Scheme

We search for the two domains $\Omega_{out}$ and $\Omega_{in}$ which minimize the criterion $J$ given by (1). Since the set of all domains has not a structure of vectorial space, we can not compute the derivative of the previous criterion according to domains. Therefore, to compute an optimal solution, a dynamical scheme is introduced where each domain becomes continuously dependent on an evolution parameter $\tau$. Such a method has also been used in [12,13] using the distribution theory but region-dependent descriptors have not be taken into account. To formalize this idea, we may suppose that the evolution process is totally determined by the existence of a family of applications $T_\tau$ that transforms the initial domains $\Omega_{in}(0)$ and $\Omega_{out}(0)$ into the current domains $\Omega_{in}(\tau)$ and $\Omega_{out}(\tau)$:

$$\begin{array}{l} \Omega_i(0) \xrightarrow{T_\tau} \Omega_i(\tau) \\ \Gamma(0) \xrightarrow{T_\tau} \Gamma(\tau) \end{array} \quad \text{where} \quad i = in \text{ or } out \; .$$

Thus the regions evolve towards an optimal solution. The triplet $\{\Omega_{out}(\tau), \Omega_{in}(\tau), \Gamma(\tau)\}$ must act as a minimizing sequence for $J(\Omega_{out}, \Omega_{in}, \Gamma)$ as $\tau$ evolves. The functional $J(\Omega_{out}(\tau), \Omega_{in}(\tau), \Gamma(\tau))$ has to converge towards the minimum value of $J$ as $\tau \to \infty$. The criterion then becomes:

$$J(\tau) = \iint\limits_{\Omega_{out}(\tau)} k^{(out)}(x,y,\tau)dxdy + \iint\limits_{\Omega_{in}(\tau)} k^{(in)}(x,y,\tau)dxdy + \int\limits_{\Gamma(\tau)} k^{(b)}(x,y)ds \; . \quad (2)$$

The functional $J(\Omega_{out}(\tau), \Omega_{in}(\tau), \Gamma(\tau))$ is noted $J(\tau)$, and the descriptors $k^{(in)}(x,y,\Omega_{in}(\tau))$ and $k^{(out)}(x,y,\Omega_{out}(\tau))$ are respectively noted $k^{(in)}(x,y,\tau)$ and $k^{(out)}(x,y,\tau)$.

In contrast with other active contours approaches, the dynamical scheme is here directly introduced in the criterion. With such a scheme, the introduction of an active contour evolving with $\tau$ is straightforward. Hence we consider that $\Gamma(\tau)$ is modelled as an active contour that converges towards the final expected segmentation. Let $\Gamma(0)$ be the initial curve defined by the user. We recall that we search for $\Gamma(\tau)$ as a curve evolving according to the following PDE:

$$\frac{\partial \Gamma(s,\tau)}{\partial \tau} = \boldsymbol{v} \quad \text{with} \quad \Gamma(0) = \Gamma_0 \; , \quad (3)$$

where $\boldsymbol{v}$ is the velocity vector of the active contour and $s$ may be the arc length of the curve. The main problem lies in finding the velocity $\boldsymbol{v}$ from the criterion (1) in order to get the fastest curve evolution towards the final segmentation.

### 2.2 Computation of the Derivative of the Criterion

In order to obtain the evolution equation, the criterion $J(\tau)$ must be differentiated with respect to $\tau$. The integral bounds depend on $\tau$ and the descriptors $k^{(out)}()$ and $k^{(in)}()$ may also depend on $\tau$.

Let us define the functional $k(x, y, \tau)$ such that:

$$k(x, y, \tau) = \begin{cases} k^{(out)}(x, y, \tau) & \text{if } (x, y) \in \Omega_{out}(\tau) \\ k^{(in)}(x, y, \tau) & \text{if } (x, y) \in \Omega_{in}(\tau) \end{cases} . \tag{4}$$

The functional $k^{(out)}()$ and $k^{(in)}()$ are defined on the whole image domain $\Omega$. Then, the criterion $J(\tau)$ writes as:

$$J(\tau) = \iint_{\Omega} k(x, y, \tau) dx dy + \int_{\Gamma(\tau)} k^{(b)}(x, y) ds = J_1(\tau) + J_2(\tau) . \tag{5}$$

In order to compute the derivative of the region integral $J_1(\tau)$, we first recall a general theorem concerning the derivative of a time-dependent criterion.
Let us define $\Omega_i(\tau)$ as a region included into $\Omega$.

**Theorem 1.** *Let $k_i$ be a smooth function on $\overline{\Omega} \times (0, T)$, and let $J(\tau) = \iint_{\Omega_i(\tau)} k_i(x, y, \tau) dx dy$ , then:*

$$\frac{dJ}{d\tau} = J'(\tau) = \iint_{\Omega_i(\tau)} \frac{\partial k_i}{\partial \tau} dx dy - \int_{\partial \Omega_i(\tau)} k_i (\boldsymbol{V} \cdot \boldsymbol{N_{\partial \Omega_i}}) ds$$

*where $\boldsymbol{V}$ is the velocity of $\partial \Omega_i(\tau)$ and $\boldsymbol{N_{\partial \Omega_i}}$ is the unit inward normal to $\partial \Omega_i(\tau)$.*

The derivative of $J$ with respect to $\tau$ is the Eulerian derivative of $J(\Omega_i)$ in the direction of $\boldsymbol{V}$ whose computation is given in appendix A. The variation of $J$ is due to the variation of the functional $k_i(x, y, \tau)$ and to the motion of the integral domain $\Omega_i(\tau)$. As a corollary of Theorem 1, we get:

**Corollary 1.** *Let us suppose that the domain $\Omega(\tau)$ is made up of two parts, $\Omega_{in}(\tau)$ and $\Omega_{out}(\tau)$, separated by a moving interface $\Gamma(\tau)$ whose velocity is $\boldsymbol{v}$. The function $k(x, y, \tau)$ is supposed to be separately continuous in $\Omega_{in}(\tau)$ and $\Omega_{out}(\tau)$ but may be discontinuous across $\Gamma(\tau)$. We note $k^{(in)}$ and $k^{(out)}$, the value of $k$ in respectively $\Omega_{in}(\tau)$ and $\Omega_{out}(\tau)$. Thus the derivative of $J(\tau)$ writes as:*

$$J'(\tau) = \iint_{\Omega(\tau)} \frac{\partial k}{\partial \tau} dx dy - \int_{\partial \Omega(\tau)} k (\boldsymbol{w} \cdot \boldsymbol{N_{\partial \Omega}}) ds + \int_{\Gamma(\tau)} [[k]] (\boldsymbol{v} \cdot \boldsymbol{N}) ds$$

*where $[[k]]$ represents the jump of $k$ across $\Gamma(\tau)$: $[[k]] = k^{(out)} - k^{(in)}$, $\boldsymbol{N}$ the unit normal of $\Gamma(\tau)$ directed from $\Omega_{out}(\tau)$ to $\Omega_{in}(\tau)$, $\boldsymbol{N_{\partial \Omega}}$ the unit inward normal to $\Omega(\tau)$ and $\boldsymbol{w}$ the velocity vector of $\Omega(\tau)$.*

**Proof**: We can apply Theorem 1 to the domain $\Omega_{in}(\tau)$ and to the domain $\Omega_{out}(\tau)$. Adding the two equations, we obtain the corollary.
It is now straightforward to get the derivative of the criterion $J_1(\tau)$: we take $\Omega(\tau) = \Omega$ the image domain (Fig.2). Then, by explicitly taking the discontinuities into account thanks to the corollary, the derivative of $J_1(\tau)$ is given by:

$$J_1'(\tau) = \int_{\Gamma(\tau)} [[k]] (\boldsymbol{v} \cdot \boldsymbol{N}) ds - \int_{\partial \Omega} k (\boldsymbol{w} \cdot \boldsymbol{N_{\partial \Omega}}) ds + \iint_{\Omega_{in}(\tau)} \frac{\partial k^{(in)}}{\partial \tau} + \iint_{\Omega_{out}(\tau)} \frac{\partial k^{(out)}}{\partial \tau} \tag{6}$$

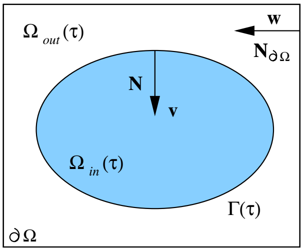



**Fig. 2.** The domains and the vectors involved in the derivation

Obviously, the second term of the derivative (6) is zero since the external boundary $\partial\Omega$ of the image is fixed. The derivative of $J_2(\tau)$ is classical [3] and thus, replacing $[[k]]$ by its expression, we find the derivative of the whole criterion:

$$J'(\tau) = \iint_{\Omega_{in}(\tau)} \frac{\partial k^{(in)}}{\partial \tau} dxdy + \iint_{\Omega_{out}(\tau)} \frac{\partial k^{(out)}}{\partial \tau} dxdy \qquad (7)$$
$$+ \int_{\Gamma(\tau)} (k^{(out)} - k^{(in)} - k^{(b)} \cdot \kappa + \nabla k^{(b)} \cdot \boldsymbol{N})(\boldsymbol{v} \cdot \boldsymbol{N}) ds$$

where $\kappa(x, y, \tau)$ is the curvature of $\Gamma(x, y, \tau)$.

In order to compute the velocity vector $\boldsymbol{v}$, we have to make it appear in the first two domain integrals of (7) by expressing them as functions of the velocity $\boldsymbol{v}$. Here we take the general case where the descriptors may depend on features globally attached to the region and so may depend on $\tau$. We model each descriptor as a combination of features globally attached to the evolving regions $\Omega_{in}(\tau)$ or $\Omega_{out}(\tau)$:

$$k^{(in)}(x, y, \tau) = g^{(in)}(x, y, G_1^{(in)}(\tau), G_2^{(in)}(\tau), ..., G_p^{(in)}(\tau))$$
$$k^{(out)}(x, y, \tau) = g^{(out)}(x, y, G_1^{(out)}(\tau), G_2^{(out)}(\tau), ..., G_m^{(out)}(\tau)) \qquad (8)$$

where: $G_j^{(\cdot)} = \iint_{\Omega.(\tau)} \psi_j^{(\cdot)}(x, y, \tau) dxdy$ with $(\cdot) = (in)$ or $(out)$

Let first compute the derivative of $k^{(in)}$ according to $\tau$. We find:

$$\frac{\partial k^{(in)}}{\partial \tau} = \sum_{j=1}^{p} \frac{\partial g^{(in)}}{\partial G_j^{(in)}} (x, y, G_1^{(in)}(\tau), ..., G_p^{(in)}(\tau)) \frac{\partial G_j^{(in)}}{\partial \tau}(\tau) \ . \qquad (9)$$

We have now to compute the derivative of $G_j^{(in)}$ according to $\tau$. We apply the Theorem 1 and we find:

$$\frac{\partial G_j^{(in)}}{\partial \tau} = \iint_{\Omega_{in}(\tau)} \frac{\partial \psi_j^{(in)}}{\partial \tau} dxdy - \int_{\Gamma(\tau)} \psi_j^{(in)}(\boldsymbol{v} \cdot \boldsymbol{N}) ds \ . \qquad (10)$$

Replacing the derivatives of $G_j^{(in)}$ by their expressions and computing the derivatives of $G_j^{(out)}$ in the same manner, we obtain the general expression for the derivative of $J(\tau)$:

$$\begin{aligned}
J'(\tau) = & \int\limits_{\Gamma(\tau)} (k^{(out)} - k^{(in)} - k^{(b)} \cdot \kappa + \nabla k^{(b)} \cdot \boldsymbol{N})(\boldsymbol{v} \cdot \boldsymbol{N}) ds \\
& - \sum_{j=1}^{p} A_j^{(in)} \int\limits_{\Gamma(\tau)} \psi_j^{(in)} (\boldsymbol{v} \cdot \boldsymbol{N}) ds + \sum_{j=1}^{m} A_j^{(out)} \int\limits_{\Gamma(\tau)} \psi_j^{(out)} (\boldsymbol{v} \cdot \boldsymbol{N}) ds \\
& + \sum_{j=1}^{p} A_j^{(in)} \iint\limits_{\Omega_{in}(\tau)} \frac{\partial \psi_j^{(in)}}{\partial \tau} dxdy + \sum_{j=1}^{m} A_j^{(out)} \iint\limits_{\Omega_{out}(\tau)} \frac{\partial \psi_j^{(out)}}{\partial \tau} dxdy
\end{aligned} \tag{11}$$

where:

$$\begin{cases}
A_j^{(in)} = \iint\limits_{\Omega_{in}(\tau)} \frac{\partial g^{(in)}}{\partial G_j^{(in)}} (x, y, G_1^{(in)}(\tau), ..., G_p^{(in)}(\tau)) dxdy \\
A_j^{(out)} = \iint\limits_{\Omega_{out}(\tau)} \frac{\partial g^{(out)}}{\partial G_j^{(out)}} (x, y, G_1^{(out)}(\tau), ..., G_m^{(out)}(\tau)) dxdy
\end{cases} .$$

### 2.3 From the Derivative Towards the Evolution Equation of the Active Contour

In order to deduce the velocity vector $\boldsymbol{v}$ of the active contour from the derivative of the criterion, we have to make the velocity vector appear in the last two domain integrals by expressing them as boundary integrals. This can easily be done, in the same manner as it has been done for the functionals $k^{(in)}$ and $k^{(out)}$, since domain integrals can always be expressed as boundary integrals. However, for simplicity, let us consider the cases where these last two integrals are equal to zero, which happens for most region-dependent descriptors. In that case, according to the inequality of Cauchy-Schwartz, the fastest decrease of $J(\tau)$ is obtained from (11) by choosing $\boldsymbol{v} = F\boldsymbol{N}$. The following evolution equation is obtained:

$$\begin{aligned}
\frac{\partial \Gamma(\tau)}{\partial \tau} = & [\ k^{(in)} - k^{(out)} + k^{(b)} \cdot \kappa - \nabla k^{(b)} \cdot \boldsymbol{N} \\
& + \sum_{j=1}^{p} A_j^{(in)} \psi_j^{(in)} - \sum_{j=1}^{m} A_j^{(out)} \psi_j^{(out)}\ ]\ \boldsymbol{N} .
\end{aligned} \tag{12}$$

We can notice that the dependence of the descriptors on $\tau$, and so on the curve evolution, induces additional terms in the evolution equation of the active contour. The additional terms found in (12) have not been previously computed. Thanks to these additional terms, we ensure the fastest evolution of the curve towards a minimum of the criterion. These additional terms may also be computed by reducing the whole problem to boundary-based terms and then computing the

Gâteaux derivative but the computation is much more difficult and less natural since the region formulation is not kept.

In this general framework, we can model the active contour by using an explicit parameterization (Lagrangian formulation) or an implicit one (Eulerian formulation) to implement the evolution equation of the active contour. See [17] and [18] for an interesting comparison between the two methods. Another interesting review on the different active contour methods is provided in [19]. Here, we use the level set method approach proposed by Osher and Sethian [20] and further developed by Caselles et al [21].

### 2.4 Importance of the Additional Terms

Region-dependent descriptors based on the minimization of the variances of the two considered regions are implemented for the segmentation of homogeneous regions. We show that the additional terms found in (12) have to be considered in order to make the active contour evolve towards a minimum of the criterion and so to achieve the expected segmentation.

The intensity for greyscale images is represented by a function $I : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$. We propose to minimize the variances of the two considered homogeneous regions in order to segment them. The following descriptors are suggested:

$$k^{(out)} = \varphi(\sigma^2_{out}), \quad k^{(in)} = \varphi(\sigma^2_{in}) \quad \text{and} \quad k^{(b)} = \lambda. \tag{13}$$

where $\varphi(r)$ is a positive function of class $C^1(\mathbb{R})$, $\lambda$ a positive constant and:

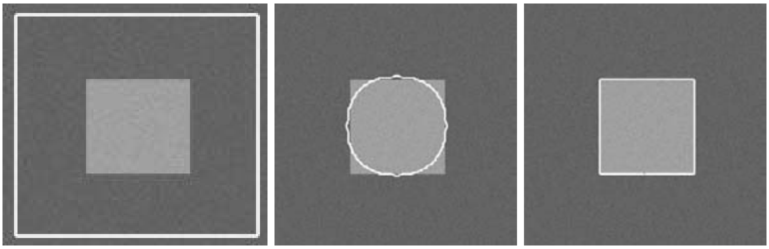
$$\begin{cases} \mu_{\cdot}(\tau) = \frac{1}{V_{\cdot}} \iint\limits_{\Omega_{\cdot}(\tau)} I dxdy \quad \text{with} \quad V_{\cdot} = \iint\limits_{\Omega_{\cdot}(\tau)} dxdy \\ \sigma^2_{\cdot}(\tau) = \frac{1}{V_{\cdot}} \iint\limits_{\Omega_{\cdot}(\tau)} (I - \mu_{\cdot}(\tau))^2 dxdy \end{cases} \quad \text{with} \quad \cdot = in \ \text{ or } \ out \ .$$

This velocity vector is computed by replacing the terms of (11) by their expressions, with $(\cdot) = (in)$ or $(out)$:

$$\begin{cases} k^{(\cdot)}(x,y,\tau) = g^{(\cdot)}(x,y,G_1^{(\cdot)}(\tau),G_2^{(\cdot)}(\tau)) = \varphi(\frac{G_1^{(\cdot)}}{G_2^{(\cdot)}}) \\ G_1^{(\cdot)}(\tau) = \iint\limits_{\Omega_{\cdot}(\tau)} \psi_1^{(\cdot)}(x,y,\tau)dxdy \quad \text{with} \quad \psi_1^{(\cdot)}(x,y,\tau) = (I(x,y) - \mu_{\cdot}(\tau))^2 \\ G_2^{(\cdot)}(\tau) = \iint\limits_{\Omega_{\cdot}(\tau)} \psi_2^{(\cdot)}(x,y,\tau)dxdy \quad \text{with} \quad \psi_2^{(\cdot)}(x,y,\tau) = 1 \end{cases}$$

Since the domain integrals of $\frac{\partial \psi_1^{(\cdot)}}{\partial \tau}$ and $\frac{\partial \psi_2^{(\cdot)}}{\partial \tau}$ are equal to zero, we can directly compute the velocity vector of the active contour from (12) and we find:

$$\begin{aligned} \frac{\partial \Gamma(\tau)}{\partial \tau} &= [\varphi(\sigma^2_{in}) - \varphi(\sigma^2_{out}) + \lambda\kappa + \varphi'(\sigma^2_{in})((I - \mu_{in})^2 - \sigma^2_{in}) \\ &\quad - \varphi'(\sigma^2_{out})((I - \mu_{out})^2 - \sigma^2_{out}) \,]\boldsymbol{N} \ . \end{aligned} \tag{14}$$



a. Initial contour b. Final contour PDE (15) c. Final contour PDE (14)

**Fig. 3.** Figure a: The initial curve,
Figure b: The final contour obtained using the PDE (15) without the additional terms,
Figure c: The final contour obtained using the PDE (14) including the additional terms.

The single parameter we need to adjust is the smoothing parameter $\lambda$.

In order to evaluate the importance of the additional terms for the segmentation, the active contour evolves through the PDE (14), including the additional terms. On the other hand, we also make it evolve with the following wrong evolution equation that does not include the additional terms:

$$\frac{\partial \Gamma(\tau)}{\partial \tau} = [\varphi(\sigma_{in}^2) - \varphi(\sigma_{out}^2) + \lambda\kappa\ ]\boldsymbol{N}\ . \tag{15}$$

For the experiments, we take $\varphi(r) = log(1+r)$ which gives $\varphi'(r) = 1/(1+r)$ and we choose $\lambda = 10$. We use a synthetic image made up of an homogeneous square of intensity 120 on a background of intensity 160. A gaussian noise of variance 20 is added to this synthetic image. The initial contour is given in Fig.3.a. The final contour obtained using the PDE (14) is given in Fig. 3.c, while the one obtained using (15) is given in Fig. 3.b. We can remark that when the PDE includes the additional terms, the square is well segmented, whereas when the PDE does not include these additional terms we obtain a circle instead of the expected square. The previous results show that it is of prime necessity to take the additional terms into account in order to reach a true minimum of the criterion.

## 3 Segmentation of Homogeneous Regions Based on the Covariance Matrix for Color Images: Application to Face Detection in Video Sequences

This part deals with the segmentation of homogeneous regions in multispectral images [22,23,24]. As an example, we show that the covariance matrix can be a very powerful tool for segmentation of homogeneous color regions. In fact, the determinant of the covariance matrix proves to be a relevant color region homogeneity measurement [25]. Minimizing this quantity means that we want to decrease the complexity of a region [26,27]. This tool may be used for face detection in image sequences.

The intensity of color images is represented by a function $\boldsymbol{I} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3$ where $\boldsymbol{I} = [I^1, I^2, I^3]^T$. By analogy with the study for greyscale images, we propose to minimize the determinant of the covariance matrix of the considered regions. Let us note $\Sigma_{in}$ and $\Sigma_{out}$ the covariance matrices of respectively $\Omega_{in}(\tau)$ and $\Omega_{out}(\tau)$:

$$\Sigma_{\cdot} = \begin{pmatrix} \sigma_{\cdot}^{11} & \sigma_{\cdot}^{12} & \sigma_{\cdot}^{13} \\ \sigma_{\cdot}^{21} & \sigma_{\cdot}^{22} & \sigma_{\cdot}^{23} \\ \sigma_{\cdot}^{31} & \sigma_{\cdot}^{32} & \sigma_{\cdot}^{33} \end{pmatrix} \tag{16}$$

$$\text{where: } \begin{cases} \sigma_{\cdot}^{ij} = \frac{1}{V_{\cdot}} \iint\limits_{\Omega_{\cdot}(\tau)} (I^i - \mu_{\cdot}^i)(I^j - \mu_{\cdot}^j)dxdy \\ \mu_{\cdot}^i \ = \frac{1}{V_{\cdot}} \iint\limits_{\Omega_{\cdot}(\tau)} I^i(x,y)dxdy \end{cases} \qquad \text{with } \ \cdot = in \text{ or } \ out \ .$$

We obviously have $\sigma_{\cdot}^{ij} = \sigma_{\cdot}^{ji}$.
As a color region homogeneity measurement, a function of the covariance matrix determinant is used as region descriptors. We choose the following set of descriptors:

$$k^{(out)} = \Phi(\det(\Sigma_{out})), \quad k^{(in)} = \Phi(\det(\Sigma_{in})) \quad \text{and} \quad k^{(b)} = \lambda \tag{17}$$

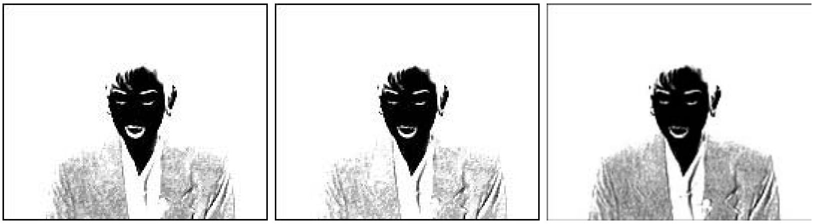
where $\Phi(r)$ is a positive function of class $C^1(\mathbb{R})$ and $\lambda$ a positive constant. Let us now compute the evolution equation of the active contour by computing the different terms of (11) [28]. We obtain the following evolution equation:

$$\begin{aligned} \frac{\partial \Gamma(\tau)}{\partial \tau} = [\ & \Phi(\det(\Sigma_{in})) - \Phi(\det(\Sigma_{out})) + \lambda\kappa \\ & + \Phi'(\det(\Sigma_{in})) \left[ \sum_{k,l=1}^{3} (I^k - \mu_{in}^k)(I^l - \mu_{in}^l) \det(M_{in}^{kl})(-1)^{k+l} \right] \\ & - \Phi'(\det(\Sigma_{out})) \left[ \sum_{k,l=1}^{3} (I^k - \mu_{out}^k)(I^l - \mu_{out}^l) \det(M_{out}^{kl})(-1)^{k+l} \right] \\ & - 3\det(\Sigma_{in})\Phi'(\det(\Sigma_{in})) + 3\det(\Sigma_{out})\Phi'(\det(\Sigma_{out}))\ ]\boldsymbol{N} \ . \end{aligned} \tag{18}$$

The matrix $M_{\cdot}^{kl}$ is deduced from the covariance matrix $\Sigma_{\cdot}$ by suppressing the $k^{th}$ row and the $l^{th}$ column. The last four lines of the equation are the additional terms coming from the variation of the descriptors with $\tau$. As it has been previously proved for greyscale images, these terms have to be considered in order to make the active contour converge towards a minimum of the criterion.

For the experiments, we take $\Phi(r) = log(1 + r^2)$ which gives $\Phi'(r) = 2r/(1 + r^2)$. We choose $\lambda = 10$. The color space selected is $(Y, C_b, C_r)$, where

a. Initial contour b. Iteration 500 c. Final contour



d. Initial velocity e. Iteration 500 f. Final velocity

**Fig. 4.** Visualization of the evolution of the contour {a,b,c} and the velocity {d,e,f} (without the boundary-based term) for face segmentation using the covariance matrix.

($I^1 = Y$) represents the luminance and ($I^2 = C_b$) and ($I^3 = C_r$) represent the two chrominances. We propose to use the descriptors based on the covariance matrix to detect human faces in video sequences. This detection may be used for video coding to encode selectively the human face. For a given compression ratio, the face can be transmitted with a higher rate to the detriment of the background. this interesting property is valuable for videoconferences, where the most important and most variable information is located on the face [29].

In order to segment an homogeneous region, we start with an initial curve inside the region of interest. The curve is then supposed to expand until it reaches the boundary of the homogeneous region.

The algorithm using the covariance matrix is performed on an image of the video sequence "akiyo" in order to detect speaker's face. The evolution of the curve is given in Fig.4 a,b,c. The contour evolves and finally converges on the boundaries of the face (Fig.4.c). The face is then accurately segmented. The amplitude of the velocity is also given in Fig.4 d,e,f and we can observe that the face region is well separated from other regions. The amplitude of the velocity is normalized between 0 and 255.

In order to track the face on the whole sequence, we first initialize the first frame by a circle inside the face to track. Then, we make the active contour evolve using (18). The final contour of the previous image is then chosen as an initial curve for the next image. The results for the sequence "akiyo" are given in Fig.5 ($\lambda = 10$) and, for the sequence "erik" are given in Fig.6 ($\lambda = 20$). The face is well detected and tracked on the whole sequence.

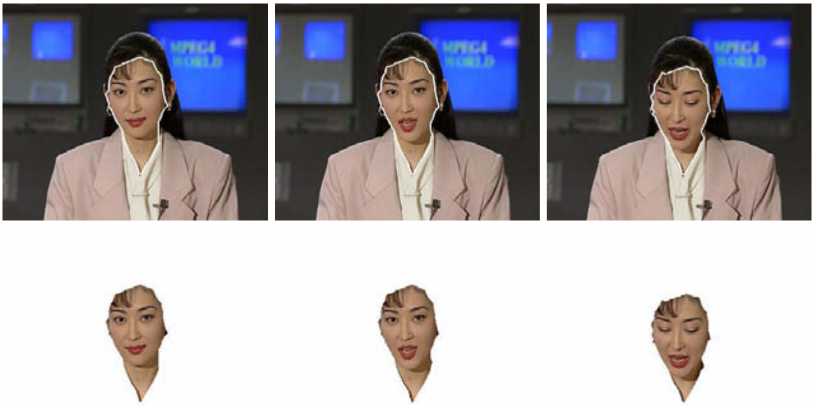


**Fig. 5.** Detection of the face on the video sequence "akiyo" (final contour and extracted face).

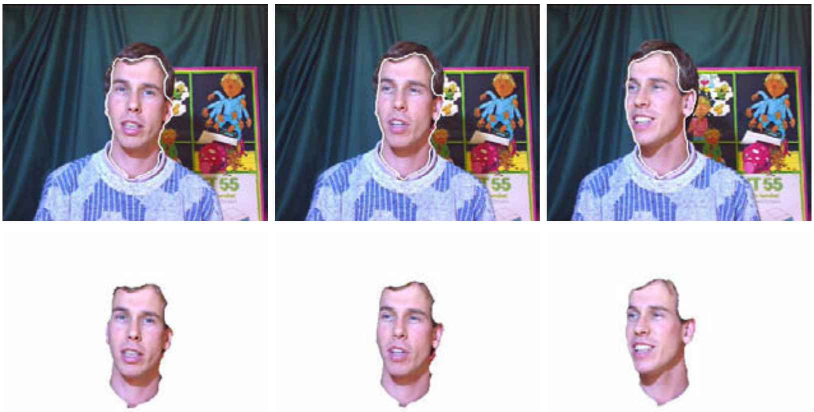


**Fig. 6.** Detection of the face on the video sequence "erik" (final contour and extracted face).

## 4 Conclusion

In this paper, we propose a new Eulerian minimization method to compute the velocity vector of an active contour that ensures its fastest evolution towards a minimum of a criterion including both region-based and boundary-based terms. With our approach, we can readily take into account the case of region-dependent descriptors that are globally attached to the evolving regions and so that vary during the propagation of the curve. We show that the variation of these descriptors induces additional terms in the evolution equation of the active contour. As far as homogeneous color regions segmentation is concerned, we take the covariance matrix determinant as a descriptor. This statistical region-dependent descriptor appears to be a very relevant tool and so it has been successfully applied for face detection in video sequences.

## Appendix A.

The issue is to differentiate the following domain functional:

$$J(\Omega_i) = \iint_{\Omega_i} k_i(x, y, \Omega_i) dx dy$$

Since the set of all domains has not a structure of vectorial space, let us make the regions evolve through a family of transformations $(T(\tau, \cdot))_{\tau \geq 0}$ smooth and bijective. For a point $p = [x, y]^T$, we note:

$$p(\tau) = T(\tau, p) \quad \text{with } T(0, p) = p, \text{ and } \Omega_i(\tau) = T(\tau, \Omega_i) \quad \text{with } T(0, \Omega_i) = \Omega_i$$

Let us then define the velocity vectors field $\boldsymbol{V}$ such that $\boldsymbol{V}(\tau, p(\tau)) = \frac{\partial T}{\partial \tau}(\tau, p)$. As we are interested in small deformations for the computation of the derivative, we expand the transformation according to first order Taylor formula:

$$T(\tau, p) = T(0, p) + \tau \frac{\partial T}{\partial \tau}(0, p) = p + \tau \boldsymbol{V}(p) \quad \text{where} \quad \boldsymbol{V}(p) = \frac{\partial T}{\partial \tau}(0, p)$$

We then introduce three main definitions:

1. The Eulerian derivative of $J(\Omega_i)$ (in the direction of $\boldsymbol{V}$):

$$dJ(\Omega_i, \boldsymbol{V}) = \lim_{\tau \to 0} \frac{J(\Omega_i(\tau)) - J(\Omega_i)}{\tau} \tag{19}$$

2. The material derivative of $k_i(p, \Omega_i)$:

$$\dot{k}_i(p, \Omega_i, \boldsymbol{V}) = \lim_{\tau \to 0} \frac{k_i(p + \tau \boldsymbol{V}(p), \Omega_i + \tau \boldsymbol{V}(p)) - k_i(p, \Omega_i)}{\tau} \tag{20}$$

3. The shape derivative of $k_i(p, \Omega_i)$:

$$k_i'(p, \Omega_i, \boldsymbol{V}) = \lim_{\tau \to 0} \frac{k_i(p, \Omega_i + \tau \boldsymbol{V}(p)) - k_i(p, \Omega_i)}{\tau} \tag{21}$$

Obviously, by expanding (20) according to first order Taylor formula, we have:

$$\dot{k}_i(p, \Omega_i, \boldsymbol{V}) = k_i'(p, \Omega_i, \boldsymbol{V}) + \nabla k_i(p, \Omega_i) \cdot \boldsymbol{V}(p) \tag{22}$$

We then compute more precisely the Eulerian derivative of $J(\Omega_i)$. We have:

$$\frac{J(\Omega_i(\tau)) - J(\Omega_i)}{\tau} = \frac{1}{\tau} \left[ \iint_{\Omega_i(\tau)} k_i(p(\tau), \Omega_i(\tau)) dp - \iint_{\Omega_i} k_i(p, \Omega_i) dp \right] \tag{23}$$

In the first integral, we make the variable change $p(\tau) = p + \tau \boldsymbol{V}(p)$ where $\boldsymbol{V}(p) = [V_1(x,y), V_2(x,y)]^T$, which gives:

$$\iint_{\Omega_i(\tau)} k_i(p(\tau), \Omega_i(\tau))dp = \iint_{\Omega_i} k_i(p + \tau \boldsymbol{V}(p), \Omega_i + \tau \boldsymbol{V}(p))|\det J_\tau(p)|dp$$

where $J_\tau(p)$ is the following Jacobian matrix:

$$J_\tau(p) = \begin{pmatrix} 1 + \tau \frac{\partial V_1}{\partial x} & \tau \frac{\partial V_1}{\partial y} \\ \tau \frac{\partial V_2}{\partial x} & 1 + \tau \frac{\partial V_2}{\partial y} \end{pmatrix}$$

and so, we have: $\det J_\tau(p) = 1 + \tau \mathrm{div}(\boldsymbol{V}(p)) + \tau^2 \det(\nabla \boldsymbol{V}(p))$.
Then we obtain $\lim_{\tau \to 0} \frac{\det J_\tau(p) - 1}{\tau} = \mathrm{div}(\boldsymbol{V}(p))$. The equation (23) then becomes:

$$\begin{aligned} \frac{J(\Omega_i(\tau)) - J(\Omega_i)}{\tau} = \frac{1}{\tau} \ [ & \iint_{\Omega_i} k_i(p + \tau \boldsymbol{V}(p), \Omega_i + \tau \boldsymbol{V}(p))|det J_\tau(p)|dp \\ & - \iint_{\Omega_i} k_i(p, \Omega_i)dp \ ] \end{aligned} \tag{24}$$

We can suppose that $\det(J_\tau(p)) \neq 0 \ \forall \tau \ \forall p$. We may then take $\det(J_\tau(p)) > 0$, and we develop the expression (24) as following by adding a term in the second integral while suppressing the same term in the first integral:

$$\frac{J(\Omega_i(\tau)) - J(\Omega_i)}{\tau} = I_1 + I_2 \tag{25}$$

where:

$$I_1 = \iint_{\Omega_i} \frac{k_i(p + \tau \boldsymbol{V}(p), \Omega_i + \tau \boldsymbol{V}(p)) - k_i(p, \Omega_i)}{\tau} \det(J_\tau(p))dp \tag{26}$$

$$I_2 = \iint_{\Omega_i} k_i(p, \Omega_i) \ \frac{\det(J_\tau(p)) - 1}{\tau} \ dp \tag{27}$$

Let us make $\tau$ tend towards 0. Using (22) and Definitions (20,21), we get:

$$\lim_{\tau \to 0} I_1 = \iint_{\Omega_i} \dot{k}_i(p, \Omega_i, \boldsymbol{V})dp = \iint_{\Omega_i} k_i'(p, \Omega_i, \boldsymbol{V})dp + \iint_{\Omega_i} \nabla k_i(p, \Omega_i) \cdot \boldsymbol{V}(p)dp$$

$$\lim_{\tau \to 0} I_2 = \iint_{\Omega_i} k_i(p, \Omega_i)\mathrm{div}(\boldsymbol{V})dp$$

And so for the Eulerian derivative, we find:

$$\begin{aligned} dJ(\Omega_i, \boldsymbol{V}) &= \iint_{\Omega_i} k_i'(p, \Omega_i, \boldsymbol{V})dp + \iint_{\Omega_i} (\nabla k_i(p, \Omega_i) \cdot \boldsymbol{V}(p) + k_i(p, \Omega_i)\mathrm{div}(\boldsymbol{V}(p)))dp \\ &= \iint_{\Omega_i} k_i'(p, \Omega_i, \boldsymbol{V})dp + \iint_{\Omega_i} \mathrm{div}(k_i \ \boldsymbol{V})dp \end{aligned} \tag{28}$$

When applying the Green-Riemann theorem in (28), we finally obtain:

$$dJ(\Omega_i, \boldsymbol{V}) = \iint_{\Omega_i} k_i'(p, \Omega_i, \boldsymbol{V})dp - \int_{\partial\Omega_i} k_i(\boldsymbol{V} \cdot \boldsymbol{N_{\partial\Omega_i}})ds \qquad (29)$$

where $\boldsymbol{N_{\partial\Omega_i}}$ is the unit inward normal to $\partial\Omega_i$. The Eulerian derivative is noted $J'(\tau)$ in the paper and the shape derivative $k_i'$ is noted $\frac{\partial k_i}{\partial \tau}$.

**Remark:** For simplicity, we choose the following variation in the proof: $p(\tau) = p + \tau \boldsymbol{V}(p)$. The proof may also be developed with more general variations such as: $p(\tau) = f(p, \tau)$ with $f$ a smooth function and $f(\cdot, \tau)$ bijective.

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