The Anatomy of a Geometric Algorithm*

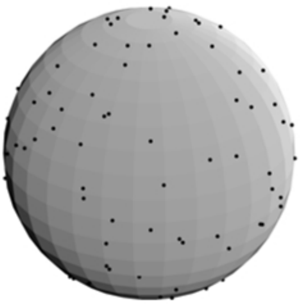
Jiří Matoušek

Department of Applied Mathematics, Charles University
Prague, Czech Republic
matousek@kam.ms.mff.cuni.cz

What is computational geometry?

(examples rather than a definition)

An n-point set in $\mathbf{R}^3$:



- What is the diameter of this point set?
- What is the smallest ball enclosing it?
- What is the "best fit" by a sphere?
- What is a "nice surface" defined by these points?

⋮

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J. Kratochvíl (Ed.): GD'99, LNCS 1731, pp. 1–7, 1999.

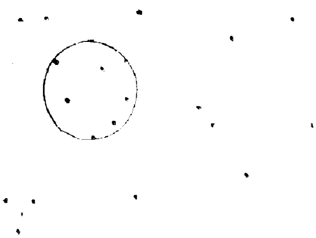
OUR PARTICULAR

PROBL-FA4 • •

INPUT! AX POINTS,
in the plane
, ≤ n

F|M): A SMALLEST CIftCIT

k POINTS



WHILE DISCUSSINe AN ALGO-
B.TTHM | WE WU, (MEPT

ft &E-Vt.:Ti.ftL, Fi Wt "ETCNV QUES

- Reformulation in terms of arra.,wge R1*« fcs
- ∧SIVI1 combinatorial-geometric observations
- randomized algorithms
- parametric search (and how it works o,voie(i:*)
- "n²-hard" problems
- geometric optimization framework (LP-type problems)

• TV PICAL OBSTACLES TO IMPLE>iENTATION

TRICK. •• I-.00k AT
FIXED SIZE W0 6LEM

Given n points and r ≥ 0, is there a circle of radius r enclosing 5? 4 (seitt'ts 3.

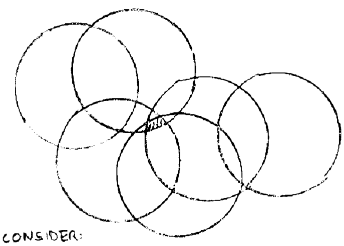
"TestINS" given r.

"CONVERSION" BY GENERAL TECHNIQUES (PARAMETRIC 5EAMH & OTHERS)
TO
"SEARCHING" for min {r ≥ 0 : test positive}

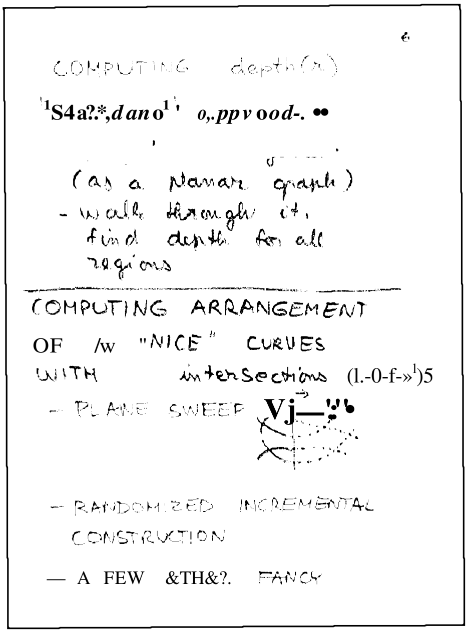
R E F o R fM.ut.AT:' ONJ
WITH ARRANGEMENTS

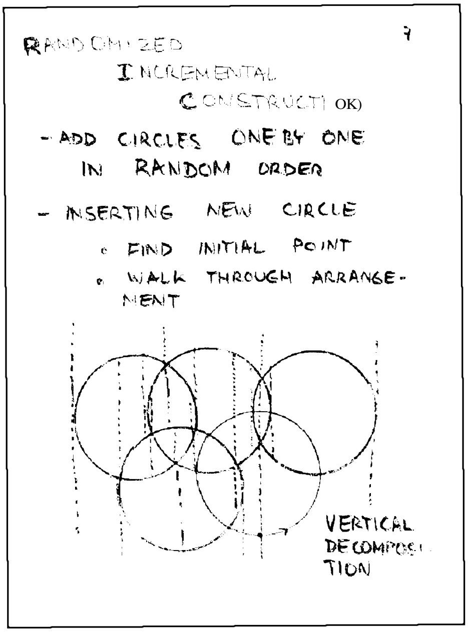
3 U-diSC <SIAt£Wviq ≥ k

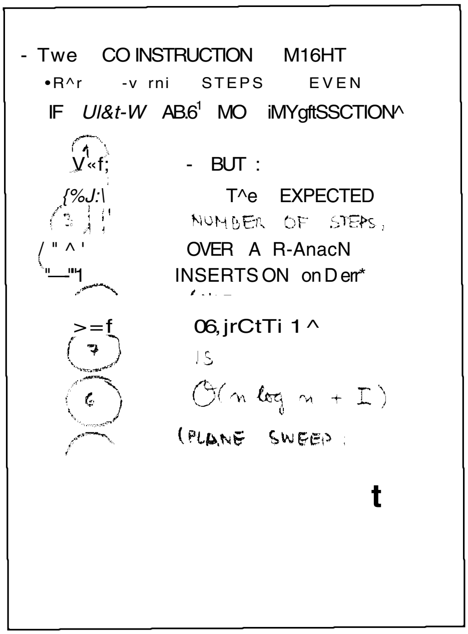
⟺

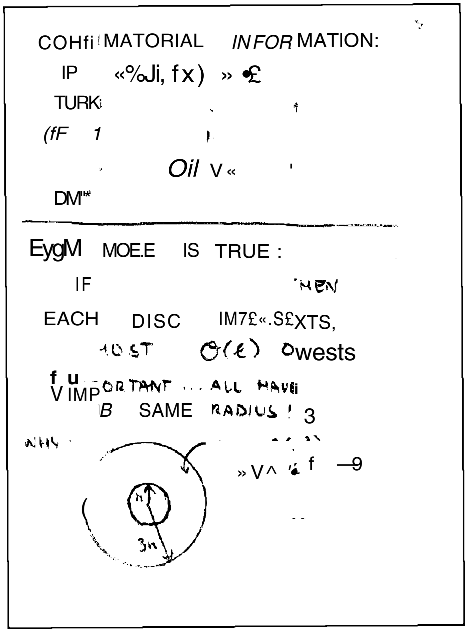


CONSIDER:
n disks of radius r around the given points
∃ point of DEPTH ≥ k
(depth = # of containing discs)


6
COMPUTING depth(x)
(as a planar graph)
- walk through it,
find depth for all
regions
COMPUTING ARRANGEMENT
OF /w "NICE" CURVES
WITH intersections
- PLANE SWEEP
- RANDOMIZED INCREMENTAL
CONSTRUCTION
— A FEW FANCY


7
RANDOMIZED
INCREMENTAL
CONSTRUCTION
- ADD CIRCLES ONE BY ONE
IN RANDOM ORDER
- INSERTING NEW CIRCLE
FIND INITIAL POINT
WALK THROUGH ARRANGE-
MENT
VERTICAL
DECOMPOSI-
TION


- Twe CO INSTRUCTION M16HT
STEPS EVEN
- BUT :
EXPECTED
NUMBER OF STEPS,
OVER A
IS
$O(n \log n + I)$
(PLANE SWEEP


MATORIAL INFORMATION:
IS TRUE :
IF
THEN
EACH DISC
$O(\ell)$
ORTANT ... ALL HAVE
SAME RADIUS !
WHY
n
3n

10

M CVM Vb£' C ftM T KT

GIVEN x (COMPUTE DEPTH(x))

MOW Do \wE' F/M]>≥

"S"?ftNt>ARX)" C Cl,$r\)&& !!(") APPROACH:

PARAMETRIC SEARCH

[_D g ^ j ^ TAltE SOMF AlGtmiTNH G

SUCH AS: test depth(x) ≥ k

SIMULATE t n ^ Exrcuno/v

r>p ^ PoK. ^ = H *

$x^* \overset{\le}{\underset{>}{=}} q$ ← given

depth(q) < k ⇒ $x^* > q$

depth(q) ≥ k ⇒ $x^* \le q$

[ignore equality for now]

TO RESOLVE 1 COMPARISON $x \overset{?}{} q$
NEED TO COMPUTE depth(q) 1x

^ ^ } i & ^ A'NV COM ^rRtfe 0 MJ5.

)Vr ONCE C * 1 t ^ I ; - . . ^

SEA rt CN

→ PARAM. SEARCH ALGORITHMS
ONLY
ASVNPT. QUITE FAbT (i.oes ioiT)

12

PfsRftM • SfftfStM ETC'

, n) algorithm
(probably impractical)

OTHER AppaoACMFs.:

— k^{th} ORDER VOliOMo I DIAGRAMS
SUBDIVISION OP TUB PLANE
REGION = POINTS WITH THE
SAME k-tuple os-
^tAa.'R,T POTOTi IN 7WP
GIVEN SET

KNOWN complexity

CONSTRUCTION TIME ctose
TO $O(k(n-k))$ [log n]

— ANALYZE "EVENTS" IN PARAM.
SEARCH,
(PfiMvH. iEACCH BtCLfttEP B'i
RANDOMIZED SEARCH - PROBLEM-SPECIFIC)

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PHASE 1

FINO x_0 S i * WITH

g; ∸ 1ST OF O f t) POTBK'TIA'L

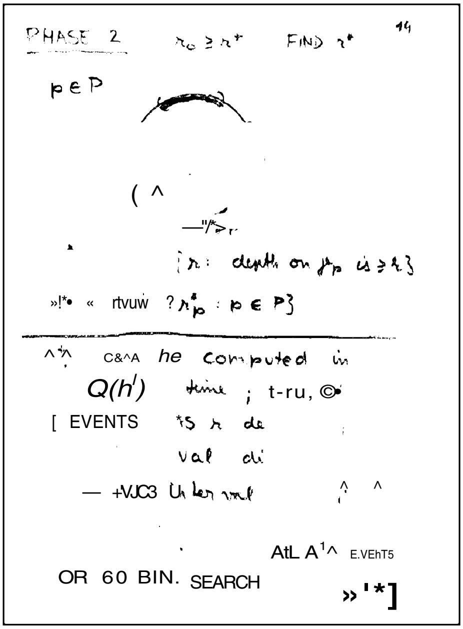
N

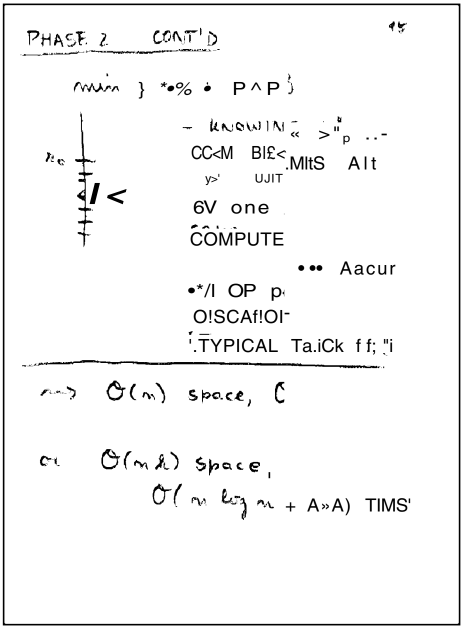
TRICK: APPROXIMATE depth(x)
BY grid-depth(x)

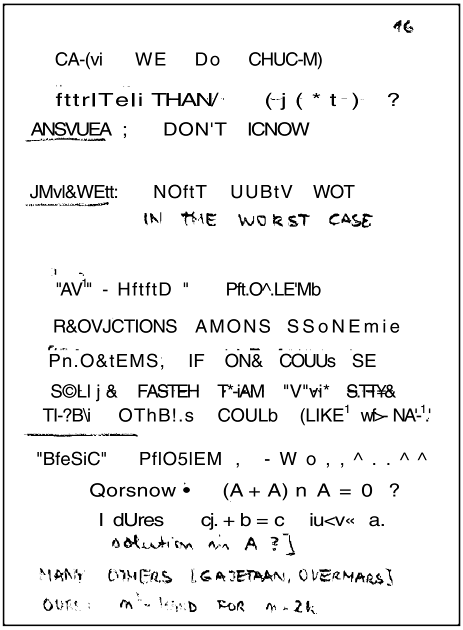
/*/./ ;/f*

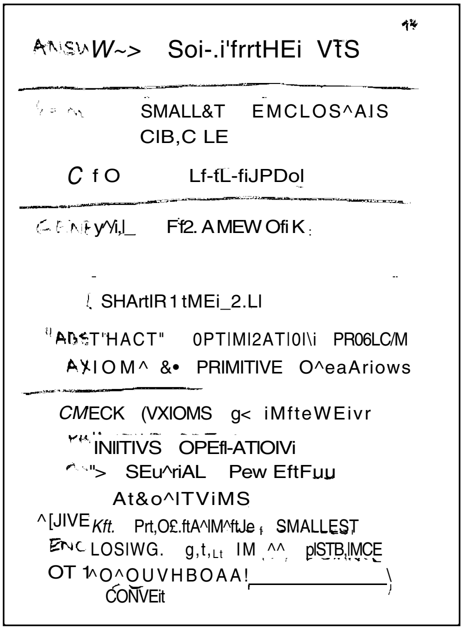
volume argument:
grid-depth(x) ≤ depth(x)
≤ const · grid-depth(x)

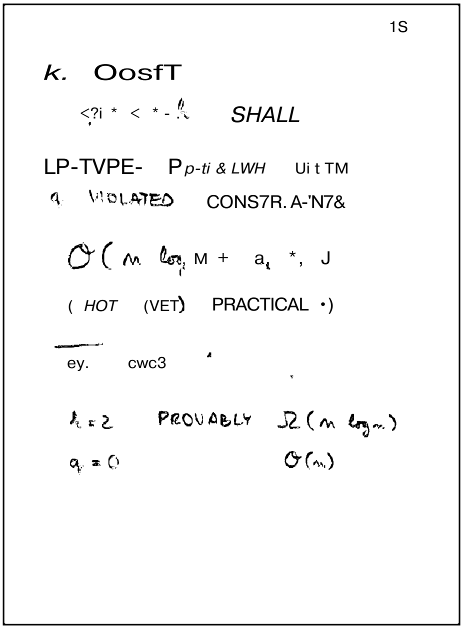
B.ESUUT-, PHASE 1 CAM &e DONE
IN O(n LKih) TIME S- 6f-n) SPACE

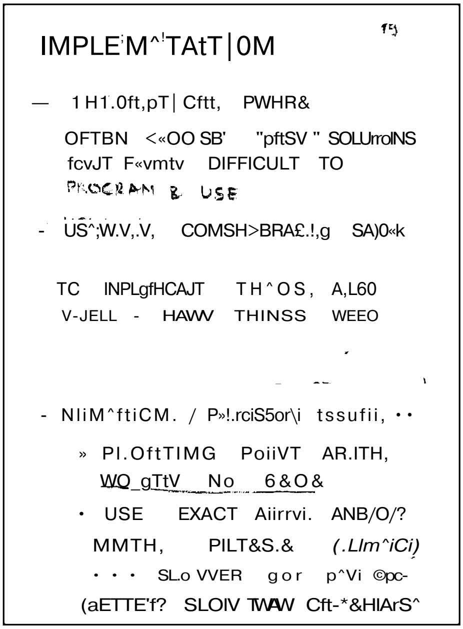

PHASE 2
FIND
p ∈ P
depth on
he computed in
EVENTS
OR BIN. SEARCH


PHASE 2 CONT'D
COMPUTE
TYPICAL
O(n) space,
O(nk) space,


WE Do
DON'T
IN THE WORST CASE
MANY OTHERS [GAJETAAN, OVERMARS]


SMALLEST
PRIMITIVE
SMALLEST


1S
k. OosfT
<?i * < * -.k SHALL
LP-TVPE- P p-ti & LWH Ui t TM
q VIOLATED CONS7R. A-'N7&
O(m log M + a *, J
(HOT (VET) PRACTICAL •)
ey. cwc3
k = 2 PROVABLY Ω (m log m)
q = 0 O(m)


IMPLE M^!TAtT|0M
— 1H1.0ft,pT| Cftt, PWHR&
OFTBN <«OO SB' "pftSV " SOLUrrolNS
fcvJT F«vmtv DIFFICULT TO
PROGRAM & USE
- US^;W.V,.V, COMSH>BRA£.!,g SA)0«k
TC INPLgfHCAJT T H ^ O S , A,L60
V-JELL - HAWV THINSS WEEO
- NliM^ftiCM. / P»!.rciS5or\i tssufii, ••
» PI.OftTIMG PoiiVT AR.ITH,
WO_gTtV No 6&O&
• USE EXACT Aiirrvi. ANB/O/?
MMTH, PILT&S.& (.Llm^iCi)
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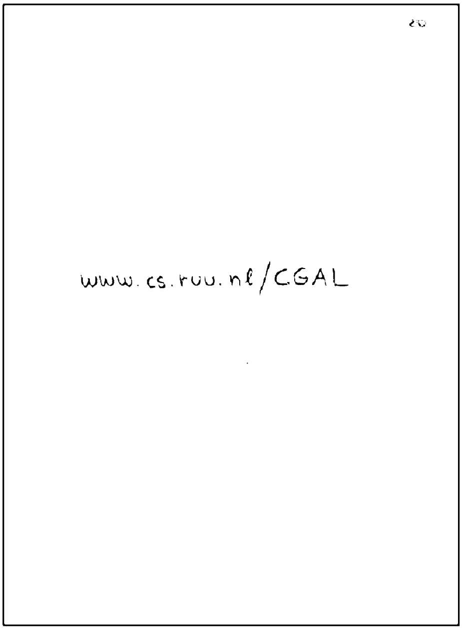
www.cs.ruu.nl/CGAL

Appendix

The purpose of this small appendix to the slides is to cite sources for the results directly mentioned in the talk, and to point to a few general-purpose references on computational geometry.

There are several introductory textbooks by now; one of them is [2]. Randomized incremental algorithms and applications of arrangements can be found there. There are two handbooks of computational geometry [6], [11]. Recent activity in the field, with increasing emphasis on more practically oriented studies, can be monitored in proceedings of the Annual ACM Symposia on Computational Geometry.

A parametric search algorithm for the considered problem is from [3]. Parametric search was formulated in [10]. Other papers on the problem are [4], [1]. The algorithm discussed in some detail is from [9]. The N^2-hard problems are collected in [5]. The LP-type problems were introduced in [8], and the application on circles enclosing all but q points is in [7].

References

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