# A Web-Based Problem Solving Environment for Solution of Option Pricing Problems and Comparison of Methods

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**Abstract.** In this paper we present a Problem Solving Environment (PSE) for solving option pricing problems and comparing methods used for this purpose. An open underlying library of methods has been developed to support the functionality of the proposed PSE. PHP, a server-side, cross-platform, HTML embedded scripting language, is exploited to make the proposed environment available through the World Wide Web. The PSE is not addressed to expert users only. It is simple in use, fast and interactive, proposing dynamically selections depending on user's input. The output is returned in "real time", in either simple or graphical representation.

## 1 Introduction

In recent years a very important progress in the area of option pricing methodologies and implementations has been observed [3]. Large financial institutions and many researchers have developed their own methods to solve a wide range of option pricing problems, as fast and accurately as possible. However, availability of these methods for the single user/investor is in question, as there are obviously great benefits for those institutions/investors using the best methods.

In the work presented in this paper, a significant number of option pricing algorithms have been implemented, providing a library of solution methods. In more detail, this library contains popular, widely applied methods [5, 17], as well as new state-of-the-art methods, for some categories of options, proposed recently by the authors [11–13]. To make such a library useful to a wide community of users, we need tools to provide remote access. For this reason, a Problem Solving Environment (PSE) has been developed, providing services for remote submission and monitoring of results from a Web-browser. The proposed PSE is not addressed to expert users only. The end-user poses the problem through a series of high-level definitions. The PSE facilitates this procedure providing the necessary guidance. After processing the input data, the server returns the results in simple arithmetic form (e.g. option prices) or through graphical representations (e.g. charts). A PHP platform [1] is used to enable communication between the PSE and the underlying option pricing library.

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The structure of the paper is as follows. In Section 2 we provide some option pricing basics. The main characteristics of the proposed PSE are discussed in Section 3, while the underlying library of methods supporting the PSE is described in Section 4. The detailed definition and solution process is given in Section 5. Finally, conclusions and future work are stated in the last Section.

## 2 Option Pricing Basics

Options are kind of financial derivative products, which are based on one or more underlying assets [17]. The underlying assets include stocks, indices, foreign currencies, commodities and other. Stock options were first traded on an organized exchange in 1973. Since then there has been a dramatic growth in options markets and they are now traded on many exchanges throughout the world. Huge volumes of options are also traded by banks and financial institutions.

There are two basic types of option contracts, calls and puts. A call (put) option gives the holder the right, not the obligation, to buy (sell) the underlying asset, by a certain date, for a certain price. The price of the contract is known as the exercise price; the date in the contract is known as the expiration or exercise date. American options can be exercised at any time up to the expiration date, while European options can be exercised only on the expiration date itself. Most of the options that are traded on exchanges are American.

The process of calculating the fair price for which an option should be offered in the financial markets is known as the option valuation or option pricing problem. A number of input parameters are required and play significant role in this calculation, namely risk-free interest rate, volatility of the option, dividend yield, exercise price and expiration date.

In the current work we concentrate on stock options. A mathematical formulation commonly used for this class of options, is based on the well-known Black Scholes model [2]. Most of the times it is difficult to calculate closed form solutions [4, 17], (e.g. for American options) which are of course favorable whenever they exist. In case that closed form solutions can not be obtained, numerical methods comprise an efficient approach [3, 17, 5, 8]. Other approaches include binomial or multinomial trees and simulation methods [5, 17].

## 3 Proposed Problem Solving Environment

Generally speaking, a PSE must satisfy a number of basic requirements [9]:

1. *Ease of use even by non-expert end-users.*
2. *Wide applicability.*
3. *Guided and user-friendly problem definition, including on-line help, validity checks, dynamic PSE-user interaction and error handling.*
4. *Presentation and analysis of results in a meaningful and understandable way.*

A significant number of environments exist for solving option pricing problems, each of which imposes its own restrictions, has its own advantages and uses its own underlying methods. Many of them are intended for professional use by stock-exchange companies, financial investment houses, etc. and as a result they are not offered for use by the single user/investor.

However, individual researchers, and a small number of financial companies, have developed PSEs that can be accessed through the World Wide Web. Their use and applicability are most of the times very restricted, because they deal with a small number of cases. Morever, the methods used in some cases may become slow and inefficient or even unable to achieve the desirable accuracy. Representative underlying platforms supporting existing environments include Java, JavaScript, or even Plain Html.

The PSE presented in this paper, first of all satisfies all basic requirements mentioned above. More than that it has a number of very important advantages over the existing environments just mentioned:

1. *Underlying library of methods:* An underlying library of algorithms constitutes the basic component of the proposed PSE. A wide range of methods from traditional to new, state-of-the-art, methods, have been implemented and comprise the basis of this library. The user is given the ability of selecting methods and defining not only financial but also, "numerical" parameters, interfering in this way directly with the underlying library.
2. *Availability through the Web:* An efficient web technology is used to support implementation of the proposed PSE under the World Wide Web. More specifically, a PHP platform enables communication between the underlying option pricing library and the end-user working on a Web-browser. The availability of the PSE through the web, facilitates access from different places and long distances. At the same time, due to the PHP platform used, user submissions are satisfied in "real time". It should be mentioned that the proposed PSE is independent of the browser platform used and has been tested and works properly with the most common browsers.
3. *Simplicity / Guidance / Functionality:* The user defines his problem through a series of simple steps. The PSE provides the necessary guidance, interacts dynamically with him/her and proposes selections that depend on input. Morever, mechanisms like on-line help or a variety of validity checks ensure for the proper functionality of the PSE.
4. *Graphical Representation of the Output:* The results returned to the user are not exclusively of numeric type. Most of the times, a graphical representation of the output is displayed.
5. *Open Architecture:* The proposed PSE is open. The underlying library of methods can be enriched by addition of new ones. The web front-end can be easily extended to support new methods.

As a drawback, one can mention the absence of an underlying expert system. The PSE proposes a list of methods, not the best one. To overcome this deficiency, a comparison menu is provided through which the end-user has the option to compare methods of the same type for a given problem. The resulting charts of execution times lead directly to conclusions about the efficiency of the compared methods.

The proposed PSE can be accessed through a typical Web-browser. Its architecture is apparent in Figure 1. First the user is guided to define the problem and choose from a variety of proposed solution methods. At the same time, parameters that are critical for the solution process must be entered. After all necessary input has been provided, the problem is directed to the server for solution. There, a Web-server takes all responsibility to call the required methods from the underlying library and pass the necessary input parameters to them. The output produced is finally returned to the browser. Typical output consists of option prices and other valuable information like execution times. As already mentioned, communication between the browser and the server is based on an underlying PHP platform the graphical capabilities [1] of which are exploited to produce chart representations.

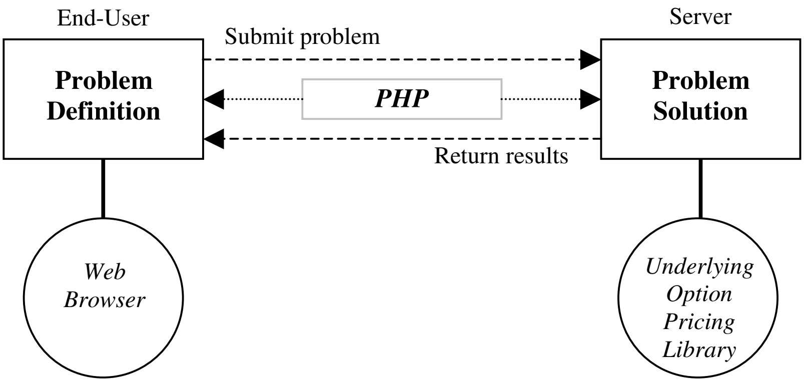


**Fig. 1.** Proposed Problem Solving Environment Architecture

## 4 Underlying Option Pricing Library

A plethora of stock option pricing methods have been developed to support the proposed PSE. Generally speaking, they can be enlisted in two basic categories: popular, widely used methods that can be found in the literature of the past three decades, and new methods, proposed recently by the authors. Up to this time the library contains methods for pricing European and American options on stocks. A categorization of these methods is given in Figure 2.

For European options, three basic categories of methods have been developed, namely closed form, grid and tree. Closed form solutions come directly from the Black-Scholes Partial Differential Equation [2]. The other categories are essentially based on numerical approximations. Tree methods (binomial and multinomial) are very easy to implement but they are generally inefficient. Grid

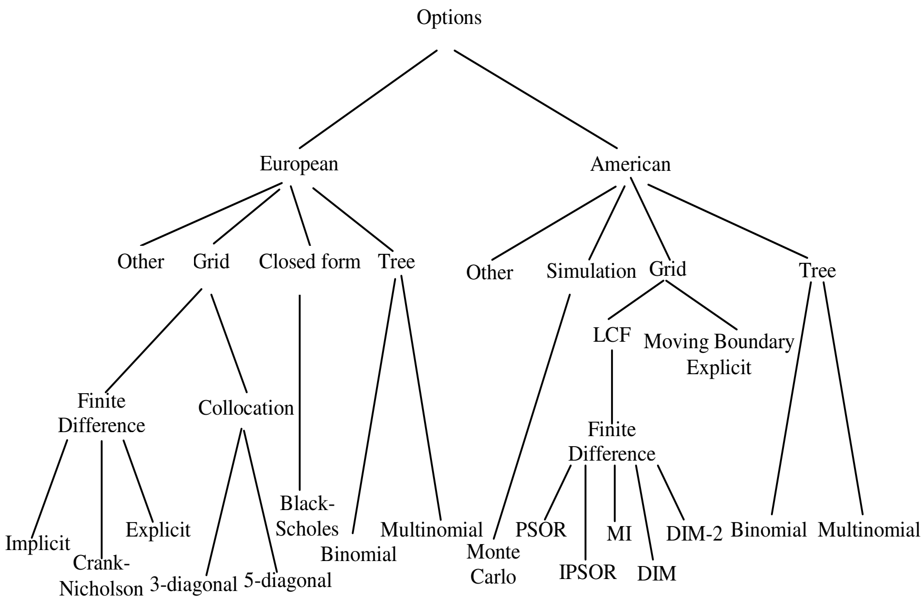


**Fig. 2.** Categorization of methods supported by the underlying option pricing library

methods have the advantage of providing option prices for a range of underlying asset prices based on a preselected grid; a discretization method plays significant role in the efficiency and performance of these methods. Discretizations supported by the underlying library up to now include Finite differences [17] (explicit, implicit and Crank Nicholson) and Collocation [15] (3-diagonal and 5-diagonal).

For American options, again grid methods, either based on Linear Complementarity Formulation (implicit) [6, 10] or making explicit use of the moving boundary (explicit) [14, 16] as well as tree methods (binomial and multinomial) [17, 5] have been implemented. The library contains five implicit grid methods: two iterative techniques, namely Projected Successive OverRelaxation (PSOR) [7] and Improved PSOR (IPSOR)[12], and three direct techniques, namely Moving Index (MI) [11], Direct Inverse Multiplication (DIM) and Stable DIM (DIM-2) [13]. Monte Carlo simulation can be also used, although it is not an efficient approach [17].

As already mentioned, the underlying library is open, and new methods can be added at any time to cover other kinds of financial derivatives (e.g. options on indices, currencies or commodities with simple or exotic payoff functions [5]).

## 5 Problem Definition and Solution Process

In this Section, we describe the sequence of the steps followed for problem definition and solution. As described in Figure 3, the user is first prompted to select the type of problem (exercise policy and option type), enter the necessary input parameters and choose operation (pricing and comparison). In response, the PSE proposes a list of available methods. Selecting one of them, the user is prompted again to define more parameters and make further selections, which depend on the specific method (e.g. choose a discretization method and define a grid, if a grid method has been selected). In case that a grid method has been selected, the output provided is a chart of option prices as a function of the underlying asset price and the user has the capability of getting "immediately" option prices for every value of the underlying asset price. For another PSE function, the user may select comparison of methods for a given problem, getting execution time charts. In this case, the PSE again is responsible for proposing a list of available methods based on selections and data provided by the user.

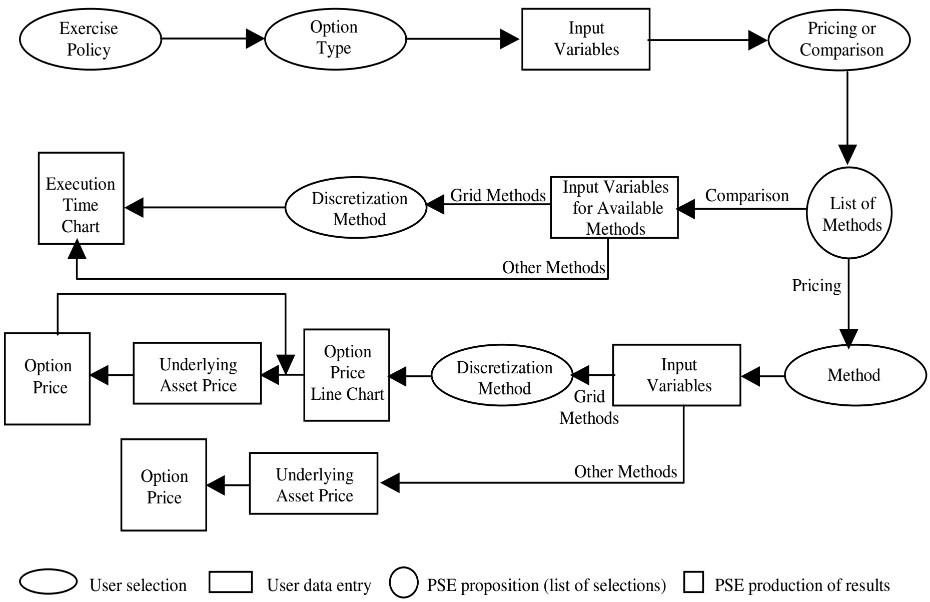


**Fig. 3.** Problem definition and solution process

For reasons of clarity, we now describe in more detail an indicative problem solution, giving exactly the steps followed through a web-browser. First of all, the end-user must determine if the option is European or American, call or put, if he/she is going to execute simple option pricing or comparison and finally enter the necessary input parameters, i.e. exercise price, interest rate, volatility, dividend and time to expiration. This is done, by making the suitable selections and completing the empty fields onto the form provided (see Figure 4).

The next step depends on the operation chosen in the first step. If "pricing" operation has been asked then the user can select only one of the available

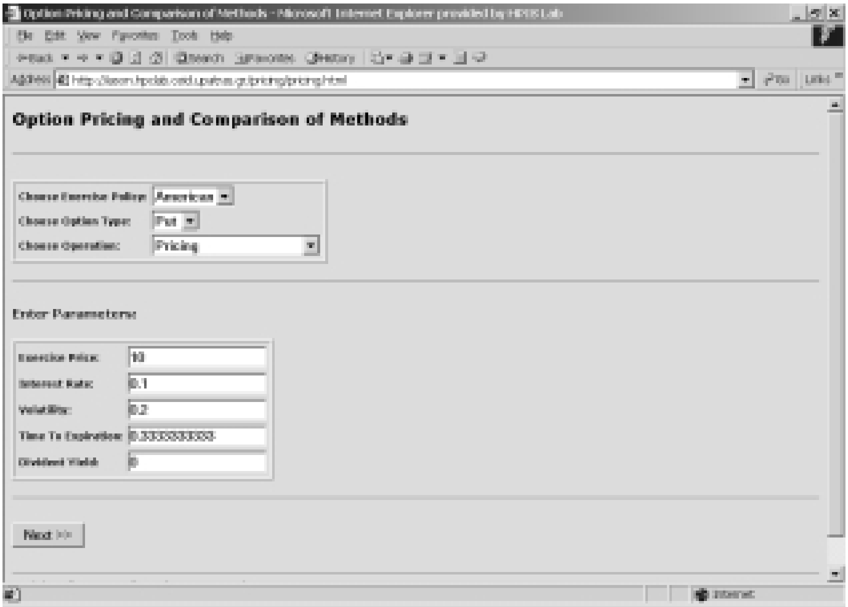


**Fig. 4.** First stage of problem definition

methods and define the required parameters. For example, if a grid method was selected a discretization method as well as a number of grid points and a number of time steps are necessary. Morever, if the method is iterative, a termination error should be entered. For the IPSOR method, an increment factor (see [12] for details), must also be defined. For the binomial method, only the number of time steps and the underlying asset price must be entered (other fields become non-input fields). If operation "comparison" was selected in the first step, then the user has the capability of comparing methods of the same type for the solution of the same problem. In this case he/she must select the methods to be compared and define the necessary input parameters. Only methods of the same type can be compared, otherwise the comparison does not make sense. For example, it is meaningful to compare MI with PSOR and DIM, because they are all grid methods, but none of them can be compared with the binomial method because it is a tree method. For this reason the PSE proposes available methods for comparison, depending on the problem which is currently considered for solution. An instance of the second stage of problem definition, if a pricing operation has been selected, is given in Figure 5.

For reasons of reporting, in all cases, the output has the form of a summary containing both information of problem definition and the results obtained. The form of the output primarily depends on the operation chosen in the first step and the method selected in the second step. For example, for grid methods, it has the form of a chart and the user has the capability of getting option prices

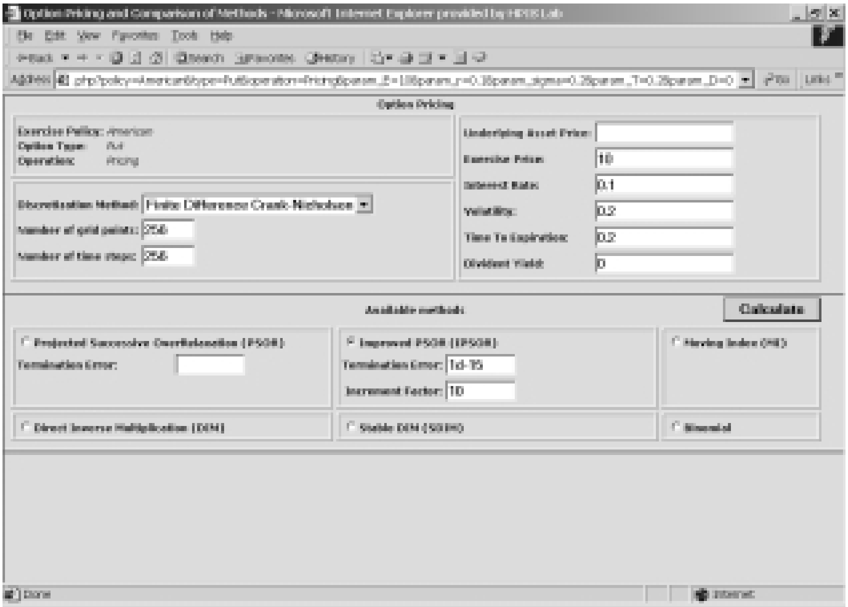


**Fig. 5.** Sample screen of the second stage of problem definition

for various values of the underlying asset. Figure 6 shows a sample screen for this case.

If a comparison were executed, then a chart of execution times for the selected methods is returned to the user (see Figure 7).

Finally, we should mention that a significant number of validity checks are performed for every method and special care is taken when wrong input is provided. For example, if DIM method is selected, then a stability check takes place before execution. In case that a stability problem exists, the user is informed that he can not execute this method for the current input.

The proposed PSE can be accessed through the following URL:
http://hydra.hpclab.ceid.upatras.gr/pricing/pricing.html

## 6 Conclusions and Future Work

In this paper we have presented a PSE for the solution of the option pricing problem. The proposed PSE has a number of very desirable properties. First, it is very simple and easy to use even by non-expert users. Second, it is available through the World Wide Web and provides immediate answers to the user in "real time". Third, although an expert system does not support the current environment, guidance is provided to the user and response of the PSE depends on user selections; morever, a comparison function is provided in order to help the end-user decide for the "best" method in terms of speed. However, the main

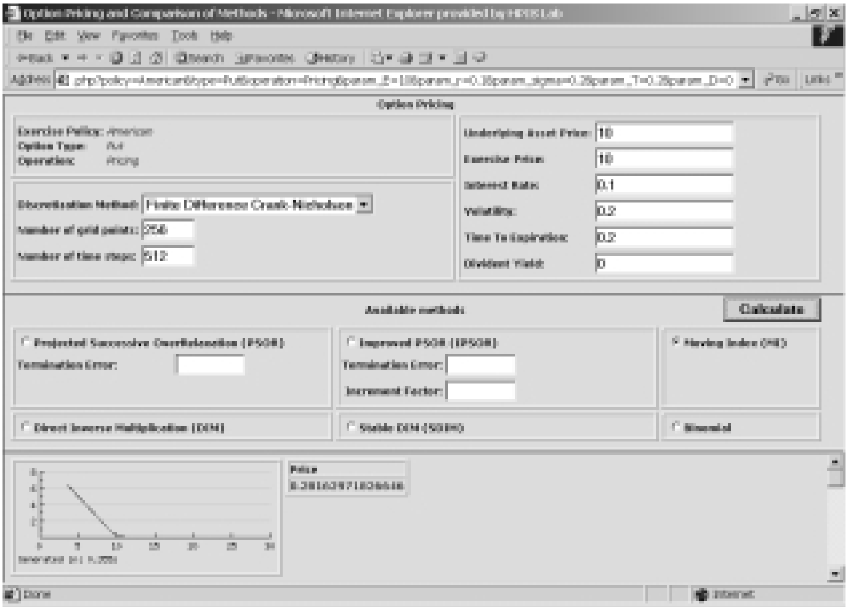


**Fig. 6.** Sample screen of the output for the case of pricing

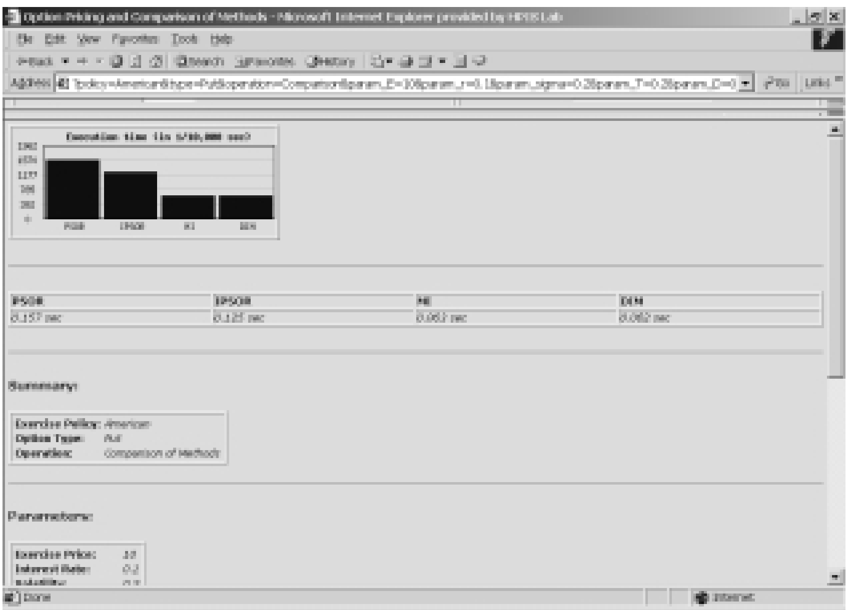


**Fig. 7.** Sample screen of the output for case of comparison of methods

advantage of the proposed PSE over other existing ones is that it is based on an open, expandable, underlying library of methods.

Implementation and addition of new methods, for the solution of option (and other derivatives) pricing problems, in the open library, is the main task of future work. Incorporation of these methods in the proposed PSE to make them available for use through the World Wide Web is another important task. Other types of comparisons are also under study and will be implemented to enrich the functions performed by the PSE.

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